

# FROM A BRANCH-AND-BOUND TO A STATE-OF-THE-ART HEURISTIC

AN EXAMPLE ON THE PERMUTATION FLOWSHOP  
PROBLEM

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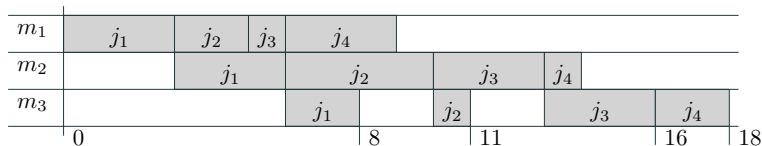
Pablo Andres Focke<sup>2</sup>, Vincent Jost<sup>2</sup>, **Luc Libralesso**<sup>1</sup>, Aurélien Secardin<sup>2</sup>  
June, 15, 2021

1. LIMOS, Clermont-Ferrand, France
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# The permutation flowshop

## Famous and fundamental problem



We study 2 objectives:

**The makespan:** completion time of the last job on the last machine

**obj: 18**

**total completion time (flowtime):** sum of completion times

**obj:  $8+11+16+18 = 53$**

**exact methods:** mostly **branch-and-bounds**

Gmys et al. [2020], Tomazella and Nagano [2020]

**meta-heuristics:** mostly **iterated-greedy algorithms** Fernandez-Viagas and Framinan [2019]

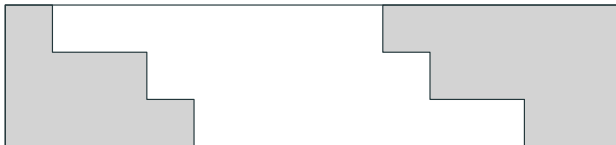
Pagnozzi and Stützle [2019]

Kizilay et al. [2019]

## Focus on branch-and-bounds: bi-directional branching

**Rationale: add jobs at the beginning and at the end**

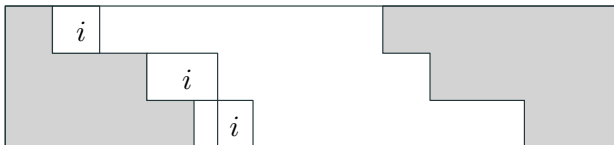
1. choose the insertion direction (forward or backward)
2. branch on jobs



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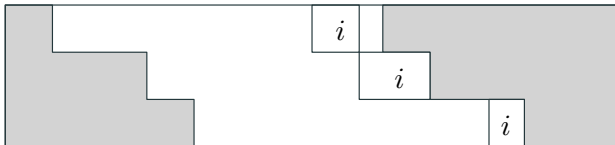
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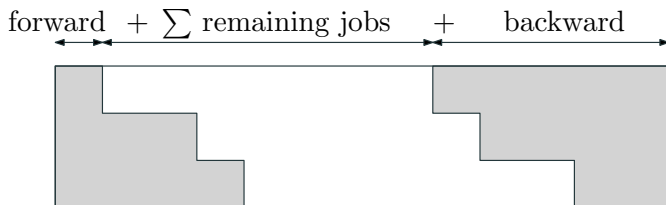
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# Focus on branch-and-bounds: bi-directional branching

**Rationale: add jobs at the beginning and at the end**

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# How to transform this branch-and-bound into an heuristic?

**Key idea:** a branch-and-bound explores a search-space

can we focus on “a-priori” good solutions first? (spoiler: yes)



# How to transform this branch-and-bound into an heuristic?

- I. The search strategy
- II. The guidance strategy

# The “classical” search strategy: Depth First Search

- small memory requirements
- simple

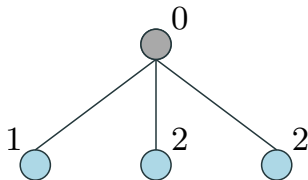
## **BUT**

- gets usually stuck in “early bad decisions”

