

Exploration in Interactive Personalized Music Recommendation: A Reinforcement Learning Approach

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Current music recommender systems typically act in a greedy manner by recommending songs with the highest user ratings. Greedy recommendation, however, is suboptimal over the long term: it does not actively gather information on user preferences and fails to recommend *novel* songs that are potentially interesting. A successful recommender system must balance the needs to *explore* user preferences and to *exploit* this information for recommendation. This paper presents a new approach to music recommendation by formulating this exploration-exploitation trade-off as a reinforcement learning task. To learn user preferences, it uses a Bayesian model which accounts for both audio content and the novelty of recommendations. A piecewise-linear approximation to the model and a variational inference algorithm help to speed up Bayesian inference. One additional benefit of our approach is a single unified model for both music recommendation and playlist generation. We demonstrate the strong potential of the proposed approach with simulation results and a user study.

Categories and Subject Descriptors: H.3.3 [Information Search and Retrieval] Information filtering; H.5.5 [Sound and Music Computing] Modeling, Signal analysis, synthesis and processing

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1. INTRODUCTION

A music recommendation system recommends songs from a large database by matching songs with a user's preferences. An *interactive* recommender system adapts to the user's preferences online by incorporating user feedback into recommendations. Each recommendation thus serves two objectives:

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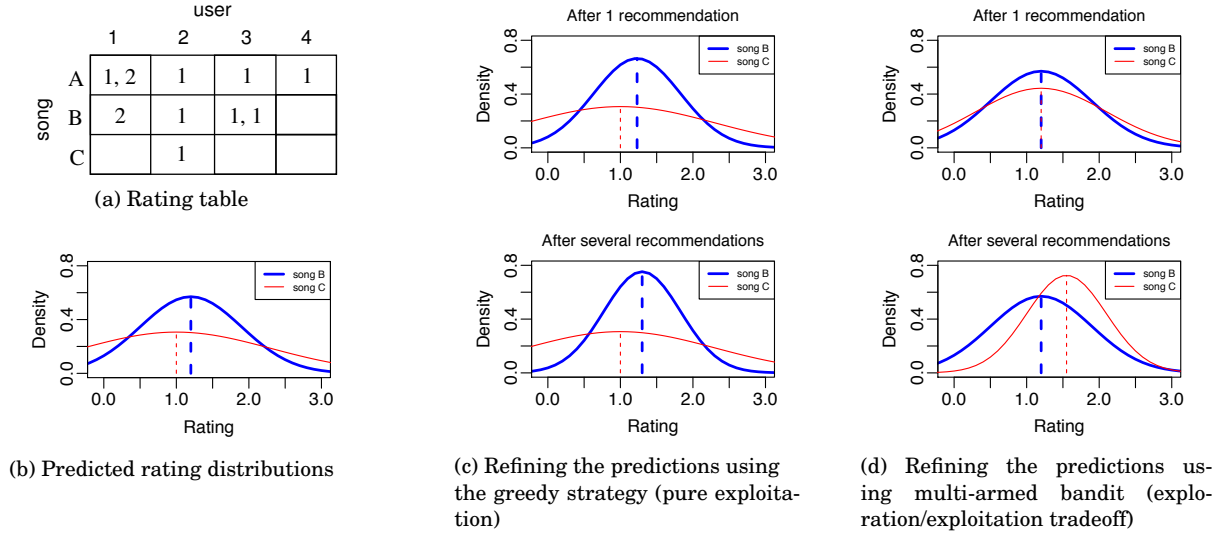


Fig. 1: Uncertainty in recommendation

(i) satisfy the user's current musical need, and (ii) elicit user feedback in order to improve future recommendations.

Current recommender systems typically focus on the first objective, while completely ignoring the other. They recommend songs with the highest user ratings. Such a greedy strategy, which does not actively seek user feedback, often results in suboptimal recommendations over the long term. Consider the simple example in Figure 1. The table contains the ratings for three songs by four users (Figure 1a), with 3 being the highest and 1 being the lowest. For simplicity, let us assume that the recommender chooses between two songs B and C only. The target user is 4, whose true ratings for B and C are 1.3 and 1.6, respectively. The true rating is the expected rating of a song by the user. It is a real number, because a user may give the same song different ratings as a result of external factors. In this case, a good recommender should choose C . Since the true user ratings are unknown to the recommender, it may approximate the rating distributions for B and C as Gaussians, P_B and P_C (Figure 1b), respectively, using the data in Figure 1a. The distribution P_B has mean 1.2. The distribution P_C has mean 1.0. P_B has much lower variance than P_C , because B has more rating data. A greedy recommender (including the highly successful collaborative filtering (CF) approach) would recommend B , the song with the highest mean rating. In response to this recommendation, user 4 gives a rating, whose expected value is 1.3. The net effect is that the mean of P_B likely shifts towards 1.3 and its variance further reduces (Figure 1c). Consequently the greedy recommender is even more convinced that user 4 favors B and will always choose B for all future recommendations. It will never choose C and find out its true rating, resulting in clearly suboptimal performance.

To overcome this difficulty, the recommender must take into account *uncertainty* in the mean ratings. If it considers both the mean and the variance of the rating distribution, the recommendation will change. Consider again Figure 1b. Although P_C has slightly lower mean than P_B , it has very high variance. It may be worthwhile to recommend C and gather additional user feedback in order to reduce the variance. User 4's rating on C has expected value 1.6. Therefore, after one recommendation, the mean of P_C will likely shift towards 1.6 (Figure 1d). By recommending C several times and gathering user feedback, we will then find out user 4's true preference C .

This example illustrates that a good interactive music recommender system must *explore* user preferences actively rather than merely *exploit* the rating information available. Balancing exploration and exploitation is critical, especially when the system is faced with a *cold start*, i.e., when a new user or a new song appears.

Another crucial issue for music recommendation is playlist generation. People often listen to a group of related songs together and may repeat the same song multiple times. This is unique to music recommendation and rarely occurs in other recommendation domains such as newspaper articles or movies. A playlist is a group of songs arranged in a suitable order. The songs in a playlist have strong interdependencies. For example, they share the same genre [Chen et al. 2012] or have a consistent mood [Logan 2002], but are diversified at the same time [Zhang et al. 2012]. They may repeat, but are not repetitive. Existing recommender systems based on CF or audio content analysis typically recommend one song at a time and do not consider their interdependencies during the recommendation process. They divide playlist generation into two distinct steps [Chen et al. 2012]. First, choose a set of favored songs through CF or content analysis. Next, arrange the songs into a suitable order in a process called *automatic playlist generation* (APG).

