MTH 231

Selected Review Problems
(pp. 111-113)

1. Let p be the proposition "I will do every exercise in this book" and q be the proposition "I will get an A in this course". Express each of these as a combination of p and q.

   (a) I will get an A in this course only if I do every exercise in this book.

   (b) I will get an A in this course and I will do every exercise in this book.

   (c) Either I will not get an A in this course or I will not do every exercise in this book.

   (d) For me to get an A in this course, it is necessary and sufficient that I do every exercise in this book.

2. Find the truth table of the compound proposition $(p \vee q) \rightarrow (p \wedge \neg r)$.

3. Show that these compound propositions are tautologies.

   (a) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

   (b) $((p \vee q) \wedge \neg p) \rightarrow q$

4. Find a compound proposition involving the propositional variables p, q, r, and s that is true when exactly three of these propositional variables are true and is false otherwise.

5. Show that these statements are inconsistent: "If Miranda does not take a course in discrete mathematics, then she will not graduate." "If Miranda does not graduate, then she is not qualified for the job." "If Miranda reads this book, she is qualified for the job." "Miranda does not take a course in discrete mathematics, but she reads this book."

6. Hearken back to the times of knights and knaves where knaves always lie and knights always tell the truth. Suppose you meet three people, Anita, Boris, and Carmen. What are Anita, Boris, and Carmen if Anita says, "I am a knave and Boris is a knight" and Boris says, "Exactly one of the three of us is a knight."

7. Let $P(x)$ be the statement "Student x knows calculus" and let $Q(y)$ be the statement "Class y contains a student who knows calculus." Express each of these as quantifications of $P(x)$ and $Q(y)$.

   (a) Some students know calculus.

   (b) Not every student knows calculus.

   (c) Every class has a student in it who knows calculus.

   (d) Every student in every class knows calculus.

   (e) There is at least one class with no students who know calculus.

8. Let $P(m, n)$ be the statement "m divides n", where the domain for both variables consists of all possible integers. (By "m divides n" we mean that $n = km$ for some integer k.) Determine the truth value of each of these statements.

   (a) $P(4,5)$

   (b) $P(2,4)$

   (c) $\forall m \forall n P(m, n)$

   (d) $\exists m \forall n P(m, n)$

   (e) $\exists n \forall m P(m, n)$

   (f) $\forall n P(1, n)$

9. Use existential and universal quantifiers to express the statement "No one has more than three grandmothers" using the propositional function $G(x, y)$, which represents "x is the grandmother of y."

10. Let $P(x)$ and $Q(x)$ be propositional functions. Show that $\exists x(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \exists Q(x)$ always have the same truth value.

11. Find the negations of these statements.

    (a) If it snows today, then I will go skiing tomorrow.

    (b) Every person in this class understands mathematical induction.

    (c) Some students in this class do not like discrete mathematics.

    (d) In every mathematics class there is some student who falls asleep during lectures. ☹

12. Express this statement using quantifiers: "Every student in this class has taken some course in every department in the school of mathematical sciences."

13. Express this statement using quantifiers: "There is exactly one student in this class who has taken exactly one mathematics class at this school" using the following the uniqueness quantifier. Then express this statement using quantifiers, without using the uniqueness quantifier.

14. Use rules of inference to show that if the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\neg R(a)$, where a is in the domain, are true, then the conclusion $\neg P(a)$ is true.

15. Prove that if x^3 is irrational, then x is irrational.

16. Prove that if x is irrational and $x \geq 0$, then $\sqrt{x}$ is irrational.