APPENDIX A

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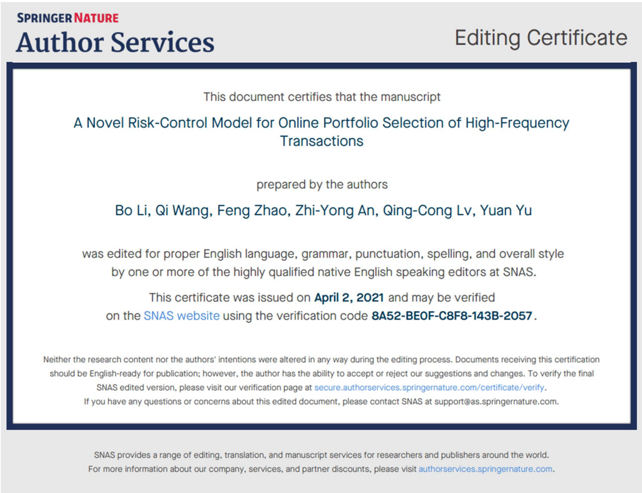

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A Novel Risk-Control Model for Online Portfolio Selection of High-Frequency Transactions

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## 1. The supplement to the paper

### (1) The generalized inverse of block matrix

Let $C(A)$ be the column space of $A$, such that:

1) $B \in C(A) \Rightarrow$ there exists a matrix $H$, $B=AH \Leftrightarrow B = AA^T B$.

2) $C(A) = C(AA^T)$.

3) $B \in C(A+BB^T)$, where $A$ is a non-negative definite matrix.

4) $B^T A \in C(B^T AB)$.

5) $B \in C(A) \Rightarrow B^T \in C(B^T A^- B)$.

Let $A_2 = A + BB^T + \frac{1}{4}BDB^T$ and $B_2 = B(I + \frac{D}{2})$, then according to 3):

$$\begin{bmatrix} I & O \\ -B_2^T A_2^- & I \end{bmatrix} \begin{bmatrix} I & \frac{B}{2} \\ O & I \end{bmatrix} \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \begin{bmatrix} I & O \\ \frac{B^T}{2} & I \end{bmatrix} \begin{bmatrix} I & -(A_2^T)^- B_2 \\ O & I \end{bmatrix}$$

$$=\begin{bmatrix} A_2 & O \\ O & D-B_2^T A_2^- B_2 \end{bmatrix} \tag{1}$$

Hence,

$$\begin{bmatrix} A & B \\ B^T & D \end{bmatrix}^+ = \begin{bmatrix} I & O \\ \frac{B^T}{2} & I \end{bmatrix} \begin{bmatrix} I & -(A_2^T)^- B_2 \\ O & I \end{bmatrix} \begin{bmatrix} A_2 & O \\ O & D-B_2^T A_2^- B_2 \end{bmatrix} \begin{bmatrix} I & O \\ -B_2^T A_2^- & I \end{bmatrix} \begin{bmatrix} I & \frac{B}{2} \\ O & I \end{bmatrix}$$

$$=\begin{bmatrix} C_1 & C_2 \\ C_2^T & C_4 \end{bmatrix} \tag{2}$$

where

$$\begin{cases} C_1 = A_2^- + (A_2^T)^- B_2 D_2^- B_2^T A_2^- \\ C_2 = C_1 \dfrac{B}{2} - (A_2^T)^- B_2 D_2^- \\ C_4 = \dfrac{B^T C_1 B}{4} - \dfrac{B^T (A_2^T)^- B_2 D_2^-}{2} - \dfrac{D_2^- B_2^T A_2^- B}{2} + D_2^- \\ D_2 = D - B_2^T A_2^- B_2 \end{cases} . \tag{3}$$

If $D = \mathbf{0}$, then according to (1) and (4):

$$\begin{cases} A_2 = A + BB^T \\ B_2 = B \\ C_1 = A_2^- [I + (A_2^T)^- B D_2^- B^T] \\ C_2 = C_1 \dfrac{B}{2} - (A_2^T)^- B D_2^- \\ C_4 = \dfrac{B^T C_1 B}{4} - \dfrac{B^T (A_2^T)^- B D_2^-}{2} - \dfrac{D_2^- B^T A_2^- B}{2} + D_2^- \\ D_2 = -B^T A_2^- B \end{cases} . \tag{4}$$

Finally, for a block matrix $M = [A \mid a]$, if $AA^- a = a$, then $M^- = \begin{bmatrix} A^- - A^- ab \\ b \end{bmatrix}$, where $b$ is an arbitrary vector. Thus:

$$[A \mid a] \begin{bmatrix} A^- - A^- ab \\ b \end{bmatrix} [A \mid a] = \begin{bmatrix} AA^- - AA^- ab + ab \end{bmatrix} [A \mid a]$$

$$= \left[ A - abA + abA \qquad AA^{-}a - aba + aba \right] = [A \mid a] = M\ . \tag{5}$$