FROM A BRANCH-AND-BOUND TO A STATE-OF-THE-ART HEURISTIC

An example on the permutation flowshop problem

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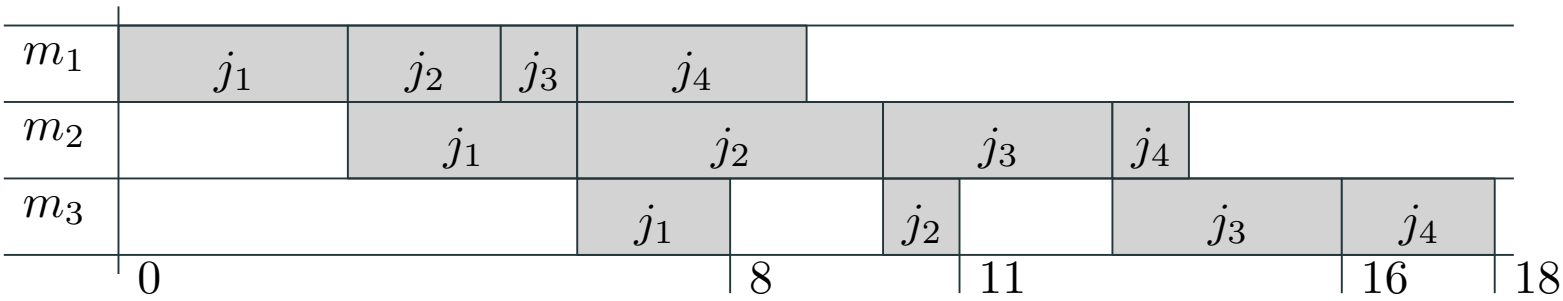
The permutation flowshop

Famous and fundamental problem



We study 2 objectives:

The makespan: completion time of the last job on the last machine

obj: 18

total completion time (flowtime): sum of completion times

obj: 8+11+16+18 = 53

The benchmarks

Makespan:

Taillard: sizes between *20 x 5* and *500 x 20*

VFR small: sizes between *10 x 5* and *60 x 20*

VFR large: sizes between *100 x 20* and *800 x 60*

Total completion time:

Taillard: sizes between *20 x 5* and *500 x 20*

Resolution methods

exact methods: mostly **branch-and-bounds**

Gmys et al. [2020], Tomazella and Nagano [2020]

meta-heuristics: mostly **iterated-greedy algorithms**

Fernandez-Viagas and Framinan [2019]

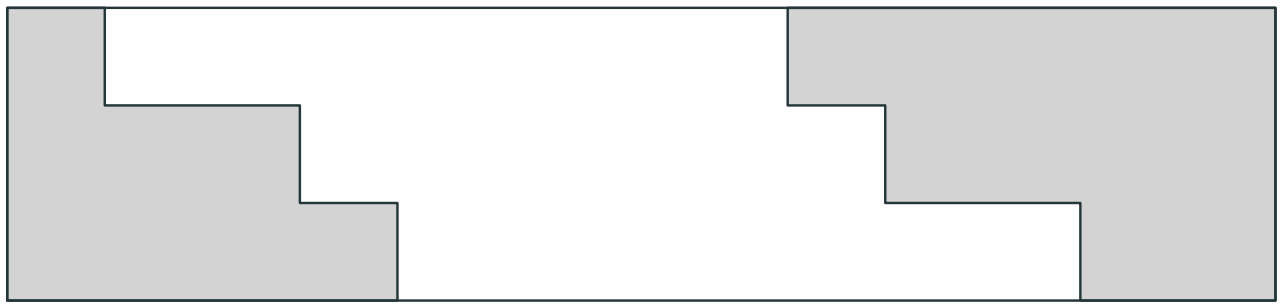
Pagnozzi and Stützle [2019]

Kizilay et al. [2019]

Focus on branch-and-bounds: bi-directional branching

Rationale: add jobs at the beginning and at the end

1. choose the insertion direction (forward or backward)
2. branch on jobs



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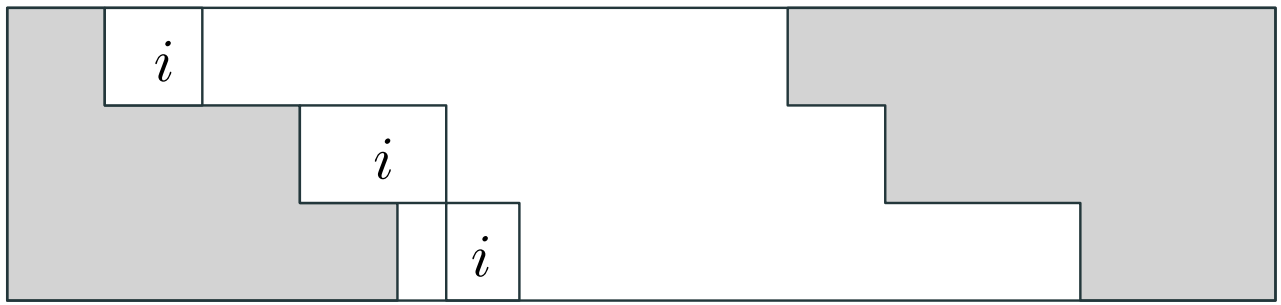

