MTH 231 Quiz 1

Name: ______________________________

Recitation Time: ______________________

Instructions:

- Answer the following questions with complete, orderly sentences (when applicable.)
- Answers without work shown will not receive credit.
- Read each question completely before attempting to solve.
- Write your name and recitation time in the blanks provided above.

1. (5 points) Determine whether $(\neg q \land (p \to q)) \to \neg p$ is a tautology and justify your answer.

True or False:

2. (8 points) Determine if the following are true or false, and justify your answer. Let the domain be all real numbers.

   (a) $\exists x(x^2 = 2)$

   (b) $\exists x(x^2 = -1)$

   (c) $\forall x(x^2 + 2 \geq 1)$

   (d) $\forall x(x^2 \neq x)$

3. (5 points) Express the following statement using predicates, quantifiers, and logical operators.

"Everyone needs coffee or sleep during finals week."

4. (7 points) Negate the following statement and translate the negated expression to an English sentence.

$$\forall x \exists y (P(x) \rightarrow Q(x, y)).$$

Let:

$P(x) =$ "x is from Washington"
$Q(x, y) =$ "x has visited y",

where the domain of x is all OSU students and the domain of y is all US states.

5. (Extra Credit: 5 points) Hat Problem: Level 1.

    There is a box containing three blue hats and two red hats. Three people sitting in a circle are blindfolded and a hat – either a red hat or a blue hat – is placed on each of their heads. The three people are not allowed to communicate with each other. The first person's blindfold is removed and after looking at the hats on the other two people he is asked to state the color of his own hat (which he cannot see.) The first person replies that he does not know the color of his hat. The second person has her blindfold removed and also states that she does not know the color of her hat. The third person, before her blindfold is removed, correctly states the color of her own hat. What is the color of the third person's hat and how does she know the color?

Useful Equivalences

$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws
$\neg(\neg p) \equiv p$	Double Negation Laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's Laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation Laws
$p \rightarrow q \equiv \neg p \vee q$ $\neg(p \rightarrow q) \equiv p \wedge \neg q$ (proved in class)	Logical Equivalences Involving Conditionals