Tree searches for Sequential Ordering Problem

Contradicting conventional wisdom

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January, 30, 2020 - RealOpt Seminar

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Before starting

Please feel free to ask me questions at any time!

About the subtitle

Reference to [2]

Closing the open shop: Contradicting conventional wisdom (2009)

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- uses a simple CP model (weighted degree)
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- learning which task is “hard” outperform sophisticated inferences.

we identify some simple components that outperform state-of-the-art (on the SOPLIB).

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Context & Methodology

Conventional wisdom - Exact Methods and Metaheuristics

- Two ways to solve a problem
- **Exact methods:** MIPs, CP, Branch and Price ...

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- **(Meta-)heuristics:** local-search, evolutionary algorithms …

Conventional wisdom - about Tree Search

Mathematical Programming Solver based on Local Search ([1]):

> “ *Tree search approaches like branch-and-bound are in essence* ***designed to prove optimality*** *[...] Moreover, tree search has an exponential behavior which makes it* ***not scalable*** *faced with real-world combinatorial problems inducing millions of binary decisions.* ”

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We believe it is false considering **anytime tree searches**

Anytime tree searches - some (trivial) definitions

Tree searches: usually constructive algorithms (explore a tree)

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Anytime: Find quickly good solutions, then later try to improve them (similar to meta-heuristics)

Anytime tree searches

Many presented in AI/planning conferences

- some famous ones: LDS, Beam Search, wA* ...
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Sometimes in Operations Research (successful in Cutting & Packing)

Not used much compared to classical meta-heuristics (tabu search, genetic algorithms ...)

why anytime tree searches are not used more?

two hypothesis (on SOP):

1. They are not efficient?
2. They are underestimated?

We believe the latter is true

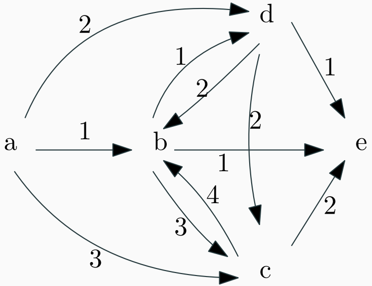
Our experiment

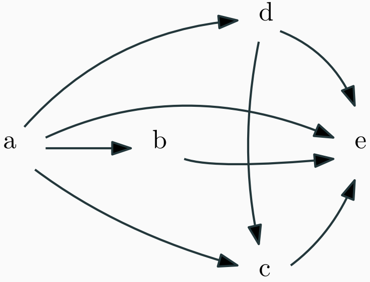
- We consider a well known benchmark (SOP)
- Apply anytime tree searches

Sequential Ordering Problem

SOP - problem definition

Asymmetric Traveling Salesman Problem with precedence constraints





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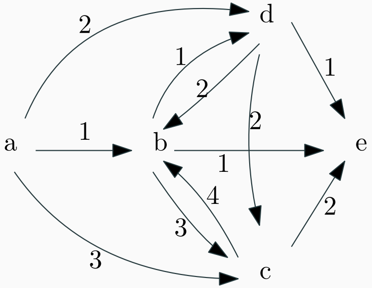

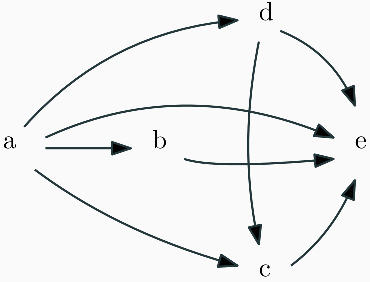

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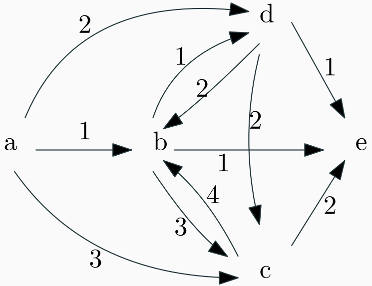

