

## Log-gases in Caeli Australi:

### Recent Developments in and Around Random Matrix Theory

The goal of this program is to bring together world-leading specialists on topics contiguous to random matrix theory for the purposes of sharing recent breakthroughs and initiating collaborations across multiple disciplines. Research activities will focus on relations between random matrix theory and the following themes: statistical mechanics of log-gases; machine learning and statistical inference; (free) probability theory; orthogonal polynomials and asymptotic analysis; and integrable systems including Painlevé equations.

#### **Organisers**

- Anas Rahman (HKU, Hong Kong)
- Jiyuan Zhang (South China University of Technology, China)
- Sylvia Serfaty (NYU Courant Institute, USA; Sorbonne Université, France)
- Alice Guionnet (ENS Lyon, France)
- Mario Kieburg (The University of Melbourne, Australia)
- Lucas Hackl (The University of Melbourne, Australia)

#### **Contact**

For emergencies, please contact Anas Rahman on +614 3093 6311.

For general inquiries concerning the program, please email [licaatmatrix2025@gmail.com](mailto:licaatmatrix2025@gmail.com).

## Program Schedule

### Week 1 Schedule - Monday to Tuesday

Time	Mon 4th	Time	Tue 5th
09:00–09:45	Registration (Coffee)	09:15–10:00	Sung-Soo Byun
09:45–10:00	Opening address	10:00–10:45	James Mingo
10:00–11:00	Gernot Akemann	10:45–11:15	Coffee
11:00–12:00	Zhenyu Liao	11:15–12:00	Thomas Wolfs
12:00–14:00	Lunch	12:00–14:00	Lunch
14:00–15:00	Arno Kuijlaars	14:00–15:00	Peter Forrester
15:00–15:30	Coffee	15:00–16:00	Cheese & Wine
15:30–16:30	Folkmar Bornemann	16:00 onwards	Discussion
16:30–17:30	Benoit Collins		

### Week 1 Schedule - Wednesday to Friday

Time	Wed 6th	Thu 7th	Fri 8th
9:15-10:00	Jacobus Verbaarschot	Daria Tieplova	Youyi Huang
10:00-10:45	Dong Wang	Dan Dai	Lu Wei
10:45-11:15	Coffee	Coffee	Coffee
11:15-12:00	Discussion	Discussion	Discussion
12:00-14:00	Lunch	Lunch	Lunch
14:00-14:45	Satya N. Majumdar	Masahiko Ito	Zhigang Bao
14:45-15:30	Gregory Schehr	Yuanyuan Xu	Giorgio Cipolloni
15:30 onwards	Discussion	Discussion	Discussion

- All 45-minute talks consist of 40 minutes for the speaker's presentation followed by 5 minutes for questions. Participants are encouraged to follow up with deeper discussions during free time.
- The conference banquet will be held **at 18:00 on Wednesday, August 6th at The American Creswick** (90 Albert St, Creswick VIC 3363, Australia).
- There will be an excursion held **at 18:00 on Wednesday, August 13th**.
- Unofficial trips to Melbourne and Ballarat will be organised on the weekend (August 9th, 10th).

### Week 2 Schedule

Time	Mon 11th	Tue 12th	Wed 13th	Thu 14th	Fri 15th
9:15-10:00	Shi-hao Li	Leslie Molag	Daniel M. George	Matthias Allard	Yeong-Gwang Jung
10:00-10:45	Fei Wei	Meng Yang	Noriyoshi Sakuma	Mathieu Yahiaoui	Eui Yoo
10:45-11:15	Coffee	Coffee	Coffee	Coffee	Coffee
11:15-12:00	Justin Ko	Nicholas Witte	Guido Mazzuca	Linfeng Wei	Yong-Woo Lee
12:00-14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00-14:45	Lun Zhang	Dang-Zheng Liu	Aron Wennman	Discussion	Airport Shuttle
14:45 onwards	Discussion	Discussion	Discussion		

