`PROBLEM:`

Consider a small hole in the top of a container (with outward normal $\widehat{z}$) containing a gas. Suppose the temperature of the gas is T, each molecule has mass m, and the number of molecules per unit volume is N.

(a) Note that the velocity distribution of particles exiting ("effusing") out of the hole differs from that inside the container. Considering the number of particles exiting the hole in time Δt, you'll see that the velocity distribution of particles exiting the hole is that inside the container, weighted by the flux. Faster particles are more likely to get out of the hole. Give the velocity distribution of particles effusing out of the hole, making sure it's correctly normalized.

(b) Calculate the mean upward speed $\langle v_z \rangle$ of the the molecules leaving the container. Also compute $\left\langle v_z^2 \right\rangle$ (note that it's indeed greater than $\left\langle v_z^2 \right\rangle$ inside the container, which is given by equipartition).

Hint: In doing integrals you might remember the Gamma function $\Gamma(z) = 2\int_0^\infty \exp(-t^2)t^{2z-1}dt$, and $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(1) = 1$, $\Gamma(z+1) = z\Gamma(z)$.

`SOLUTION:`

Assume inside the container that the gas molecules obey the Maxwell-Boltzmann distribution. The distribution of the effusing molecules is weighted by the flux, i.e. an additional factor of v_z for $v_z > 0$, and it vanishes for $v_z < 0$.

(a) So the distribution for $v_z > 0$ is: $(m^2/2\pi k^2 T^2) v_z \exp(-mv^2/2kT) d^3v$, where the prefactor ensures that it's properly normalized to integrate to 1.

(b) Integrating v_z times the velocity distribution gives $\langle v_z \rangle = (\pi kT/2m)^{1/2}$. Doing the integral for v_z^2 gives $\left\langle v_z^2 \right\rangle = 2kT/m$, which is indeed a factor of two greater than that inside the container (it's kT/m inside the container).