MTH 231

Selected Review Problems
(pp. 113, 187, 357-358, 378-379)

1. Prove that given a nonnegative integer n, there is a unique nonnegative integer m such that $m^2 \leq n < (m+1)^2$.

2. Disprove the statement that every positive integer is the sum of the cubes of eight nonnegative integers.

3. Let A be the set of English words that contain the letter x, and let B be the set of English words that contain the letter q. Express each of these sets as a combination of A and B.

   (a) The set of English words that do not contain the letter x.

   (b) The set of English words that contain both an x and a q.

   (c) The set of English words that contain a x but not a q.

   (d) The set of English words that do not contain either an x or a q.

   (e) The set of English words that contain an x or a q, but not both.

4. Show that if A is a subset of B, then the power set of A is a subset of the power set of B.

5. Suppose that A, B, and C are sets. Prove or disprove that $(A - B) - C = (A - C) - B$.

6. Show that if A and B are finite sets, then $|A \cap B| \leq |A \cup B|$. Determine when this relationship is an equality.

7. (a) For which positive integers n is $11n + 17 \leq 2^n$?

(b) Prove the conjecture you made in part (a) using mathematical induction.

8. Use mathematical induction to show that $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^n} = 1 - \frac{1}{3^n}$ whenever n is a positive integer.

9. Show that $\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\cdots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$ whenever n is a positive integer.

10. Use mathematical induction to prove that 9 divides $n^3+(n+1)^3+(n+2)^3$ whenever n is a nonnegative integer.

11. (a) Which amounts of postage can be formed using only 5-cent and 9-cent stamps?

(b) Prove the conjecture you made using strong mathematical induction.

12. Find $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if f is recursively defined by $f(0) = -1$, $f(1) = 2$, and for $n = 1, 2, \ldots$

(a) $f(n+1) = f(n) + 3f(n-1)$.

(b) $f(n+1) = f(n)^2 f(n-1)$.

(c) $f(n+1) = 3f(n)^2 - 4f(n-1)^2$.

(d) $f(n+1) = \frac{f(n-1)}{f(n)}$.

13. Give a recursive definition of
    (a) The set of odd positive integers.

(b) The set of positive integer powers of 3.

(c) The set of polynomials with integer coefficients.