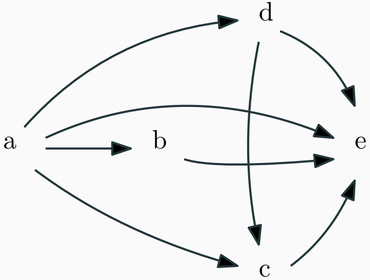

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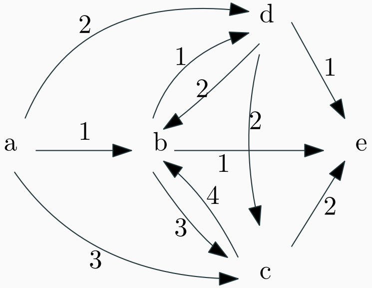

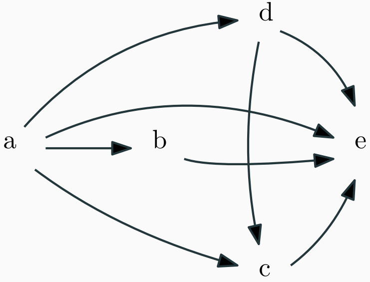

- a,d,c,b,e is a feasible and costs 10
- a,b,c,d,e is not feasible
- a,b,d,c,e is optimal and costs 6

The benchmark: SOPLIB

- proposed in 2006
- Standard for meta-heuristics
- "large" instances (200 to 700 cities)
- different densities (1, 15, 30, 60) % precedence constraints
- 15% precedence-dense instances remain open (7 instances)

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- **1% precedences** are easy (using MIP + lazy constraints)
- **15% precedences** are “hard”
- **30% and 60% precedences** are easy (solved by dynamic programming)

Literature

Many methods implemented during the 30 last years to solve SOP

Exact methods:

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- Ant Colony Optimization
- Various searches (GA, ABC, parallel roll-out, LKH ...)

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Meta-heuristics:

- Local searches (3-opt)
- Ant Colony Optimization
- Various searches (GA, ABC, parallel roll-out, LKH ...)

- Exact methods tend to build stronger bounds
- meta-heuristics strongly rely on 3-opt (local search)

Tree Search

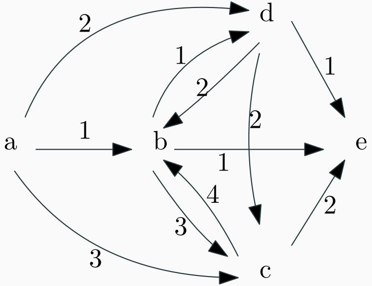
Two parts:

Implicit tree: how to branch, bounds ...

Search strategy: DFS, best-first, Beam Search ...

Implicit tree - Branching

Forward branching



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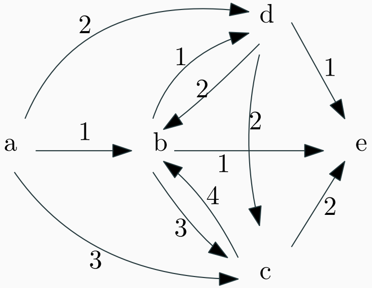

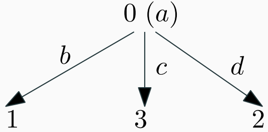

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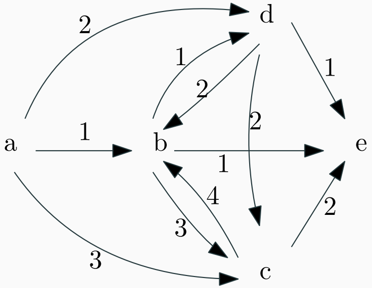

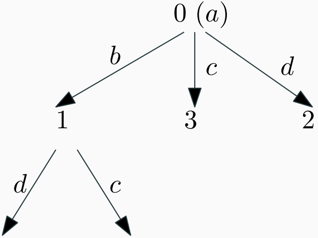

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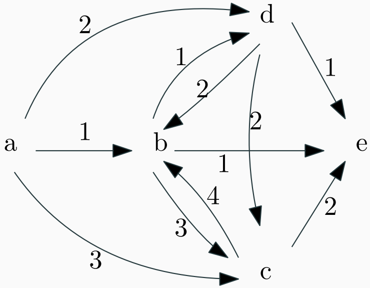

