Tree search algorithms for the sequential ordering problem

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Context & Methodology

Conventional wisdom - Exact Methods and Metaheuristics

Two ways to solve a **Operations Research** problem

- **Exact methods:** MIPs, CP, Branch and Price ...

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- **(Meta-)heuristics:** local-search, genetic algorithms, ant colony ...

Conventional wisdom - about Tree Search

Mathematical Programming Solver based on Local Search ([1]):

> *“ Tree search approaches like branch-and-bound are in essence* ***designed to prove optimality*** *[...] Moreover, tree search has an exponential behavior which makes it* ***not scalable*** *faced with real-world combinatorial problems inducing millions of binary decisions. ”*

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We believe it is false considering **anytime tree searches**

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- Some famous ones: **LDS, Beam Search, wA* ...**
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Similar in purpose and definitions to meta-heuristics

Still not used much compared to classical meta-heuristics

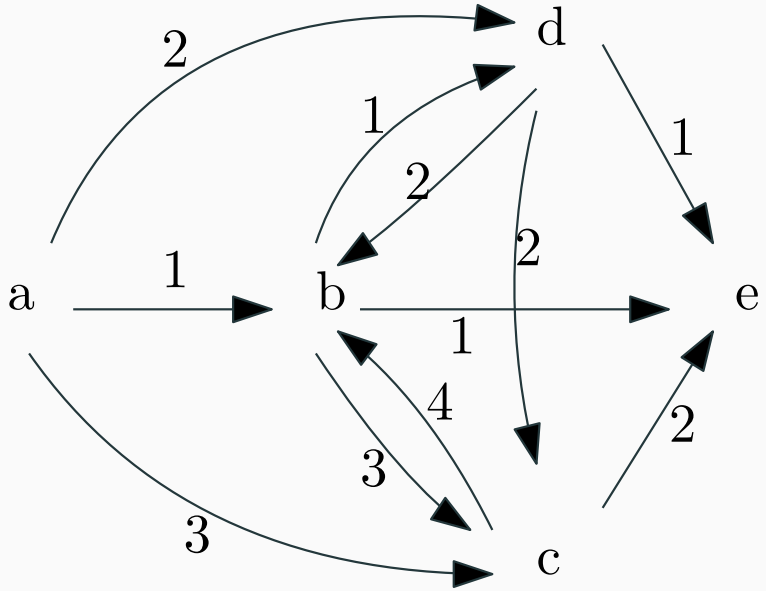
Our experiment

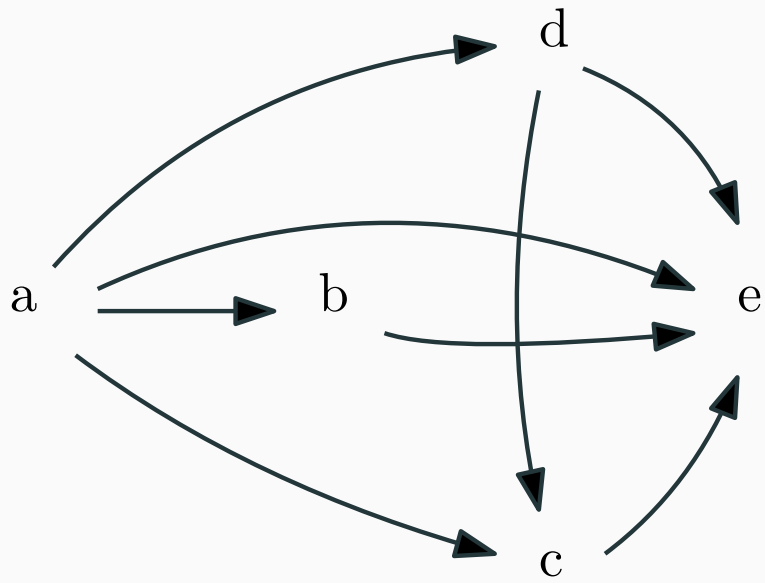
- We consider a well known benchmark (SOP)
- Apply anytime tree searches

Sequential Ordering Problem

SOP - problem definition

Asymmetric Traveling Salesman Problem with precedence constraints





The benchmark: SOPLIB

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- Some instances are almost precedence free
- Some are heavily constrained
- “in the middle” instances remain open (7 instances)

Literature

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Exact methods:

- Branch and cuts
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- Exact methods tend to build stronger bounds
- meta-heuristics strongly rely on 3-opt (local search)

Our anytime Branch-and-Bound

Tree Search

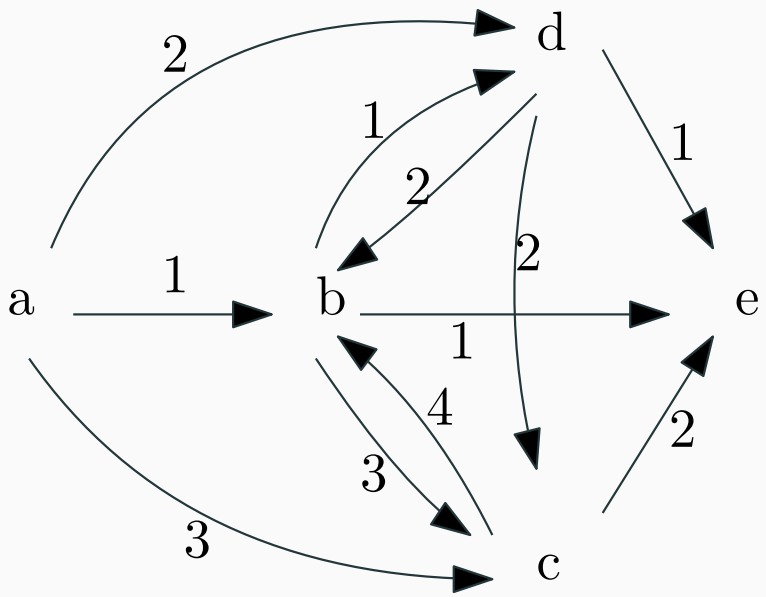
Two parts:

Implicit tree: how to branch, bounds ...

Search strategy: DFS, best-first, Beam Search ...

Implicit tree - Branching

Forward branching + Prefix bounds



$0\ (a)$

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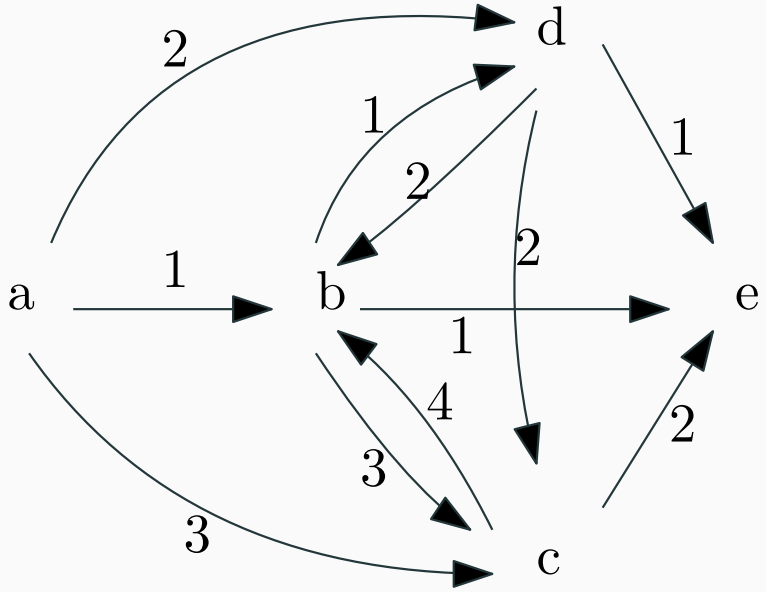

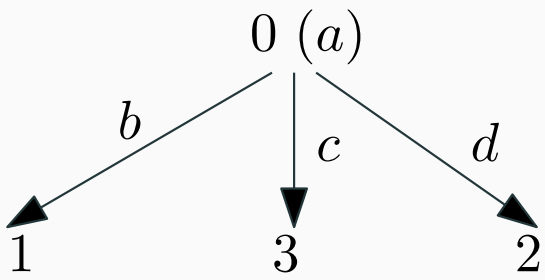

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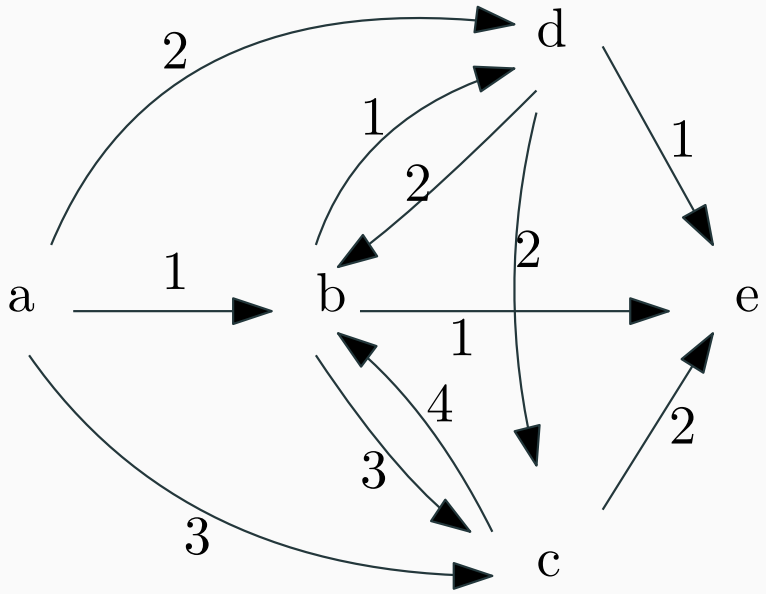