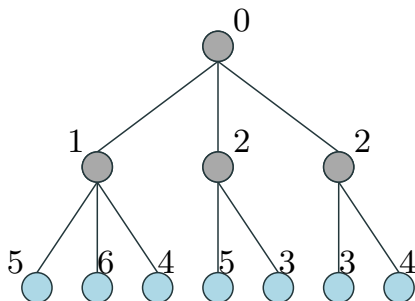
## Another search strategy: Beam Search



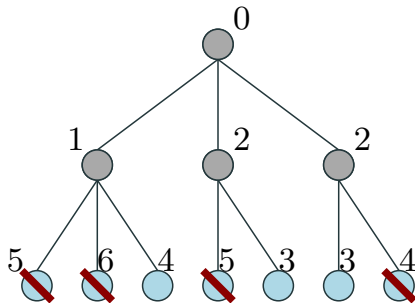
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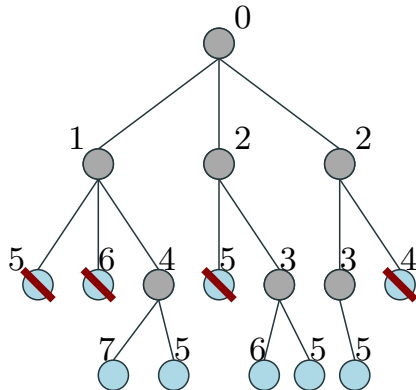
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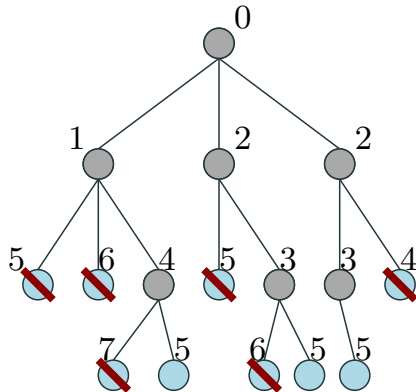
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## Another search strategy: Beam Search





# Iterative Beam Search

- Runs a beam of size 1 (greedy)
- Then runs a beam of size 2, then 4, then 8 ...

Stops when no heuristic fathoming is done (proves optimality)

# How to transform this branch-and-bound into an heuristic?

- I. The search strategy
- II. The guidance strategy**

# Guide functions: which node to choose?

which criterion?

**the bound**

**the idle time**

**a bit of both**  $(\alpha \cdot \text{bound} + (1 - \alpha) \cdot \text{idle})$

**with weighted idle time** (similar to the Liu & Reeves heuristic)

$(\alpha \cdot \text{bound} + (1 - \alpha) \cdot \text{weighted idle})$

## Results

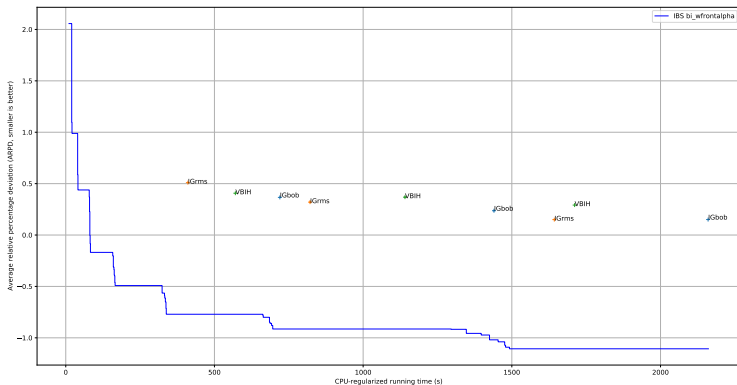
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# Makespan variant

## 101/240 new-best-known solutions

- excellent on large instances (500+ jobs, 40+ machines)
- less efficient on “small” instances (100 jobs, 60 machines)

### 800 jobs, 60 machines

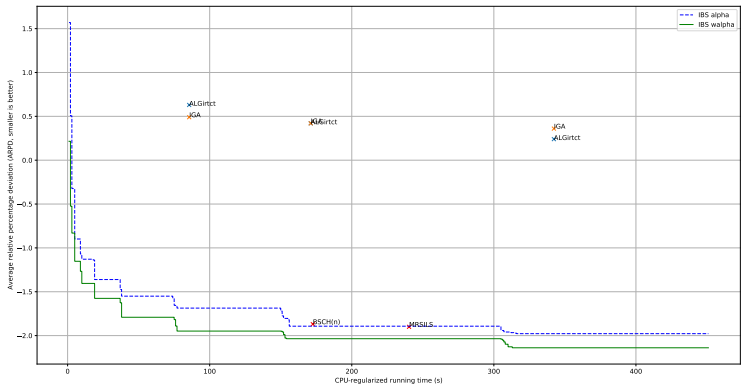


# Total completion time variant

only the forward direction

51/120 new-best-known solutions

500 jobs, 20 machines



# Conclusions

This methodology leads to interesting algorithms  
sometimes, improving upon the state-of-the-art  
used on other problems:

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used on other problems:

- **sequential ordering problem (ECAI2020, Q1)**
  - **EURO/ROADEF 2018 challenge (EJOR, Q1)**
  - general 2D cutting & packing problems
  - longest common subsequence
- 
- pre-print: [http://librallu.gitlab.io/pdfs/2020\\_pfsp\\_ibs.pdf](http://librallu.gitlab.io/pdfs/2020_pfsp_ibs.pdf)
  - code: <https://github.com/librallu/dogs-pfsp>



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## References

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