In this work, we formulate interactive, personalized music recommendation as a reinforcement learning task called the *multi-armed bandit* [Sutton and Barto 1998] and address both exploration-exploitation trade-off and playlist generation with a single unified model:

- Our bandit approach systematically balances exploration and exploitation, a central issue well studied in reinforcement learning. Experimental results show that our recommender system mitigates the difficulty of cold start and improves recommendation performance compared to the greedy approach.
- We build a single rating model that captures both user preference over audio content and the novelty of recommendations. It seamlessly integrates music recommendation and playlist generation.
- We also present an approximation to the rating model and new probabilistic inference algorithms in order to achieve real-time recommendation performance.
- Although our approach is designed specifically for music recommendation, it is possible to be generalized to other media types as well.

In the following, Section 2 describes related work. Section 3 formulates the rating model and our multi-armed bandit approach to music recommendation. Section 4 presents the approximate Bayesian models and inference algorithms. Section 5 presents evaluation of our models and algorithms. Section 6 discusses the possible generalization directions of the approach to other media types. Section 7 summarizes the main results and provides directions for future research.

2. RELATED WORK

2.1 Music recommendation

Since [Song et al. 2012; Shen et al. 2013] provide a very recent and comprehensive review of existing related works on music recommendation, we will first briefly summarize the state of the art and discuss highly relevant work in detail later. Currently, music recommender systems can be classified according to their methodologies into four categories: *collaborative filtering* (CF), *content-based* methods, *context-based* methods, and hybrid methods. Collaborative filtering recommends songs by considering those preferred by other like-minded users. The state-of-the-art method for performing CF is matrix factorization [Koren et al. 2009]. Although CF is the most widely used method, it suffers from the notorious cold-start problem, as it cannot recommend songs to new users whose preferences are unknown (the *new-user* problem) or recommend new songs to users (the *new-song* problem). Unlike CF, content-based

methods recommend songs with audio content similar to the user's preferred songs [Chen and Chen 2001]. The recommendation quality of content-based systems is largely determined by acoustic features, e.g., timbre and rhythm features [Song et al. 2012]. Content-based systems solve the new-song problem, but not the new-user problem. Context-based music recommender systems are increasingly popular. They recommend songs to match various aspects of the user context (e.g., activities, locations, environment, mood, or physiological states) or text context [Wang et al. 2012; Braunhofer et al. 2013; Kaminskas et al. 2013; Cheng and Shen 2014; Schedl and Schnitzer 2014; Eck et al. 2007; Knees and Schedl 2013]. Hybrid methods combine two or more of the above methods [Yoshii et al. 2006].

There is relatively little work combining music recommendation with playlist generation. In [Chen et al. 2012], a playlist is modeled as a Markov process whose transition probability models both user preferences and playlist coherence. In [Zheleva et al. 2010], a model similar to Latent Dirichlet Allocation is used to capture user latent taste and mood of songs. In [Aizenberg et al. 2012], a new CF model is developed to model playlists in Internet radio stations. While these earlier approaches also combine recommendation with playlist generation, our model differs in three aspects: (1) it is based on audio content while the previous three depend only on usage data; (2) our model is highly efficient, thus allowing easy online updates; (3) our model is crafted and evaluated based on real-life user interaction data not data crawled from the web. [Zhang et al. 2012] try to recommend using a linear combination of CF results with the results from an existing novelty model, which ranks songs by CF before generating the playlists according to novelty. The parameters for the linear combination are adjusted manually. Moreover, they provide only system-level control of novelty while our method provides personalized control. Other work, such as [Hu and Ogihara 2011], generates music playlists within a user's own music library. They assume that the user's preference is already known and does not need to be inferred.

2.2 Reinforcement learning

Unlike supervised learning (e.g., classification, regression), which considers only prescribed training data, a reinforcement learning (RL) algorithm *actively* explores its environment to gather information and exploits the acquired knowledge to make decisions or predictions.

The multi-armed bandit is a thoroughly studied reinforcement learning problem. For a *bandit (slot machine)* with M arms, pulling arm i will result in a random payoff r , sampled from an unknown and arm-specific distribution p_i . The objective is to maximize the *total payoff* given a number of trials. The set of arms is $\mathcal{A} = \{1 \dots M\}$, known to the player; each arm $i \in \mathcal{A}$ has a probability distribution p_i , unknown to the player. The player also knows he has n rounds of pulls. At the l -th round, he can pull an arm $I_l \in \mathcal{A}$, and receive a random payoff r_{I_l} , sampled from the distribution p_{I_l} . The objective is to wisely choose the n pulls $((I_1, I_2, \dots, I_n) \in \mathcal{A}^n)$ in order to maximize *total payoff* $\sum_{l=1}^n r_{I_l}$.

A naive solution to the problem would be to first randomly pull arms to gather information to learn p_i (exploration) and then always pull the arm that yields the maximum predicted payoff (exploitation). However, either too much exploration (the learnt information is not used much) or too much exploitation (the player lacks information to make accurate predictions) results in a suboptimal total payoff. Thus, balancing exploration and exploitation is the key issue.

The multi-armed bandit approach provides a principled solution to this problem. The simplest multi-armed bandit approach, namely ϵ -greedy, chooses the arm with the highest predicted payoff with probability $1 - \epsilon$ or chooses arms uniformly at random with probability ϵ . An approach better than ϵ -greedy is based on a simple and elegant idea called upper confidence bound (UCB) [Auer 2003]. Let U_i be the true expected payoff for arm i , i.e., the expectation of p_i ; UCB-based algorithms estimate both its expected payoff \hat{U}_i and a confidence bound c_i from past payoffs, so that U_i lies in $(\hat{U}_i - c_i, \hat{U}_i + c_i)$ with high probability. Intuitively, selecting an arm with large \hat{U}_i corresponds to exploitation, while select-

ing one with large c_i corresponds to exploration. To balance exploration and exploitation, UCB-based algorithms follow the principle of “optimism in the face of uncertainty” and always select the arm that maximizes $\hat{U}_i + c_i$.

Bayes-UCB [Kaufmann et al. 2012] is a state-of-the-art Bayesian counterpart of the UCB approach. In Bayes-UCB, the expected payoff U_i is regarded as a random variable, and the posterior distribution of U_i given the history payoffs \mathcal{D} , denoted as $p(U_i|\mathcal{D})$, is maintained, and the fixed-level *quantile* of $p(U_i|\mathcal{D})$ is used to mimic the upper confidence bound. Similar to UCB, every time Bayes-UCB selects the arm with the maximum quantile. UCB-based algorithms require an explicit form of the confidence bound, which is difficult to derive in our case, but in Bayes-UCB, the quantiles of the posterior distributions of U_i can be easily obtained using *Bayesian inference*. We therefore use Bayes-UCB in our work.