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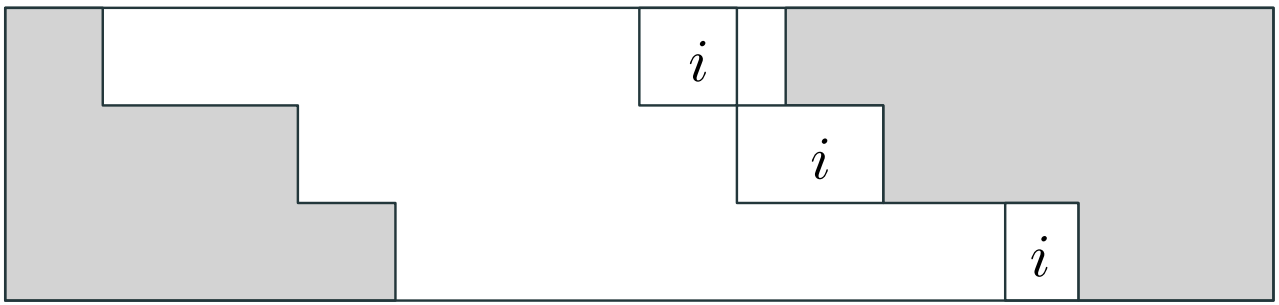

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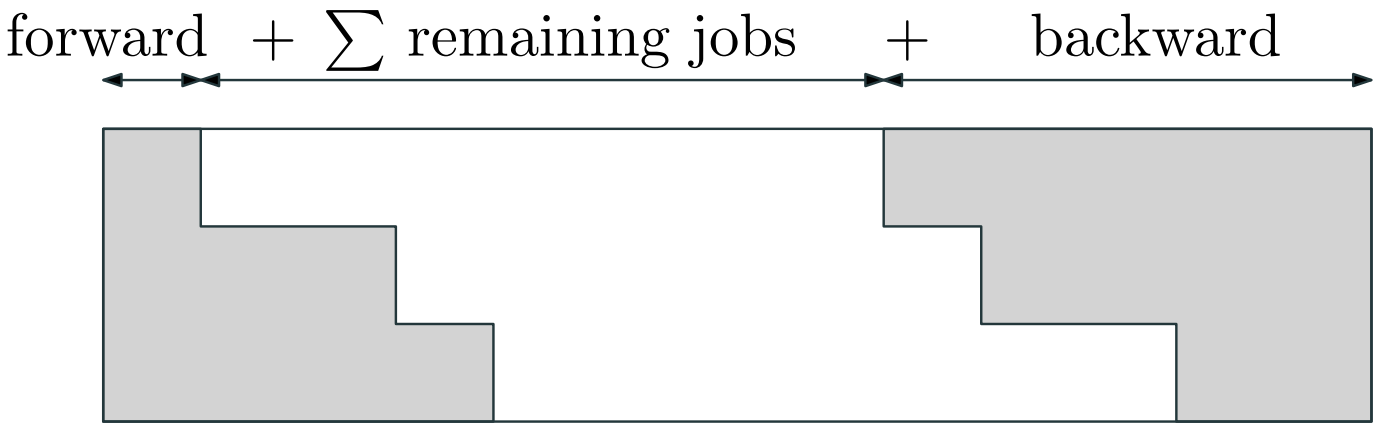


How to transform this branch-and-bound into an heuristic?

Key idea: a branch-and-bound explores a search-space

can we focus on “a-priori” good solutions first? (spoiler: yes)

How to transform this branch-and-bound into an heuristic?

I. The search strategy
II. The guidance strategy

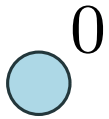
The "classical" search strategy: Depth First Search

- small memory requirements
- simple

BUT

- gets usually stuck in "early bad decisions"

Another search strategy: Beam Search



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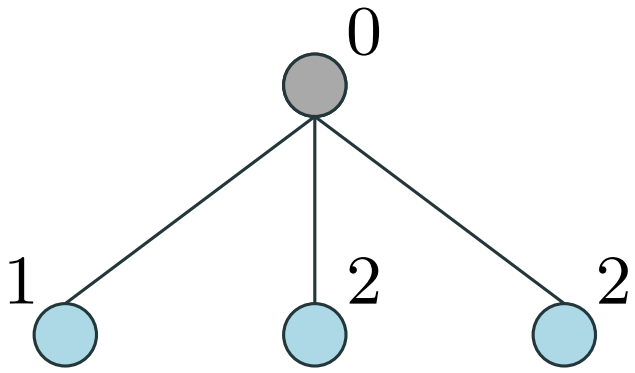