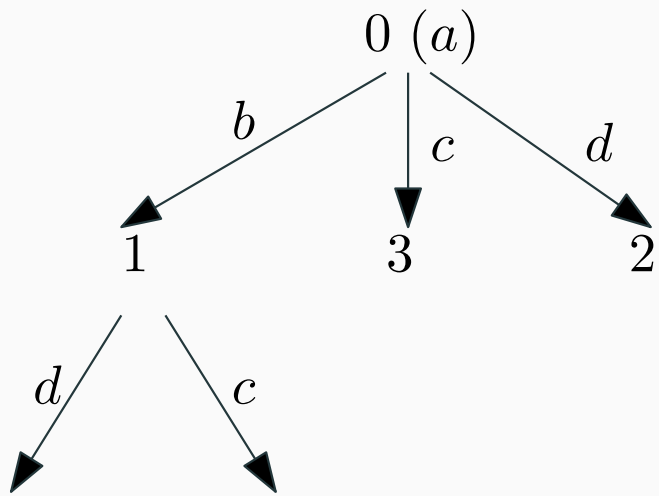

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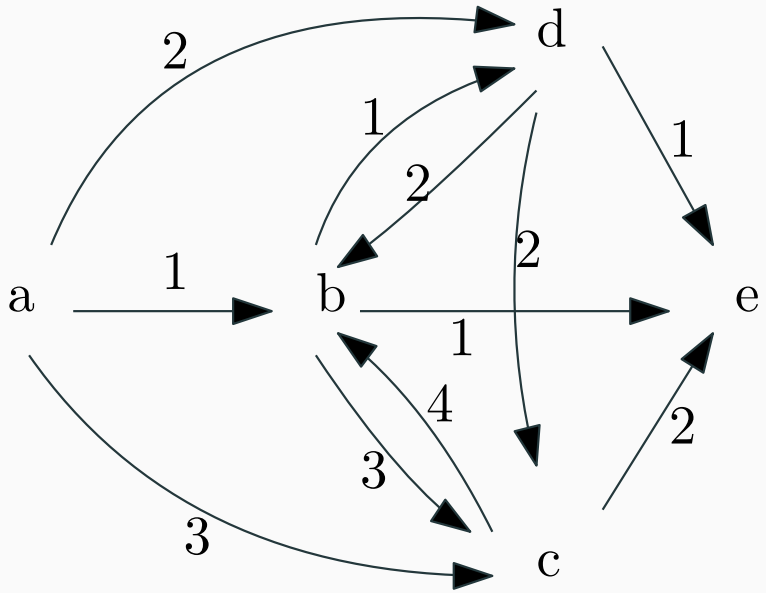

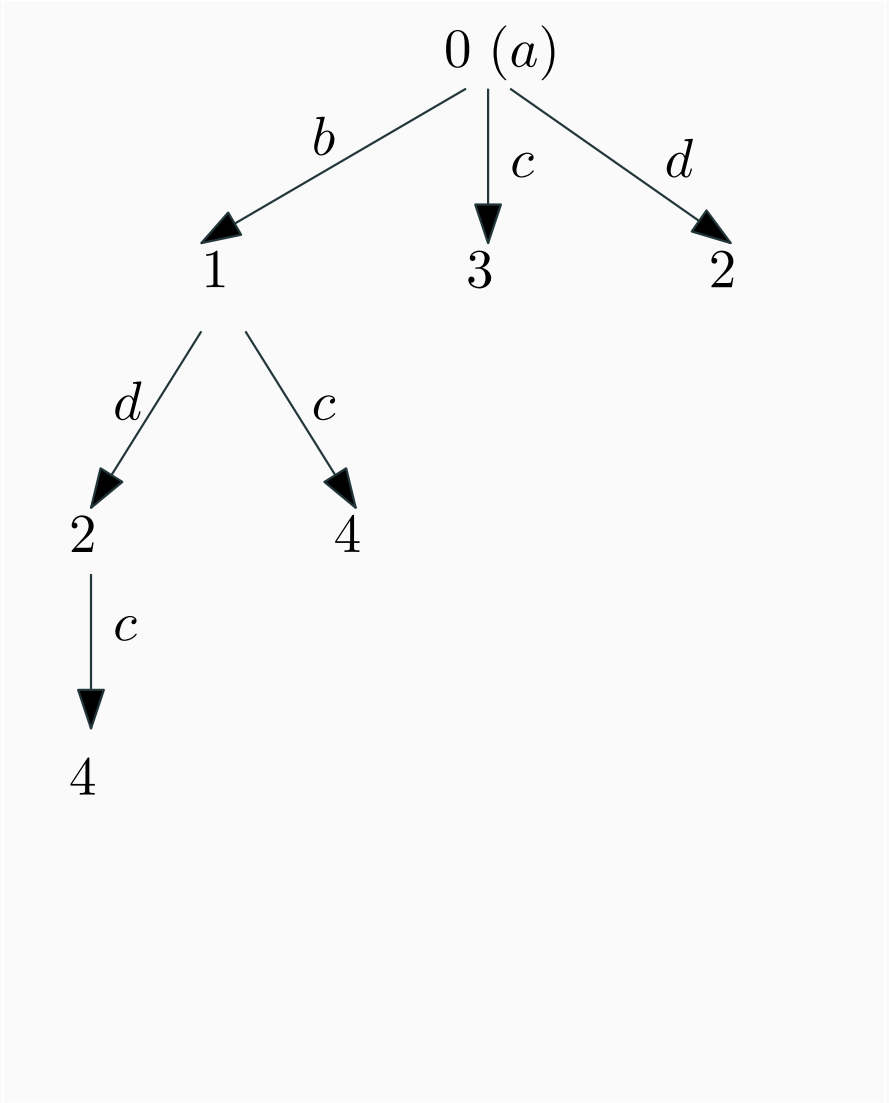

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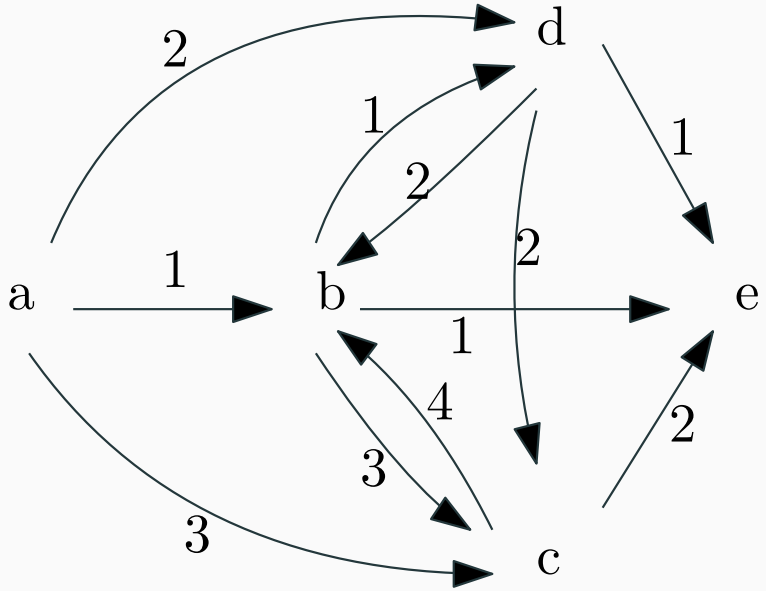