## Talk Abstracts

Monday 4th August

### **Non-Hermitian random matrices: Recent advances and open problems**

- Gernot Akemann
- Monday 4th 10:00–11:00

**Abstract:** In this talk I will give a brief overview on recent topics in non-Hermitian random matrix theory. I will start with the three Ginibre ensembles and then turn to the two classes of complex symmetric and complex self-dual random matrices. These are expected to be the only two additional classes that give different generic bulk and edge statistics, compared to the Ginibre class. The current status of these ensembles is reviewed, where tools include characteristic polynomials and a recently rediscovered technique by Yan Fyodorov. If time permits, I will also mention work in progress on derivatives of products of characteristic polynomials.

### **Random matrix theory for deep learning: Opportunities and challenges**

- Zhenyu Liao
- Monday 4th 11:00–12:00

**Abstract:** In this talk, I will review recent advances at the intersection of high-dimensional statistics, random matrix theory (RMT), and the theory of deep neural networks. Building on classical results for sample covariance matrices, I will present a systematic overview of how RMT provides valuable insights into the behavior and performance of deep learning models. The discussion follows a natural progression: from shallow to deep architectures, from simple to structured data, and from random to nonrandom models. Topics include the analysis of nonlinear neural networks, the phenomenon of double descent, and neural tangent kernels. I will also highlight the limitations of existing approaches and outline possible directions and technical challenges for applying RMT to fundamental problems in modern machine learning.

### **Riemann Hilbert problems**

- Arno Kuijlaars
- Monday 4th 14:00–15:00

**Abstract:** I am to give an overview on the use of Riemann Hilbert problems for asymptotics of orthogonal polynomials.

### **Fredholm determinants and Painlevé transcendents: A pragmatist's perspective**

- Folkmar Bornemann
- Monday 4th 15:30–16:30

**Abstract:** In the classical integrable cases of random matrix theory, one encounters an abundance of options for expressing quantities either as Fredholm determinants or as Painlevé transcendents and their various siblings. Serving as a kind of user's guide, we highlight the respective merits of each representation for tasks such as numerical evaluation, series expansions, asymptotics, and the identification of equalities. With a computer at your side, the journey is surprisingly enjoyable.

### **Recent developments around strong convergence for random matrices**

- Benoît Collins
- Monday 4th 16:30–17:30

**Abstract:** We will introduce the notion of strong convergence for random matrix models, describe some of our contributions and some other recent progress by other sets of authors.

## Tuesday 5th August

### Recent progress on free energy expansions of two-dimensional Coulomb gases

- Sung-Soo Byun
- Tuesday 5th 09:15–10:00

**Abstract:** The free energy expansions of two-dimensional Coulomb gases are classical and fundamental topics in the field. In this talk, I will explain how structural features of the Coulomb gas model—such as its potential-theoretic, topological, and conformal-geometric aspects—emerge naturally in these expansions. I will also present recent developments, including some of the most recent findings on anomalous terms that arise in multi-component or degenerate regimes.

### Real infinitesimal free independence

- James Mingo
- Tuesday 5th 10:00–10:45

**Abstract:** We introduce a new kind of free independence, called real infinitesimal freeness. We show that independent orthogonally invariant with infinitesimal laws are asymptotically real infinitesimally free. The theory is necessary to account for the extra structures encountered in ensembles which are orthogonally invariant. We introduce new cumulants, called real infinitesimal cumulants and show that real infinitesimal freeness is equivalent to vanishing of mixed cumulants. We prove the formula for cumulants with products as entries.

This joint work with Guillaume Cébron.

### Multiple orthogonal polynomial ensembles of derivative type

- Thomas Wolfs
- Tuesday 5th 11:15–12:00

**Abstract:** I will provide a classification for the biorthogonal ensembles that are both a multiple orthogonal polynomial ensemble and a polynomial ensemble of derivative type (also called a Pólya ensemble). We will consider the notions of derivative type from Kieburg–Kösters (2016) and Kuijlaars–Róman (2019) that typically appear in connection to the squared singular values of products of invertible random matrices and the eigenvalues of sums of Hermitian random matrices. Afterwards, I will explain how these notions are connected to the finite free multiplicative, respectively additive, convolution from free probability. As a consequence, we will obtain families of multiple orthogonal polynomials that (de)compose naturally using these convolutions. I will end my talk by discussing how one can aim to develop discrete notions of derivative type that are compatible with certain non-intersecting path models induced by random tilings.