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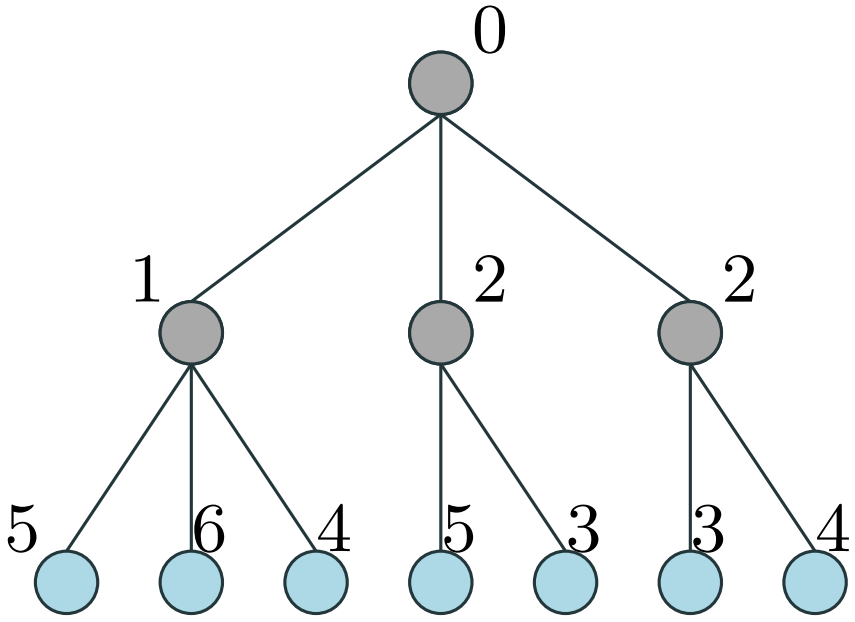

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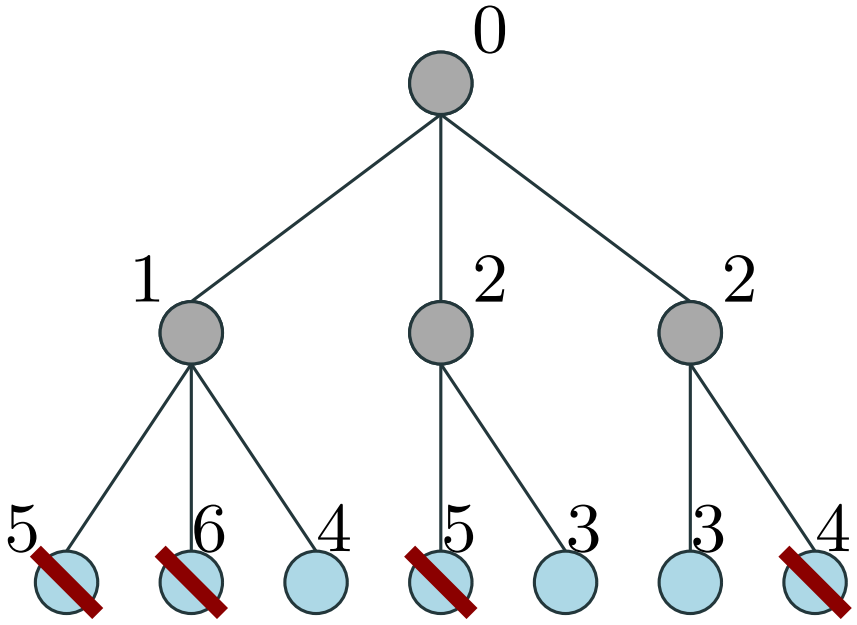

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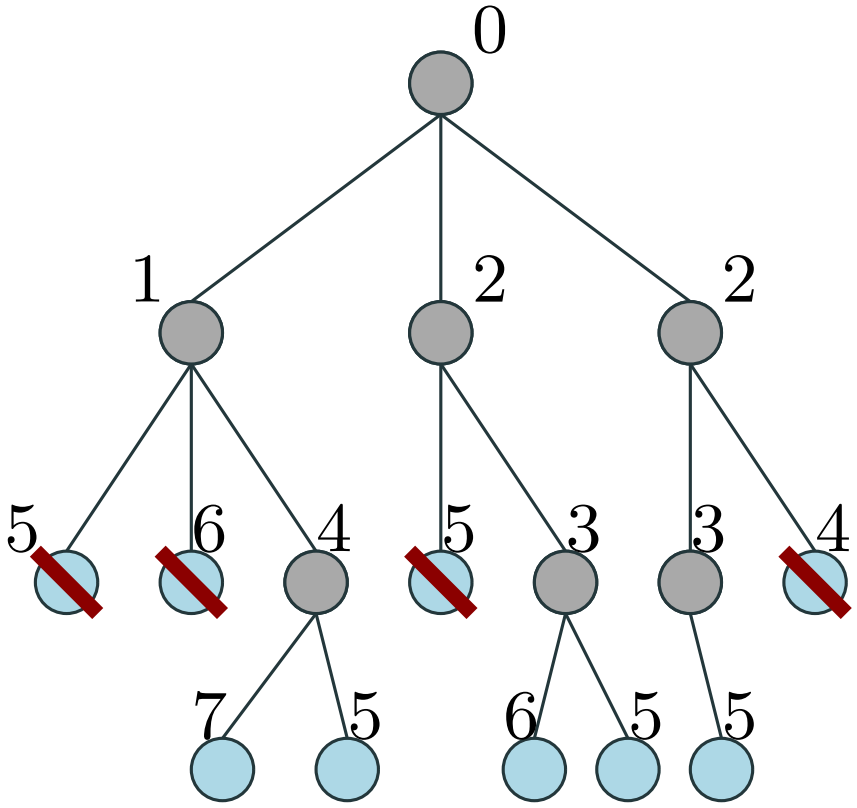