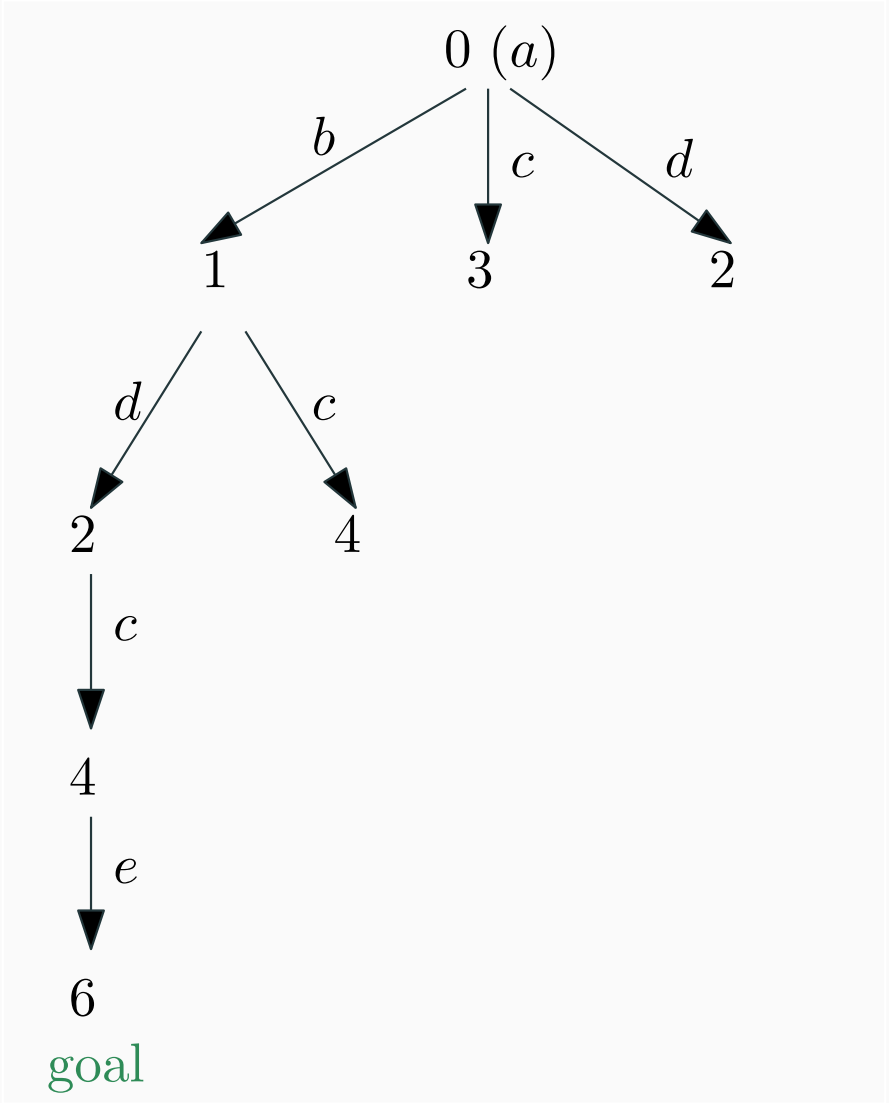

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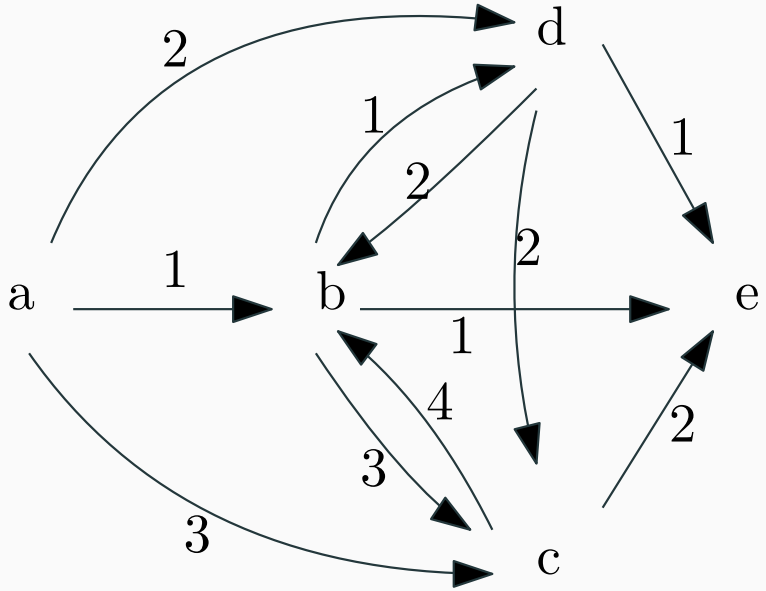

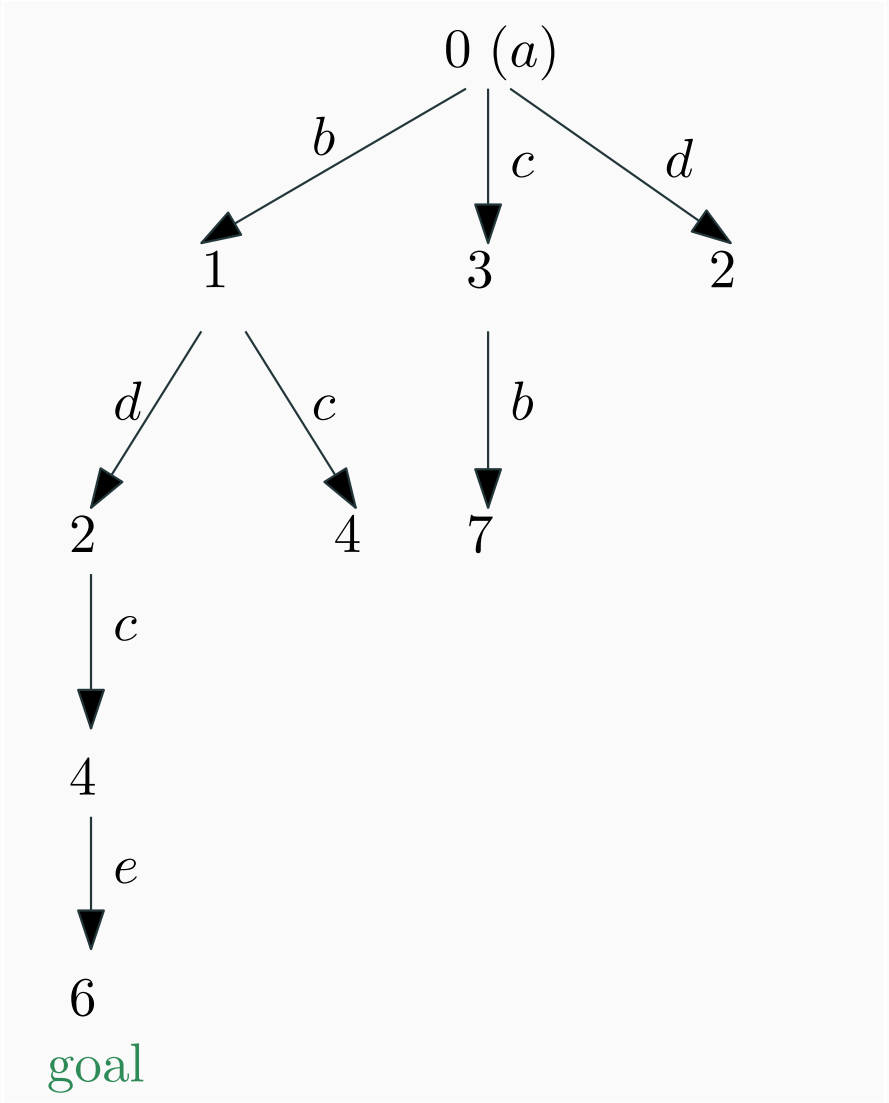

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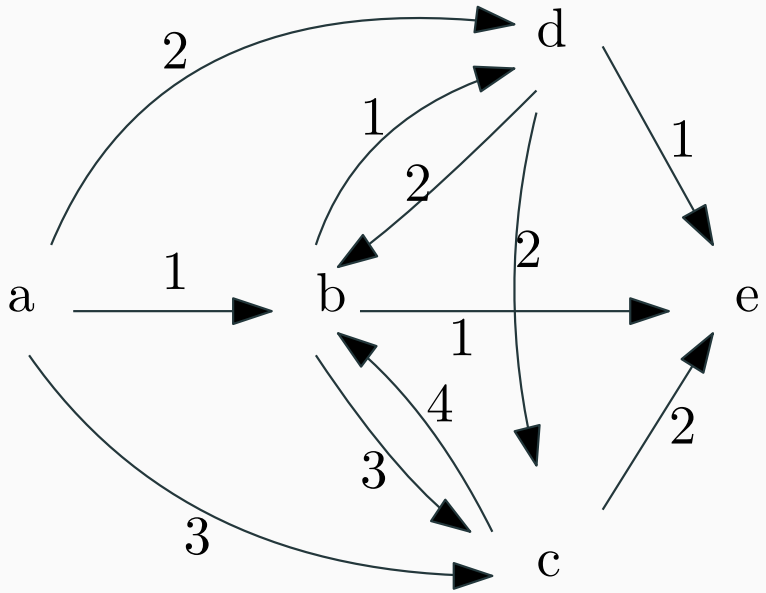