### On and around large $x, N$ , small $k$ expansions for log-gases and random matrices

- Peter Forrester
- Tuesday 5th 14:00–15:00

**Abstract:** The expansions in the title will be developed in relation to the two-component log-gas, gap probabilities, the spectral form factor, and finite size corrections for distribution functions.

## Wednesday 6th August

### Non-Hermitian random matrix theories and integrability

- Jacobus Verbaarschot
- Wednesday 6th 9:15–10:00

**Abstract:** We start this lecture with a review of the physics background of recent developments in non-Hermitian random matrix theories and their applications to many-body physics. In the bulk of this lecture, based on work by Forrester and Witte, we will discuss in detail the application of integrable techniques to relate universal spectral properties to the Toda lattice equation. The lecture will be concluded with some remarks on topology in non-Hermitian random matrix theory.

### Asymptotics of biorthogonal polynomials related to Muttalib–Borodin ensemble and Hermitian random matrix with external source

- Dong Wang
- Wednesday 6th 10:00–10:45

**Abstract:** The Muttalib–Borodin ensemble is a typical biorthogonal ensemble, and its correlation kernel is expressed by biorthogonal polynomials. In this talk we consider the limit of these biorthogonal polynomials, and the limit of the correlation kernel for the Muttalib–Borodin ensemble with an integer  $\theta$  parameter. We show that the limits are related to Painlevé-type equations in the hard-to-soft transition regime. Our result generalizes the result for Laguerre type random matrix model that is the  $\theta = 1$  specialization of Muttalib–Borodin ensemble, in which the limiting correlation kernel is related to the Painlevé XXXIV equation in the hard-to-soft transition regime.

Moreover, a variation of the biorthogonal polynomials mentioned above is related to a special type of Hermitian random matrix model with external source. We show that the limit of these biorthogonal polynomials yields the Pearcey limit and a higher critical limit of the external source model.

This talk is based on joint work with Shui-Xia Xu.

### Dynamically emergent strong correlations via stochastic resetting

- Satya N. Majumdar
- Wednesday 6th 14:00–14:45

**Abstract:** I will first discuss the equilibrium properties of a gas of  $N$  interacting particles on a line. This will include, for example, the log-gas in random matrix theory (RMT) and the Riesz gas which is a generalisation of the log-gas. I will then discuss some examples of stationary point processes that are out of equilibrium. As a simple example, I will introduce a model of  $N$  independent Brownian particles that are subjected to simultaneous stochastic resetting with rate  $r$ . The simultaneous resetting generates an effective dynamical all-to-all attractions between particles that persist even at long times in its nonequilibrium stationary state (NESS). Despite the presence of strong correlations, many physical observables such as the average density, extreme statistics, order and gap/spacing statistics, full counting statistics etc. (the standard observables of interest in RMT) can be computed exactly in the NESS and they exhibit rich and interesting behaviors. The physical mechanism built in this simple model allows it to generalise and invent a whole class of solvable strongly correlated out of equilibrium point processes, some of which are experimentally realisable in optical trap systems.