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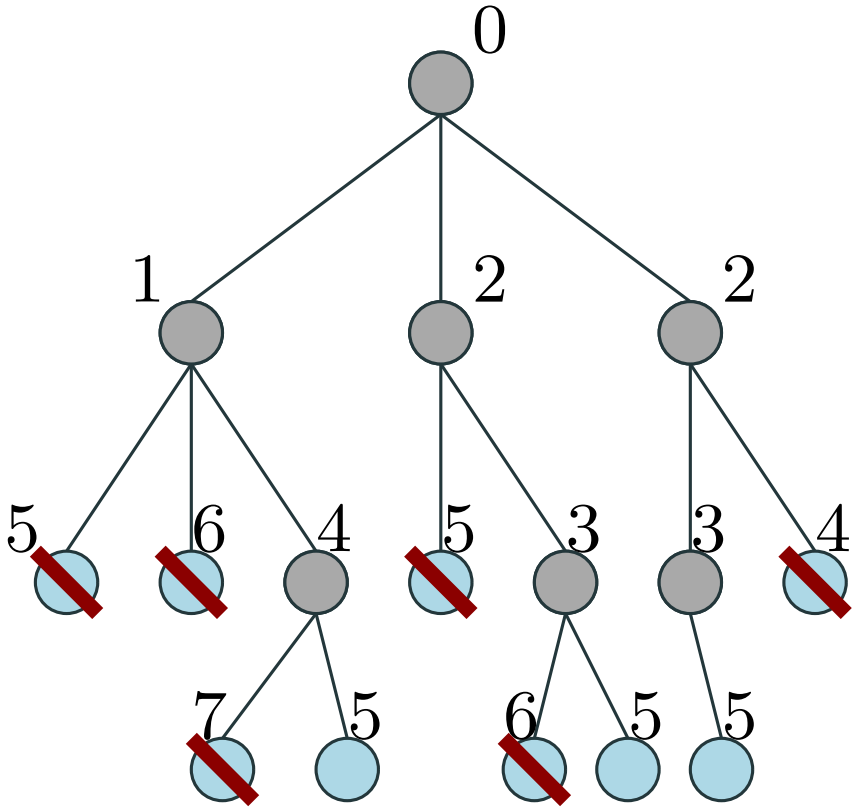


Iterative Beam Search

- Runs a beam of size 1 (greedy)
- Then runs a beam of size 2, then 4, then 8 ...

Stops when no heuristic fathoming is done (proves optimality)

How to transform this branch-and-bound into an heuristic?

I. The search strategy
II. The guidance strategy

Guide functions: which node to choose?

which criterion?

the bound

the idle time

a bit of both $(\alpha \,.\, \text{bound} \;+\; (1-\alpha) \,.\, \text{idle})$

with weighted idle time (similar to the Liu & Reeves heuristic)