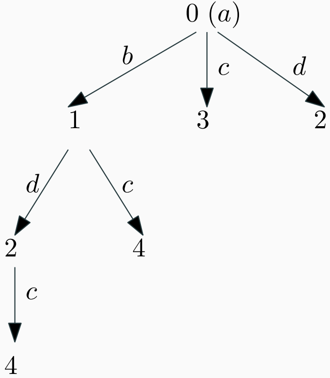

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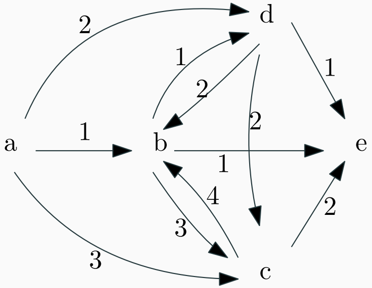

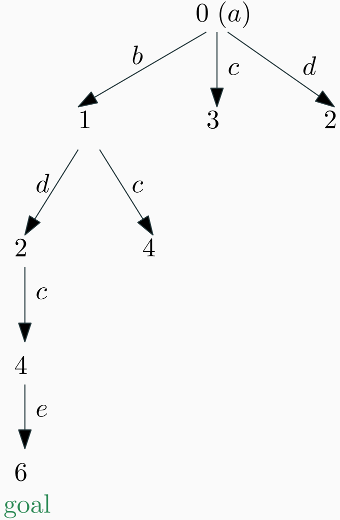

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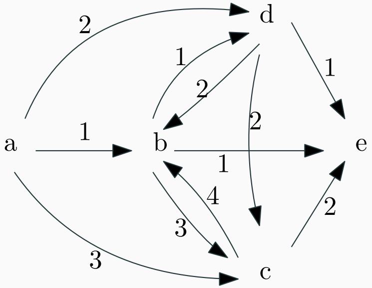

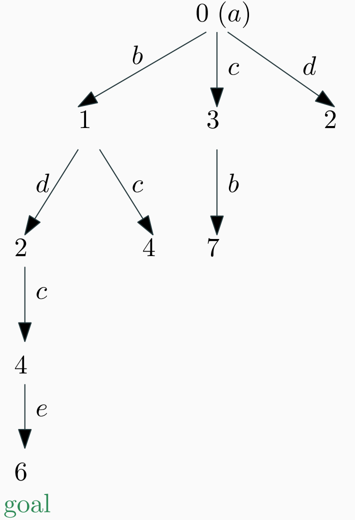

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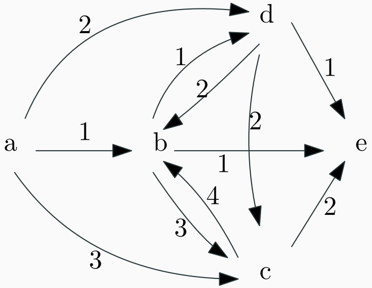

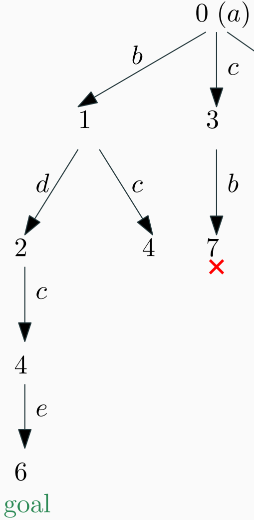

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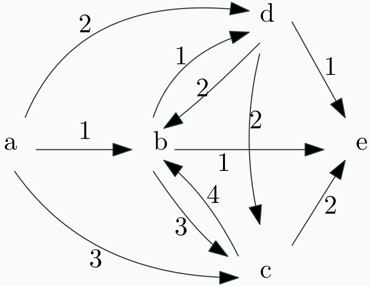

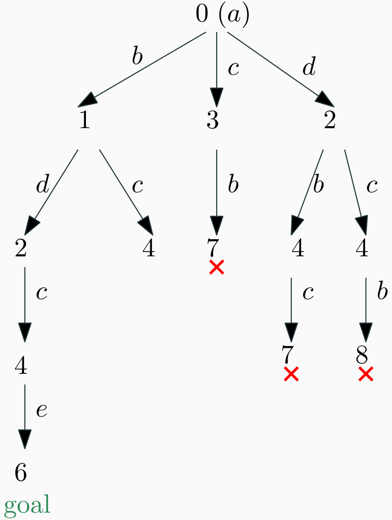

Bounds

We consider 3 (simple) bounds:

Prefix bound: only arcs between selected vertices (previous example)

ingoing/outgoing: adding minimum ingoing/outgoing arc for not-selected vertices

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Out of our experiments, the prefix bound (despite its simplicity) provides the same guiding quality as other bounds. For simplicity, we only show results using it.

Depth First Search

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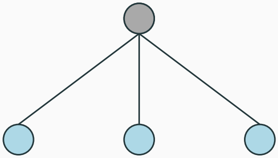

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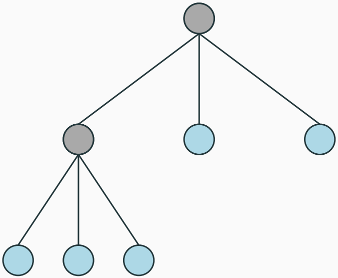

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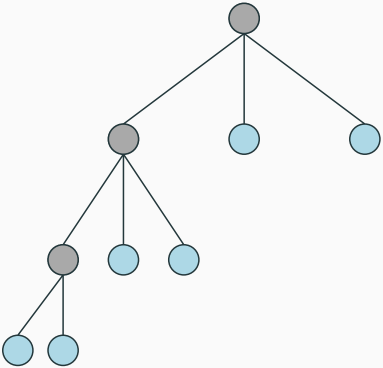

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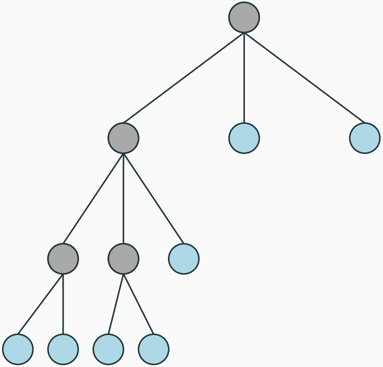

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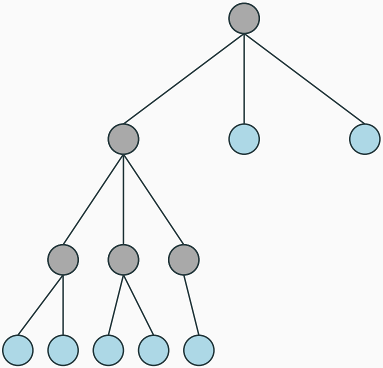
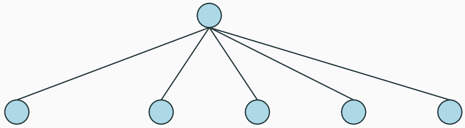


Depth First Search - Drawbacks

Stuck in early sub-optimal choices near the root

Limited Discrepancy Search (LDS) - key idea

Correct DFS drawback: early bad decisions



Explore more the most promising but still keep exploring others

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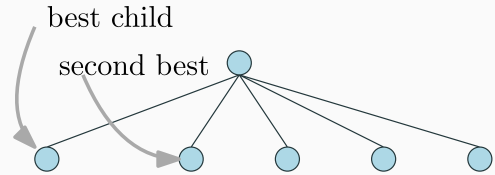