### Higher-order cumulants of linear statistics in Coulomb and Riesz gases

- Gregory Schehr
- Wednesday 6th 14:45–15:30

**Abstract:** I will first consider a system of  $N$  classical particles interacting via the Coulomb potential in spatial dimension  $d$ , in the presence of an external confining potential, and at thermal equilibrium. For large  $N$ , the particles are confined within a droplet of finite size. The main focus will be on smooth linear statistics, i.e., fluctuations of observables of the form  $\mathcal{L}_N = \sum_{i=1}^N f(\mathbf{x}_i)$ , where the  $\mathbf{x}_i$  denote the positions of the particles, and  $f(\mathbf{x})$  is a sufficiently regular test function. I will present explicit results for the higher-order cumulants of  $\mathcal{L}_N$  in the large- $N$  limit, in the case where both the function  $f(\mathbf{x}) = f(|\mathbf{x}|)$  and the confining potential are rotationally invariant. A remarkable feature of our results is that the higher-order cumulants depend only on the values of  $f'(|\mathbf{x}|)$  and its higher-order derivatives *evaluated precisely at the boundary of the droplet*, which in this case is a  $d$ -dimensional sphere. I will then extend these results to the linear statistics of one-dimensional trapped Riesz gases, consisting of  $N$  particles with positions  $x_i$  interacting via a repulsive power-law potential of the form  $\propto 1/|x_i - x_j|^k$ , with  $k > -2$ , and confined by an external potential of the form  $V(x) \sim |x|^n$ .

Thursday 7th August

## Information-theoretic reduction of deep neural networks to linear models in the overparametrized proportional regime

- Daria Tieplova
- Thursday 7th 09:15–10:00

**Abstract:** In this talk, I will present a rigorous analysis of fully-trained deep neural networks in the proportional scaling regime (high-dimensional regime), where both the number of training samples and the width of each layer grow proportionally. We establish an information-theoretic equivalence between a Bayesian neural network and a generalized linear model trained under the same conditions. This result proves the recently conjectured deep Gaussian equivalence principle, providing an exact computation of the optimal generalization error for these networks.

## The multiplicative constant in asymptotics of higher-order analogues of the Tracy–Widom distribution

- Dan Dai
- Thursday 7th 10:00–10:45

**Abstract:** In this paper, we are concerned with higher-order analogues of the Tracy–Widom distribution, which describe the eigenvalue distributions in unitary random matrix models near critical edge points. The associated kernels are constructed by functions related to the even members of the Painlevé I hierarchy  $P_I^{2k}$ ,  $k \in \mathbb{N}^+$ , and are regarded as higher-order analogues of the Airy kernel. We present a novel approach to establish the multiplicative constant in the large gap asymptotics of the distribution, resolving an open problem in the work of Clayes, Its and Krasovsky. An important new feature of the expression is the involvement of an integral of the Hamiltonian associated with a special, real, pole-free solution for  $P_I^{2k}$ . In addition, we show that the total integral of the Hamiltonian vanishes for all  $k$ , and establish a transition from the higher-order Tracy–Widom distribution to the classical one in the asymptotic regime. Our approach can also be adapted to calculate similar critical constants in other problems arising from mathematical physics.

This is a joint work with Wen-Gao Long, Shuai-Xia Xu, Lu-Ming Yao and Lun Zhang.

## Trigonometric and elliptic Selberg integrals

- Masahiko Ito
- Thursday 7th 14:00–14:45

**Abstract:** I will talk about  $q$ -analog and elliptic analog of the Selberg integrals associated with root systems.

## Optimal decay of eigenvector overlap for non-Hermitian random matrices

- Yuanyuan Xu
- Thursday 7th 14:45–15:30

**Abstract:** We consider the standard overlap of any bi-orthogonal family of left and right eigenvectors of a large random matrix with centred i.i.d. entries, and we prove that it decays as an inverse second power of the distance between the corresponding eigenvalues. This extends similar results for the complex Gaussian ensemble from Bourgade and Dubach, as well as Benaych–Georges and Zeitouni, to any i.i.d. matrix ensemble in both symmetry classes. Based on a joint work with Giorgio Cipolloni and László Erdős.



Friday 8th August

### Cumulant structures of entanglement entropy over Hilbert–Schmidt ensemble

- Youyi Huang
- Friday 8th 09:15–10:00

**Abstract:** We review existing methods in deriving exact cumulants of entanglement entropy over the Hilbert–Schmidt ensemble. The existing methods require simplifying nested summations, which become prohibitively tedious as the order of cumulant increases. These difficulties are circumvented by a new summation-free method, an example of which for the variance computation will be presented.