$$(\alpha \,.\, \text{bound} \;+\; (1-\alpha) \,.\, \text{weighted idle})$$

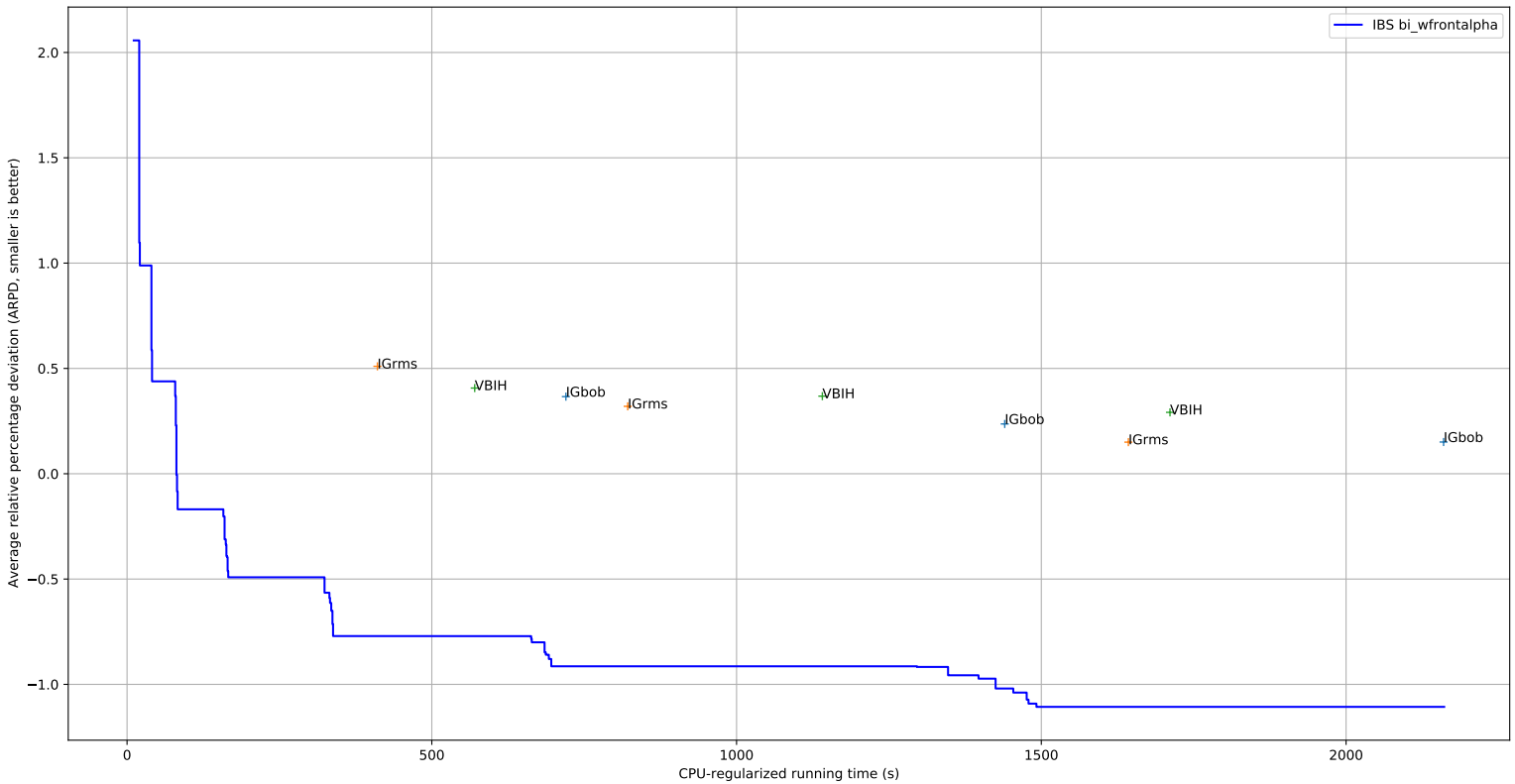
Results

Makespan variant

101/240 new-best-known solutions

- excellent on large instances (500+ jobs, 40+ machines)
- less efficient on “small” instances (100 jobs, 60 machines)