There are more sophisticated RL methods such as Markov Decision Process (MDP) [Szepesvári 2010], which generalizes the bandit problem by assuming that the states of the system can change following a Markov process. Although MDP can model a broader range of problems than the multi-armed bandit, it requires much more data to train and is often more expensive computationally.

2.3 Reinforcement learning in recommender systems

Previous work has used reinforcement learning to recommend web pages, travel information, books, news, etc. For example, [Joachims et al. 1997] use Q-learning to guide users through web pages. In [Golovin and Rahm 2004], a general framework is proposed for web recommendation, where user implicit feedback is used to update the system. [Zhang and Seo 2001] propose a personalized web-document recommender where each user profile is represented as vector of terms whose weights of the terms are updated based on the temporal difference method using both implicit and explicit feedback. In [Srivihok and Sukonmanee 2005], a Q-learning-based travel recommender is proposed, where trips are ranked using a linear function of several attributes including trip duration, price and country, and the weights are updated according to user feedback. [Shani et al. 2005] use a MDP to model the dynamics of user preference in book recommendation, where purchase history is used as the states and the generated profit the payoffs. Similarly, in a web recommender [Taghipour and Kardan 2008], browsing history are used as the states, and web content similarity and user behavior are combined as the payoffs. [Chen et al. 2013] consider the exploration/exploitation tradeoff in the rank aggregation problem — aggregating partial rankings given by many users into a global ranking list. This global ranking list can be used for unpersonalized recommenders but is of very limited use for personalized ones.

In the seminal work done by [Li et al. 2012], news articles are represented as feature vectors; the click-through rates of articles are treated as the payoffs and assumed to be a linear function of news feature vectors. A multi-armed bandit model called LinUCB is proposed to learn the weights of the linear function. Our work differs from this work in two aspects. Fundamentally, music recommendation is different from news recommendation due to the sequential relationship between songs. Technically, the additional novelty factor of our rating model makes the reward function nonlinear and the confidence bound difficult to obtain. Therefore we need the Bayes-UCB approach and the more sophisticated Bayesian inference algorithms (Section 4). Moreover, we cannot apply the offline evaluation techniques developed in [Li et al. 2011], because we assume that ratings change dynamically over time. As a result, we must conduct online evaluation with real human subjects.

Although we believe reinforcement learning has great potential in improving music recommendation, it has received relatively little attention and found only limited application. [Liu et al. 2009] use MDP to recommend music based on a user’s heart rate to help the user maintain it within the normal range. States are defined as different levels of heart rate, and biofeedback is used as payoffs. However,

(1) parameters of the model are not learnt from exploration, and thus exploration/exploitation tradeoff is not needed; (2) the work does not disclose much information about the evaluation of the approach. [Chi et al. 2010] uses MDP to automatically generate playlist. Both SARSA and Q-learning are used to learn user preference, and, states are defined as mood categories of the recent listening history similar to [Shani et al. 2005]. However, in this work, (1) exploration/exploitation tradeoff is not considered; (2) mood or emotion, while useful, can only contribute so much to effective music recommendation; and (3) the MDP model cannot handle long listening history, as the state space grows exponentially with history length; as a result, too much exploration and computation will be required to learn the model. Independent of and concurrent with our work, [Liebman and Stone 2014] build a DJ agent to recommend playlists based on reinforcement learning. Their work differs from ours in that: (1) exploration/exploitation tradeoff is not considered; (2) the reward function does not consider the novelty of recommendations; (3) their approach is based on a simple tree-search heuristic, while ours the thoroughly studied multi-armed bandit; (4) not much information about the simulation study is disclosed, and no user study is conducted.

The active learning approach [Karimi et al. 2011] *only explores* songs in order to optimize the predictive performance on a pre-determined test dataset. Our approach, on the other hand, requires no test dataset and balances *both* exploration and exploitation to optimize the entire interactive recommendation process between the system and users. Since many recommender systems in reality do not have test data or at least have no data for new users, our bandit approach is more realistic compared with the active learning approach.

Our work is the first to balance exploration and exploitation based on reinforcement learning and particularly multi-armed bandit in order to improve recommendation performance and mitigate the cold-start problem in music recommendation.

3. A BANDIT APPROACH TO MUSIC RECOMMENDATION

3.1 Personalized user rating model

Music preference is the combined effect of many factors including music audio content, novelty, diversity, moods and genres of songs, user emotional states, and user context information [Wang et al. 2012]. As it is unrealistic to cover all the factors in this paper, we focus on audio content and novelty.

Music Audio Content - Whether a user likes or dislikes a song is highly related to the audio content of the song. We assume that the music audio content of a song can be described as a feature vector \mathbf{x}^1 . Without considering other factors, a user's overall preference in this song can be represented as a linear function of \mathbf{x} as:

$$U_c = \theta' \mathbf{x} \quad (1)$$

where the parameter vector θ represents user preference in different music features. Different users may have different preferences and thus different values of θ . To keep the problem simple, we assume that a user's preference is invariant, i.e. θ remains constant over time, and we will address the case of changing θ in future work.

Although the idea of exploration/exploitation tradeoff can be applied to collaborative filtering (CF) as long as the rating distribution can be estimated as shown in Figure 1, we choose to work on the content-based approach instead of CF for a number of reasons. First, we need a posterior distribution of U_c in order to use Bayes-UCB as introduced in Section 2.2, so non-Bayesian CF methods cannot be used. Second, existing Bayesian methods for matrix factorization [Salakhutdinov and Mnih 2008; Silva and

¹Please refer to Table I in the appendix for a summary of the notations used in this paper.

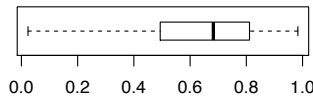


Fig. 2: Proportion of repetitions in users' listen history

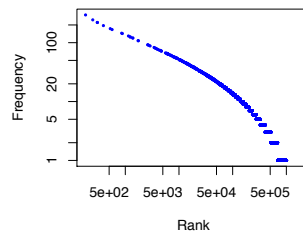


Fig. 3: Zipf's law of song repetition frequency

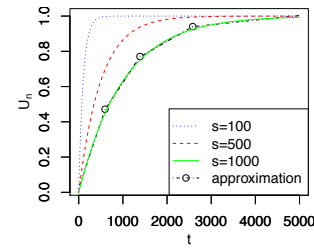


Fig. 4: Examples of $U_n = 1 - e^{-t/s}$. The line marked with circles is a 4-segment piecewise linear approximation.