### Entropic cumulant structures of random state ensembles

- Lu Wei
- Friday 8th 10:00–10:45

**Abstract:** This talk is a continuation of the preceding talk by Youyi Huang that focuses on existing methods in deriving cumulant expressions of entanglement entropy. In this talk, results on new cumulant structures of the Hilbert–Schmidt ensemble will be presented first before discussing on-going effort in finding cumulant structures of other random state ensembles. Some open problems in this direction will also be discussed.

### Law of fractional logarithm for random matrices

- Zhigang Bao
- Friday 8th 14:00–14:45

**Abstract:** In this talk, I will present a complete resolution of the Paquette-Zeitouni law of fractional logarithm (LFL) for the extreme eigenvalues of Wigner matrices for both symmetry classes. To achieve this, we develop a robust and natural framework to handle the absence of direct comparison with the Gaussian case, as the previous results were known only for the GUE. Our approach involves two key components: (1) a martingale technique that captures the strong correlation between the largest eigenvalue of an  $N \times N$  Wigner matrix and its  $(N - k) \times (N - k)$  minor in the regime  $k \ll N^{2/3}$ , and (2) a dynamical approach to show that this correlation is weak in the regime  $k \gg N^{2/3}$ .

This talk is based on joint work with Giorgio Cipolloni, László Erdős, Joscha Henheik, and Oleksii Kolupaiev.

### Universality of the spectral form factor

- Giorgio Cipolloni
- Friday 8th 14:45–15:30

**Abstract:** We prove that the *spectral form factor* of general Wigner matrices exhibits the universal dip-ramp-plateau phenomenon, a signature of quantum chaos widely used in physics.

Joint with Paul Bourgade.

Monday 11th August

### Multiple skew orthogonal polynomials and applications

- Shi-Hao Li
- Monday 11th 09:15–10:00

**Abstract:** It is well known that orthogonal polynomials have wide connections with random matrices, classical integrable systems, etc.. In this talk, I will introduce a concept called multiple skew orthogonal polynomials, and make applications into non-intersecting Brownian motions, integrable systems, lattice path enumeration, and so on.

### On the moments of the derivative of CUE characteristic polynomials inside the unit disc

- Fei Wei
- Monday 11th 10:00–10:45

**Abstract:** In this talk, I will introduce some results on the moments of the derivative of characteristic polynomials from the CUE, evaluated at points inside the unit disc. I will talk about our results in three different regimes: the global, mesoscopic, and microscopic regimes. This is based on joint work with Nick Simm. Toward the end of the talk, I will also introduce some open questions related to this topic.

### On the phase diagram of extensive-rank symmetric matrix denoising beyond rotational invariance

- Justin Ko
- Monday 11th 11:15–12:00

**Abstract:** Matrix denoising is central to signal processing and machine learning. Its analysis when the matrix to infer has a factorized structure with a rank growing proportionally to its dimension remains a challenge, except when it is rotationally invariant. In this case the information theoretic limits and a Bayes-optimal denoising algorithm, called the rotational invariant estimator, are known. Beyond this setting few results can be found. The reason is that the model is not a usual spin system because of the growing rank dimension, nor a matrix model due to the lack of rotation symmetry, but rather a hybrid between the two. In this talk we make progress towards the understanding of Bayesian matrix denoising when the hidden signal is a factored matrix that is not rotationally invariant.

We first review the rigorous results in the case when the matrix is low rank or sublinear rank. In the extensive rank regime, we provide evidence of the existence of a denoising-factorisation transition separating a phase where denoising using the rotational invariant estimator remains Bayes-optimal due to universality properties of the same nature as in random matrix theory, from one where universality breaks down and better denoising is possible by exploiting the signal's prior and factorised structure, though algorithmically hard. On the theoretical side, we combine mean-field techniques in an interpretable multiscale fashion in order to access the minimum mean-square error and mutual information.

