Tree Search and the EURO/ROADEF challenge

Midi ROSP

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March 21, 2019

G-SCOP

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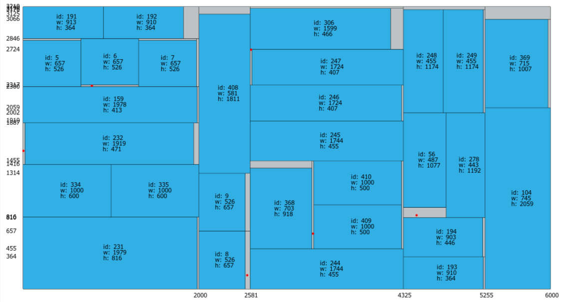
Glass Cutting Challenge ?

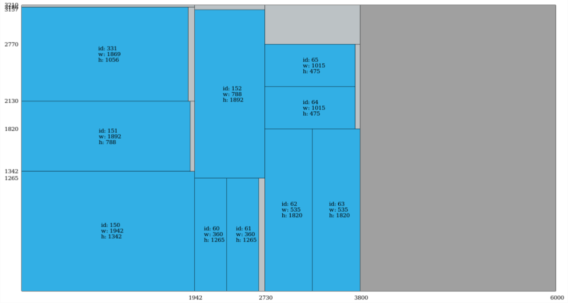
Glass cutting Problem

Proposed by *Saint Gobain*.

Cut rectangular glass items from big glass plates (Plates)

One of our solutions





Glass cutting Problem

Objective:

minimize waste

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CONSTRAINTS:

- **guillotine constraint**

Guillotine constraint

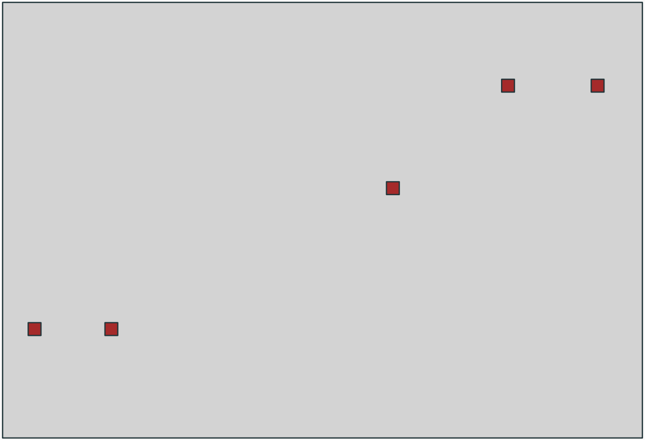


Figure 1: Example of a solution

Guillotine constraint

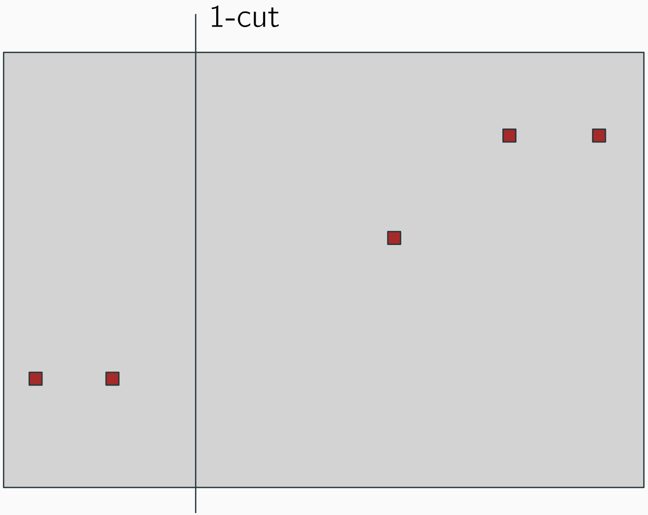


Figure 2: Example of a solution

Guillotine constraint

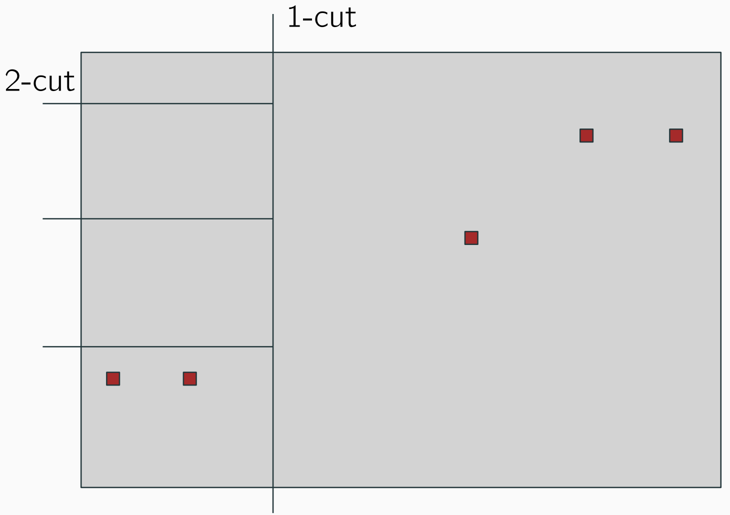


Figure 3: Example of a solution

Guillotine constraint

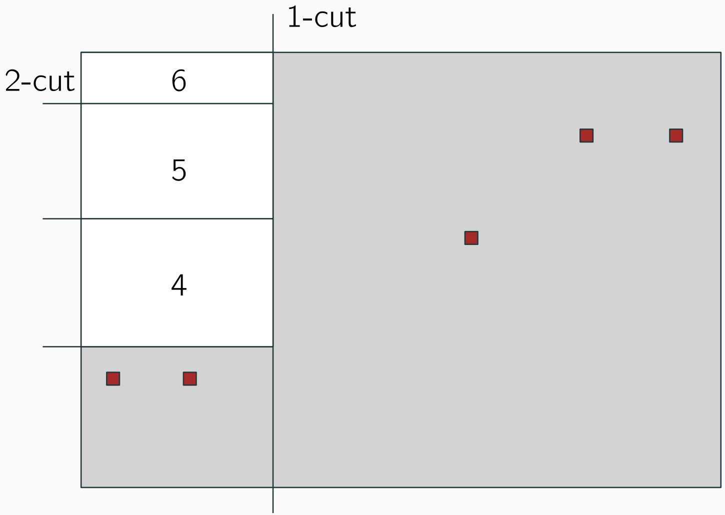


Figure 4: Example of a solution

Guillotine constraint

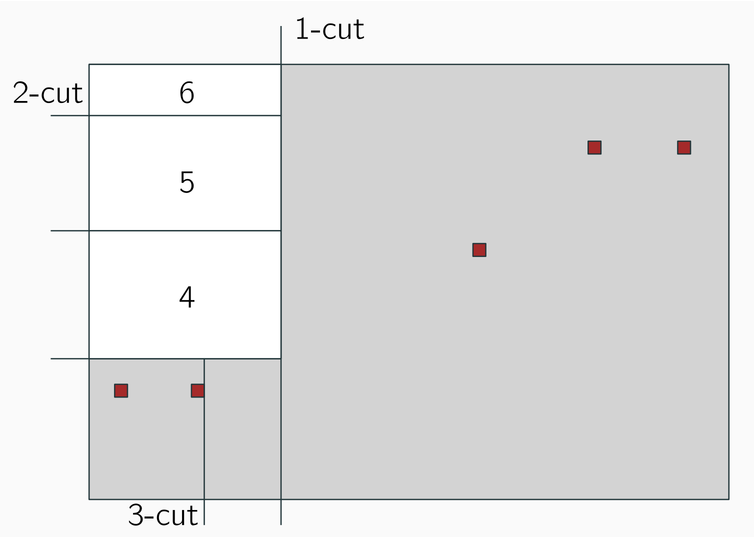


Figure 5: Example of a solution

Guillotine constraint

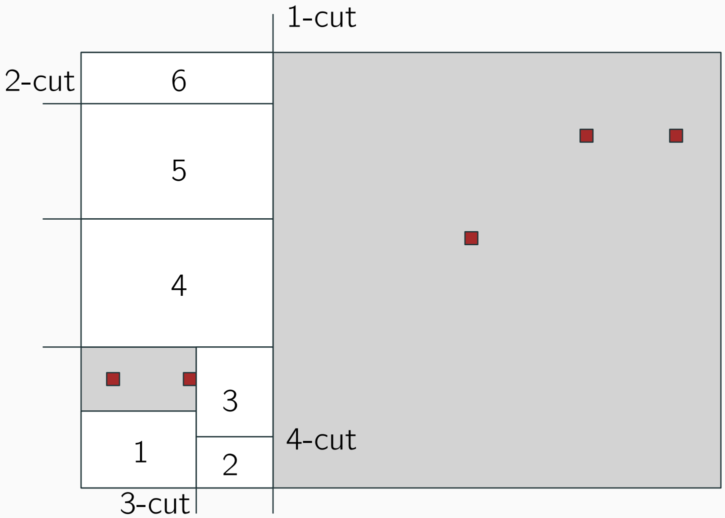
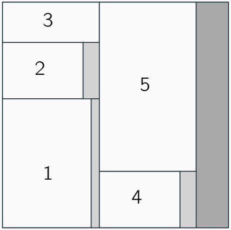


Figure 6: Example of a solution

guillotine cuts and not allowed cuts



guillotine

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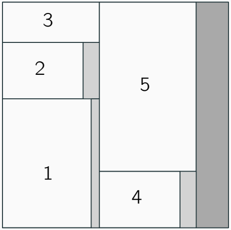
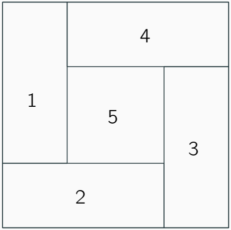
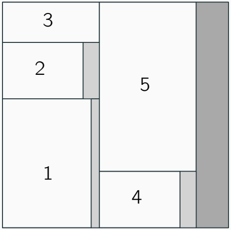


guillotine

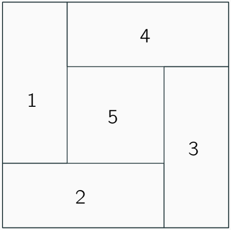


non-guillotine

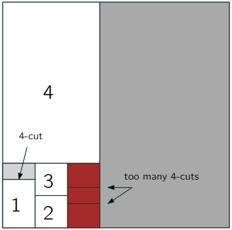
guillotine cuts and not allowed cuts



guillotine



non-guillotine



too many 4-cuts

Precedence constraints

OBJECTIVE:

minimize waste

DATA:

- Items (defined width and height, rotation possible)
- Stacks (chain precedence constraints)
- 100 Plates (6m x 3m) with defects