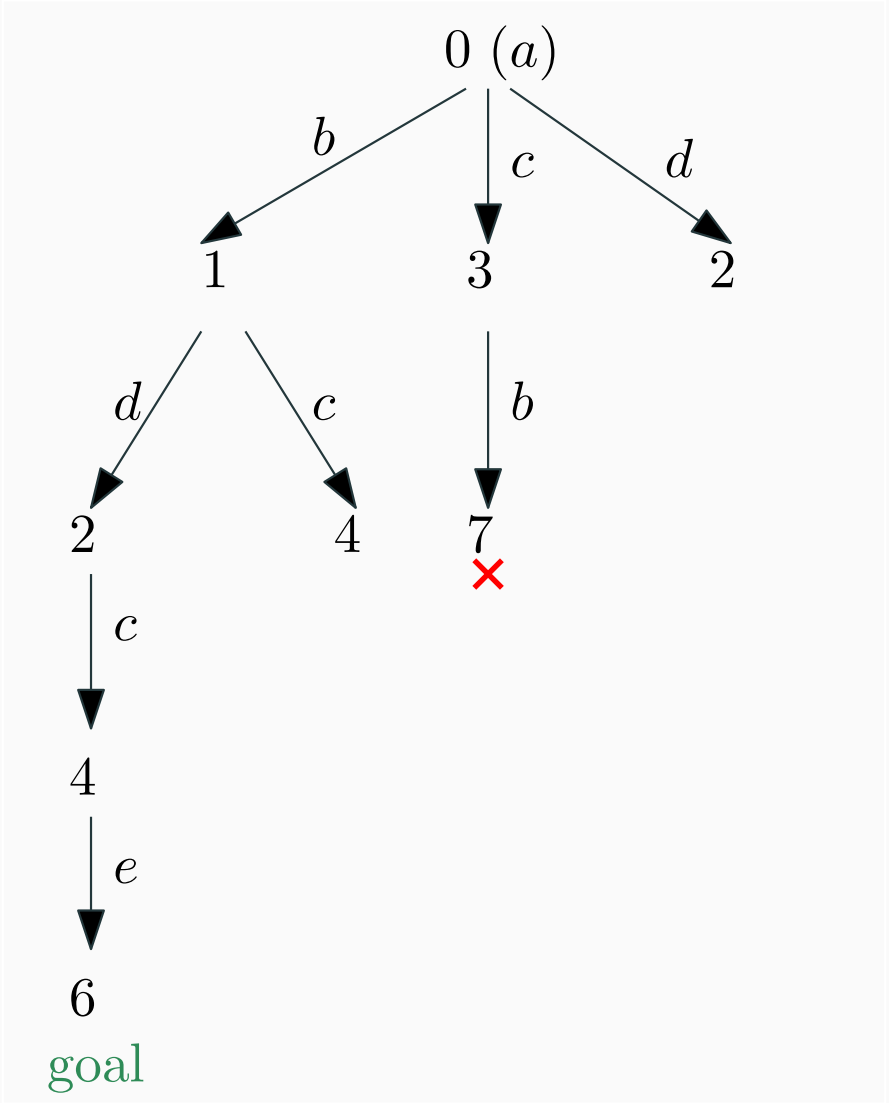

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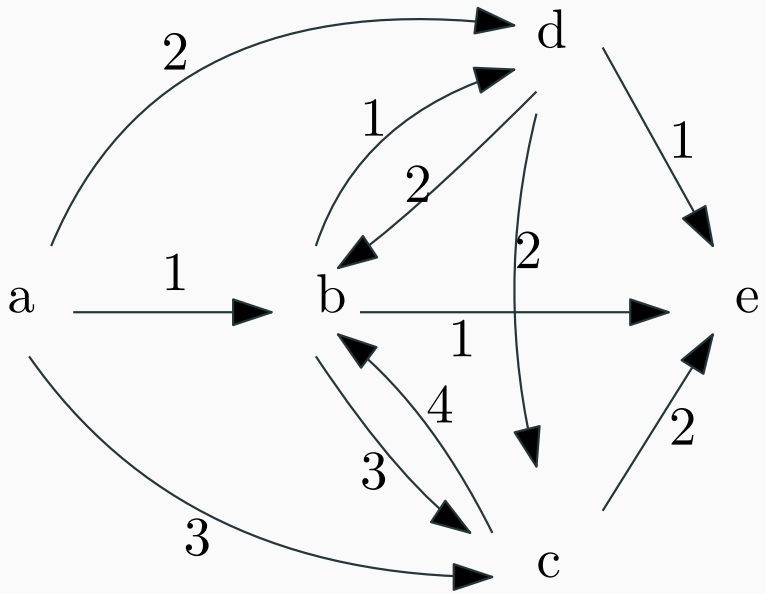