Carin 2012] are much more complicated than our linear model and also require large amounts of training data. These render the user study costly and cumbersome. Third, our bandit approach requires the model to be updated once a new rating is obtained, but existing Bayesian matrix factorization methods are inefficient for online updating [Salakhutdinov and Mnih 2008; Silva and Carin 2012]. Fourth, CF suffers from the new-song problem while the content-based method does not. Fifth, CF captures correlation instead of causality and thus does not explain why a user likes a song. In contrast, the content-based approach captures one important aspect of the causality, i.e. music content.

Novelty - Inspired by [Gunawardana and Shani 2009], we seek to measure novelty by first examining the repetition distributions of 1000 users' listening histories collected from Last.fm². The box plot in Figure 2 shows the proportion of repetitions, which is defined as: $1 - \frac{\text{number of unique songs}}{\text{listening history length}}$. Note that since Last.fm does not record the user's listening histories outside Last.fm, the actual proportion is expected to be even larger than the median 68.3% shown here. Thus, most of the songs a user listens to are repeats. We also studied the song repetition frequency distribution of every individual user's listening history. The frequencies of songs were first computed for every user. Then, all users' frequencies were ranked in decreasing order. Finally, we plotted frequencies versus ranks on a log-log scale (Figure 3). The distribution approximately follows the Zipf's law [Newman 2005]—only a small set of songs are repeated most of the time while all the rest are repeated much less often. Most other types of recommenders do not follow Zipf's law. For instance, recommending books that have been bought or movies that have been watched makes little sense. In music recommendation, however, it is critically important to repeat songs appropriately.

One problem with existing novelty models is that they do not take the time elapsed since previous listening into consideration [Gunawardana and Shani 2009; Lathia et al. 2010; Castells et al. 2011; Zhang et al. 2012]. As a result, songs listened to a year ago and just now have the same likelihood to be recommended. Inspired by [Hu and Ogihara 2011], we address this issue by assuming that the novelty of a song decays immediately after it is listened to it and then gradually recovers. Let t be the time elapsed since the last listening of the song, the novelty recovers according to the function:

$$U_n = 1 - e^{-t/s} \quad (2)$$

where s is a parameter indicating the recovery speed. The higher the value of s , the slower the recovery of novelty. Figure 4 shows a few examples of U_n with different values of s . Note that the second term of Equation (2), $e^{-t/s}$, is the well-established forgetting curve proposed by [Ebbinghaus et al. 1913], which measures the user's memory retention of a song. Novel songs are thus assumed to be those of which a user has little or no memory.

²<http://ocelma.net/MusicRecommendationDataset/lastfm-1K.html>

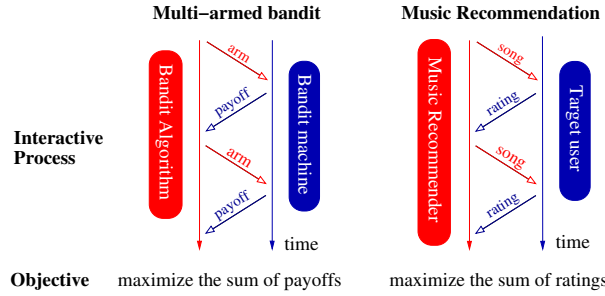


Fig. 5: Relationship between the multi-armed bandit problem and music recommendation

Different users can have different recovery rates s . As can be seen from the widespread distribution in Figure 2, some users may repeatedly listen to their favorite songs, while the others may be keen to exploring songs they have not listened to previously. Therefore, s is an unknown user parameter to be learnt from user interactions.

Combined Model - A user's preference of a recommendation can be represented as a rating; the higher the rating, the more the user likes the recommendation. Unlike traditional recommenders, which assume a user's ratings are static, we assume that a rating is the combined effect of the user's preference of the song's content and the dynamically changing novelty. Thus, a song rated as 5 last time could be rated as 2 later because the novelty has decreased. Finally, we define the combined rating model as:

$$U = U_c U_n = \theta' \mathbf{x} \left(1 - e^{-t/s}\right). \quad (3)$$

In this model, the more the user likes a particular song, the more likely it will be repeated due to the larger U_c value. Also, given that the user's favorites comprise a small subset of his/her library, the U model behaves in accordance with Zipf's Law and ensures that only a small proportion of songs will be repeated frequently. This property of the model will be verified in Section 5.3.2.

In Section 5.3.1, we will show that the product form of Equation (3) leads to significantly better performance than the alternative linear combination $U = aU_c + bU_n$.

Other factors - We note that, besides novelty, the repetition of songs can also be affected in other ways. When a user comes to a song of great excitement, he may listen to it again and again. When his interest changes, he may discard songs that he has been frequently repeating. Sometimes, the user finds a song boring initially but repeats it frequently later, while in other cases he may stop repeating a song because he is bored. Understanding and modeling all these factors and precisely predicting when to repeat a song for a particular user would make very interesting follow-up studies.

3.2 Interactive music recommendation

Under our rating model, each user is represented by a set of parameters $\Omega = \{\theta, s\}$. If we knew the values of Ω of a user, we could simply recommend the songs with the highest rating according to Equation (3). **However, Ω is hidden and needs to be estimated from historical data, and thus uncertainty always exists.** In this case, the greedy strategy used by traditional systems is suboptimal, and it is necessary to take the uncertainty into account and balance exploration and exploitation.

The multi-armed bandit approach introduced in Section 2.2 offers a way to do so for the interactive music recommendation process. As illustrated in Figure 5, we treat songs as arms and user ratings as

ALGORITHM 1: Recommendation using Bayes-UCB

```

for  $l = 1$  to  $n$  do
  for all song  $k = 1, \dots, |S|$  do
    compute  $q_k^l = Q(1 - 1/l, \lambda_k^{l-1})$ 
  end for
  recommend song  $k^* = \arg \max_{k=1 \dots |S|} q_k^l$  and gather rating  $r_l$ ; update  $p(\Omega|\mathcal{D}_l)$  and  $\lambda_k^l$ 
end for

```

payoffs³. The music recommendation problem is then transformed into a multi-armed bandit problem, and the objective of the music recommender is also changed to maximizing the sum of the ratings given by the target user *over the long term*. We argue that the cumulative rating is a more realistic objective than the *myopic* predictive accuracy used by traditional music recommenders, because users generally listen to songs for a long time instead of focusing on one individual song.

