The Classical Indian Just Intonation Tuning System

with 22 SRUTI-s defining the 7 SWARA-s of Hindu Classical Music

combining the three different kinds of SRUTI which are understood as
PRAMANA ("measuring" or "standard") SRUTI = Syntonic Comma (81/80) = 21.5 cents
NYUNA ("deficient") SRUTI = Minor Chroma (25/24) = 70.7 cents
PURNA ("fullfilling") SRUTI = Pythagorean Limma (256/243) = 90.2 cents
(where PURNA SRUTI may also, enharmonically, be interpreted as the sum of PRAMANA SRUTI and NYUNA SRUTI = Major Chroma (135/128 = 81/80 * 25/24) = 92.2 cents)

Wolfgang von Schweinitz
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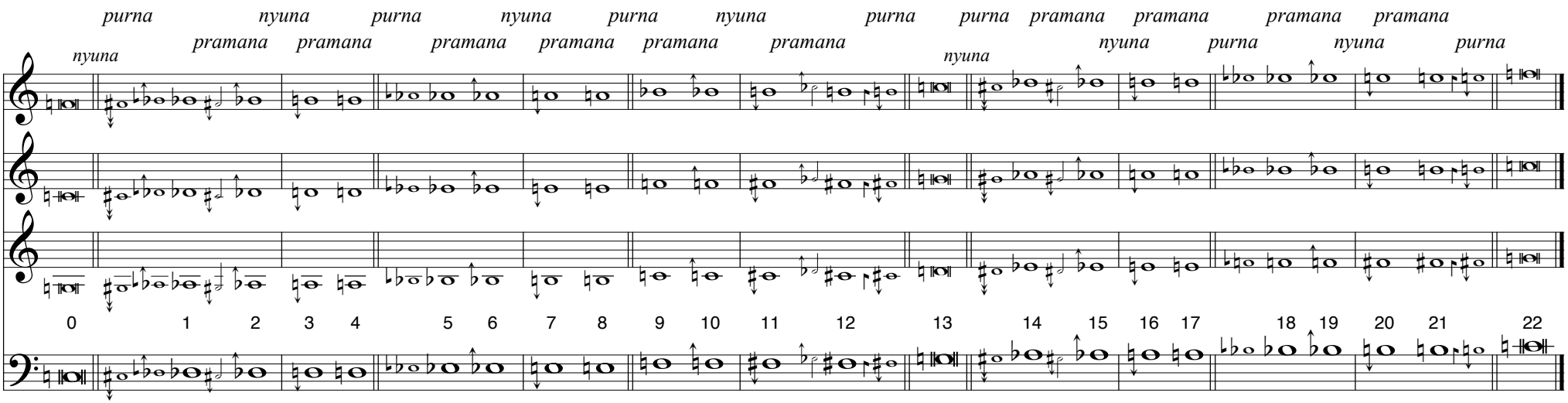

	Sa		ri		ri	Ri	Ri		ga	ga	Ga	Ga	ma	ma	Ma		Ma		Pa		dha		dha	Dha	Dha		ni	ni	Ni	Ni		Sa
Ratio	1	25	21	256	135	16	10	9	7	32	6	5	81	4	27	45	64	729	10	3	25	128	405	8	5	27	7	16	9	15	243	40	2
	1	24	20	243	128	15	9	8	6	27	5	4	64	3	20	32	45	512	7	2	16	81	256	5	3	16	4	9	5	8	128	21	1
Cents	0	70.7		90.2	92.2	111.7	182.4	203.9		294.1	315.6	386.3	407.8	498.0	519.6	590.2	609.8	611.7		702.0		792.2	794.1	813.7	884.4	905.9		996.1	1017.6	1088.3	1109.8		1200
		84.5							266.9										617.5		772.7						968.8				1115.5		

CARNATIC NAMES:
Ekasruti Rishabha | *Trisruti Rishabha* | *Suddha Gandhara, Komal Sadharana Ga* | *Antara Gandhara* | *Suddha Madhyama* | *Prati Madhyama* | *Chyuta Panchama Madhyama* | *Ekasruti Dhaivata* | *Trisruti Dhaivata* | *Suddha Nishada, Komal Kaisiki Nishada* | *Kakali Nishada*