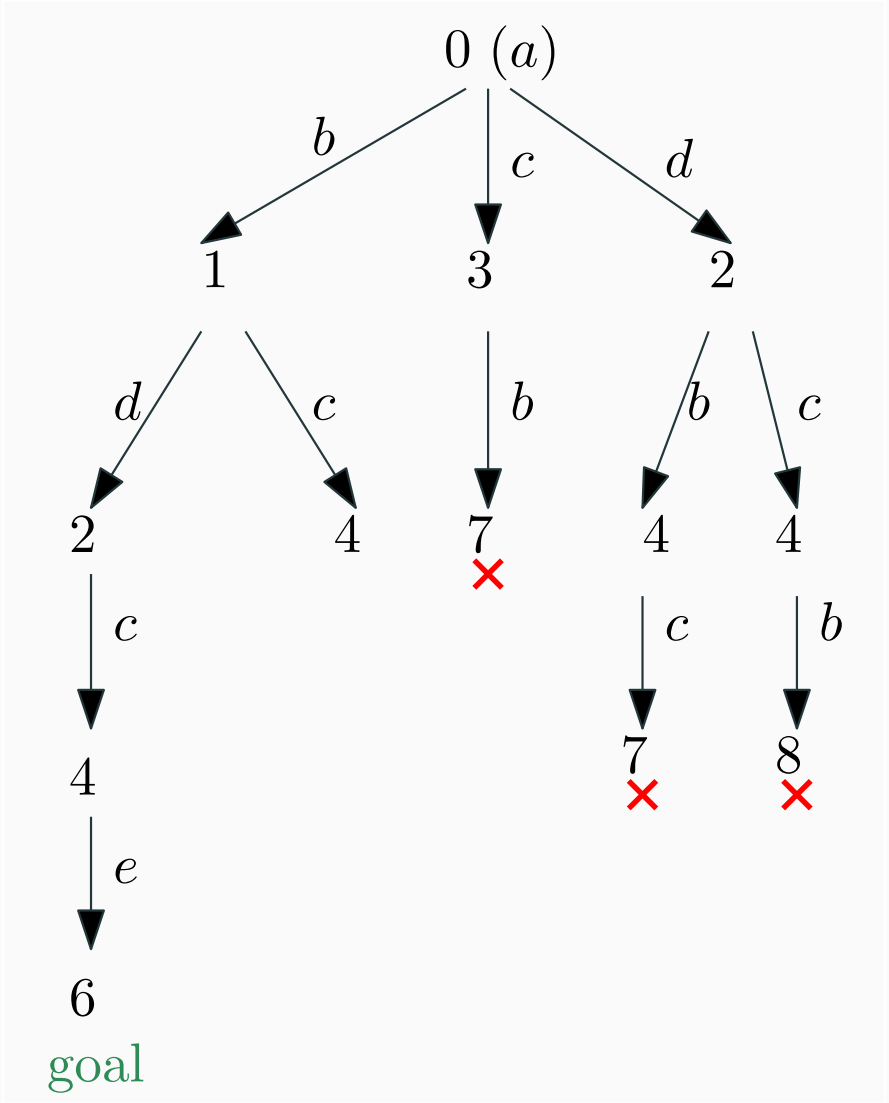

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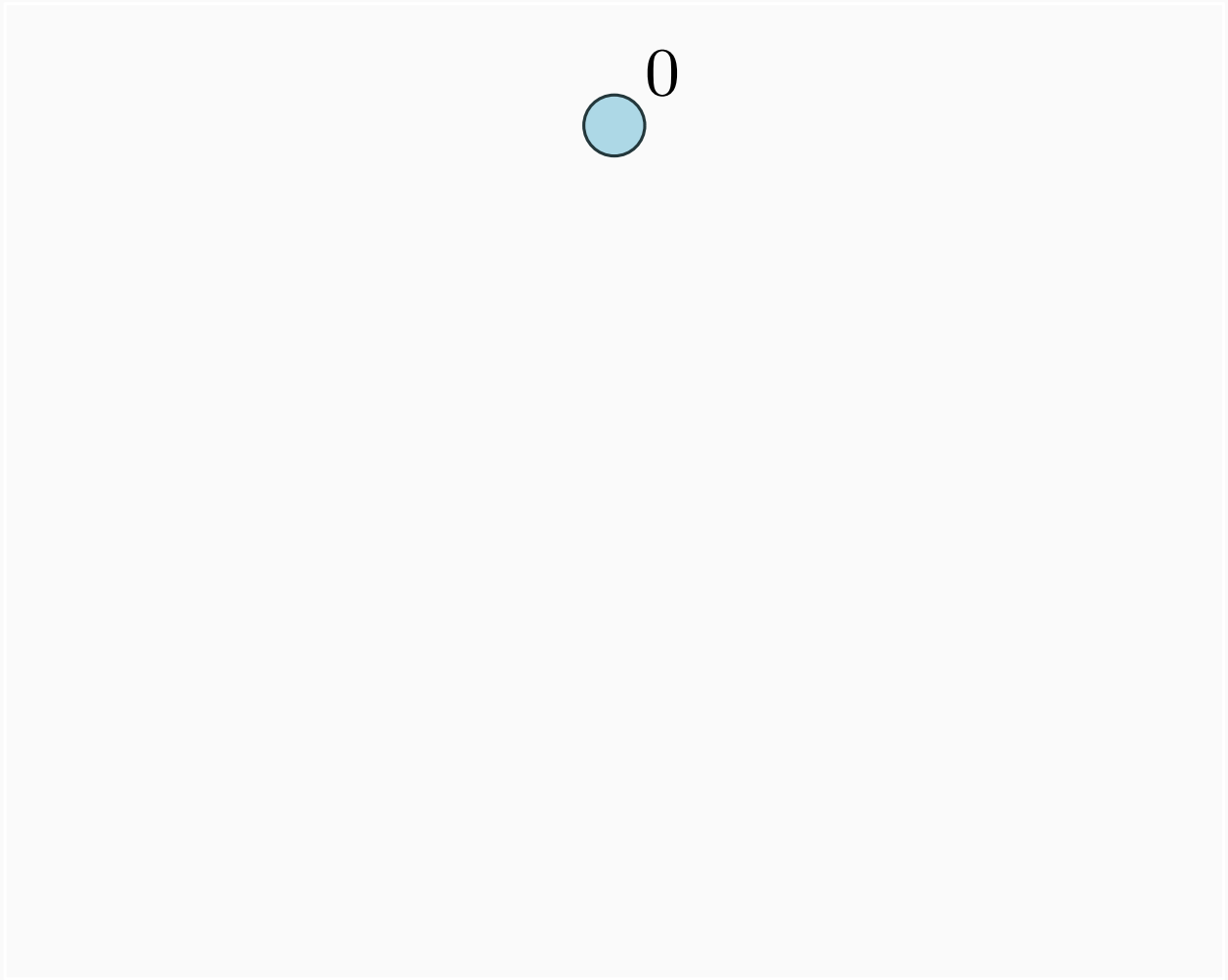
- **Iterative Beam Search** (next slide)

Beam Search ($D = 3$)



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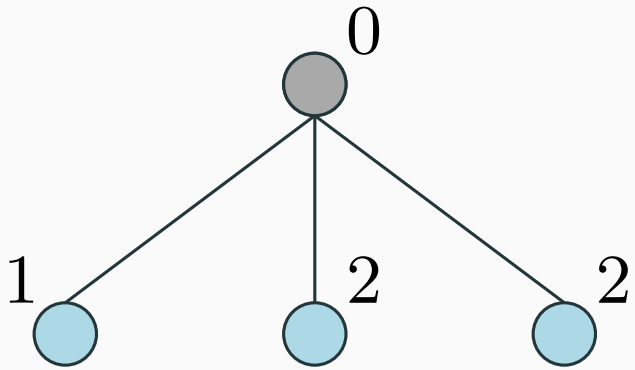

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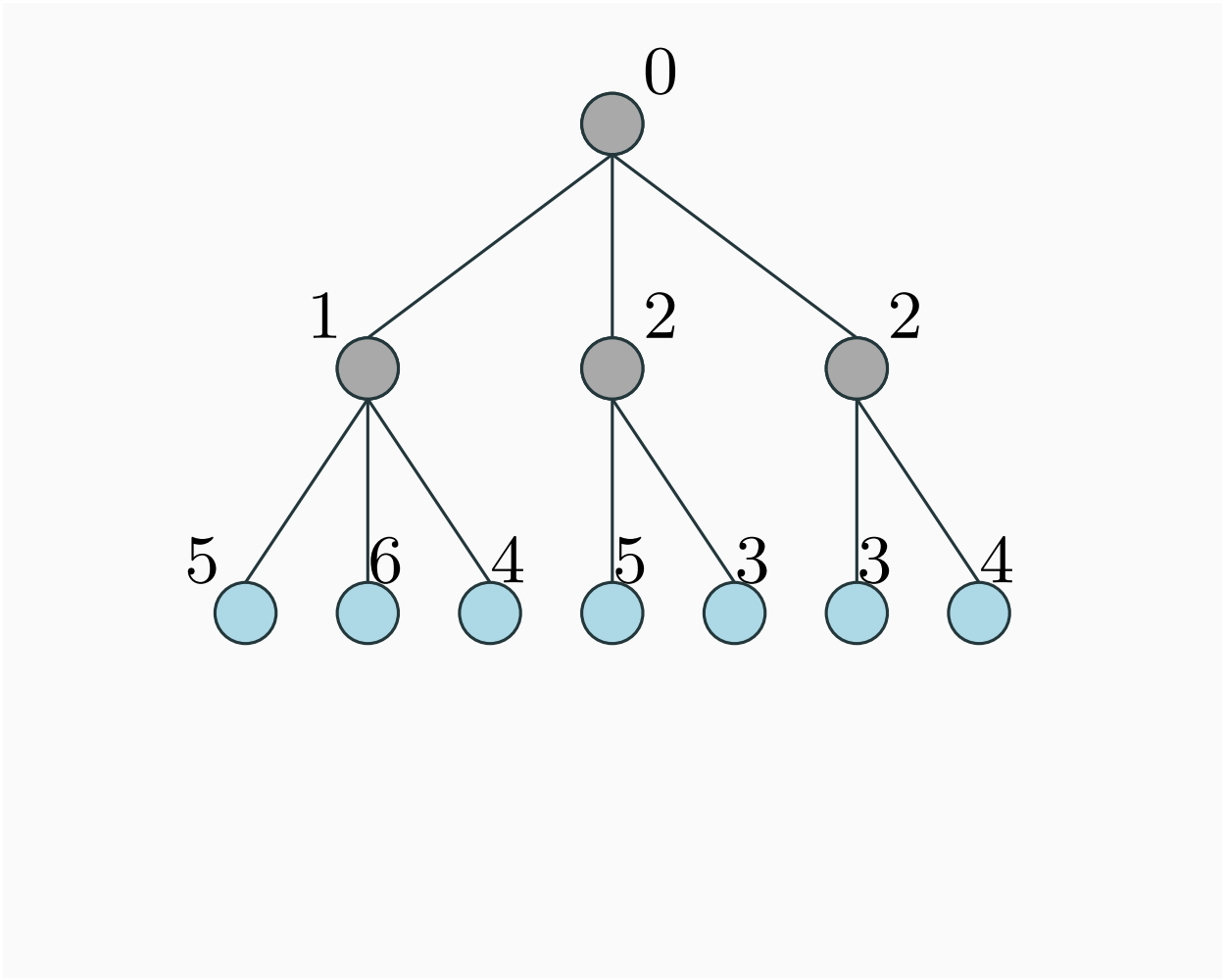