We adopt the Bayes-UCB algorithm introduced in Section 2.2 for our recommendation task. We denote the rating given by the target user to a recommendation i as a random variable R_i , and the **expectation of R_i** is U given the feature vector (\mathbf{x}_i, t_i) :

$$\mathbb{E}[R_i] = U_i = \theta' \mathbf{x}_i \left(1 - e^{-t_i/s}\right) \quad (4)$$

Then, we develop Bayesian models to estimate the posterior distribution of U given the history recommendation and user ratings. We sketch the framework here and explain it in greater detail in Section 4. We assume that the **prior distribution of Ω is $p(\Omega)$** and that, at the $(l+1)$ th recommendation, we have accumulated l history recommendations $\mathcal{D}_l = \{(\mathbf{x}_i, t_i, r_i)\}_{i=1}^l$ as training samples, where r_i is the rating given by the user to the i -th recommendation. The posterior distribution of Ω can then be obtained based on the Bayes' rule:

$$p(\Omega|\mathcal{D}_l) \propto p(\Omega)p(\mathcal{D}_l|\Omega) \quad (5)$$

Consequently, the expected rating of song k , denoted as U_k can be predicted as:

$$p(U_k|\mathcal{D}_l) = \int p(U_k|\Omega)p(\Omega|\mathcal{D}_l)d\Omega \quad (6)$$

Henceforth, we will use λ_k^l to denote $p(U_k|\mathcal{D}_l)$ for simplicity.

Finally, to balance exploration and exploitation, Bayes-UCB recommends song k^* , which maximizes the quantile function: $k^* = \arg \max_{k=1 \dots |S|} Q(\alpha, \lambda_k^l)$ where Q satisfies $\mathbb{P}[U_k \leq Q(\alpha, \lambda_k^l)] = \alpha$ and S is all the songs in the database. We set $\alpha = 1 - \frac{1}{l+1}$. The detailed recommendation algorithm is described in Algorithm 1.

The cold-start problem is caused by the lack of information required for making good recommendations. There are many ways for mitigating the cold-start problem, most of which rely on additional information about the users or songs, e.g., popularity/metadata information about the songs [Hariri et al. 2012], context/demographic information about the users [Wang et al. 2012]. Although music audio content is required by U_c , it is usually easy to obtain from industry. Our bandit approach addresses the cold-start problem without relying on additional information about users and songs. Instead, it seeks to appropriately explore and exploit information during the whole interactive process. Thus, the

³Although in reality users usually do not give explicit feedback (i.e. ratings) to every recommended song, implicit feedback (e.g. skipping a song, listening to a song fully) can be obtained much more easily. In this paper, we focus on explicit feedback to keep the problem simple.



Fig. 6: Graphical representation of the Bayesian models. Shaded nodes represent observable random variables, while white nodes represent hidden ones. The rectangle (plate) indicates that the nodes and arcs inside are replicated for N times.

bandit approach presents a fundamentally different solution to the cold-start problem, yet it can be used in conjunction with the existing methods.

There are other Bayesian multi-arm bandit algorithms such as Thompson sampling [Agrawal and Goyal 2012] and optimistic Bayesian sampling [May et al. 2012]. Thompson sampling is based on the probability matching idea, i.e. selecting the song according to its probability of being optimal. Optimistic Bayesian sampling uses an exploitative function on top of Thompson sampling. Which of the three is superior? Theoretically, this remains an open question. However, they have been shown to perform comparably well in practice [May et al. 2012]. Existing studies provide little guidance on our selection between them. In this paper, we focus on the exploration/exploitation tradeoff principle and simply choose the most recent Bayes-UCB in our implementation. Nevertheless, our system could easily adapt to the other two algorithms, as they are also based on the posterior rating distributions.

4. BAYESIAN MODELS AND INFERENCE

4.1 Exact Bayesian model

To compute Equations (5) and (6), we develop the following Bayesian model (see Figure 6a for graphical representation).

$$\begin{aligned}
 R|\mathbf{x}, t, \boldsymbol{\theta}, s, \sigma^2 &\sim \mathcal{N}\left(\boldsymbol{\theta}'\mathbf{x}\left(1 - e^{-t/s}\right), \sigma^2\right) & \boldsymbol{\theta}|\sigma^2 &\sim \mathcal{N}(\mathbf{0}, c_0\sigma^2\mathbf{I}) \\
 s &\sim \mathcal{G}(d_0, e_0) & \tau = 1/\sigma^2 &\sim \mathcal{G}(f_0, g_0)
 \end{aligned}$$

Every part of the model defines a probabilistic dependency between the random variables. $\mathcal{N}(\cdot, \cdot)$ is a (multivariate) Gaussian distribution with the mean and (co)variance parameters, and $\mathcal{G}(\cdot, \cdot)$ is a Gamma distribution with the shape and rate parameters. The rating R is assumed to be normally distributed following the convention of recommender systems. A gamma prior is put on s because s takes positive values. Following the conventions of Bayesian regression models, a normal prior is put on $\boldsymbol{\theta}$ and a Gamma prior on τ . We assume that $\boldsymbol{\theta}$ depends on σ^2 because it leads to better convergence in the simulation study.

Since there is no closed-form solution to Equation (5) under this model, Markov Chain Monte Carlo (MCMC) is used as the approximate inference procedure. Directly evaluating Equation (6) is also infeasible. Thus we use Monte Carlo simulation to obtain λ_k^l : for every sample obtained from the MCMC procedure, we substitute it into Equation (4) to obtain a sample of U_i , and then use the histogram of the samples of U_i to approximate λ_k^l .

This approach is easy to understand and implement. However, it is very slow, and users could wait for up to a minute until the Markov chain converges. To make the algorithm more responsive, we develop an approximate Bayesian model and a highly efficient variational inference algorithm in the following sections.

4.2 Approximate Bayesian model

4.2.1 Piecewise Linear approximation . It is very difficult to develop better inference algorithms for the exact Bayesian model because of the irregular form of the function $U_n(t)$. Fortunately, U_n can be well approximated by a piecewise linear function (as shown in Figure 4), which enables us to develop an efficient model.

For simplicity, we discretize time t into K predetermined intervals: $[0, \xi_1), [\xi_1, \xi_2), \dots, [\xi_{K-1}, +\infty)$, and only consider the class of piecewise linear functions whose consecutive line segments intersect at the boundaries of these intervals. This class of functions can be compactly represented as a linear function [Hastie et al. 2009]. We first map t into a vector $\mathbf{t} = [(t - \xi_1)_+, \dots, (t - \xi_{K-1})_+, t, 1]$, where $(t - \xi)_+ = \max(t - \xi, 0)$, and then approximate $U_n(t)$ as $U_n(t) \approx \beta' \mathbf{t}$, where $\beta = [\beta_1, \dots, \beta_{K+1}]'$ is a vector of parameters to be learnt from training data. Now, we can represent U as the product of two linear functions: $U = U_c U_n \approx \theta' \mathbf{x} \beta' \mathbf{t}$.

