Individual Coursework

The submission of this coursework includes:

- Written report with individual answers to each question (pdf);
- Simulink file(s) (.slx) for question 1b), 1d), 1e), 2c)
- Matlab file (.m) for question 1f)

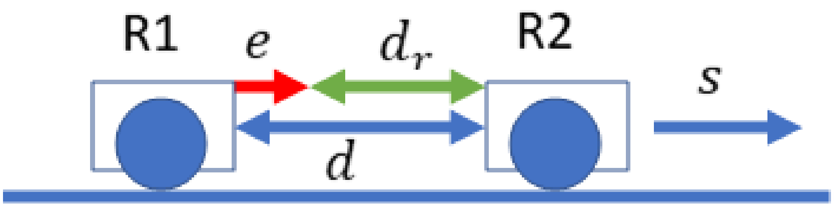
IMPORTANT: Only in questions that are marked with [MATLAB] you can use Matlab results/plots/code to justify your answers. For all other questions, justify your answers based on mathematical derivations and/or written argument as necessary.

This coursework aims at controlling the distance $d(t)$ of a robot R1 relative to a robot R2 moving at a speed $s(t)$, so that it follows a reference $d_r(t)$.

All quantities are in SI units (metres, seconds, m/s, etc).

Assume the following:

- the distance $d(t)$ is measured and can be used for control feedback
- the initial conditions are: $d(0) = 1,\ \ \dot{d}(0) = 0,\ \ s(0) = 0$



Questions

1) **[70 marks]** Assume the velocity of R1 is directly and instantly controlled by an input $u(t)$.

   a) **[8 marks]** Model the system as a continuous state-space representation, considering as inputs the control of R1 and the speed of R2 (respectively $u(t), s(t)$), and as output the distance $d(t)$.

   b) **[20 marks][MATLAB]** Implement a Simulink model that simulates the described system. Consider that:

      - the speed of R2 (which is unknown to R1) follows:

$$s(t) = \begin{cases} 0.1t, & 0 \leq t < 4 \\ 0.4, & 4 \leq t < 8 \\ 0.8 - 0.05t, & 8 \leq t < 12 \\ 0.2, & 12 \leq t < \infty \end{cases}$$

      - the reference distance is constant $d_r = 1$
      - the input of R1 $u(t)$ is generated by a continuous PID controller. Tune this controller so that $d(t)$ follows the reference d_r, justify your choice of parameters, making sure the system is stable.

      Generate plots of R1's velocity and the distance $d(t)$ over 16 seconds

   c) **[8 marks]** Suppose we want to implement a discrete, digital version of the PID controller from the previous question, with a sampling time of 0.2 seconds. Derive its expression in the form of a discrete digital filter.

   d) **[8 marks] [MATLAB]** Re-implement the Simulink model from 1b), but now with the digital PID controller derived in 1c) represented as a discrete transfer function. Generate plots of R1's velocity $u(t)$ and the distance $d(t)$ over 16 seconds. Comment on the performance of the digital PID controller, when compared to the continuous controller implemented in 1b).

   e) **[8 marks] [MATLAB]** Re-implement the Simulink model from 2c), adding random Gaussian noise of 0.01 standard deviation to the control signal $u(t)$ and also to the sensor measurement of $d(t)$. Compare the performance of the controller against the noise-free scenario in d), and comment on the differences.

   f) **[18 marks] [MATLAB]** In a Matlab .m script, re-implement a simulation of the noisy system in 1e), together with the discrete digital filter derived in 1c). Compare the results against the Simulink simulation in e).

2) **[30 marks]** Assume now that the input of R1 $u(t)$ controls its acceleration instead. Also assume that R2 is stopped ($s(t) = 0$).

a) **[8 marks]** Determine the open-loop transfer function of this system considering as input the reference $u(t)$ and as the output the distance $d(t)$.

b) **[4 marks] [MATLAB]** Investigate the viability of using a proportional controller for this system, through a root locus analysis.

c) **[18 marks] [MATLAB]** Implement a Simulink model that simulates the described system. Consider that:

- The reference distance is variable and should follow:

$$d_r(t) = \begin{cases} 1.5, & 0 \le t < 4 \\ 2.3 - 0.2t, & 4 \le t < 8 \\ 0.7, & 8 \le t < 12 \end{cases}$$

- The input of R1 $u(t)$ is generated by a lead or a lag compensator. Tune its parameters, by conforming to suitable stability margins.

Generate plots of R1's velocity and the distance $d(t)$ over 16 seconds.