CONSTRAINTS:

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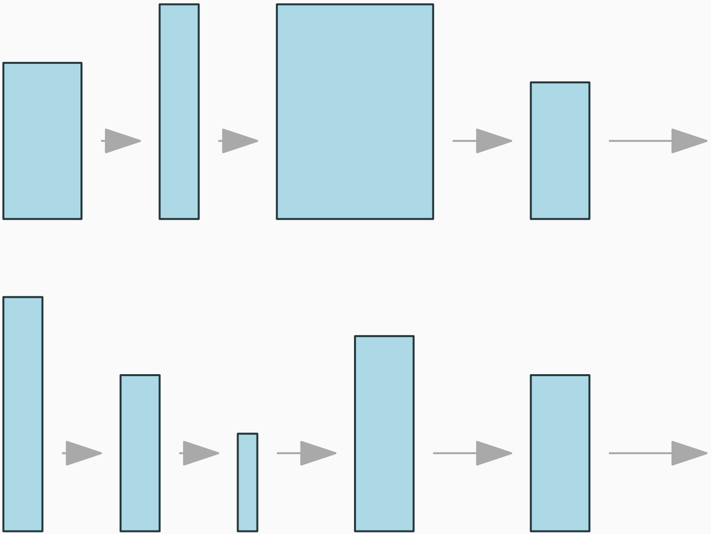
Precedence Constraint



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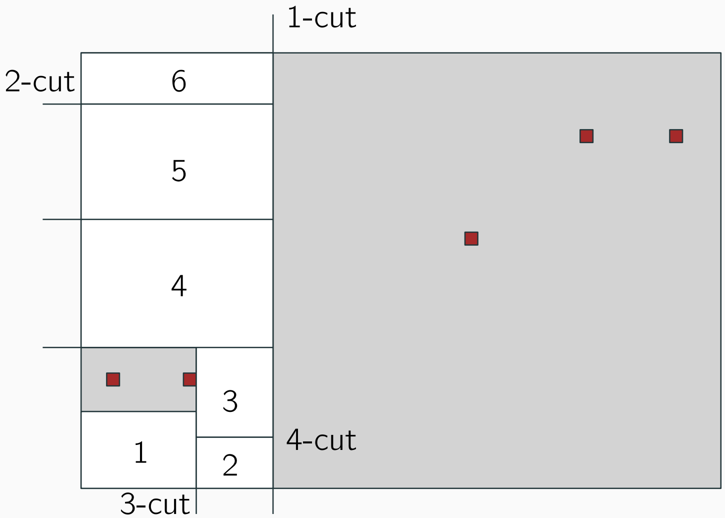

Defect avoidance

Objective:

minimize waste

Data:

- Items (defined width and height, rotation possible)
- Stacks (chain precedence constraints)
- 100 Plates (6m x 3m) with defects

Constraints:

- guillotine constraint
- all items produced in a valid order
- **no defects in items**
- **no cut on a defect**

minimum/maximum cut size

Objective:

minimize waste

Data:

- Items (defined width and height, rotation possible)
- Stacks (chain precedence constraints)
- 100 Plates (6m x 3m) with defects

Constraints:

- guillotine constraint
- all items produced in a valid order
- no defects in items
- no cut on a defect
- **min/max constraints on cuts and waste**

Min-waste constraint

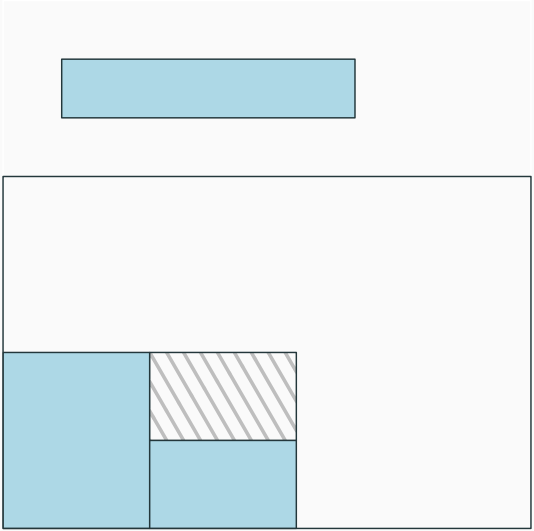

Figure 7: Min waste: easy case

Min-waste constraint

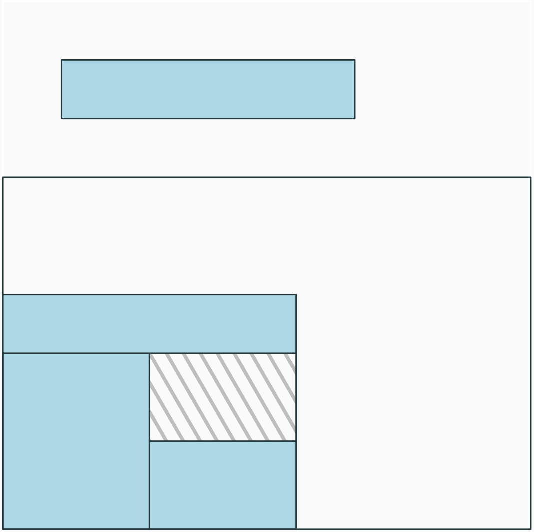

Figure 8: Min waste: easy case

Min-waste constraint

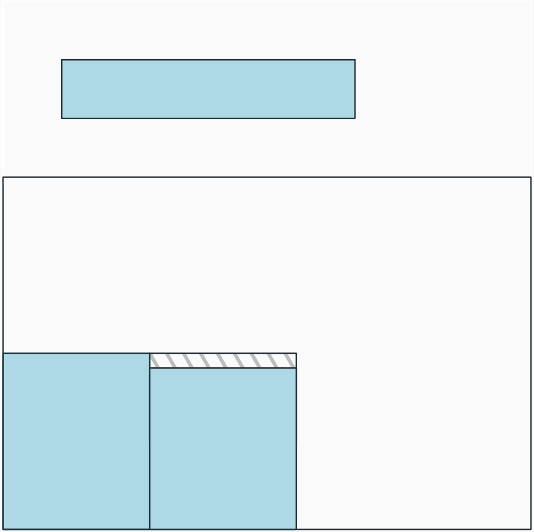

Figure 9: Min waste: more difficult

Min-waste constraint

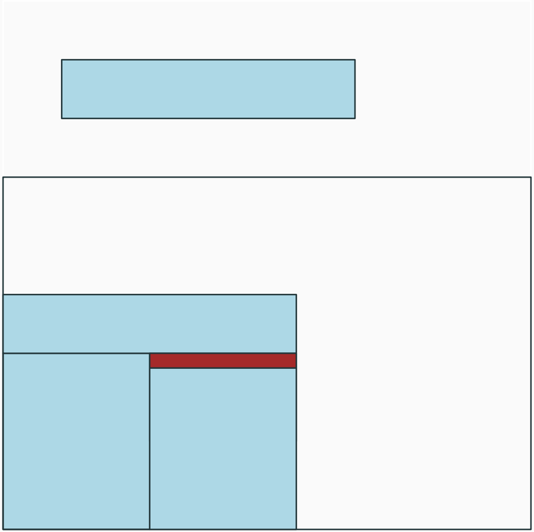

Figure 10: Min waste: more difficult

Min-waste constraint

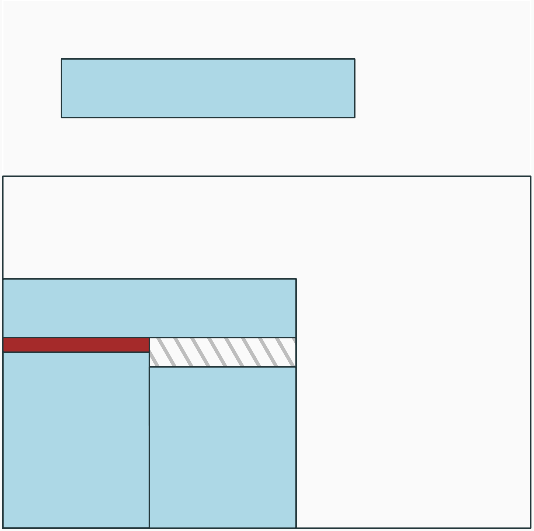

Figure 11: Min waste: more difficult

Min-waste constraint

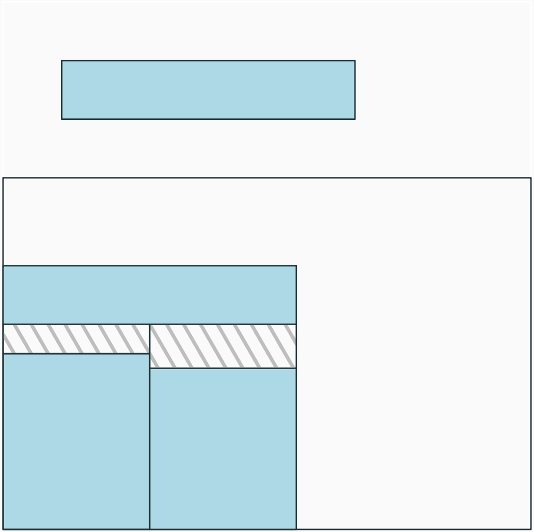

Figure 12: Min waste: more difficult

Glass cutting Problem

The problem is $\mathcal{NP}$-Hard.

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Difficult problem and big instances

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Difficult problem and big instances

We use anytime algorithms (meta-heuristics)

In this talk

We generate an implicit search tree. (next section)
It is called **Branching Scheme**

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we use **anytime tree searches**

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Results & conclusions & perspectives

Branching Scheme

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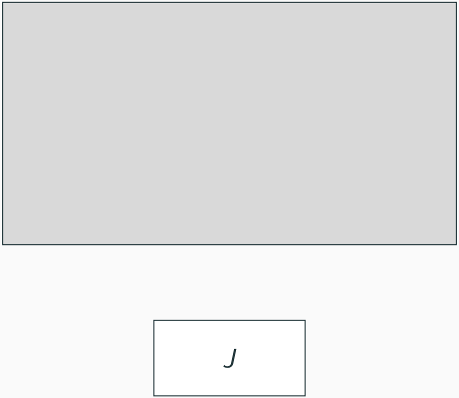
- Children: where to place items?