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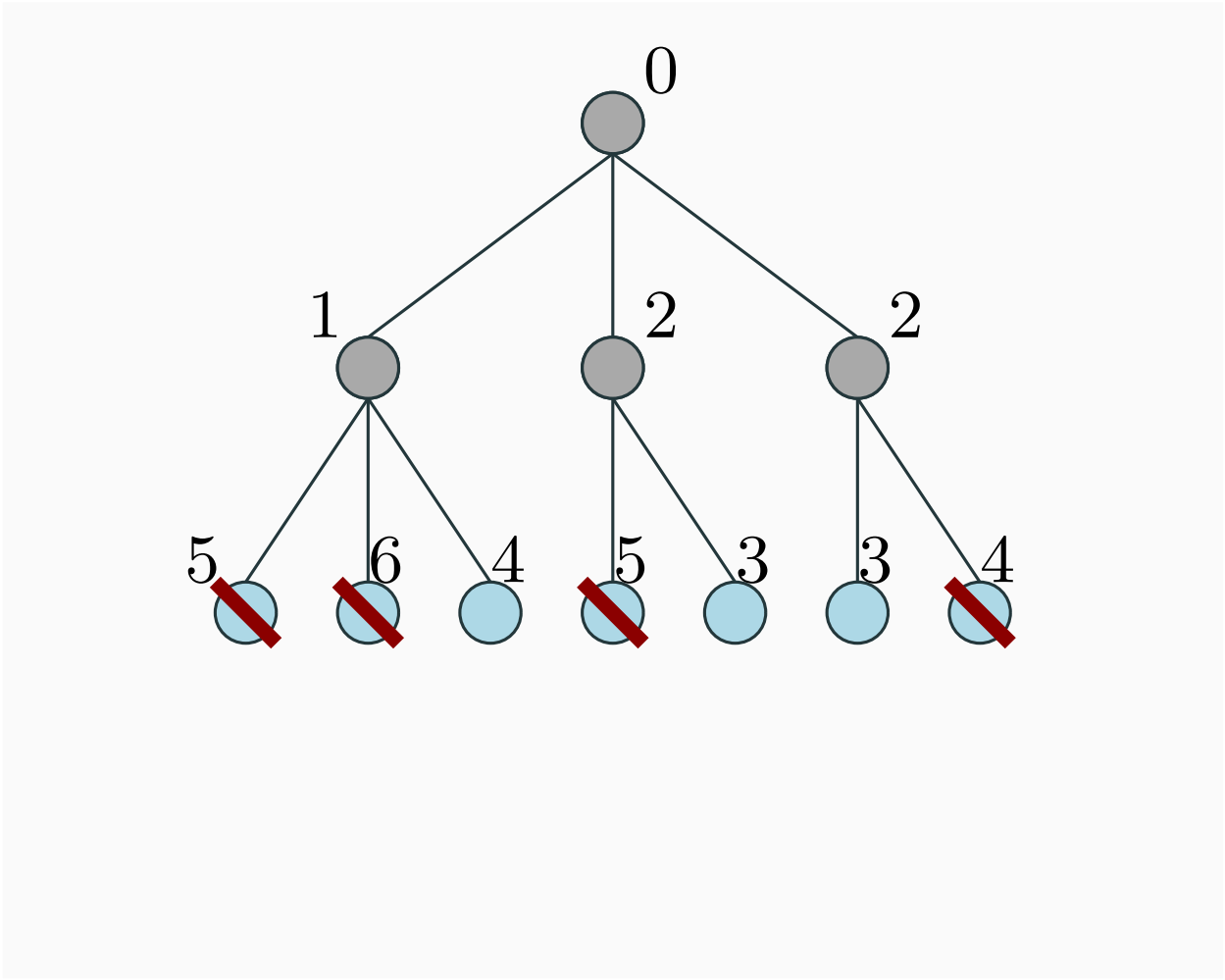

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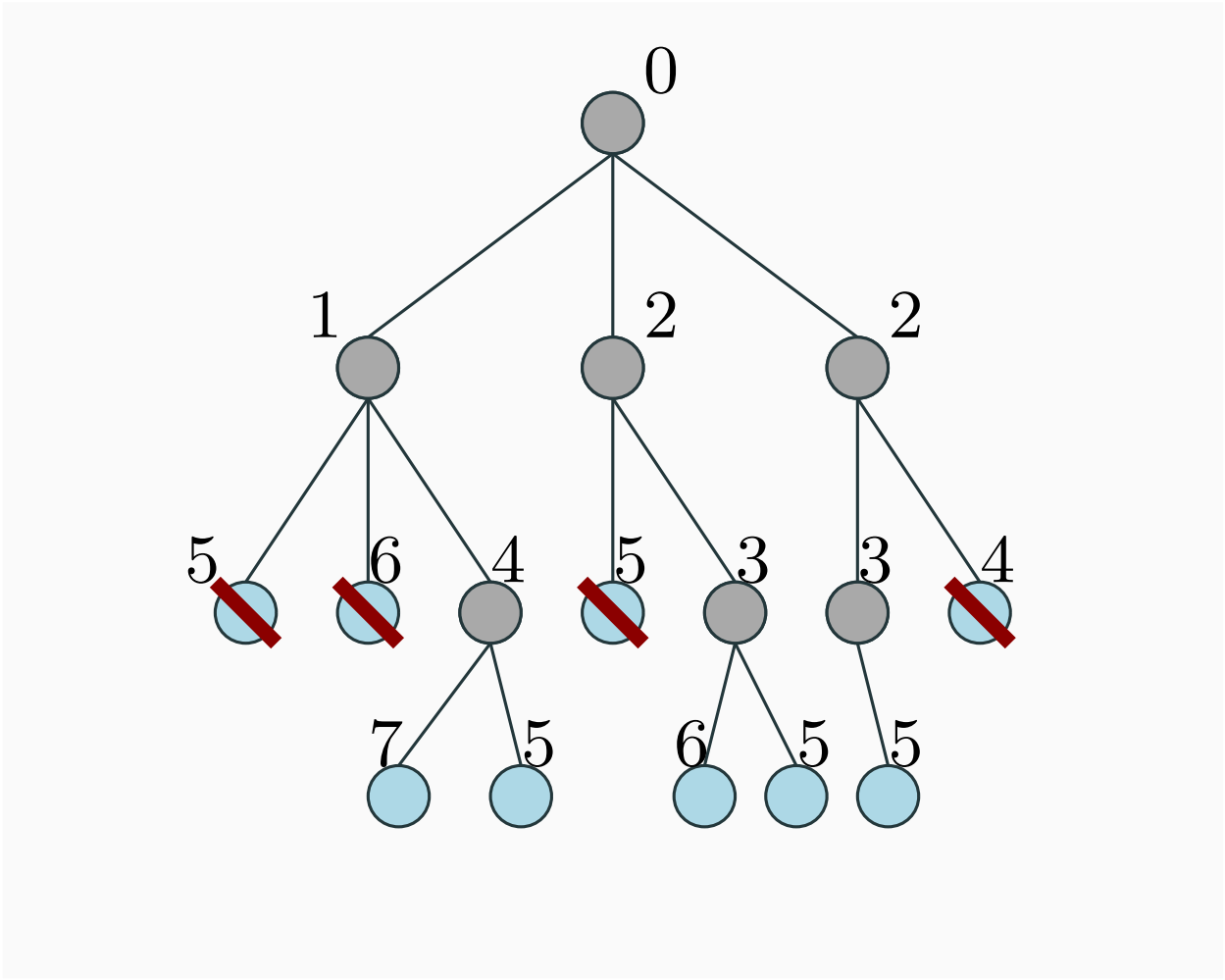