This is based on a series of joint work with Jean Barbier, Francesco Camilli, Jonathan Husson, Koki Okajima, and Anas Rahman.

### Asymptotics for the noncommutative Painlevé II equation

- Lun Zhang
- Monday 11th 14:00–14:45

**Abstract:** In this talk, we are concerned with the following noncommutative Painlevé II equation

$$\mathbf{D}^2\beta_1 = 4\mathbf{s}\beta_1 + 4\beta_1\mathbf{s} + 8\beta_1^3,$$

where  $\beta_1 = \beta_1(\vec{s})$  is an  $n \times n$  matrix-valued function of  $\vec{s} = (s_1, \dots, s_n)$ ,  $\mathbf{s} = \text{diag}(s_1, \dots, s_n)$  and  $\mathbf{D} = \sum_{j=1}^n \frac{\partial}{\partial s_j}$ . If  $n = 1$ , it reduces to the classical Painlevé II equation up to a scaling. Given an arbitrary  $n \times n$  constant matrix  $C = (c_{jk})_{j,k=1}^n$ , a remarkable result due to Bertola and Cafasso asserts that there exists a unique solution  $\beta_1(\vec{s}) = \beta_1(\vec{s}; C)$  of the noncommutative PII equation such that its  $(k, l)$ -th entry behaves like  $-c_{kl}\text{Ai}(s_k + s_l)$  as  $S = \frac{1}{n} \sum_{i=1}^n s_i \rightarrow +\infty$ , where  $\text{Ai}$  stands for the standard Airy function. For a class of structured matrices  $C$ , we establish asymptotics of the associated solutions as  $S \rightarrow -\infty$ , which particularly include the so-called connection formulas. In the present setting, it comes out that the solution exhibits a hybrid behavior in the sense that each entry corresponds to either an extension of the Hastings–McLeod solution or an extension of the Ablowitz–Segur solution for the PII equation. It is worthwhile to emphasize the asymptotics of the  $(k, l)$ -th entry as  $S \rightarrow -\infty$  cannot be deduced solely from its behavior as  $S \rightarrow +\infty$  in general, which actually also depends on the positive infinity asymptotics of the  $(l, k)$ -th entry. This new and intriguing phenomenon disappears in the scalar case. Based on a joint work with Junwen Liu and Luming Yao.

Tuesday 12th August

## Universality for fluctuations of counting statistics of random normal matrices

- Leslie Molag
- Tuesday 12th 09:15–10:00

**Abstract:** The random normal matrix model, depending on a potential  $V$ , describes a two-dimensional Coulomb gas at inverse temperature 2 consisting of  $n$  particles. We establish, for general potentials  $V$ , an explicit limiting formula for the number variance associated with sets  $A$  compactly contained in the droplet. After appropriate normalization by  $\sqrt{n}$ , this variance converges to an integral over the boundary of  $A$ , with an integrand determined by  $V$ .

In addition, we derive a limiting formula in the case where the set  $A$  is a microscopic dilation which approaches the droplet. Key to this result is an asymptotic formula for the kernel near the droplet boundary, which is a hybrid version of Hedenmalm–Wennman and Ameur–Cronvall.

Our results significantly generalize previous work by Akemann–Byun–Ebke and Lacroix-A-Chez-Toine–Majumdar–Schehr.

This is joint work with Jordi Marzo and Joaquim Ortega-Cerdà.

## Planar orthogonal polynomials and their applications

- Meng Yang
- Tuesday 12th 10:00–10:45

**Abstract:** Planar orthogonal polynomials play an important role in studying the statistical behavior of the eigenvalues of random normal matrix ensembles. In this talk, I will present the strong asymptotics of planar orthogonal polynomials for the Gaussian weight with logarithmic singularities, and then describe their applications in computing correlation kernels and partition functions. This talk is based on recent works with Seung-Yeop Lee, Torben Kruger, Sung-Soo Byun, and Seong-Mi Seo.