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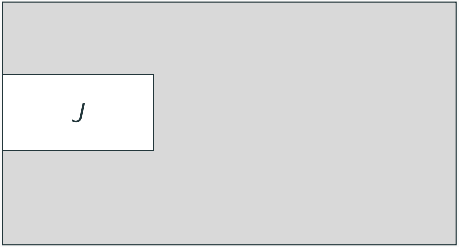

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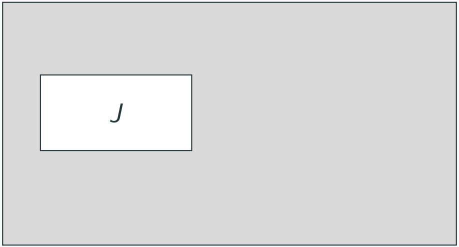

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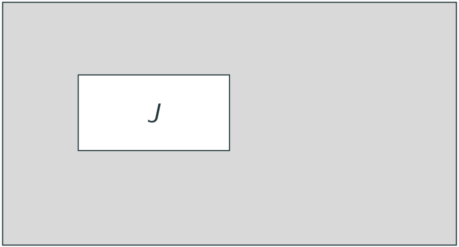

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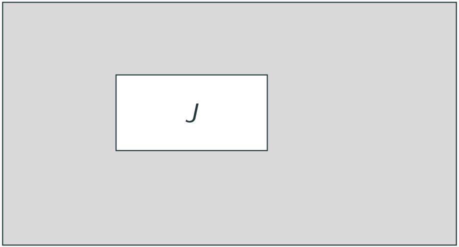

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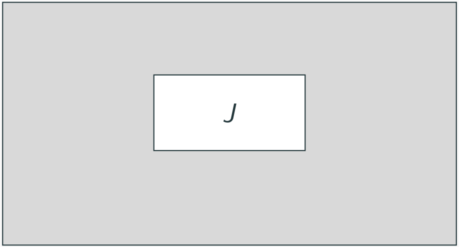
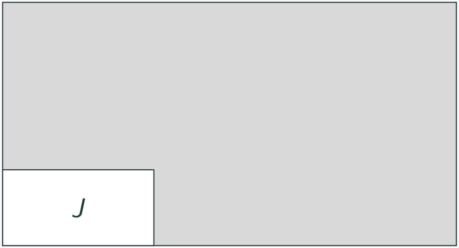


Branching scheme

- Root node (initial solution): empty solution, no item placed.
- Children: where to place items?
- In a corner?



Dominant sets

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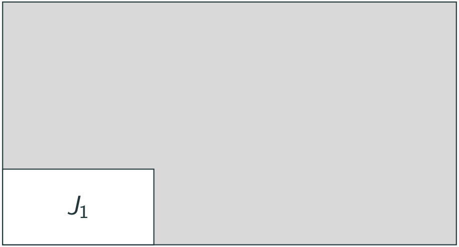

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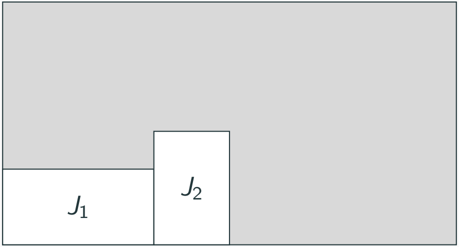

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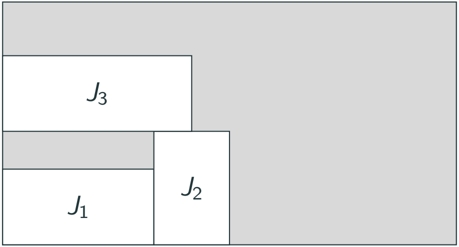

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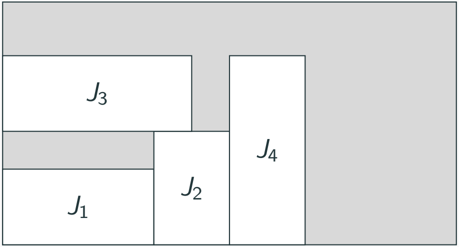

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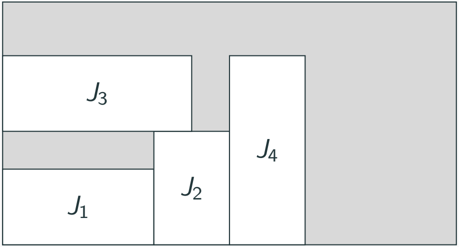

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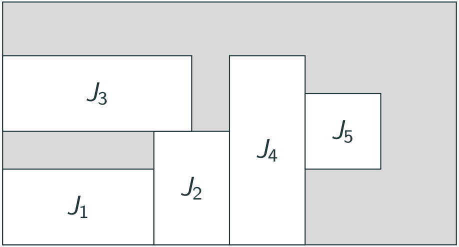

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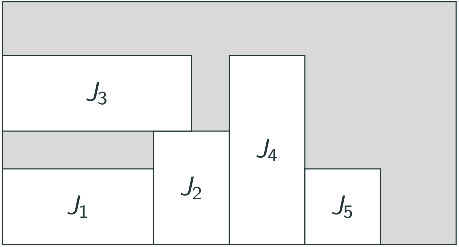

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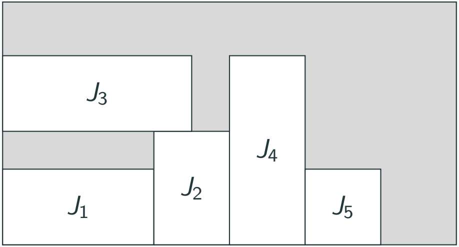
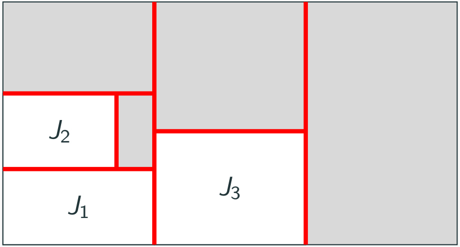


- There exists an optimal solution such that for each item of the solution, its left and its bottom touch either another item or a border.
- Does the property hold in our problem?

Dominant sets

- The property holds for the guillotine two-dimensional packing problem.



Dominant sets

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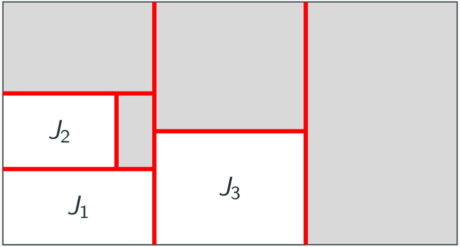
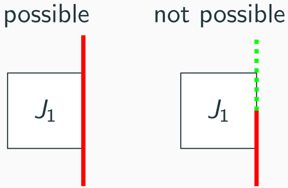


- There exists an optimal solution such that every left side of its vertical cuts and every bottom sides of its horizontal cuts touch an item.

Dominant sets

- The border of an item corresponds to one unique cut (or border)



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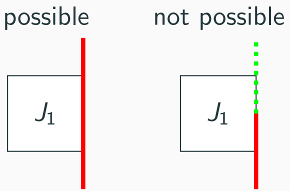
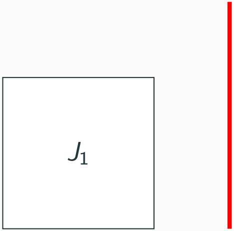


- Then the proof is trivial



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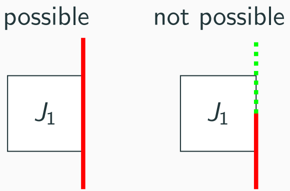

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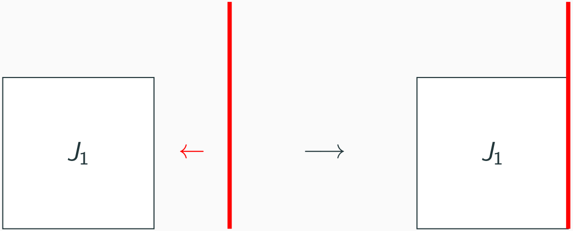

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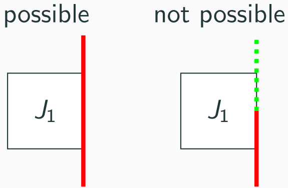

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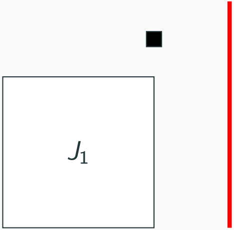

- And with defects?

Dominant sets

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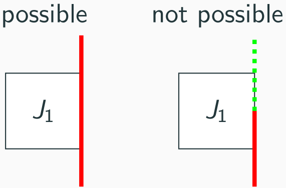

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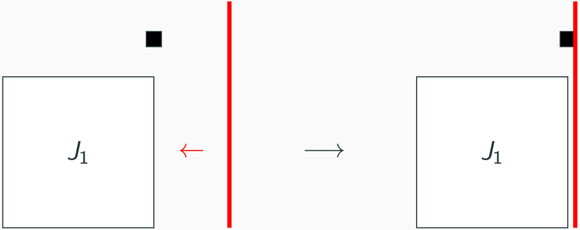

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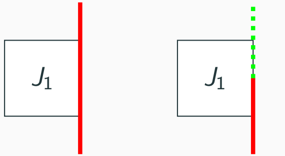

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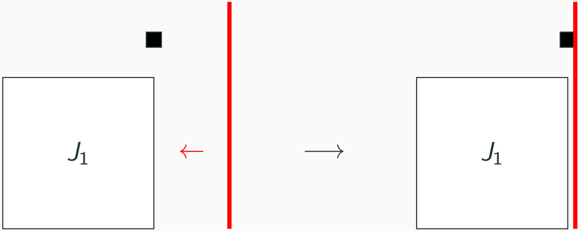

- And with defects? And with precedences?

Dominant sets

- No defects, no precedences, but minimum waste.