800 jobs, 60 machines



Makespan variant

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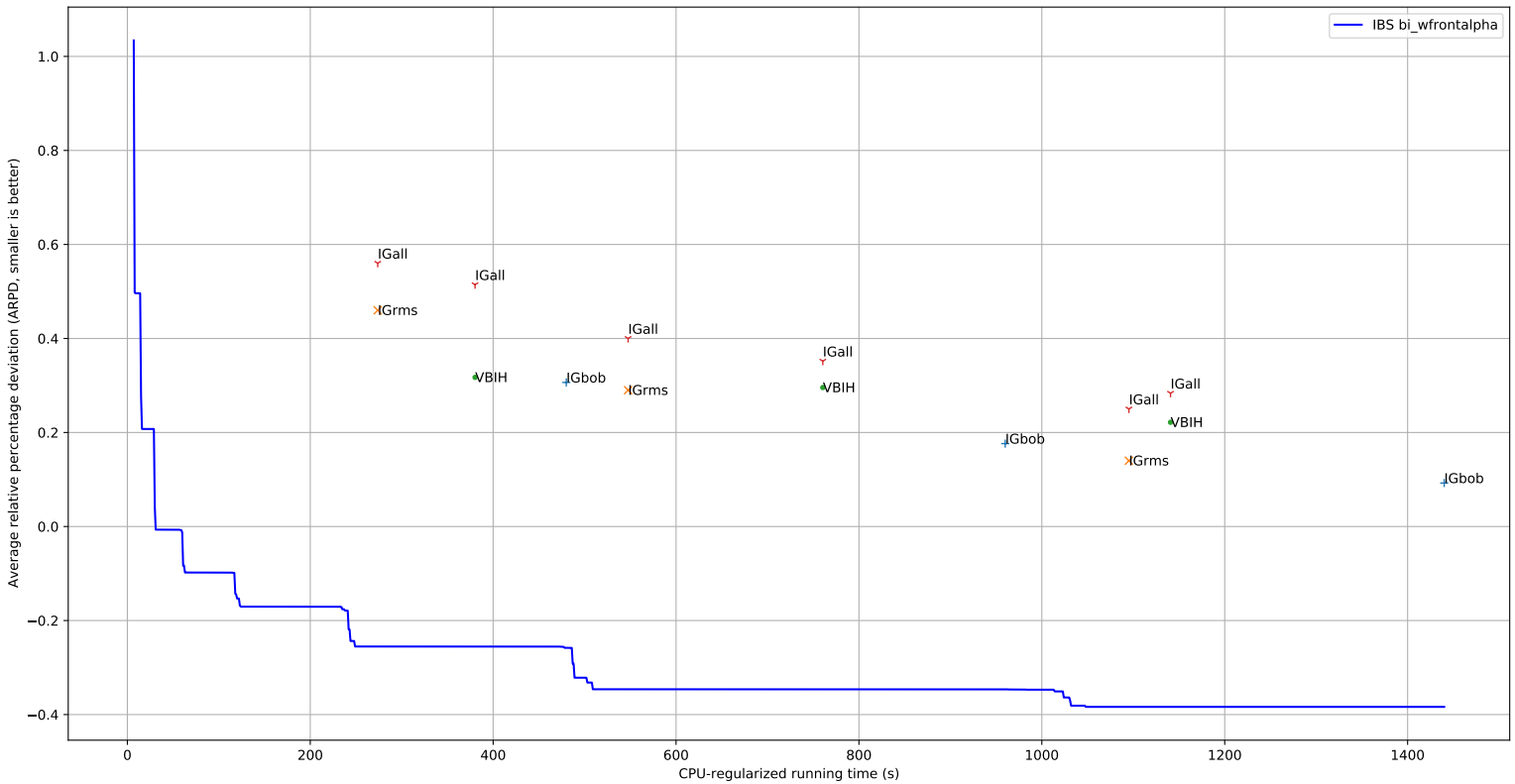
800 jobs, 40 machines