## TBA

- Nicholas Witte
- Tuesday 12th 11:15–12:00

**Abstract:** TBA

## Edge statistics for random band matrices

- Dang-Zheng Liu
- Tuesday 12th 14:00–14:45

**Abstract:** We consider Hermitian and symmetric random band matrices on the  $d$ -dimensional lattice  $(\mathbb{Z}/L\mathbb{Z})^d$  with bandwidth  $W$ , focusing on local eigenvalue statistics at the spectral edge in the limit  $W \rightarrow \infty$ . Our analysis reveals a critical dimension  $d_c = 6$  and identifies the critical bandwidth scaling as  $W_c = L^{(1-d/6)+}$ .

In the Hermitian case, we establish the Anderson transition for all dimensions  $d < 4$  and GUE edge universality when  $d \geq 4$  under the condition  $W \geq L^{1/3+\epsilon}$  for any  $\epsilon > 0$ . In the symmetric case, we also establish parallel but more subtle transition phenomena after tadpole diagram renormalization. These findings extend Sodin's pioneering work [Ann. Math. 172, 2010], which was limited to the one-dimensional case and did not address the critical phenomena. Joint work with Guangyi Zou (arXiv: 2401.00492v2).

## Wednesday 13th August

### Third order cumulants of products

- Daniel Muñoz George
- Wednesday 13th 09:15–10:00

**Abstract:** Free cumulants are objects that characterize the concept of freeness in a straightforward manner. This property, also known as the vanishing of mixed cumulants, allows one to study freeness from a purely combinatorial point of view. Naturally, understanding the cumulants of products  $a_1 a_2 \cdots a_n$  in terms of the cumulants of the individual variables  $a_i$  is desirable. It was Krawczyk and Speicher who discovered the formula relating these two objects. Later, the concept of freeness was extended from the level of expectations to the level of fluctuations. This extended notion, now called higher order freeness, led to the introduction of the corresponding higher order cumulants. Mingo, Speicher, and Tan addressed the analogue of Krawczyk and Speicher’s formula for the second-order cumulants. More recently, I and co-authors Arizmendi and Sigarreta extended these results to the third order case. In this talk, I will introduce the formula, discuss its applications, and outline a sketch of the proof.

### On generalized free gamma distributions

- Noriyoshi Sakuma
- Wednesday 13th 10:00–10:45

**Abstract:** TBA

### Generalized hydrodynamics for the Volterra lattice

- Guido Mazzuca
- Wednesday 13th 11:15–12:00

**Abstract:** TBA

### An equivariant Weierstrass theorem

- Aron Wennman
- Wednesday 13th 14:00–14:45

**Abstract:** Weierstrass’ classical theorem for entire function states that for any locally finite multi-set  $\Lambda$  in the plane, there is an entire function  $F_\Lambda$  with zero set  $\Lambda$ , counting multiplicities. This assignment is by no means unique, and it is natural to ask if it can be done in some particularly nice way. In particular, can the map be made *equivariant*, i.e., commute with translation

$$F_{\Lambda-w}(z) = F_\Lambda(z + w)?$$

In addition, can it be made measurable, or even continuous (with respect to the standard topologies)?

With Konstantin Slutsky and Mikhail Sodin we recently found that, excluding periodic zero sets, there exists a measurable equivariant map  $\Lambda \mapsto F_\Lambda$ , but not a continuous one. As a corollary, it follows that any non-periodic translation-invariant point process in dimension 2 can be realized as the zero set of a *translation-invariant random entire function*. I plan to explain this result and discuss how we came to this question from work in statistical physics.

The talk is based on the two papers (arXiv:2507.12058) and (arXiv:2210.09882).