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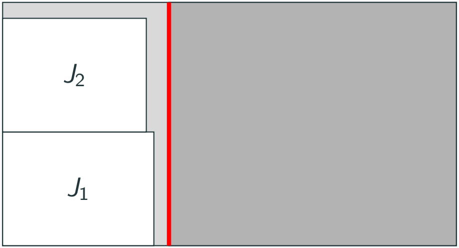
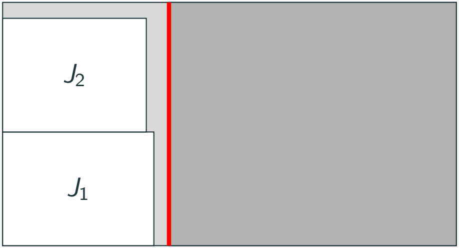


Dominant sets

- No defects, no precedences, but minimum waste.
- There exists an optimal solution such that every left side of its vertical cuts and every bottom sides of its horizontal cuts touch an item **or is exactly at $w_{\mathbf{min}}$ from an item.**



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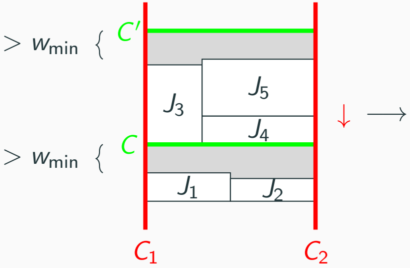

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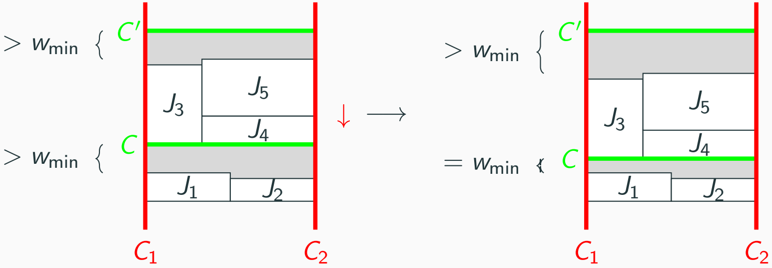

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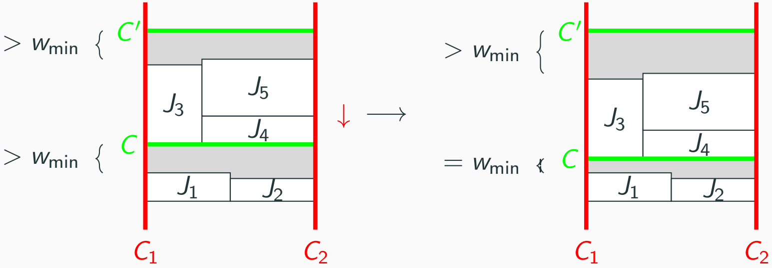
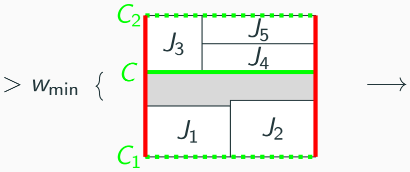
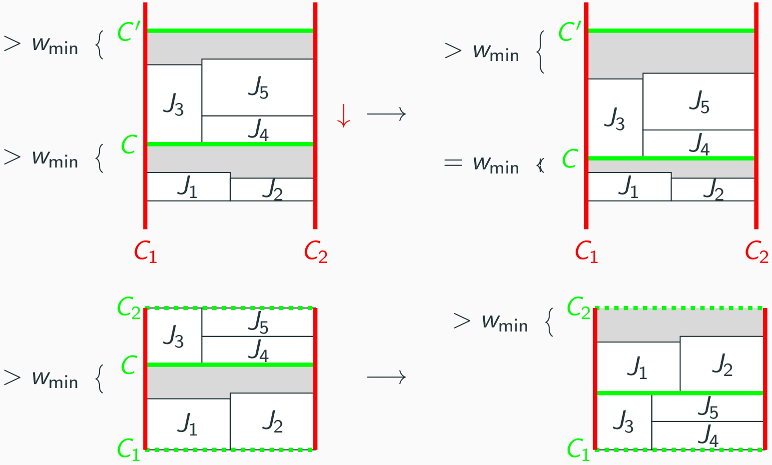




Dominant sets



Dominant sets

- But what happens with minimum waste and precedences? or with minimum waste and defects?

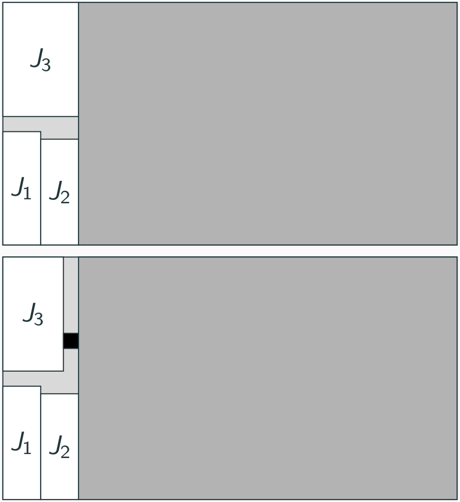
Dominant sets

- But what happens with minimum waste and precedences? or with minimum waste and defects?

$J_1 \prec J_3$

Dominant sets

- But what happens with minimum waste and precedences? or with minimum waste and defects?



$J_1 \prec J_3$

Dominant sets

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Dominant sets

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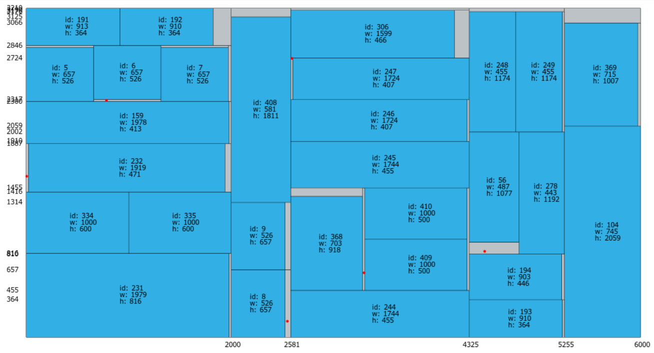
$\Longrightarrow$ We base our branching scheme on it anyway.



id: 191 w: 913 h: 364
id: 192 w: 910 h: 364
id: 306 w: 1599 h: 466
id: 248 w: 455 h: 1174
id: 249 w: 455 h: 1174
id: 369 w: 715 h: 1007
id: 5 w: 657 h: 526
id: 6 w: 657 h: 526
id: 7 w: 657 h: 526
id: 408 w: 581 h: 1811
id: 247 w: 1724 h: 407
id: 246 w: 1724 h: 407
id: 159 w: 1978 h: 413
id: 232 w: 1919 h: 471
id: 245 w: 1744 h: 455
id: 56 w: 487 h: 1077
id: 278 w: 443 h: 1192
id: 334 w: 1000 h: 600
id: 335 w: 1000 h: 600
id: 9 w: 526 h: 657
id: 368 w: 703 h: 918
id: 410 w: 1000 h: 500
id: 409 w: 1000 h: 500
id: 104 w: 745 h: 2059
id: 194 w: 903 h: 446
id: 231 w: 1979 h: 816
id: 8 w: 526 h: 657
id: 244 w: 1744 h: 455
id: 193 w: 910 h: 364
3066
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455
364
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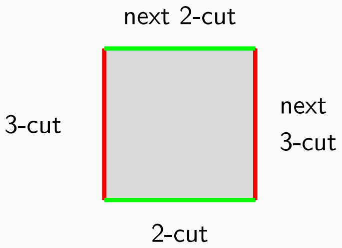
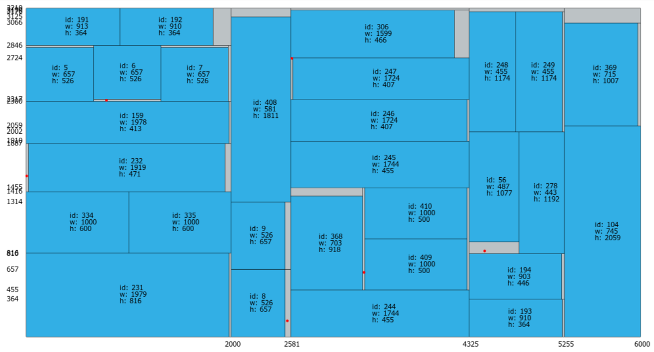


next 2-cut
3-cut
next
3-cut
2-cut



id: 191 w: 913 h: 364
id: 192 w: 910 h: 364
id: 5 w: 657 h: 526
id: 6 w: 657 h: 526
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id: 334 w: 1000 h: 600
id: 335 w: 1000 h: 600
id: 231 w: 1979 h: 816
id: 408 w: 581 h: 1811
id: 9 w: 526 h: 657
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id: 244 w: 1744 h: 455
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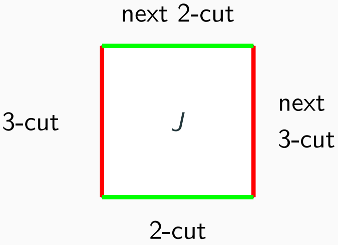
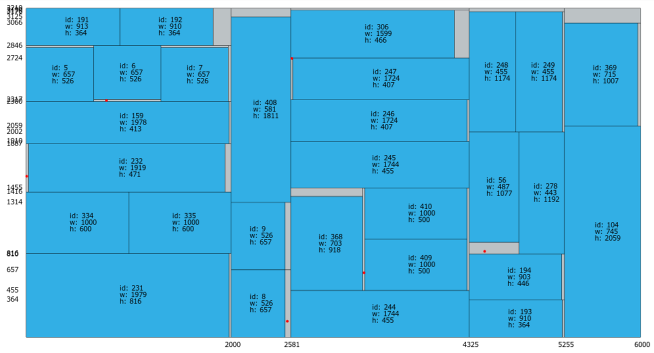
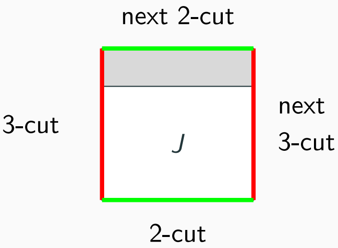
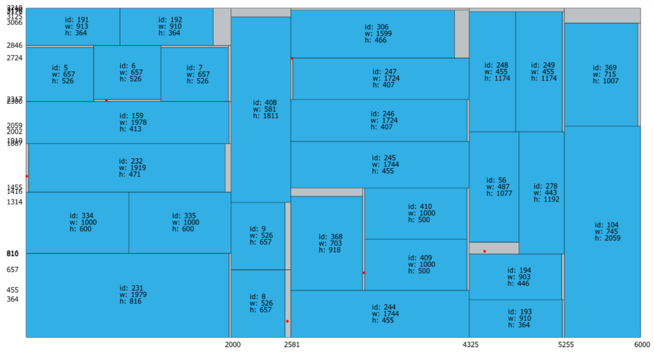


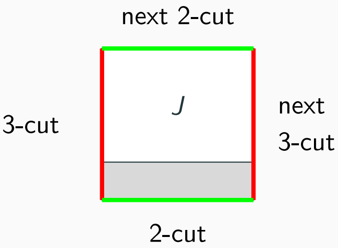
next 2-cut
3-cut
J
next
3-cut
2-cut



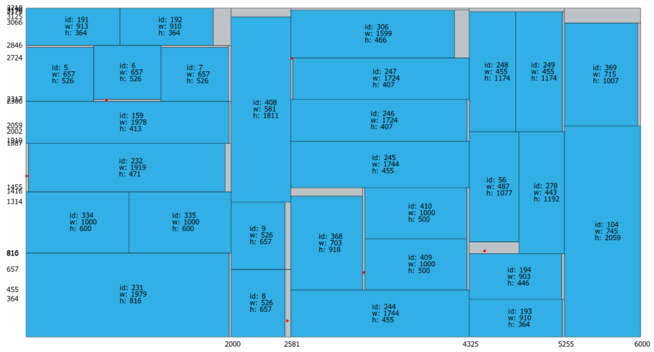


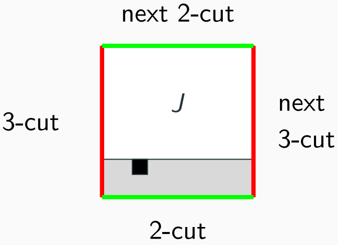
- 1 item at the bottom of the block (231, 334, 335...)