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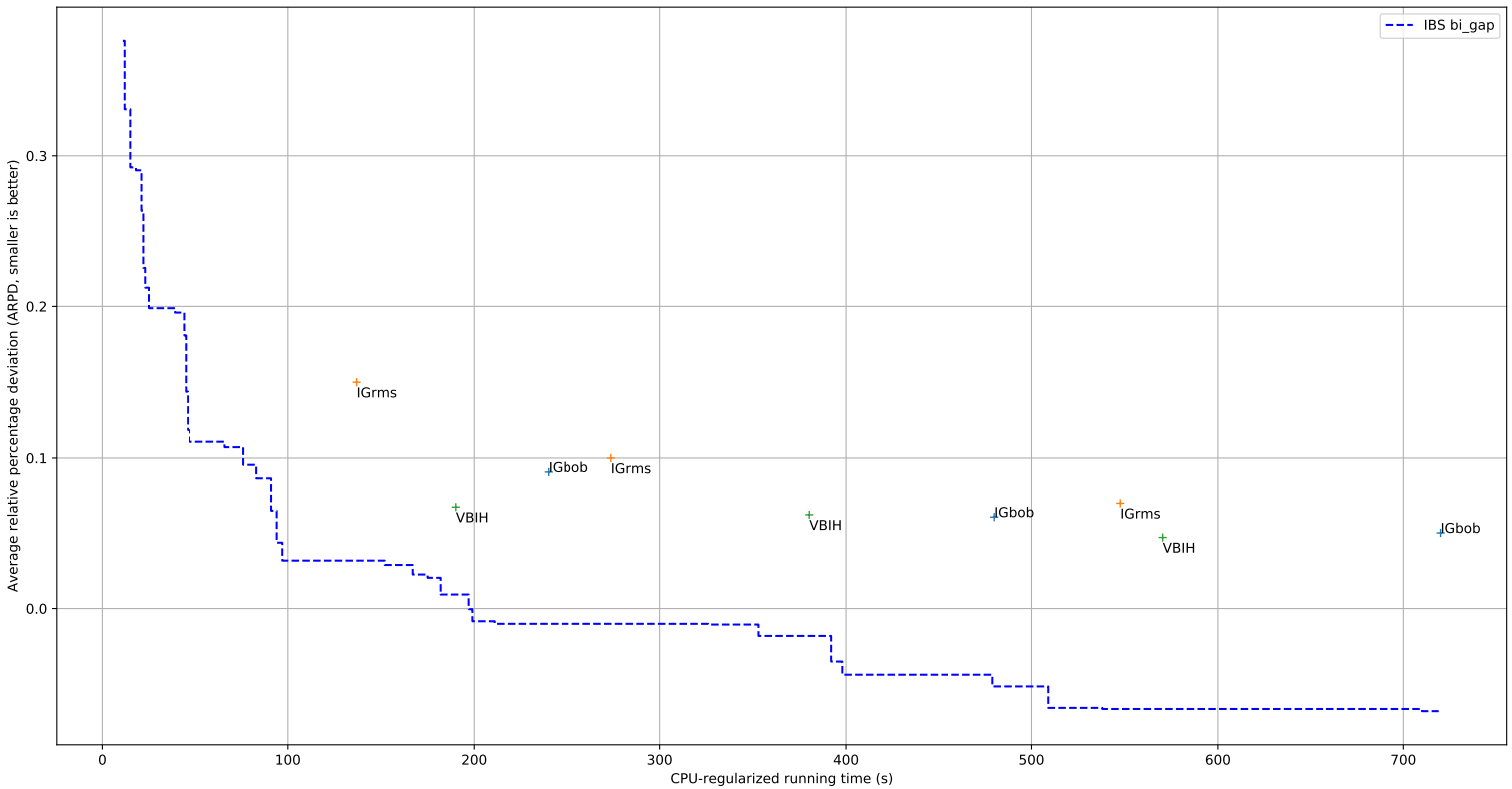
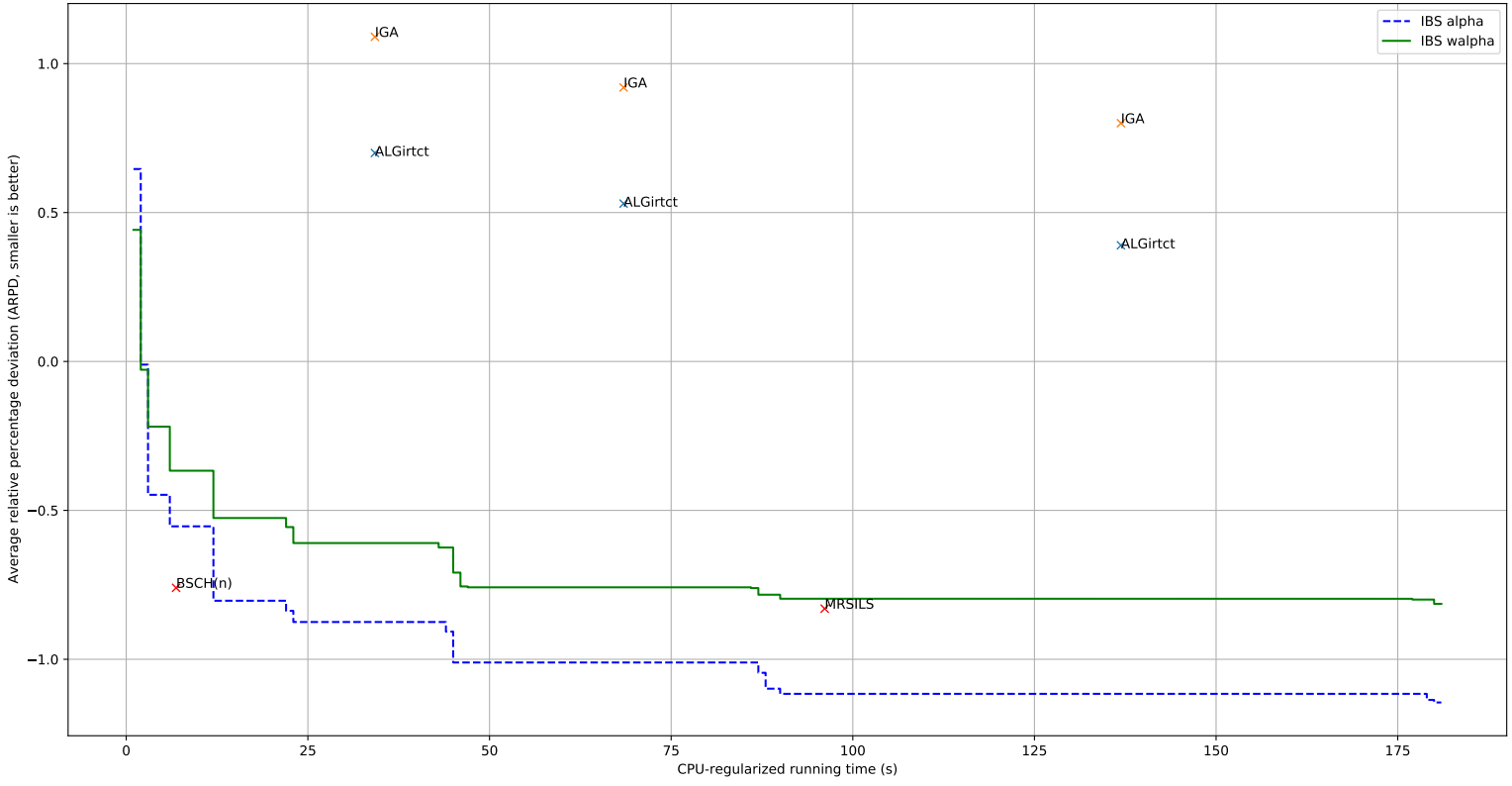