Based on this approximation, we approximate the distributions of R and the parameters of the exact Bayesian model as follows:

$$\begin{aligned} R|\mathbf{x}, \mathbf{t}, \theta, \beta, \sigma^2 &\sim \mathcal{N}(\theta' \mathbf{x} \beta' \mathbf{t}, \sigma^2), & \theta|\sigma^2 &\sim \mathcal{N}(\mu_{\theta 0}, \sigma^2 \mathbf{D}_0) \\ \beta|\sigma^2 &\sim \mathcal{N}(\mu_{\beta 0}, \sigma^2 \mathbf{E}_0), & \tau = 1/\sigma^2 &\sim \mathcal{G}(a_0, b_0) \end{aligned} \quad (7)$$

where θ, β, τ are parameters. $\mathbf{D}_0, \mathbf{E}_0, \mu_{\theta 0}, \mu_{\beta 0}, a_0, b_0$ are the hyperparameters of the priors to be specified beforehand; \mathbf{D}_0 and \mathbf{E}_0 are positive definite matrices. The graphical representation of the model is shown in Figure 6b. We use conjugate priors for θ, β, τ , which make the variational inference algorithm described later very efficient.

4.2.2 Variational inference . Recall that our objective is to compute the posterior distribution of parameters Ω (now it is $\{\theta, \beta, \tau\}$) given the history data $\mathcal{D} = \{(\mathbf{x}_i, \mathbf{t}_i, r_i)\}_{i=1}^N$, i.e., $p(\theta, \beta, \tau|\mathcal{D})$. Using piecewise linear approximation, we now develop an efficient variational inference algorithm.

Following the convention of the mean-field approximation [Friedman and Koller 2009], we assume that the joint posterior distribution can be approximated by a restricted distribution $q(\theta, \beta, \tau)$, which consists of three independent factors [Friedman and Koller 2009]:

$$p(\Omega|\mathcal{D}) = p(\theta, \beta, \tau|\mathcal{D}) \approx q(\theta, \beta, \tau) = q(\theta)q(\beta)q(\tau).$$

Because of the choice of the conjugate priors, it is easy to show that the restricted distributions $q(\theta)$, $q(\beta)$, and $q(\tau)$ take the same parametric forms as the prior distributions. Specifically,

$$q(\theta) \propto \exp\left(-\frac{1}{2}\theta' \Lambda_{\theta N} \theta + \eta'_{\theta N} \theta\right), \quad q(\beta) \propto \exp\left(-\frac{1}{2}\beta' \Lambda_{\beta N} \beta + \eta'_{\beta N} \beta\right), \quad q(\tau) \propto \tau^{a_N-1} \exp(-b_N \tau).$$

To find the values that minimize the KL-divergence between $q(\theta, \beta, \tau)$ and the true posterior $p(\theta, \beta, \tau|\mathcal{D})$ for the parameters $\Lambda_{\theta N}, \eta_{\theta N}, \Lambda_{\beta N}, \eta_{\beta N}, a_N$, and b_N , we use the coordinate descent method. Specifically, we first initialize the parameters of $q(\theta)$, $q(\beta)$, and $q(\tau)$, and then iteratively update $q(\theta)$, $q(\beta)$, and $q(\tau)$ until the variational lower bound \mathcal{L} (elaborated in the Appendix C) converges. Further explanation about the principle can be found in [Friedman and Koller 2009]. The detailed steps are described in Algorithm (2), where p and K are the dimensionalities of \mathbf{x} and \mathbf{t} , respectively; the moments of θ, β, τ are derived in the Appendix B.

4.2.3 Predict the posterior distribution $p(U|\mathcal{D})$. Because $q(\theta)$ and $q(\beta)$ are normal distributions, $\theta' \mathbf{x}$ and $\beta' \mathbf{t}$ are also normally distributed:

$$p(\theta' \mathbf{x}|\mathbf{x}, \mathbf{t}, \mathcal{D}) \approx \mathcal{N}(\mathbf{x}' \Lambda_{\theta N}^{-1} \eta_{\theta N}, \mathbf{x}' \Lambda_{\theta N}^{-1} \mathbf{x}), \quad p(\beta' \mathbf{t}|\mathbf{x}, \mathbf{t}, \mathcal{D}) \approx \mathcal{N}(\mathbf{t}' \Lambda_{\beta N}^{-1} \eta_{\beta N}, \mathbf{t}' \Lambda_{\beta N}^{-1} \mathbf{t})$$

ALGORITHM 2: Variational inference

input: $\mathcal{D}, \mathbf{D}_0, \mathbf{E}_0, \boldsymbol{\mu}_{\theta 0}, \boldsymbol{\mu}_{\beta 0}, a_0, b_0$
 initialize $\boldsymbol{\Lambda}_{\theta N}, \boldsymbol{\eta}_{\theta N}, \boldsymbol{\Lambda}_{\beta N}, \boldsymbol{\eta}_{\beta N}, a_N, b_N$
repeat
 update $q(\theta)$: $\boldsymbol{\Lambda}_{\theta N} \leftarrow \mathbb{E}[\tau] \left(\mathbf{D}_0^{-1} + \sum_{i=1}^N \mathbf{x}_i \mathbf{t}_i' \mathbb{E}[\boldsymbol{\beta} \boldsymbol{\beta}'] \mathbf{t}_i \mathbf{x}_i' \right), \quad \boldsymbol{\eta}_{\theta N} \leftarrow \mathbb{E}[\tau] \left(\mathbf{D}_0^{-1} \boldsymbol{\mu}_{\theta 0} + \sum_{i=1}^N r_i \mathbf{x}_i \mathbf{t}_i' \mathbb{E}[\boldsymbol{\beta}] \right)$
 update $q(\beta)$: $\boldsymbol{\Lambda}_{\beta N} \leftarrow \mathbb{E}[\tau] \left(\mathbf{E}_0^{-1} + \sum_{i=1}^N \mathbf{t}_i \mathbf{x}_i' \mathbb{E}[\boldsymbol{\theta} \boldsymbol{\theta}'] \mathbf{x}_i \mathbf{t}_i' \right), \quad \boldsymbol{\eta}_{\beta N} \leftarrow \mathbb{E}[\tau] \left(\mathbf{E}_0^{-1} \boldsymbol{\mu}_{\beta 0} + \sum_{i=1}^N r_i \mathbf{t}_i \mathbf{x}_i' \mathbb{E}[\boldsymbol{\theta}] \right)$
 update $q(\tau)$: $a_N \leftarrow \frac{p+K+N}{2} + a_0,$
 $b_N \leftarrow \frac{1}{2} [\text{tr} [\mathbf{D}_0^{-1} (\mathbb{E}[\boldsymbol{\theta} \boldsymbol{\theta}'])] + (\boldsymbol{\mu}'_{\theta 0} - 2\mathbb{E}[\boldsymbol{\theta}']) \mathbf{D}_0^{-1} \boldsymbol{\mu}_{\theta 0}] + \frac{1}{2} [\text{tr} [\mathbf{E}_0^{-1} (\mathbb{E}[\boldsymbol{\beta} \boldsymbol{\beta}'])] + (\boldsymbol{\mu}'_{\beta 0} - 2\mathbb{E}[\boldsymbol{\beta}']) \mathbf{E}_0^{-1} \boldsymbol{\mu}_{\beta 0}]$
 $+ \frac{1}{2} \sum_{i=1}^N \left(r_i^2 + \mathbf{x}_i' \mathbb{E}[\boldsymbol{\theta} \boldsymbol{\theta}'] \mathbf{x}_i \mathbf{t}_i' \mathbb{E}[\boldsymbol{\beta} \boldsymbol{\beta}'] \mathbf{t}_i \right) - \sum_{i=1}^N r_i \mathbf{x}_i' \mathbb{E}[\boldsymbol{\theta}] \mathbf{t}_i' \mathbb{E}[\boldsymbol{\beta}] + b_0$
until \mathcal{L} converges
return $\boldsymbol{\Lambda}_{\theta N}, \boldsymbol{\eta}_{\theta N}, \boldsymbol{\Lambda}_{\beta N}, \boldsymbol{\eta}_{\beta N}, a_N, b_N$