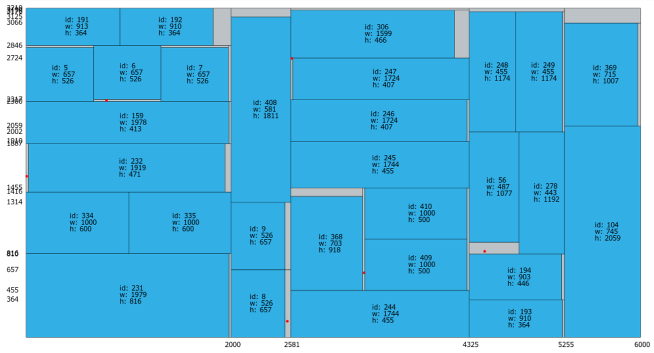


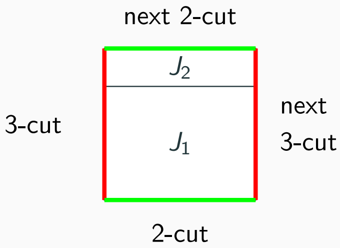
- 1 item at the bottom of the block (231, 334, 335. . .)
- 1 item at the top of the block (6, 56)



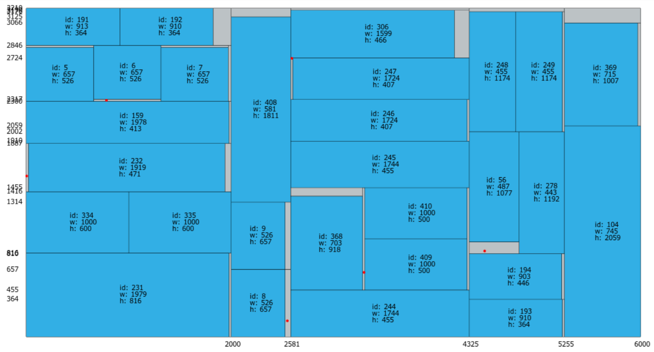


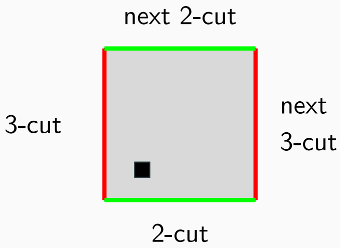
- 1 item at the bottom of the block (231, 334, 335. . .)
- 1 item at the top of the block (6, 56)



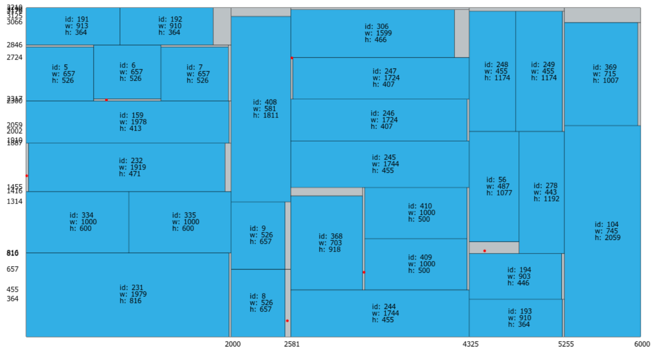


- 1 item at the bottom of the block (231, 334, 335. . .)
- 1 item at the top of the block (6, 56)
- 2 items (409 and 410)



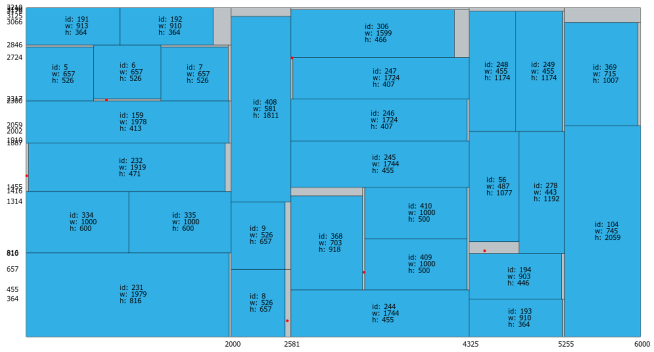


- 1 item at the bottom of the block (231, 334, 335…)
- 1 item at the top of the block (6, 56)
- 2 items (409 and 410)
- a defect (before 232, before 409 and 410, before 247)



Four depths of insertions:

- 0: on a new plate (231)
- 1: new current 1-cut (8, 244, 193, 104)
- 2: new 2-cut (334, 159, defect before 232...)
- 2: new 3-cut (335, 6, 7, defect before 409 and 410...)



Four depths of insertions:

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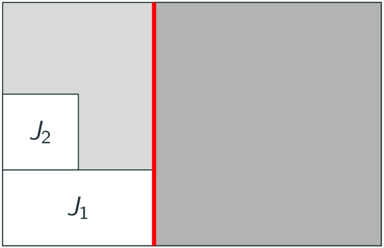
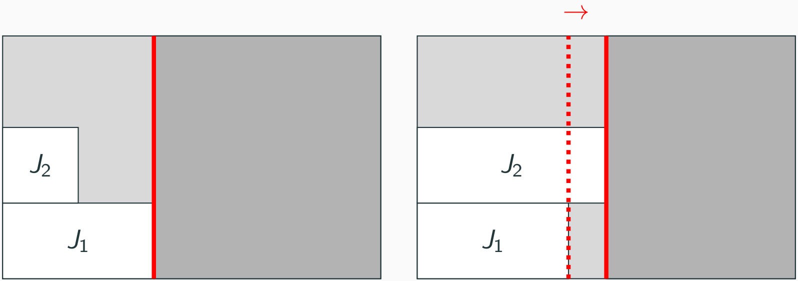
Note that items may be rotated.