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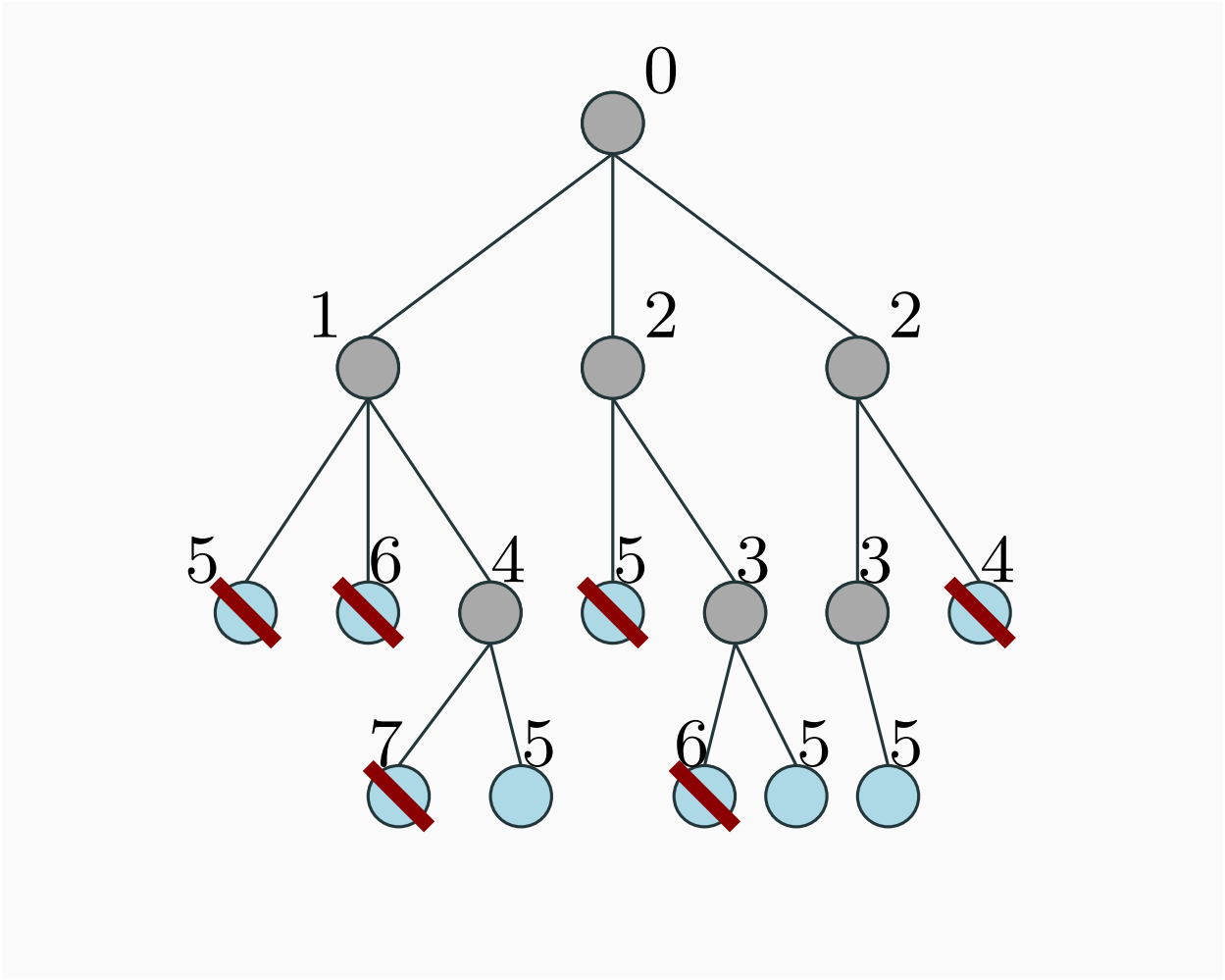

Iterative Beam Search

- Runs a beam of size 1 (greedy)
- Then runs a beam of size 2, then 4, then 8 ...

Stops when no heuristic fathoming is done (proves optimality)

Dynamic Programming inspired prunings

Example, two equivalent partial solutions:

1. **a,b,c,d** cost 10
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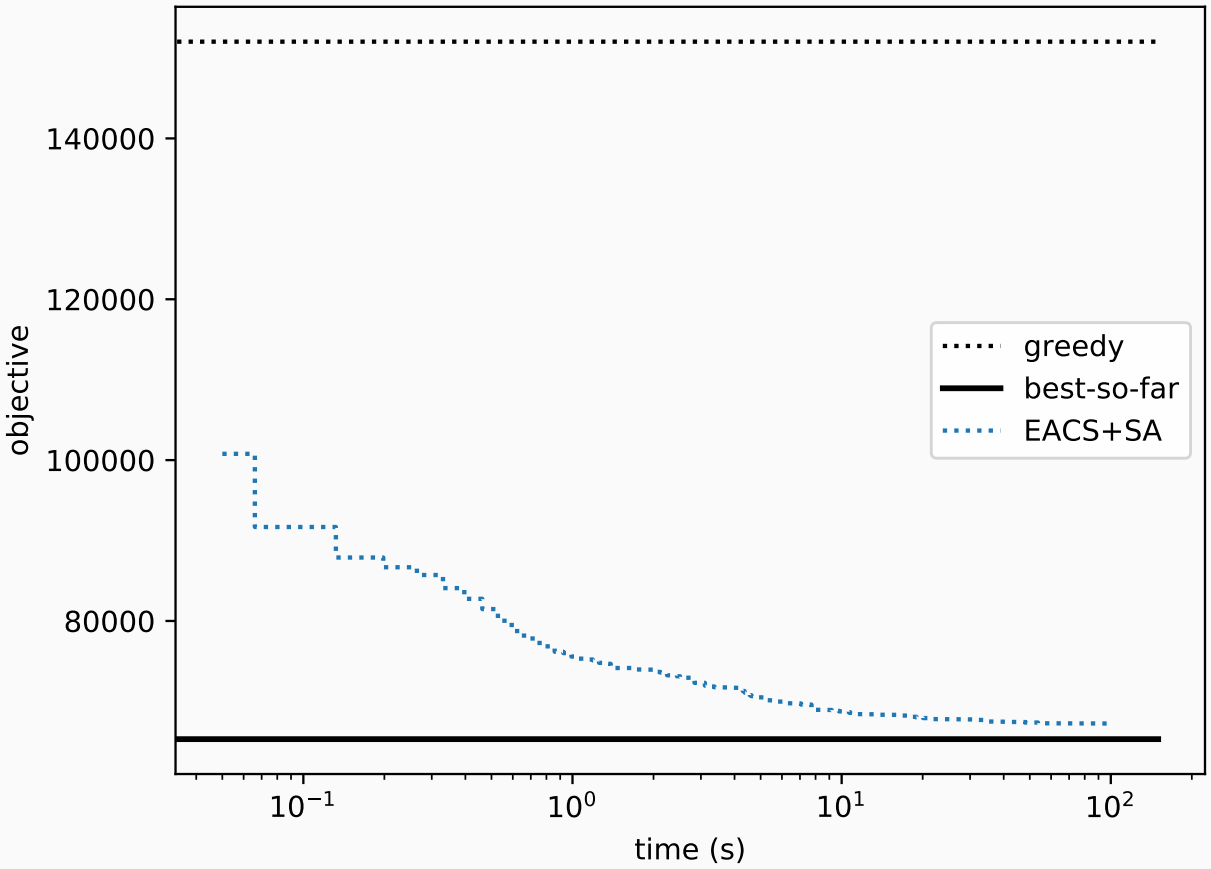
Discard (2) as it is “dominated” by (1).

Numerical Results

Results - Performance profiles on R.700.1000.15

state-of-the-art:

- Enhanced Ant Colony System and Simulated Annealing (EACS+SA)
- best-so-far LKH3 with 100.000 seconds run (≈ 27h)



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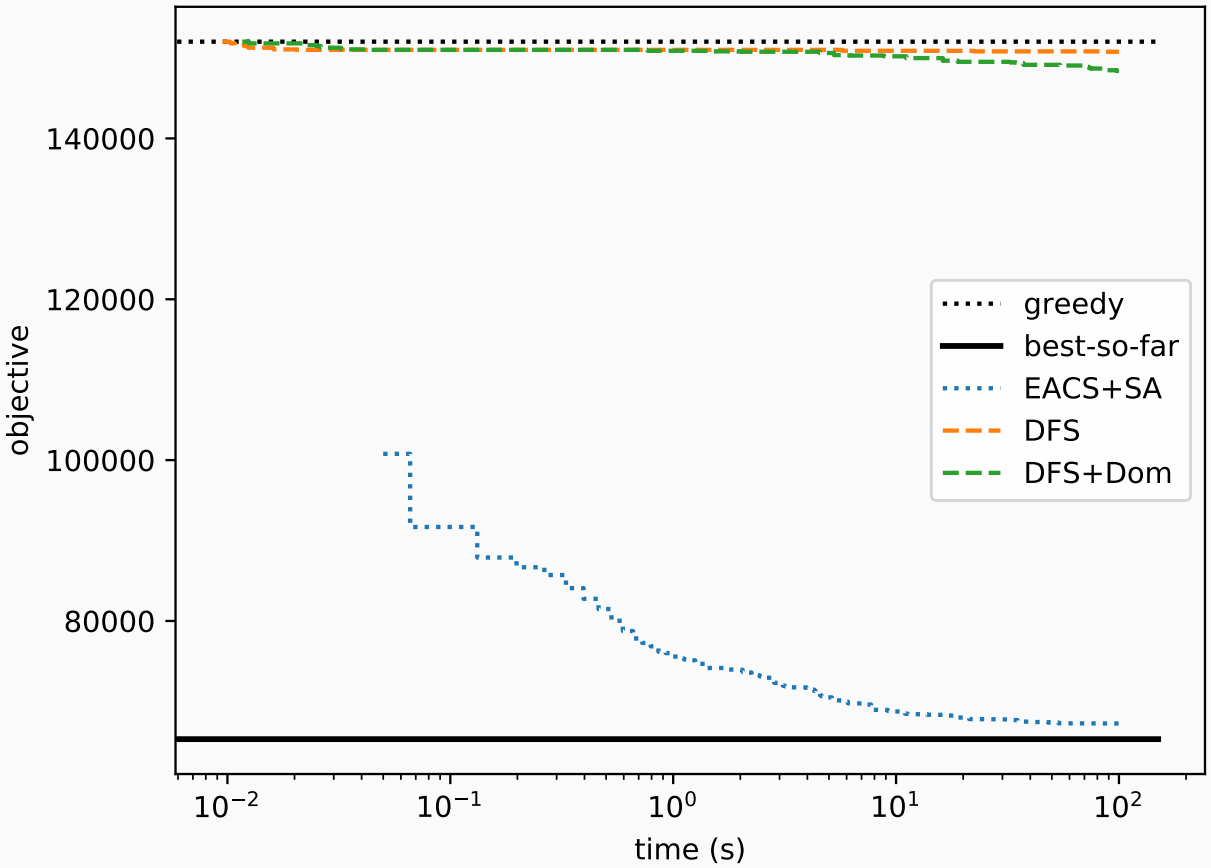