and the posterior distribution of U in Equation (6) can be computed as:

$$p(U|\mathbf{x}, \mathbf{t}, \mathcal{D}) = p(\boldsymbol{\theta}' \mathbf{x} \boldsymbol{\beta}' \mathbf{t} | \mathbf{x}, \mathbf{t}, \mathcal{D}) = \int p(\boldsymbol{\theta}' \mathbf{x} = a | \mathbf{x}, \mathbf{t}, \mathcal{D}) p(\boldsymbol{\beta}' \mathbf{t} = \frac{U}{a} | \mathbf{x}, \mathbf{t}, \mathcal{D}) da.$$

Since there is no closed-form solution to the above integration, we use Monte Carlo simulation. Namely, we first obtain one set of samples for each of $\boldsymbol{\theta}' \mathbf{x}$ and $\boldsymbol{\beta}' \mathbf{t}$ and then use the element-wise products of the two group of samples to approximate the distribution of U . Because $\boldsymbol{\theta}' \mathbf{x}$ and $\boldsymbol{\beta}' \mathbf{t}$ are normally distributed univariate random variables, the sampling can be done very efficiently. Moreover, the prediction for different songs is trivially parallelizable and is thus scalable.

4.2.4 Integration of other factors. Although the approximate model considers music audio content and novelty only, it is easy to incorporate other factors as long as they can be approximated by linear functions. For instance, diversity is another important factor for a playlist. We could measure the diversity that a song contributes to a playlist as d and assume user preference of d follows a function that can be approximated by a piecewise linear function. Following the method in Section 4.2.1, we can map d into a vector \mathbf{d} and modify the approximate Bayesian model in Section (4.2.1) by adding an additional term $\boldsymbol{\gamma}' \mathbf{d}$ to Equation (7) and putting a prior on $\boldsymbol{\gamma}$. As shown in the following,

$$R|\mathbf{x}, \mathbf{t}, \mathbf{d}, \sigma^2, \boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\gamma} \sim \mathcal{N}(\boldsymbol{\theta}' \mathbf{x} \boldsymbol{\beta}' \mathbf{t} \boldsymbol{\gamma}' \mathbf{d}, \sigma^2), \quad \boldsymbol{\gamma} | \sigma^2 \sim \mathcal{N}(\boldsymbol{\mu}_{\gamma 0}, \sigma^2 \mathbf{F}_0).$$

Given the symmetry between \mathbf{x} , \mathbf{t} , and \mathbf{d} , we can modify Algorithm 2 without further derivation.

Similarly, we could incorporate more factors such as coherence of mood and genre into the model. Moreover, the model can also be applied to other regression tasks as long as the regression function can be factorized into the product of a few linear functions.

5. EXPERIMENTS

We compare the results from our evaluations of 6 recommendation algorithms in this section. Extensive experimental evaluations of both efficiency and effectiveness of the algorithms and models have been conducted, and the results show significant promise from both aspects.

5.1 Experiment setup

5.1.1 Compared recommendation algorithms. To study the effectiveness of the exploration/exploitation tradeoff, we introduced two baselines, Random and Greedy. The Random approach represents pure

exploration and recommends songs uniformly at random. The Greedy approach represents pure exploitation and always recommends the song with the highest predicted rating. Therefore, the Greedy approach simulates the strategy used by the traditional recommenders, where the parameters $\{\theta, s\}$ were estimated by minimizing the mean square error using the L-BFGS-B algorithm [Byrd et al. 1995].

To study the effectiveness of the rating model, we introduced a baseline using LinUCB, a bandit algorithm which assumes that the expected rating is a linear function of the feature vector [Li et al. 2012]. In LinUCB, ridge regression served as the regression method, and upper confidence bound is used to balance exploration and exploitation.

Two combinations of the factors U_c and U_n were evaluated: U_c and $U_c U_n$. We denote them as C and CN for short, where C and N indicate content and novelty, respectively. For example, Bayes-UCB-CN contains both content and novelty. Furthermore, Bayes-UCB-CN corresponds to the exact Bayesian model with the MCMC inference algorithm (Section 4.1), and Bayes-UCB-CN-V the approximate model with the variational inference algorithm (Section 4.2).

We evaluated 6 recommendation algorithms, which were combinations of the four approaches and three factors: Random, LinUCB-C, LinUCB-CN, Bayes-UCB-CN, Bayes-UCB-CN-V, and Greedy-CN. Because LinUCB-CN cannot handle nonlinearity and thus cannot directly model $U_c U_n$, we combined the feature vector x in U_c and the time variable t in U_n as one vector and treated the expected rating as a linear function of the combined vector. Greedy-C was not included because it was not related to our objective. As discussed in Section 3.2, the bandit approach can also combine with existing methods to solve the cold-start problem. We plan to study the effectiveness of such combinations in future works.