Shadja | *Suddha Rishabha, Dvisruti Rishabha* | *Chatussruti Rishabha* | *Shatsruti Rishabha, Sadharana Gandhara* | *Chyuta Madhyama Gandhara, Tivra Antara Gandhara* | *Tivra Suddha Madhyama, Begada Madhyama* | *Panchama* | *Suddha Dhaivata, Dvisruti Dhaivata* | *Chatussruti Dhaivata* | *Shatsruti Dhaivata, Tivra Kaisiki Nishada* | *Kakali Nishada Chyuta Shadja Nishada*

HINDUSTANI NAMES:
Komal Rishabha | *Suddha Rishabha* | *Ati-komal Gandhara* | *Suddha Gandhara* | *Suddha Madhyama* | *Tivra Madhyama* | *Panchama* | *Ati-komal Dhaivata* | *Suddha Dhaivata* | *Ati-komal Nishada* | *Suddha Nishada*

Shadja | *Madhya Rishabha* | *Tivra Rishabha* | *Komal Gandhara* | *Tivra Gandhara* | *Ekasruti Madhyama* | *Tivratara Madhyama* | *Komal Dhaivata* | *Tivra Dhaivata* | *Komal Nishada* | *Tivra Nishada*

Annotations:

suddha = 'pure', tivra = sharp, komal = flat ('soft'), chyuta = 'fallen'. Note that the perfect forth (ma) consists of 9 Sruti-s, the perfect fifth (Pa) of 13, and the octave of 22, as related in the ancient 'NATYA-SASTRA', which also defined the size of Pramana-Sruti as the difference between four consecutive fifths and the pure major third. Later, in the medieval south Indian treatise 'SVARA-MELA-KALANIDHA' (chapter on the VINA), the author Ramamatya (while admitting a small difference in pitch) assigned only one fret to represent both Antara and Chyuta Madhyana Gandhara, and so also for Kakali and Chyuta Shadja Nishada.
The 'Extended Helmholtz-Ellis JI Pitch Notation' used for this transcription of a tuning chart presented by P. Sambamurthy in 'South Indian Music' (Book IV) was devised by Marc Sabat and Wolfgang von Schweinitz. The accidentals ♭ 𝄮 ♯ represent pitches tuned according to the series of pure fifths based on ma - Sa - Pa, and the accidentals 𝄬 𝄭 𝄯 𝄰 signify an enharmonic pitch-alteration by a Syntonic Comma or Pramana Sruti (81/80): + or - 21.5 cents. The two additional septimal accidentals 𝄾 and 𝄿 raise / lower the pitch by a Septimal Comma (64/63): + or - 27.3 cents.

Prof. P. Sambamurthy

EARLY EXPERIMENTS IN MUSIC

This commentary explaining the classical just intonation tuning system is copied from the chapter 'Early Experiments in Music' of the great anthology 'South Indian Music' Book V by musicologist Prof. P. Sambamurthy, first published in 1963, and in May 1999 in the Seventh Edition by The Indian Music Publishing House, Royapettah, Chennai

Much of the knowledge that we owe at present regarding the nature of scales and srutis (quarter-tones)[1] is due to the experiments in music carried out by scholars in ancient and medieval times. These experiments performed with great care and accuracy led them to perceive the beauties of the scale of just intonation and the frequencies of subtle srutis. The early perception of the highly concordant notes, panchama (*3/2* or 702 cents) and madhyama (*4/3* or 498 cents), led them to work out the cycles of fifths and fourths to their logical conclusions. Although the cycle of fourths is implied in the cycle of fifths, the fourth (suddha madhyama) being an inverted fifth (panchama) from the immediately higher sa, still it was found useful to work out the series of fourths as well. The knowledge of the 22 srutis was obtained by working out these two cycles. The scale of equal temperament, which became a necessity in Europe on account of the exigencies of harmony, was unknown in India.

CYCLE OF FIFTHS

Cycle of fifths or *Spiral of fifths* means a series of fifths or panchama svaras. (The panchama svara is the third harmonic note and next to the octave is the most consonantal interval.) In this process, the fifth of each note of the cycle is taken as the tonic note and its panchama determined; the relation of the new panchama to the original tonic note, shadja is then determined.