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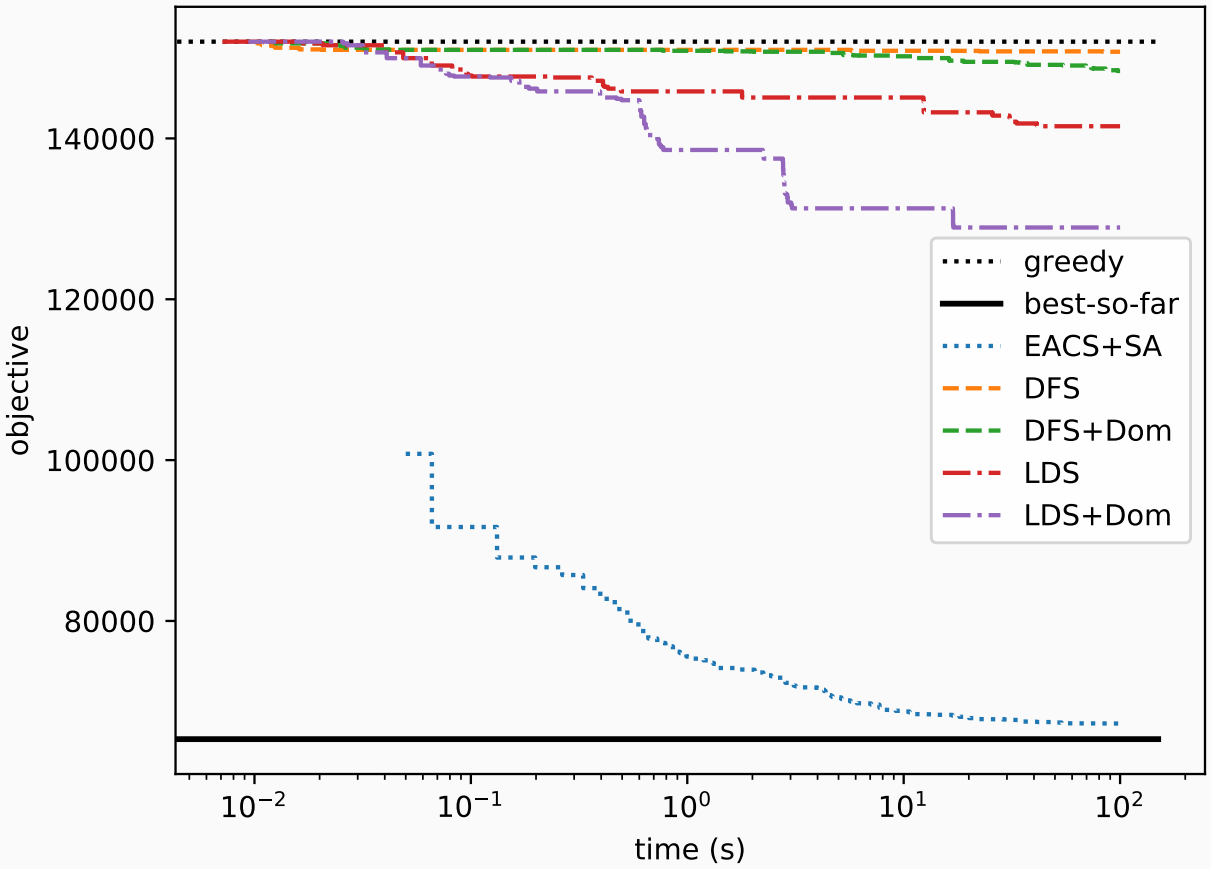

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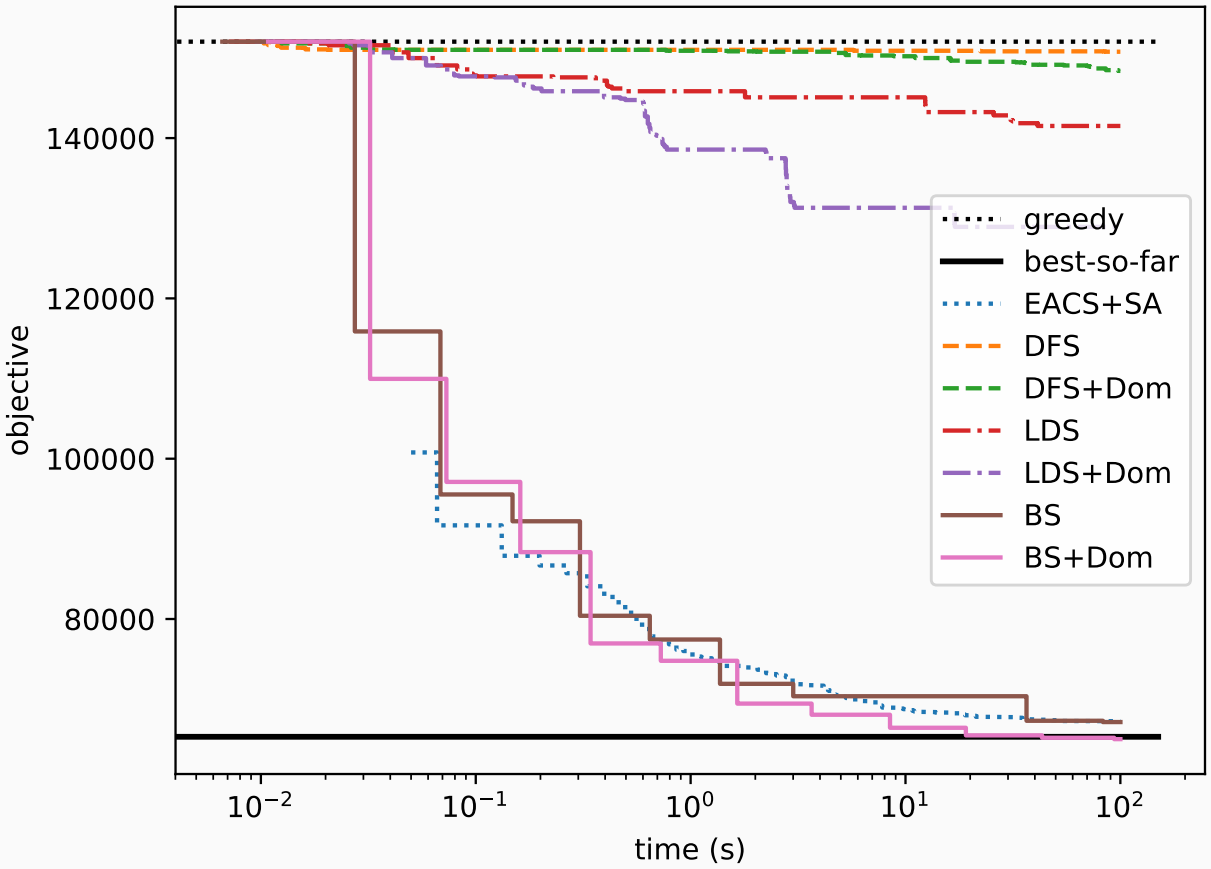

Results - New best-so-far solutions

6 over 7 new-best-so-far solutions
(the other one is probably optimal)

Instance	best known	BS+PE (600s)
R.500.100.15	5.284	**5.261**
R.500.1000.15	49.504	**49.366**
R.600.100.15	5.472	**5.469**
R.600.1000.15	55.213	**54.994**
R.700.100.15	7.021	**7.020**
R.700.1000.15	65.305	**64.777**

Results - Overview

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The SOPLIB mainly contains heavily constrained instances:

- hard for MIPs and local searches
- but (relatively) easy for constructive algorithms
- **thus the need to consider anytime tree searches**

Wrapping-up

Conclusions

- At least on the SOPLIB, anytime tree searches outperform classical meta-heuristics

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- The search-strategy choice is crucial

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References

[1] Frédéric Gardi, Thierry Benoist, Julien Darlay, Bertrand Estellon, and Romain Megel. *Mathematical programming solver based on local search*. Wiley Online Library, 2014.