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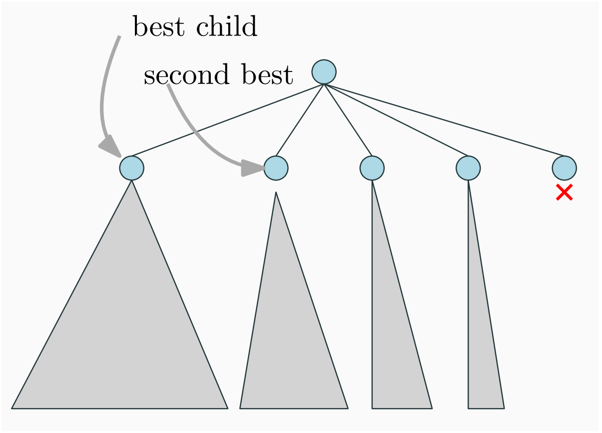
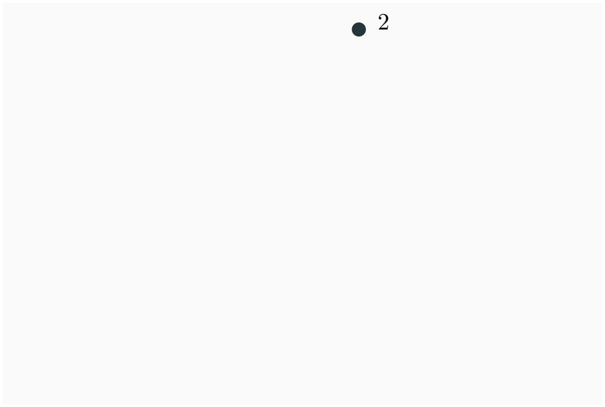


Explore more the most promising but still keep exploring others

Limited Discrepancy Search (LDS) - An example: D=2

Count the number of deviations from the best child (discrepancies)



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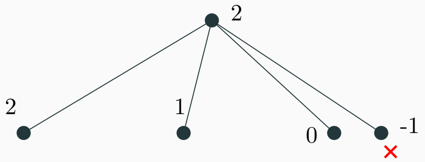

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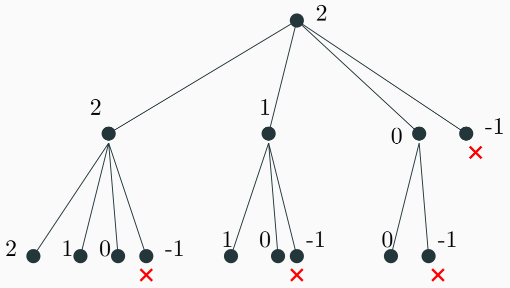

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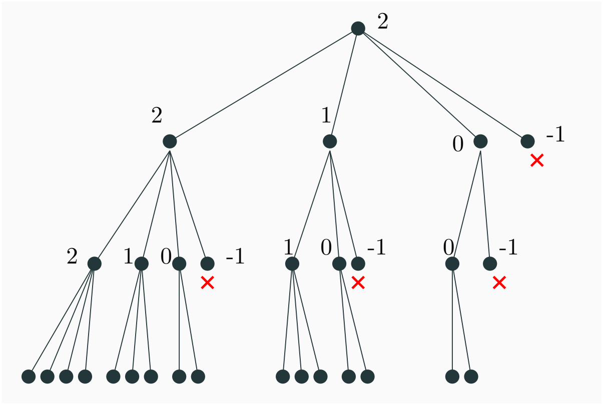
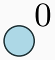


Beam Search ($D = 3$)



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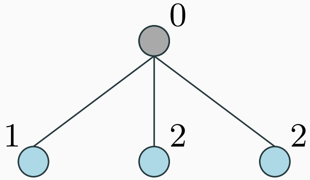

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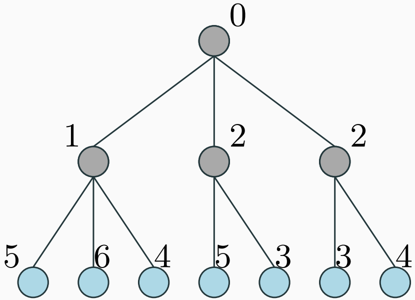

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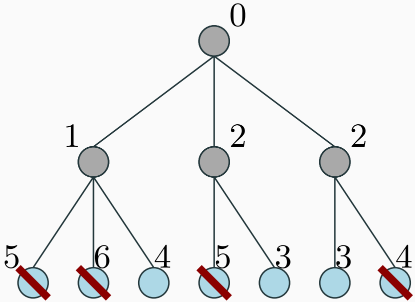