Total completion time variant

only the forward direction

51/120 new-best-known solutions

200 jobs, 20 machines



Total completion time variant

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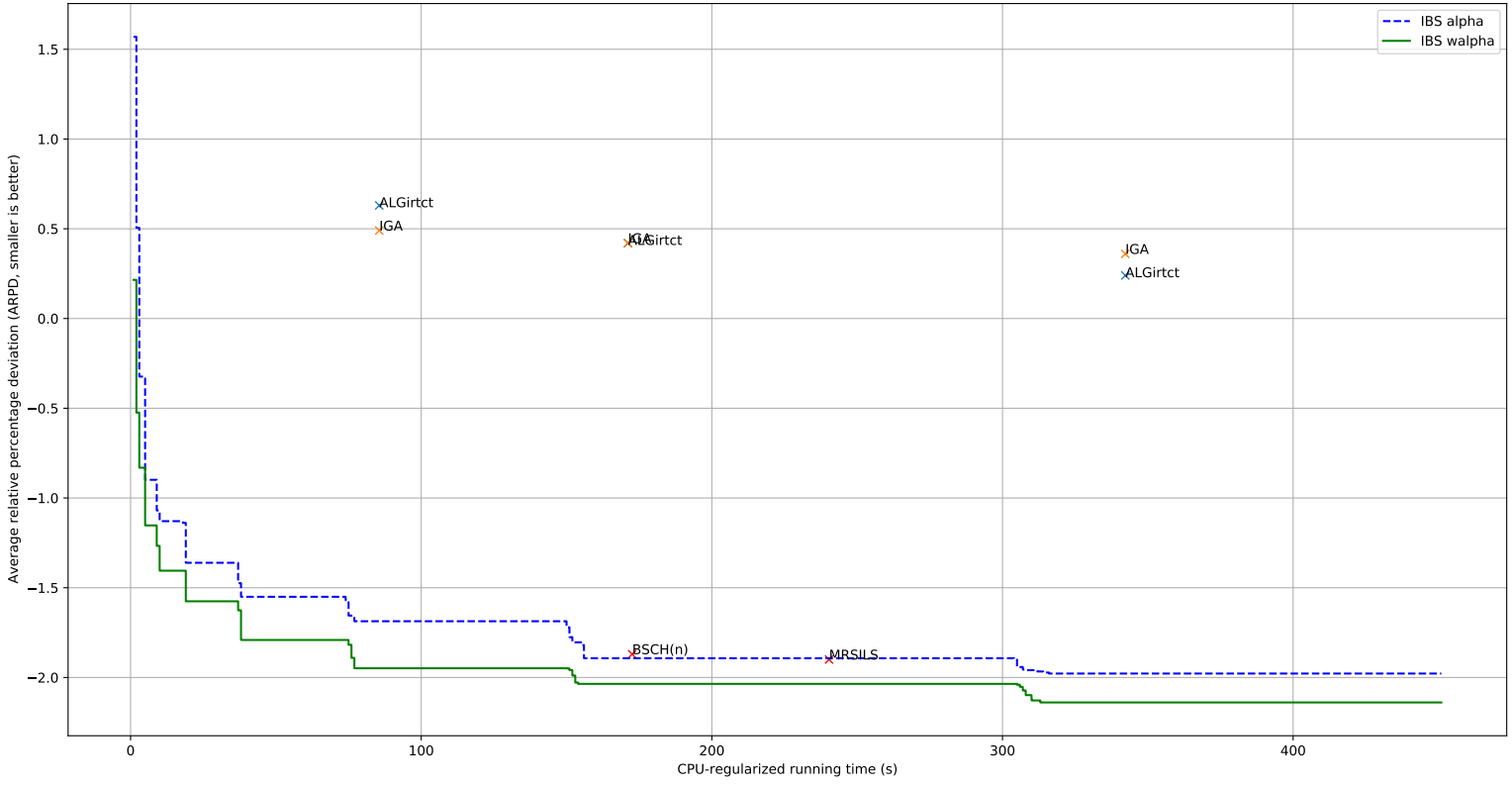
51/120 new-best-known solutions

500 jobs, 20 machines



Conclusions

This methodology leads to interesting algorithms

sometimes, improving upon the state-of-the-art

used on other problems:

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This methodology leads to interesting algorithms

sometimes, improving upon the state-of-the-art

used on other problems:

- **sequential ordering problem (ECAI2020, Q1)**
- **EURO/ROADEF 2018 challenge (EJOR, Q1)**
- general 2D cutting & packing problems
- longest common subsequence

- pre-print: *http://librallu.gitlab.io/pdfs/2020_pfsp_ibs.pdf*
- code: *https://github.com/librallu/dogs-pfsp*

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References

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