For instance, with the middle octave shadja as the starting note, we find its fifths is the panchama of the same octave, frequency *3/2*. Taking this panchama as shadja, its fifth is found to be *3/2 * 3/2 = 9/4* or the Chatussruti rishabha of the tara sthayi. The fifth or panchama of this note is found to be *9/4 * 3/2 = 27/8* or the Chatussruti dhaivata of the tara sthayi. The fifth or panchama of this note is found to be *27/8 * 3/2 = 81/16* or the Chyuta madhyama gandhara of the ati tara sthayi and so on.

[1] *or rather:* precisely tuned microtones *(Wolfgang von Schweinitz)*

The process was continued till the 12th cycle in each case when it was found that the 12th note of the cycle in one case and the 11th and 12th notes of the cycle in the other were higher or lower than shadja or panchama by the small interval of a comma or pramana sruti. These notes were ignored as not being of practical importance and the remaining 22 notes were retained and these are the 22 srutis of the ancient Indian scale. The further notes obtained in the two cycles were only of academic interest, since all the notes, important from the point of view of practical music, were already obtained.

In Table 4 (P. 5) all the notes shown on the right of the central line, belong to the cycle of fifths and those shown on the left, to the cycle of fourths. The roman numerals indicate the order in which the several notes occur in the cycles of fifths and fourths. All compound intervals arrived at in the working out of this process are reduced to the middle octave for purpose of easy comparison, the precise octave of the note however, being indicated in notation against each note.[2]

In the scale of equal temperament, the octave is divided into 1200 equal parts of cyclic cents and each semitone comprises 100 cents. Table 4 on page 4 visually shows the points of difference in the frequencies of the notes belonging to the scales of just intonation and equal temperament. Since none of the notes of the scale of equal temperament are used in Indian music, the unsuitability of the harmonium and other fixed-toned instruments of the west (tuned to the scale of equal temperament), for playing correct Indian music is obvious. The limitations of the uncultivated human ear being what they are, it is too much to expect the average person to perceive the refined distinctions in the frequencies of the notes belonging to the two scales, but nevertheless these distinctions are solid and aesthetic facts.

The note *4/3* does not come in the cycle of fifths and the note *3/2* does not come in the cycle of fourths. The idea of seven octaves was possibly suggested by the cycle of fifths since at the 12th stage, the original sa was almost again got.

Most of the conclusions arrived at by the ancient scholars can be proved by modern methods. The beauty and symmetry underlying the scale of 22 srutis is clear from the illustration. There are ten pairs of notes and these with the *sa* and *pa* give the 22 srutis of the Indian musical scale. The two notes constituting each pair are found to be uniformly separated by the interval of a comma or pramana sruti. The interval of a comma though small is still recognizable by the trained ear. Of the ten sets of twin notes, the note of the lower pitch belongs to the cycle of fourths and the note of the higher pitch, to the cycle of fifths and this is naturally so, since *ma* is a note

[2] the octave-register points above the capital note-name letter *(WvS)*

less in pitch compared to *pa*. At the sixth stage of each cycle, a small but negligible correction of 2 cents is introduced to facilitate easy calculation. In the cycle of fifths, two cents are subtracted and in the cycle of fourths two cents are added. In the cycle of fourths, the correction is made at stage VI to get at the antara gandhara *5/4* [3] a harmonic note heard in the tambura. All these delicate srutis are the pride and glory of Indian music and are carefully treasured up in the ragas and compositions of great composers in those ragas.

In the sa grama, all the notes excepting Panchama are obtained in the cycle of fourths. Even the note *40/27* is obtained in the cycle of fourths.

Two other methods of determining the notes occurring in the cycles of fifths and fourths are given below : –

1. *Cents method*. Take madhya shadja as equal to 0. Its Panchama will be equal to 702 cents. The Panchama of this Panchama is got by adding 702 to 702. The result is 1404 and this is a compound interval or a note in the tara sthayi. By subtracting from this 1200 the total number of cents for an octave, we get 204 which is the value of the note in the madhya stayi. This is the chatussruti rishabha. By adding 702 to it, we get 906 cents which is the frequency of the chatussruti dhaivata and so on. – For the cycle of fourths, add 498 in each case and proceed as mentioned above.