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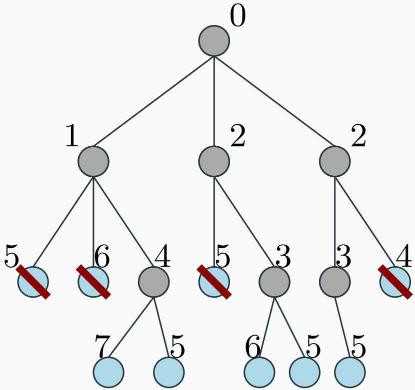

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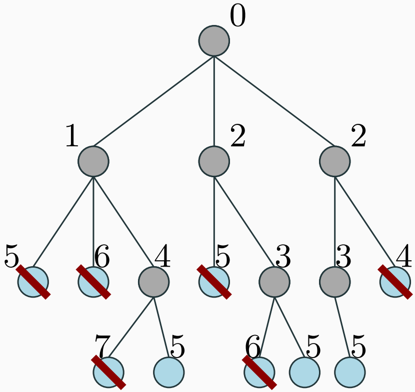


Iterative Beam Search

- Runs a beam of size 1 (greedy)
- Then runs a beam of size 2, then 4, then 8 …

Stops when no heuristic fathoming is done (proves optimality)

Prefix equivalence fathomings

Inspired from dynamic programming

Example, two partial equivalent[1] solutions:

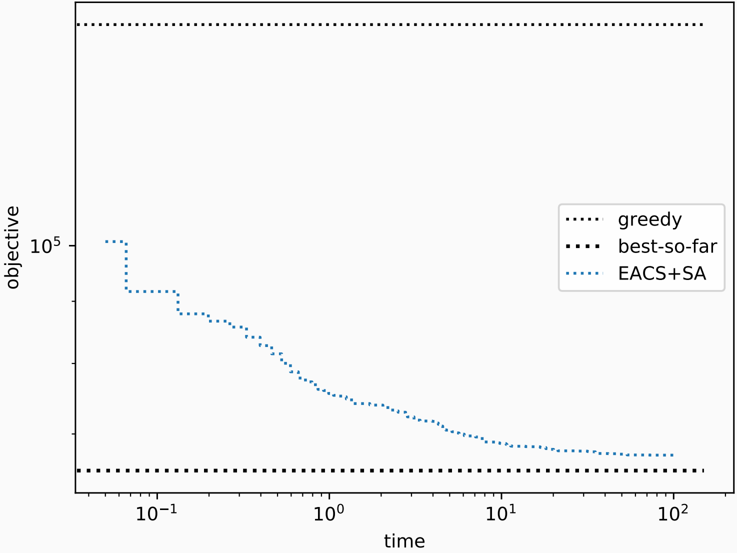
1. **a,b,c,d** cost 10
2. **a,c,b,d** cost 12

Discard (2) as it is "dominated" by (1).

[1] have the same sub-trees. The node contains the same subset of visited nodes and the same last vertex

Results - Performance profiles on R.700.1000.15

best-so-far LKH3 with 100.000 seconds run ($\approx$ 27h)



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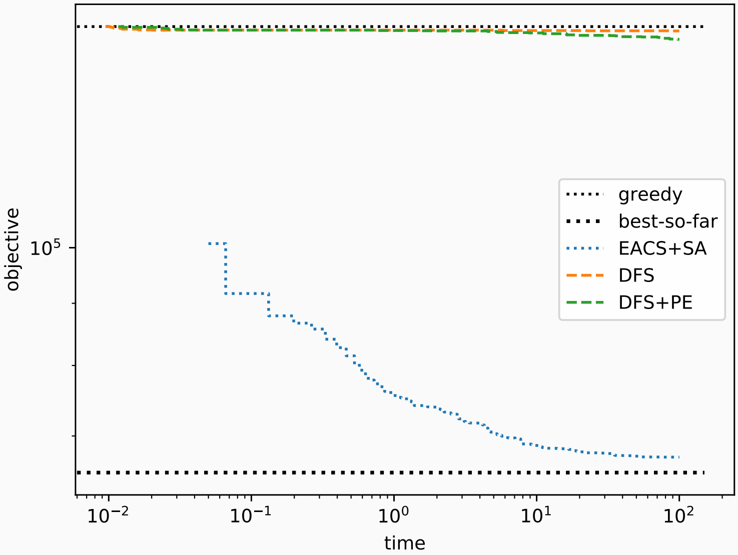