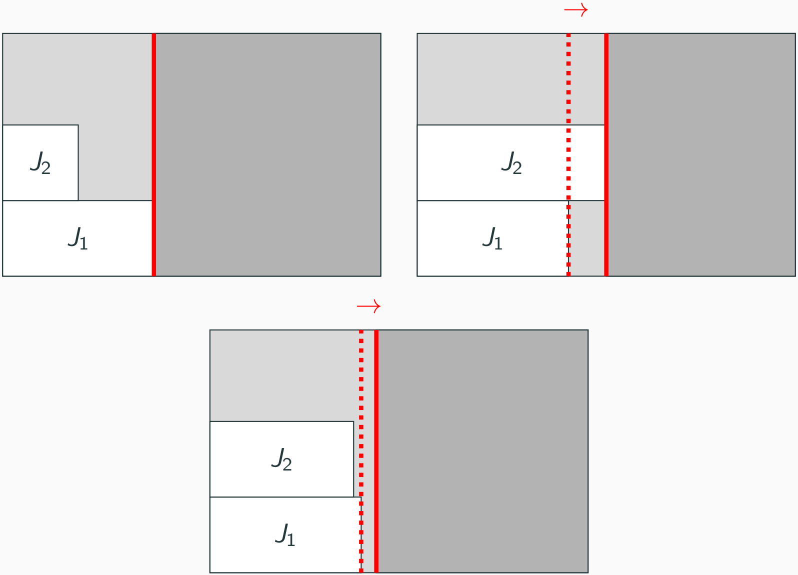
Computing last cuts positions



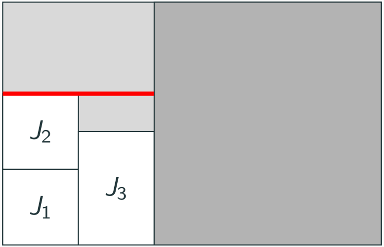
Computing last cuts positions



Computing last cuts positions

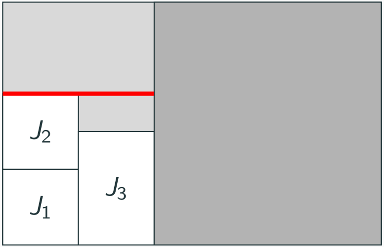


Computing last cuts positions

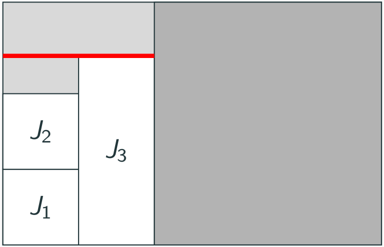


feasible

Computing last cuts positions



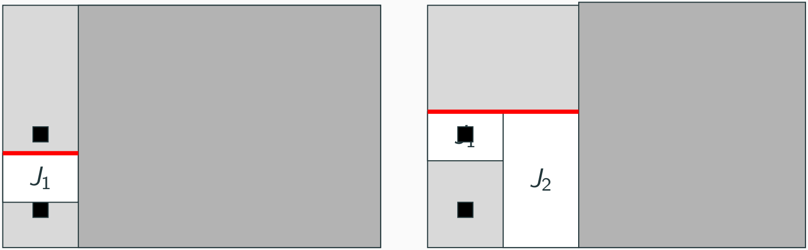
feasible



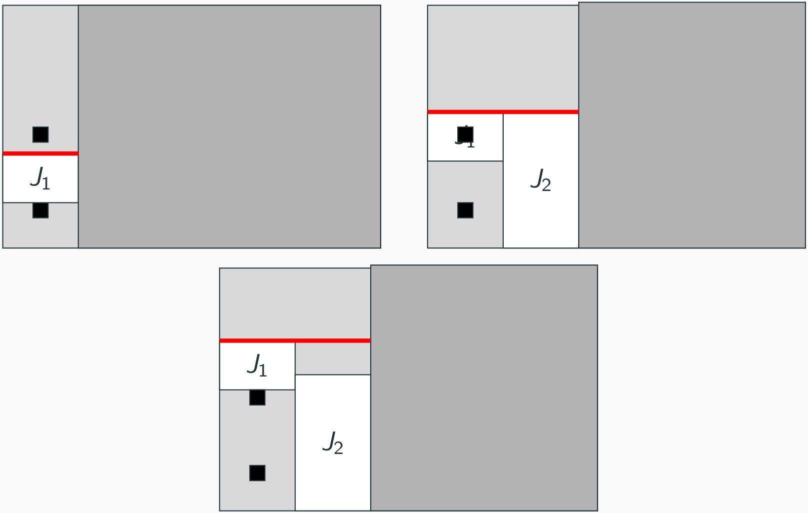
infeasible

Computing last cuts positions

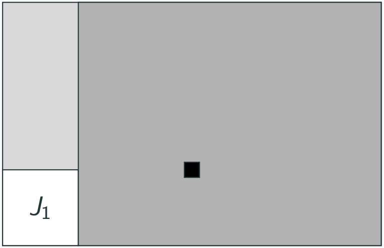
Computing last cuts positions



Computing last cuts positions

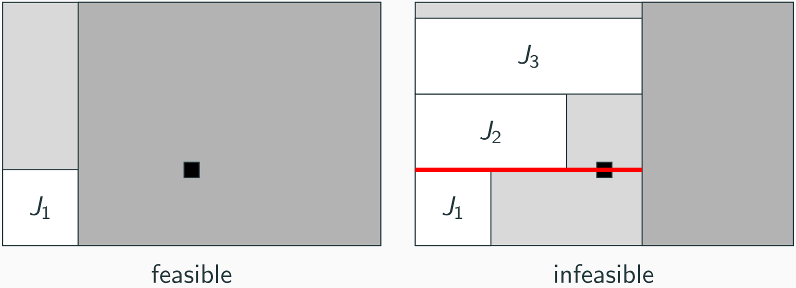


Computing last cuts positions



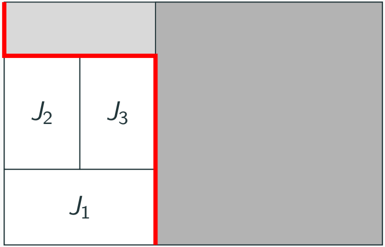
feasible

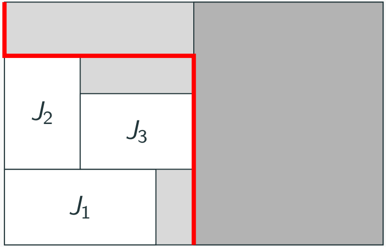
Computing last cuts positions



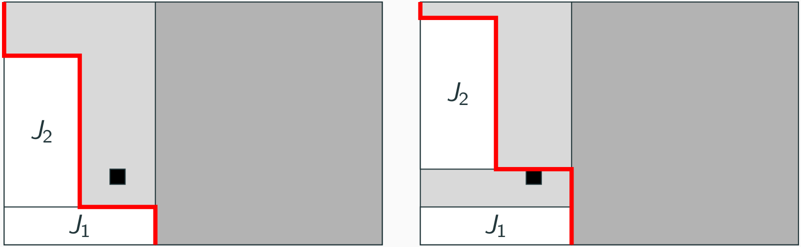
Pseudo-dominance

- Dominance rule: if two partial solutions S_1 and S_2 contain the same items and the front of S_1 is before the front of S_2, then S_1 dominates S_2.

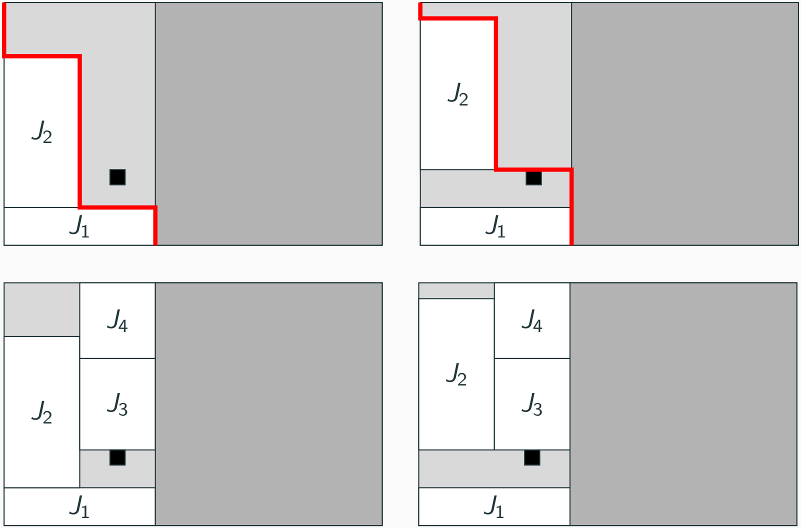




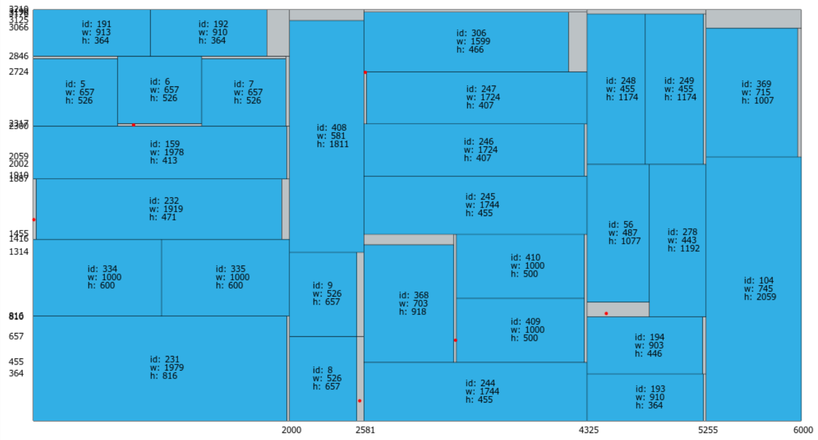
Pseudo-dominance



Pseudo-dominance



- Symmetry breaking strategy: for two consecutive blocks, the one with the smallest minimum item id comes before.



Wrapping up

Branching scheme

- not dominant, but good compromise

Wrapping up

Branching scheme

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- all constraints taken into account

Wrapping up

Branching scheme

- not dominant, but good compromise
- all constraints taken into account
- very high number of nodes

Wrapping up

Branching scheme

- not dominant, but good compromise
- all constraints taken into account
- very high number of nodes
- pseudo-dominance rules and symmetry breaking strategy

Anytime Algorithms & Tree Search

Some formalizations

Two separate parts in *Tree Search*:

Branching Scheme: The implicit search tree. Contains

- root node
- how to generate children from a node
- lower bounds
- guides (estimation of node quality)

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- guides (estimation of node quality)

Tree Search Algorithms: How to visit the tree (Branch & Bound, Greedy, *others ?*)

Anytime algorithms (meta-heuristics) - A landscape

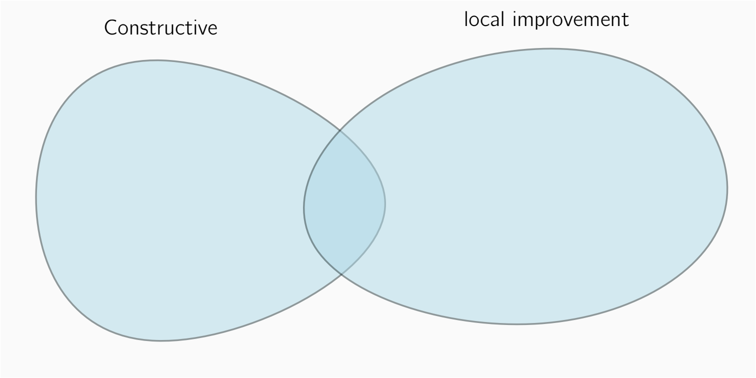


Figure 13: Anytime algorithms: a classification

Anytime algorithms (meta-heuristics) - A landscape

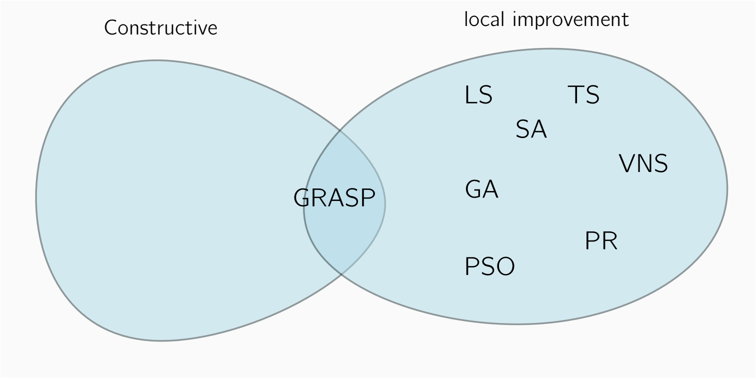


Figure 14: Anytime algorithms: a classification

Anytime algorithms (meta-heuristics) - A landscape

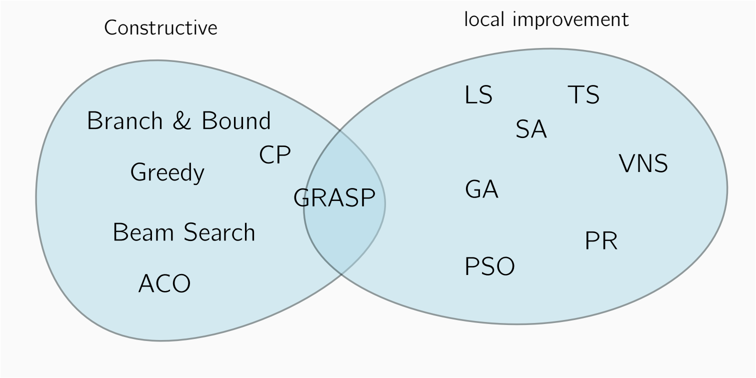


Figure 15: Anytime algorithms: a classification

Anytime algorithms (meta-heuristics)

- Many anytime algorithms are based on **Local Improvement**
- A few are **constructive**

Anytime algorithms (meta-heuristics)

- Many anytime algorithms are based on **Local Improvement**
- A few are **constructive**

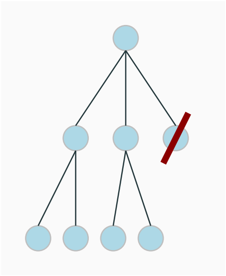
Why ?

Can constructive methods be competitive with Local Searches ?