2. *Arithmetical method*. The octave consists of 22 srutis. The panchama has 13 srutis and suddha mydhyama 9. (13 + 9) = 22. Take madhya shadja as equal to 0. Its Panchama is the 13th sruti. The Panchama of this Panchama is got by adding 13 to it. The result is 26 and this is a compound interval or a note in the higher octave. By subtracting 22 from it (the total number of srutis in an octave) we get 4 which is the value of the note in the madhya sthayi. This is the chatussruti rishabha. By adding 13 to it, we get the value 17 which is the sruti value of the chatussruti dhaivata and so on. – For the cycle of fourths, add 9 srutis in each case and proceed as mentioned above.

The Tables on pp 5–6 give the values of the notes of the cycles of fifths and fourths worked out in the above two methods. Corresponding to a reduction of 2 cents in the sixth cycle in the cycle of fifths, a reduction of one sruti is made, in the arithmetical method; likewise an addition of one sruti is made, corresponding to an addition of 2 cents in the sixth cycle in the cycle of fourths. The reasons for this subtraction and addition have already been explained on p. 4.[4]

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[3] see transcription on page 7 *(WvS)*

[4] left column, line 5-6 : alteration by an enharmonic *Schisma* of 2 cents, for the sake of the pure major thirds *(WvS)*

TABLE 4

CYCLE OF FOURTHS						CYCLE OF FIFTHS		
				sa –	– 1200			
....N	160/81	Not used	XII	1178 –				
					– 1110	V	Chyuta shadja ni	243/128	..N
..N	15/8	Kakali ni	VII	1088 –	– 1100				
					– 1018	X	Kaisiki ni	9/5	.....N
N	16/9	Bhairavi ni	II	996 –	– 1000				
					– 906	III	Chatussruti dha	27/16	.D
...D	5/3	Trisruti dha	IX	884 –	– 900				
					– 814	VIII	Suddha dha	8/5	....D
.D	128/81	Ekasruti dha	IV	792 –	– 800				
					– 702	I	Panchama	3/2	P
....M	40/27	Not used	XI	680 –	– 700				
					– 612 or 610	VI	Chyuta pa	729/512 or 64/45	...M
..M	1024/729 or 45/32	Prati ma	VI	588 or 590 –	– 600				
					– 520	XI	Begada ma	27/20	.....M
M	4/3	Suddha ma	I	498 –	– 500				
					– 408	IV	Chyuta madhyama ga	81/64	..G
...G	5/4	Antara ga	VIII	386 –	– 400				
					– 316	IX	Sadharana ga	6/5	.....G
.G	32/27	Bhairavi ga	III	294 –	– 300				
					– 204	II	Chatussruti ri	9/8	.R
....R	10/9	Trisruti ri	X	182 –	– 200				
					– 112	VII	Suddha ri	16/15	....R
..R	256/243	Gaula ri	V	90 –	– 100				
					– 22	XII	Not used	81/80	
				Sa –	– 0				

TABLE 5 CYCLE OF FIFTHS

No. of the cycle	Basic note	Resulting note	Value in Cents	Value in sruti number
1	*sa*	*pa*	702	13
2	*pa*	chatussruti *ri*	702 + 702 = 1404 − 1200 = **204**	13 + 13 = 26 − 22 = **4**
3	chatussruti *ri*	chatussruti *dha*	204 + 702 = **906**	4 + 13 = **17**
4	chatussruti *dha*	chyuta madhyama *ga*	906 + 702 = 1608 − 1200 = **408**	17 + 13 = 30 − 22 = **8**
5	chyuta madhyama *ga*	chyuta shadja *ni*	408 + 702 = **1110**	8 + 13 = **21**
6	chyuta shadja *ni*	chyuta *pa*	1110 + 702 = 1812 − 1200 = 612 or **610**	21 + 13 = 34 − 22 = 12 or **11**
7	chyuta *pa*	suddha *ri*	610 + 702 = 1312 − 1200 = **112**	11 + 13 = 24 − 22 = **2**
8	suddha *ri*	suddha *dha*	112 + 702 = **814**	2 + 13 = **15**
9	suddha *dha*	sadharana *ga*	814 + 702 = 1516 − 1200 = **316**	15 + 13 = 28 − 22 = **6**
10	sadharana *ga*	kaisiki *ni*	316 + 702 = **1018**	6 + 13 = **19**
11	kaisiki *ni*	Begada *ma*	1018 + 702 = 1720 − 1200 = **520**	19 + 13 = 32 − 22 = **10**
12	Begada *ma*	pramana sruti above *sa* and not used	520 + 702 = 1222 − 1200 = **22**	10 + 13 = 23 − 22 = **1**