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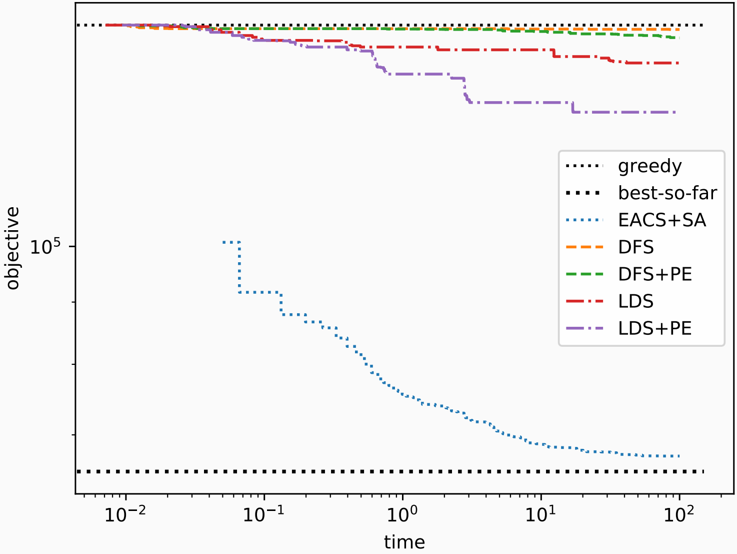

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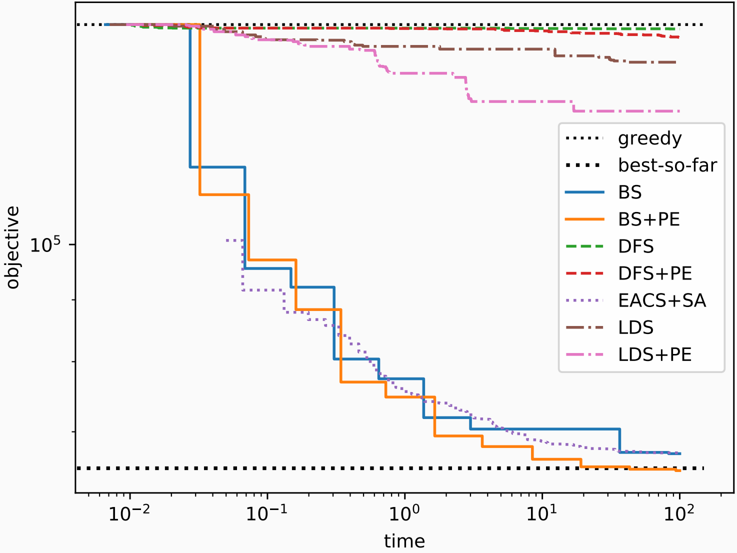

Results - New best-so-far solutions

6 over 7 new-best-so-far solutions
(the other one is probably optimal)

Instance	best known	BS+PE (600s)
R.500.100.15	5.284	5.261
R.500.1000.15	49.504	49.366
R.600.100.15	5.472	5.469
R.600.1000.15	55.213	54.994
R.700.100.15	7.021	7.020
R.700.1000.15	65.305	64.777

Results - Overview

How this simple tree search behaves on the SOPLIB:

1% precedence constraints: poor results. too many choices and too simple guides

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- hard for MIPs and local searches
- but (relatively) easy for constructive algorithms
- thus the need to consider anytime tree searches

Wrapping-up

Conclusions

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- At least on the SOPLIB, anytime tree searches outperform classical meta-heuristics
- The search-strategy choice is crucial
- An analysis of the impact of each algorithmic component leads to interesting insights

My thesis (goal)

- Study tree searches (from AI/planning, OR, and heuristics)
- Combine them and study their efficiency on OR problems

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Tree searches for Sequential Ordering Problem

Contradicting conventional wisdom

Luc Libralesso - Abdel-Malik Bouhassoun - Hadrien Cambazard - Vincent Jost

January, 30, 2020 - RealOpt Seminar

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