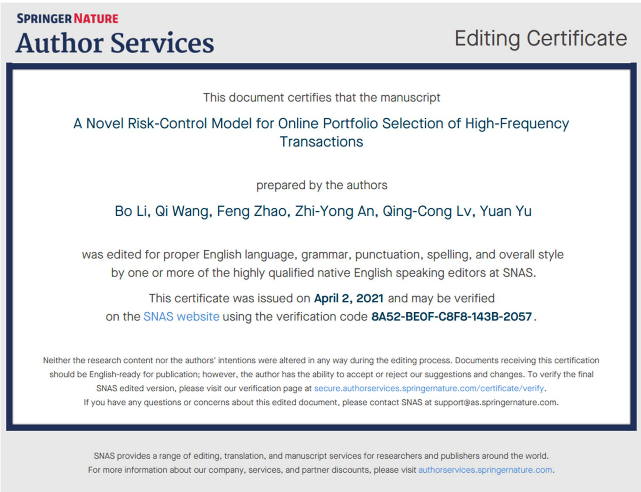
APPENDIX A

This file includes two parts: **details of the revision (page 1-2)** and **the supplement to the paper (page 3-5).**

1. **Details of the revision**

Dear editor or reviewers, it is our second time to submit this paper, we have revised this paper for three months carefully according to the reviewers' comments, details are as follows:

(1) Improve the paper's presentation quality. We have submitted the paper to the editing service of Springer (https://secure.authorservices.springernature.com/cn/login), here is the evidence: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A Novel Risk-Control Model for Online Portfolio Selection of High-Frequency Transactions

prepared by the authors

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was edited for proper English language, grammar, punctuation, spelling, and overall style by one or more of the highly qualified native English speaking editors at SNAS.

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(2) Describe the reasons more clearly about why we adopt high-frequency data, and why we do not adopt high-frequency data in a minute or even seconds. This description is in the "DISCUSSION" section.

(3) Add several recent methods for comparison according to the reviewers. They list five literatures as follows. We add three of them (literature 2, 3 and 4) for comparison, because the other two literatures (literature 1 and 5) are old.

1) T J Chang, N Meade, J E Beasley, et al. Heuristics for cardinality constrained portfolio optimization. Computers & Operations Research. 2000, 27(13): 12711302.

2) K Benidis. Y Feng, D P Palomar. Sparse portfolios for high-dimensional financial index tracking. IEEE Transactions on Signal Processing. 2018, 66(1):155170.

3) Y Zheng, B Chen, T M Hospedales, et al. Index tracking with cardinality constraints: a stochastic neural networks approach[C]. The Thirty-Fourth AAAI Conference on Artificial Intelligence. February, 2020, 1242-1249.
4) O Strub, P Baumann. Optimal construction and rebalancing of index-tracking portfolios. European Journal of Operational Research 2018，264(1):370387.
5) A Takeda, M Niranjan, J Gotoh, Y Kawahara. Simultaneous pursuit of out-of-sample performance and sparsity in index tracking portfolios. Computational Management Science. 2013, 10(1): 2149.

(4) Demonstrate the motivations more clearly in “INTRODUCTION” section.
(5) Demonstrate the contributions more clearly in “INTRODUCTION”, “DISCUSSION” and “CONCLUSION” sections.
(6) Redraw all the figures.
(7) Rearrange and reedit all the equations.
(8) Review some recent literatures on technical analysis and sentiment embedding for market trend prediction in “ INTRODUCTION” section.

2. The supplement to the paper

(1) The generalized inverse of block matrix

Let $C(A)$ be the column space of A, such that:

1) $B \in C(A) \Rightarrow$ there exists a matrix H, $B=AH \Leftrightarrow B = AA^{T}B$.

2) $C(A) = C(AA^{T})$.

3) $B \in C(A+BB^{T})$, where A is a non-negative definite matrix.

4) $B^{T}A \in C(B^{T}AB)$.

5) $B \in C(A) \Rightarrow B^{T} \in C(B^{T}A^{-}B)$.