TABLE 6 CYCLE OF FOURTHS

No. of the cycle	Basic note	Resulting note	Value in Cents	Value in sruti number
1	*sa*	suddha *ma*	498	9
2	suddha *ma*	Bhairavi *ni*	498 + 498 = **996**	9 + 9 = **18**
3	Bhairavi *ni*	Bhairavi *ga*	996 + 498 = 1494 − 1200 = **294**	18 + 9 = 27 − 22 = **5**
4	Bhairavi *ga*	Ekasruti *dha*	294 + 498 = **792**	5 + 9 = **14**
5	ekasruti *dha*	Ekasruti *ri* or Gaula *ri*	792 + 198 = 1290 − 1200 = **90**	14 + 9 = 23 − 22 = **1**
6	Gaula *ri*	Prati *ma*	90 + 498 = 588 or **590**	1 + 9 = 10 or **11**
7	prati *ma*	Kakali *ni*	590 + 498 = **1088**	11 + 9 = **20**
8	kakali *ni*	Antara *ga*	1088 + 498 = 1586 − 1200 = **386**	20 + 9 = 29 − 22 = **7**
9	antara *ga*	Trisruti *dha*	386 + 498 = **884**	7 + 9 = **16**
10	trisruti *dha*	Trisruti *ri*	884 + 498 = 1382 − 1200 = **182**	16 + 9 = 25 − 22 = **3**
11	trisruti *ri*	Pramana sruti below *pa* and not used	182 +498 = **680**	3 +9 = **12**
12	Pramana sruti below *pa*	pramana sruti below *sa* and not used	680 + 498 = **1178**	12 + 9 = **21**

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Transcription of Table 4, 5, and 6 in chapter II ('Early Experiments in Music')
of Book V of the anthology 'South Indian Music' by Prof. P. Sambamurthy
notated in the 'Extended Helmholtz-Ellis JI Pitch Notation'

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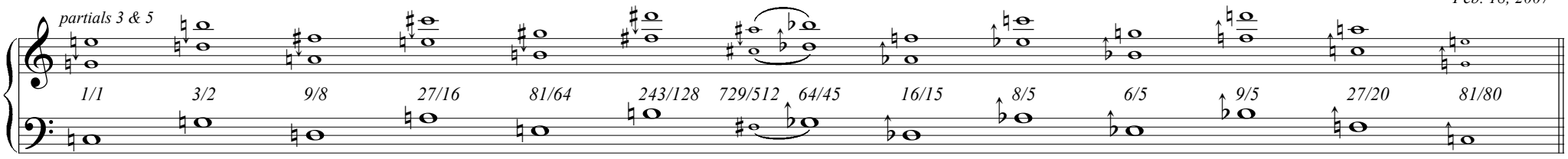
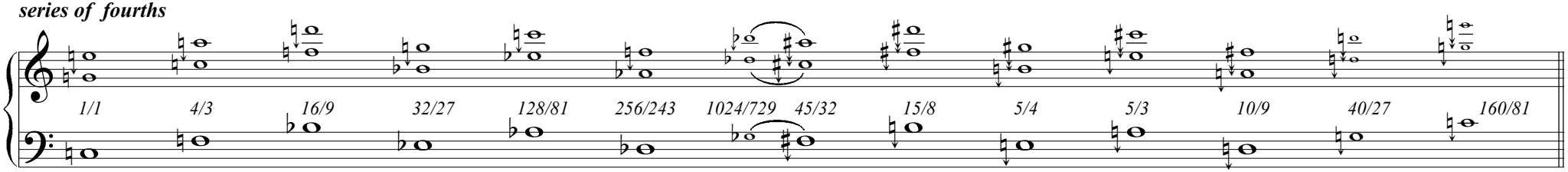
series of fifths



series of fourths



sruti scale