Focus on constructive algorithms

They explore a tree

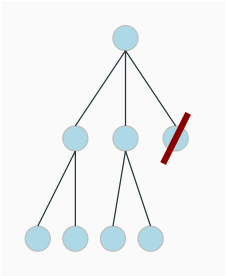
Branch & Bound



Focus on constructive algorithms

They explore a tree

Branch & Bound



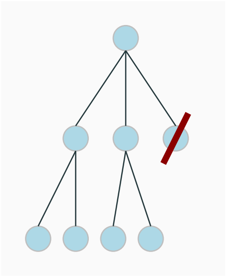
time –

quality +

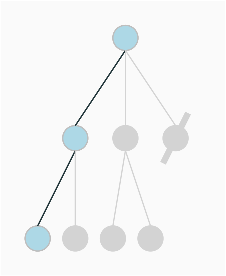
Focus on constructive algorithms

They explore a tree

Branch & Bound



Greedy



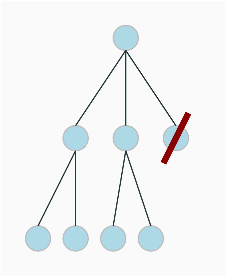
time –

quality +

Focus on constructive algorithms

They explore a tree

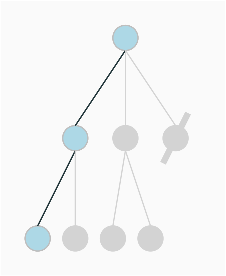
Branch & Bound



time –

quality +

Greedy



time +

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Focus on constructive algorithms

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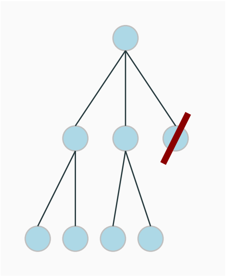
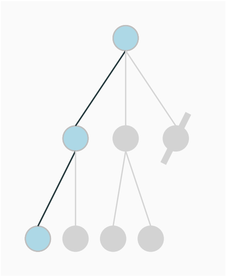
Branch & Bound



time –

quality +

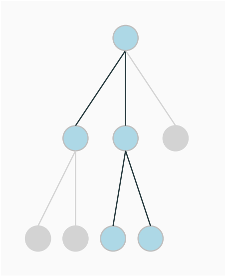
Greedy



time +

quality –

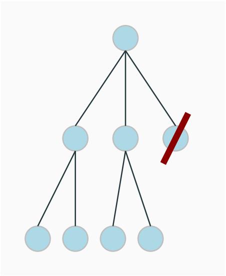
Beam Search



Focus on constructive algorithms

They explore a tree

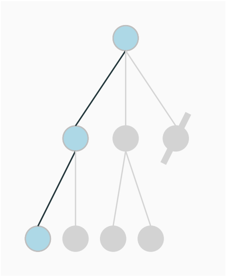
Branch & Bound



time –

quality +

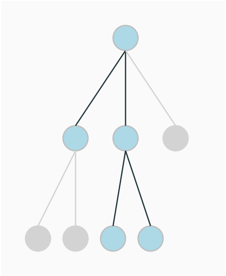
Greedy



time +

quality –

Beam Search



depends

time vs quality

Figure 16: time vs quality axis

time vs quality

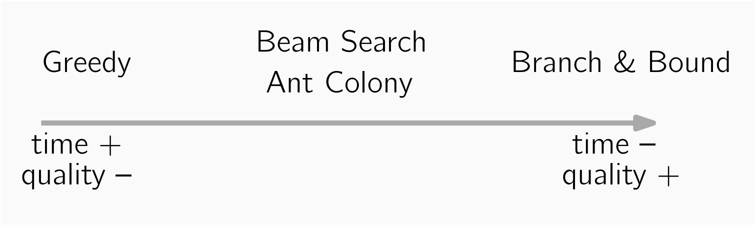


Figure 17: time vs quality axis

time vs quality

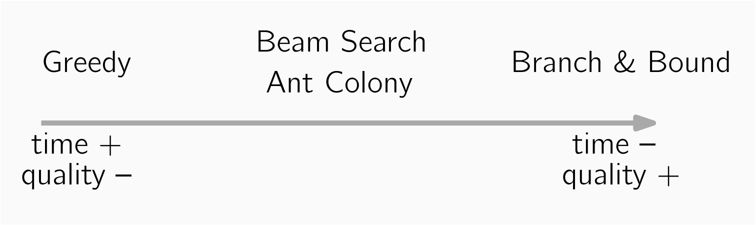


Figure 17: time vs quality axis

- *Beam Search* behaves like a BFS when the beam is big.

time vs quality

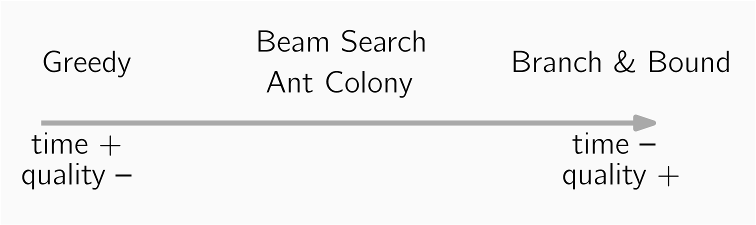


Figure 17: time vs quality axis

- *Beam Search* behaves like a BFS when the beam is big.
- *Ant Colony* depends too much on the structure of the problem

time vs quality

Figure 17: time vs quality axis

- *Beam Search* behaves like a BFS when the beam is big.
- *Ant Colony* depends too much on the structure of the problem

Are there some other algorithms ?

AI and Tree Search

Not in *Meta-heuristics* nor *Operations Research*[1]

[1] to the best of our knowledge
[2] and a bit in CP

AI and Tree Search

Not in *Meta-heuristics* nor *Operations Research*[1]

But in *AI*[2]

[1] to the best of our knowledge
[2] and a bit in CP

AI and Tree Search

Not in *Meta-heuristics* nor *Operations Research*[1]

But in *AI*[2]

We import those methods

[1] to the best of our knowledge
[2] and a bit in CP

Tree Searches in AI

Many algorithms usable in Operations Research:

Beam Stack Search ([ZH05]), **Limited Discrepancy Search** ([HG95]), **BULB** ([FK05]), others

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Our Approach *(Memory Bounded A*)*

MBA*: *A** or *Best First* with a limit on the number of nodes (like *Beam Search*)