Let $A_2 = A + BB^{T} + \frac{1}{4}BDB^{T}$ and $B_2 = B(I + \frac{D}{2})$, then according to 3):

$$\begin{bmatrix} I & O \\ -B_2^{T}A_2^{-} & I \end{bmatrix}\begin{bmatrix} I & \frac{B}{2} \\ O & I \end{bmatrix}\begin{bmatrix} A & B \\ B^{T} & D \end{bmatrix}\begin{bmatrix} I & O \\ \frac{B^{T}}{2} & I \end{bmatrix}\begin{bmatrix} I & -(A_2^{T})^{-}B_2 \\ O & I \end{bmatrix}$$

$$= \begin{bmatrix} A_2 & O \\ O & D - B_2^{T}A_2^{-}B_2 \end{bmatrix} \tag{1}$$

Hence,

$$\begin{bmatrix} A & B \\ B^{T} & D \end{bmatrix}^{+} = \begin{bmatrix} I & O \\ \frac{B^{T}}{2} & I \end{bmatrix}\begin{bmatrix} I & -(A_2^{T})^{-}B_2 \\ O & I \end{bmatrix}\begin{bmatrix} A_2 & O \\ O & D - B_2^{T}A_2^{-}B_2 \end{bmatrix}\begin{bmatrix} I & O \\ -B_2^{T}A_2^{-} & I \end{bmatrix}\begin{bmatrix} I & \frac{B}{2} \\ O & I \end{bmatrix}$$

$$= \begin{bmatrix} C_1 & C_2 \\ C_2^{T} & C_4 \end{bmatrix} \tag{2}$$

where

$$\begin{cases} C_1 = A_2^- + (A_2^T)^- B_2 D_2^- B_2^T A_2^- \\ C_2 = C_1 \dfrac{B}{2} - (A_2^T)^- B_2 D_2^- \\ C_4 = \dfrac{B^T C_1 B}{4} - \dfrac{B^T (A_2^T)^- B_2 D_2^-}{2} - \dfrac{D_2^- B_2^T A_2^- B}{2} + D_2^- \\ D_2 = D - B_2^T A_2^- B_2 \end{cases}. \tag{3}$$

If $D = \mathbf{0}$, then according to (1) and (4):

$$\begin{cases} A_2 = A + BB^T \\ B_2 = B \\ C_1 = A_2^- [I + (A_2^T)^- B D_2^- B^T] \\ C_2 = C_1 \dfrac{B}{2} - (A_2^T)^- B D_2^- \\ C_4 = \dfrac{B^T C_1 B}{4} - \dfrac{B^T (A_2^T)^- B D_2^-}{2} - \dfrac{D_2^- B^T A_2^- B}{2} + D_2^- \\ D_2 = -B^T A_2^- B \end{cases}. \tag{4}$$

Finally, for a block matrix $M = [A \mid a]$, if $AA^- a = a$, then $M^- = \begin{bmatrix} A^- - A^- ab \\ b \end{bmatrix}$, where b is an arbitrary vector. Thus:

$$[A \mid a] \begin{bmatrix} A^- - A^- ab \\ b \end{bmatrix} [A \mid a] = \begin{bmatrix} AA^- - AA^- ab + ab \end{bmatrix} [A \mid a]$$

$$= \begin{bmatrix} A - abA + abA & AA^- a - aba + aba \end{bmatrix} = [A \mid a] = M. \tag{5}$$

Table A.1. Average computational runtimes for one period (in seconds) for each of the eight datasets.

Dataset	SMEBII −30 min	GEBII −30 min	BG50I −30 min	R50I −30 min	SMEBII −60 min	GEBII −60 min	BG50I −60 min	R50I −60 min
Best Stock	0.0015	0.0011	0.0013	0.0017	0.0012	0.0017	0.0015	0.0010
UBAH	0.0017	0.0021	0.0010	0.0020	0.0016	0.0021	0.0025	0.0015
BCRP	0.0018	0.0016	0.0017	0.0025	0.0011	0.0014	0.0020	0.0016
UCRP	0.0018	0.0017	0.0018	0.0020	0.0019	0.0026	0.0027	0.0017
ONS	0.7033	0.8065	0.7057	0.8032	0.7045	0.8618	0.7475	0.6936
PAMR	0.0014	0.0012	0.0015	0.0015	0.0019	0.0013	0.0011	0.0012
CWMR	0.0027	0.0023	0.0027	0.0027	0.0020	0.0028	0.0023	0.0019
OLMAR	0.0025	0.0026	0.0029	0.0030	0.0023	0.0025	0.0026	0.0021
CORN	7.1233	9.3455	13.4732	5.4632	6.1046	5.4637	10.4756	6.3498

LOAD	0.0033	0.0031	0.0036	0.0042	0.0035	0.0031	0.0037	0.0031
NEW	2.0287	3.4030	3.4087	4.0177	2.6268	2.6361	3.8168	3.8779
ALAIT	2.6969	1.8534	2.6687	2.0855	1.4260	1.4974	3.0568	2.8612
SNN	**1216**	**1382**	**1417**	**1491**	**1236**	**1209**	**1354**	**1208**
BLSM	**917**	**846**	**829**	**1017**	**983**	**1104**	**1034**	**838**