## Thursday 14th August

### Correlation functions between singular values and eigenvalues

- Matthias Allard
- Thursday 14th 09:15–10:00

**Abstract:** The talk is about the probabilistic relations between eigenvalues and singular values of bi-unitarily invariant ensembles. We first extend the notion of  $k$ -point correlation function to  $j, k$ -point correlation functions when studying the interactions between  $j$  singular values and  $k$  singular values and, then, give an exact formula for the  $1, k$ -point correlation function. This formula simplifies drastically when assuming the singular values are drawn from a polynomial ensemble. We will show some numerical simulations to illustrate what the  $1, 1$ -point correlation function looks like for the classical cases of Laguerre and Jacobi ensembles and what it reveals about the interactions between singular values and eigenvalues. We then explore what happens in the large matrix size limit, especially around hard edges for both singular values and eigenvalues.

### Random winding numbers for determinantal curves from non-Hermitian matrix random fields

- Mathieu Yahiaoui
- Thursday 14th 10:00–10:45

**Abstract:** TBA

### Skewness of von Neumann entropy over Bures–Hall random states

- Linfeng Wei
- Thursday 14th 11:15–12:00

**Abstract:** We study the degree of entanglement, as measured by von Neumann entropy, of bipartite systems over the Bures–Hall ensemble. Closed-form expressions of the first two cumulants of von Neumann entropy over the ensemble have been recently derived in the literature. In this talk, we focus on its skewness by calculating the third cumulant that describes the degree of asymmetry of the distribution. The main result is an exact closed-form formula of the third cumulant, which leads to a more accurate approximation to the distribution of von Neumann entropy. The key to obtaining the result lies on finding a dozen of new summation identities in simplifying a large number of finite summations involving polygamma functions.

## Friday 15th August

### Spectral analysis of $q$ -deformed unitary ensembles with the Al-Salam–Carlitz weight

- Yeong-Gwang Jung
- Friday 15th 09:15–10:00

**Abstract:** Al-Salam–Carlitz polynomial is a family of basic hypergeometric orthogonal polynomials in the Askey Scheme, which arises in a natural generalization of  $q$ -deformed Gaussian unitary ensemble. In this talk, I will first introduce Flajolet and Viennot’s theory concerning the combinatorics of the spectral moments of orthogonal polynomials, which yields a sign-definite sum formula of spectral moments. This leads to an explicit description of the limiting spectral density. This talk is based on joint work with Sung-Soo Byun and Jaeseong Oh.

### Three topological phases of the elliptic Ginibre ensembles with a point charge

- Eui Yoo
- Friday 15th 10:00–10:45

**Abstract:** In the large  $N$  limit, random matrix models exhibit limiting spectra in the complex plane whose supports are called the droplets. In the 1990s, Jancovici and his collaborators proposed that topological properties of the droplet play important roles in large  $N$  expansion of 2D Coulomb gas. In this talk, we will discuss the elliptic Ginibre matrix model conditioned to have real eigenvalues with multiplicity proportional to the dimension of the matrix. We prove that the droplets are either simply connected, doubly connected, or composed of two simply connected components. Moreover, we present the explicit description of the droplet and electrostatic energies for the simply and doubly connected case. Finally, we introduce the asymptotic behavior of the moments of the characteristic polynomials of elliptic Ginibre matrices as an application. This is based on a joint work with Sung-Soo Byun (arXiv:2502.02948).

### Large deviations for the number of real eigenvalues of the elliptic GinOE

- Yong-Woo Lee
- Friday 15th 11:15–12:00

**Abstract:** The Ginibre Orthogonal Ensemble (GinOE), a real non-Hermitian random matrix model, exhibits the intriguing feature that its spectrum can contain real eigenvalues with non-trivial probability, in contrast to other classical non-Hermitian ensembles. The number of real eigenvalues is a random variable whose average scales with the square root of the matrix size, and has Gaussian fluctuation around the average. In this talk, we discuss the probability of observing an exceptionally large number of real eigenvalues in the spectrum of the GinOE. We derive sharp asymptotics for these probabilities and give potential theoretic descriptions. We extend analysis to the elliptic GinOE, a one-parameter generalization of the GinOE that interpolates between the GinOE and the Gaussian Orthogonal Ensemble by varying the asymmetry. This is based on joint work with Gernot Akemann and Sung-Soo Byun.