MBA* - an example

Figure 18: MBA* with a maximum fringe size of 4

MBA* - an example

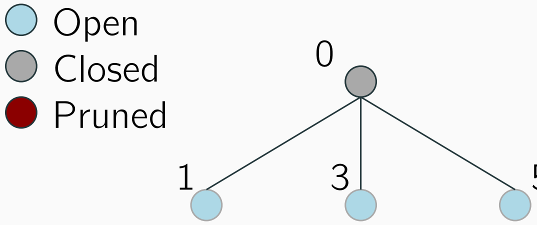

Figure 19: MBA* with a maximum fringe size of 4

MBA* - an example

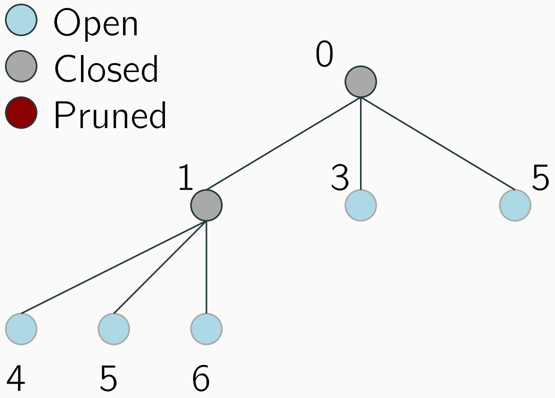

Figure 20: MBA* with a maximum fringe size of 4

MBA* - an example

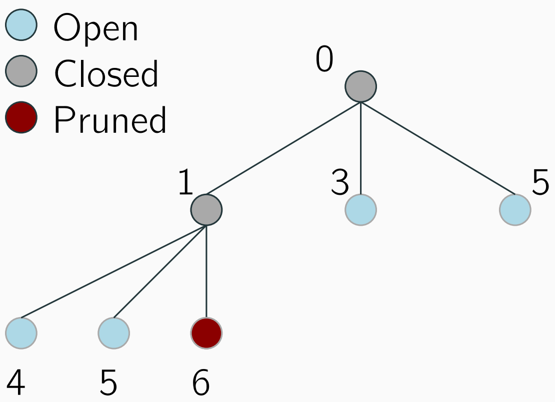

Figure 21: MBA* with a maximum fringe size of 4

MBA* - an example

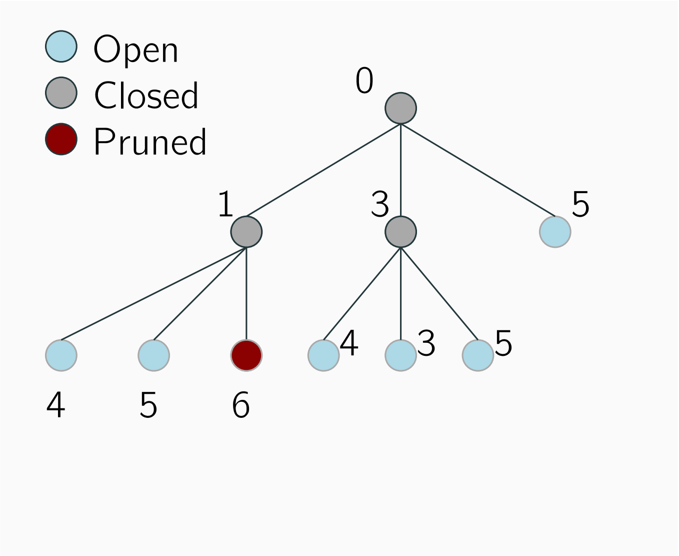

Figure 22: MBA* with a maximum fringe size of 4

MBA* - an example

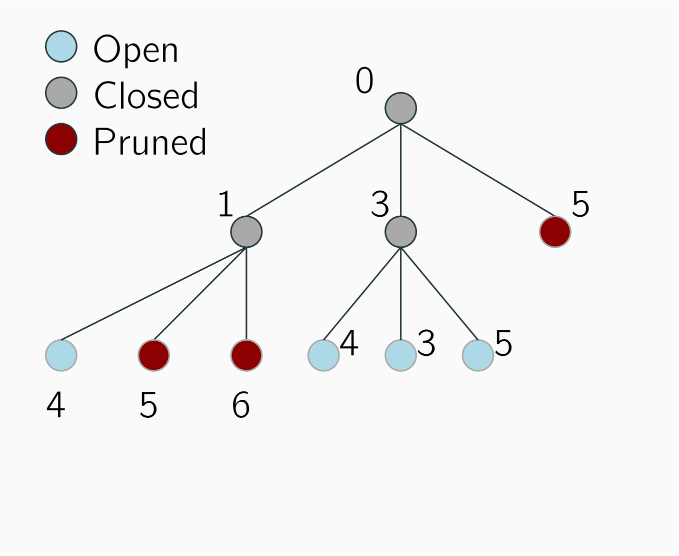

Figure 23: MBA* with a maximum fringe size of 4

MBA* - an example

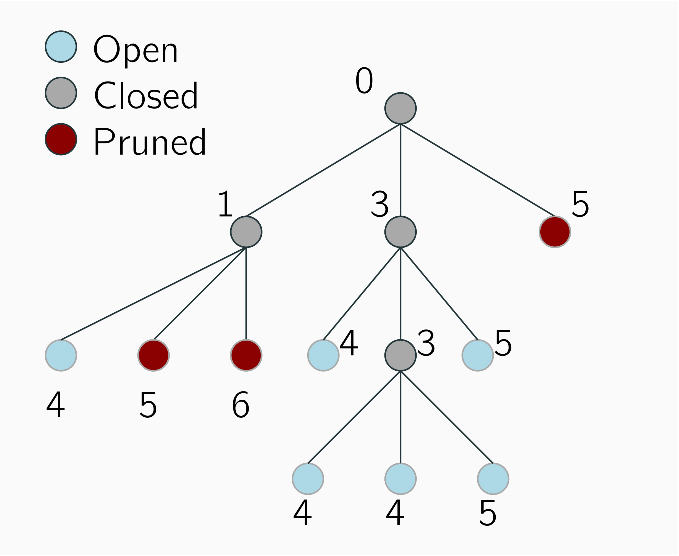

Figure 24: MBA* with a maximum fringe size of 4

MBA* - an example

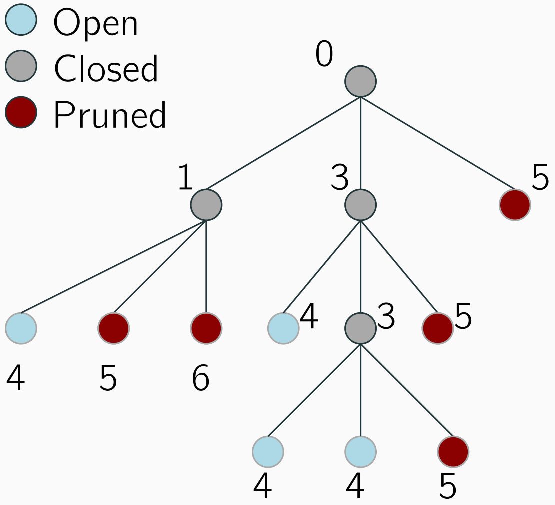

Figure 25: MBA* with a maximum fringe size of 4

A Tree Search Framework

We currently develop a framework (C++) for Tree Search

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The user defines the Branching Scheme (root, children generation, LB, guides)

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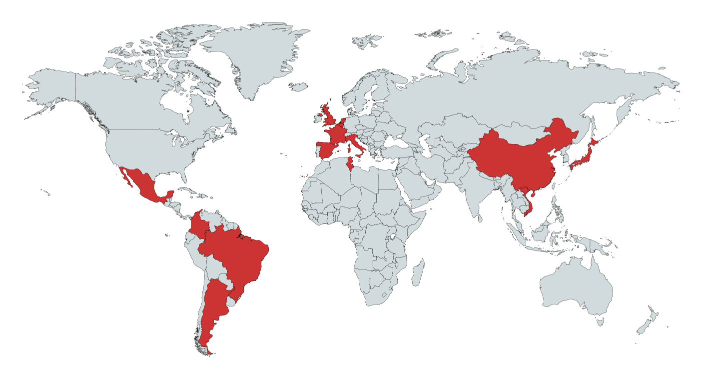
The user defines the Branching Scheme (root, children generation, LB, guides)

We provide the *Tree Search* algorithms

Results and Conclusion

- 20 over 74 international teams qualified for the Final phase.



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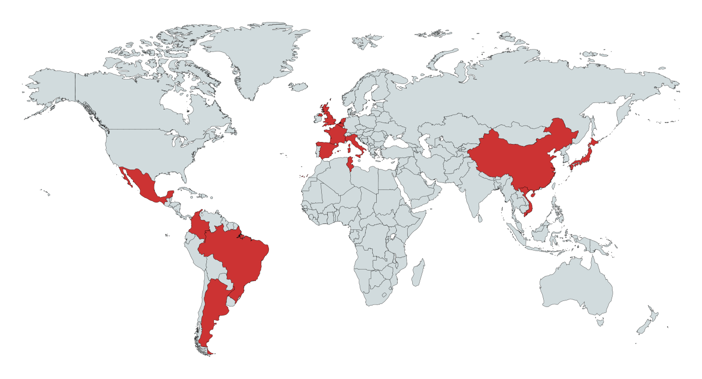

- 30 instances (15 known, 15 unknown), 1 hour running time.

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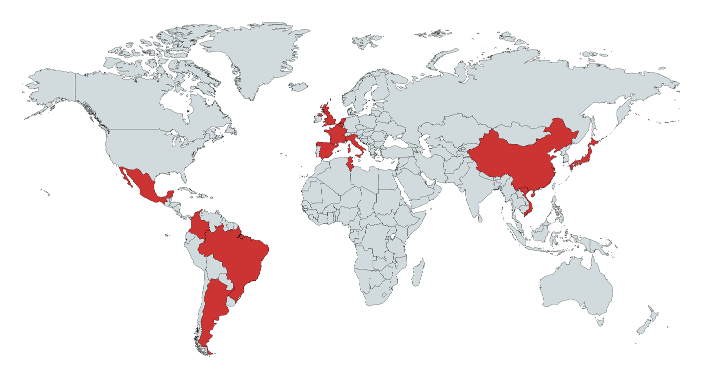
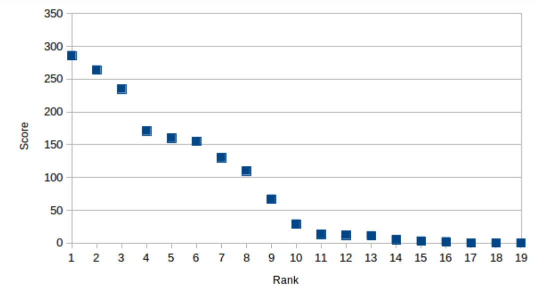
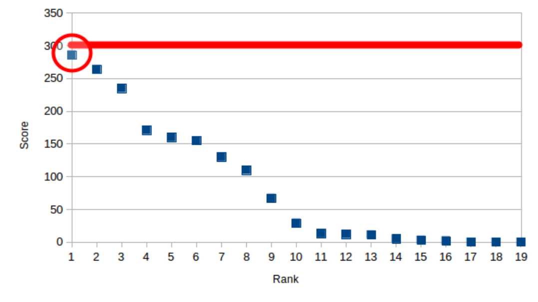
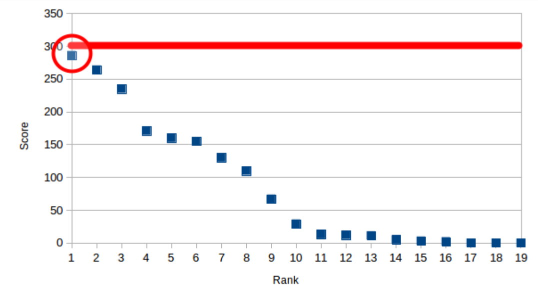


- 30 instances (15 known, 15 unknown), 1 hour running time.
- for each instance, a team earns 10 points minus the number of teams that found a better solution.

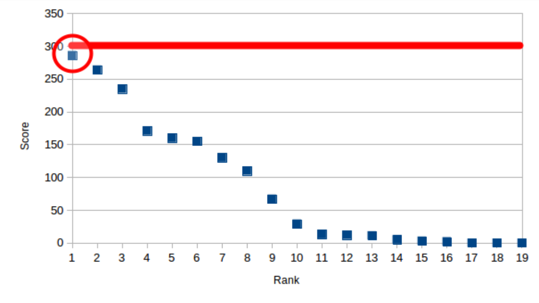

350
300
250
200
150
100
50
0
Score
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Rank



- Ranked first during the Final phase of the challenge.



- Ranked first during the Final phase of the challenge.
- Best solutions found on 20 over 30 instances.



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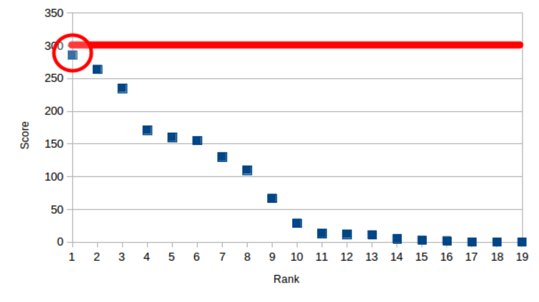
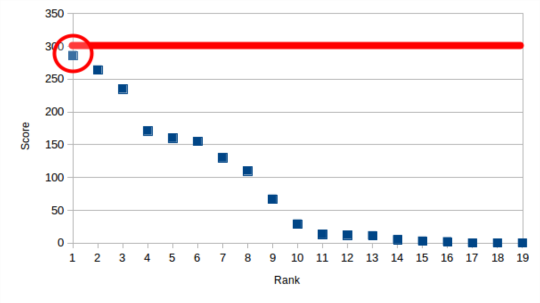
- Best solutions found on 20 over 30 instances.
- Total waste: $493M$, Total waste 2nd team: $506M$.



- Ranked first during the Final phase of the challenge.
- Best solutions found on 20 over 30 instances.
- Total waste: $493M$, Total waste 2nd team: $506M$.
- Total waste new version: $469M$; better on 26 instances compared to the challenge version.



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- Anytime algorithm

Conclusion and perspectives

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Perspectives

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- Apply method for classical and industrial problems.
- Combining tree search with local searches.

Questions or remarks ?

References i

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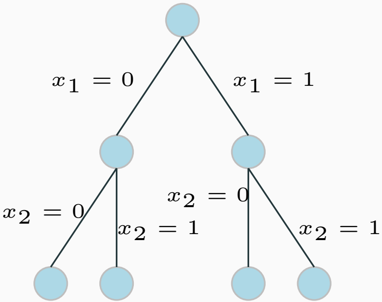
Trailing vs Copying

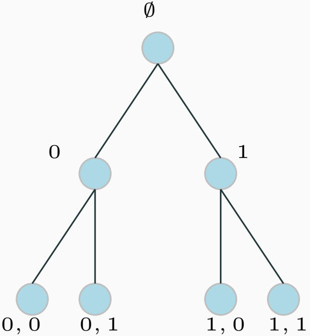
Trailing



(+) more nodes

(-) less tree searches available

Copying



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