5.1.2 Songs and Features. Ten thousand songs from different genres were used in the experiments. Videos of the songs were crawled from YouTube and converted by ffmpeg⁴ into mono channel WAV files with a 16kHz sampling rate. For every song, a 30-second audio clip was used [Wang et al. 2012]. Feature vectors were then extracted using a program developed based on the MARSYAS library⁵, in which a window size of 512 was used without overlapping. The features used (and their dimensionalities) are Zero Crossing Rate (1), Spectral Centroid (1), Spectral Rolloff (1), Spectral Flux (1), MFCC (13), Chroma (14), Spectral Crest Factor (24) and Spectral Flatness Measure (24). Detailed descriptions of these features are given in Table II in the Appendix. The features have been commonly used in the music retrieval/recommendation domain [Cano et al. 2005; Yoshii et al. 2006; Wang et al. 2012]. To represent a 30-second clip in one feature vector, we used the mean and standard deviation of all feature vectors from the clip. Next, we added the 1-dimensional feature *tempo* to the summarized feature vectors, and the resulting feature dimensionality is $79 \times 2 + 1 = 159$. Directly using the 159-dimensional features requires a large amount of data to train the models and makes user studies very expensive and time-consuming. To reduce the dimensionality, we conducted Principal Component Analysis (PCA) with 90% of variance reserved. The final feature dimensionality is thus reduced to 91.

The performance of these features in music recommendation was checked based on a dataset that we built. We did not use existing music recommendation datasets because they lack explicit ratings, and dealing with implicit feedbacks is not our focus. Fifty-two undergraduate students with various cultural backgrounds contributed to the dataset, with each student annotating 400 songs with a 5-point Likert scale from “very bad” (1) to “very good” (5). We computed the 10-fold cross-validation RMSE of U_c for each user and averaged the accuracy over all users. The resulting RMSE is 1.10, significantly lower than the RMSE (1.61) of the random baseline with the same distribution as the data. Therefore, these audio features indeed provide useful information for recommendation. The accuracy can be further improved by feature engineering [Oord et al. 2013], which we reserve for future work.

⁴<http://ffmpeg.org>

⁵<http://marsyas.sourceforge.net>

5.1.3 Evaluation protocol. In [Li et al. 2011], an offline approach is proposed for evaluating contextual-bandit approaches with the assumption that the context (including the audio features and the elapsed time of songs) at different iterations is identically independently distributed. Unfortunately, this is not true in our case because when a song is not recommended, its elapsed time t keeps increasing and is thus strongly correlated. Therefore, an online user study is the most reliable means of evaluation.

To reduce the cost of the user study, we first conducted a comprehensive simulation study to verify the approaches. We then proceeded to user study for further verification only if they passed the simulations. The whole process underwent a few iterations, during which the models and algorithms were continually refined. The results hereby presented are from the final iteration, and intermediate results are either referred to as preliminary study whenever necessary or omitted due to page limitation.

5.2 Simulations

5.2.1 Effectiveness study. $U = U_c U_n$ was used as the true model because the preliminary user studies showed that this resulted in better performance, which will be verified in Section 5.3 again. Because our model considers the impact of time, to make the simulations close to real situations, songs were rated about 50 seconds after being recommended. We treated every 20 recommendations as a recommendation session, and the sessions were separated by 4-minute gaps.

Priors for the Bayesian models were set as uninformative ones or chosen based on preliminary simulation and user studies. For the exact Bayesian model, they are: $c_0 = 10$, $d_0 = 3$, $e_0 = 10^{-2}$, $f_0 = 10^{-3}$, $g_0 = 10^{-3}$, where f_0, g_0 are uninformative and c_0, d_0, e_0 are based on preliminary studies. For the approximate Bayesian model, they are: $D_0 = E_0 = 10^{-2}I$, $\mu_{\theta 0} = \mu_{\beta 0} = 0$, $a_0 = 2$, $b_0 = 2 \times 10^{-8}$, where $\mu_{\theta 0}, \mu_{\beta 0}, a_0, b_0$ are uninformative and D_0, E_0 are based on preliminary studies; I is the identity matrix.

U_n was discretized into the following intervals (in minutes) according to the exponentially decaying characteristics of human memory [Ebbinghaus et al. 1913]: $[0, 2^{-3})$, $[2^{-3}, 2^{-2})$, \dots , $[2^{10}, 2^{11})$, $[2^{11}, +\infty)$. We defined the smallest interval as $[0, 2^{-3})$ because people usually do not listen to a song for less than 2^{-3} minutes (7.5 seconds). The largest interval was defined as $[2^{11}, +\infty)$ because our preliminary user study showed that evaluating one algorithm takes no more than 1.4 day, i.e., approximately 2^{11} minutes. Further discretization of $[2^{11}, +\infty)$ should be easy. For songs that had not been listened to by the target user, the elapsed time t was set as one month to ensure that U_n is close to 1.

We compared the performance of the six recommendation algorithms in terms of regret, which is a widely used metric in the RL literature. First we define that for the l -th recommendation, the difference between the maximum expected rating $\mathbb{E}[\hat{R}^l] = \max_{k=1 \dots |S|} U_k$ and the expected rating of the recommended song is $\Delta_l = \mathbb{E}[\hat{R}^l] - \mathbb{E}[R^l]$. Then, the cumulative regret for the n -th recommendation is: $\mathfrak{R}_n = \sum_{l=1 \dots n} \Delta_l = \sum_{l=1 \dots n} \mathbb{E}[\hat{R}^l] - \mathbb{E}[R^l]$, where a smaller \mathfrak{R}_n indicates better performance.

Different values of parameters $\{\theta, s\}$ were tested. Elements of θ were sampled from the standard normal distribution and s was sampled from the uniform distribution with the range (100, 1000), where the range was determined based on the preliminary user study. We conducted 10 runs of the simulation study. Figure 7 shows the means and standard errors of the regret of different algorithms at different number of recommendations n . From the figure, we see that the algorithm Random (pure exploration) performs the worst. The two LinUCB-based algorithms are worse than Greedy-CN because LinUCB-C does not capture the novelty and LinUCB-CN does not capture the nonlinearity within U_c and U_n although both LinUCB-C and LinUCB-CN balance exploration and exploitation.

Bayes-UCB-based algorithms performed better than Greedy-CN because Bayes-UCB balances exploration and exploitation. In addition, the difference between Bayes-UCB and Greedy increases very fast when n is small. This is because small n means a small number of training samples and results in high uncertainty, i.e., the cold-start stage. Greedy algorithms, which are used by most existing recommendation systems, do not handle the uncertainty well, while Bayes-UCB can reduce the uncertainty

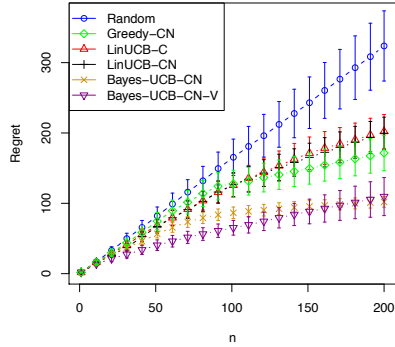


Fig. 7: Regret comparison in simulation

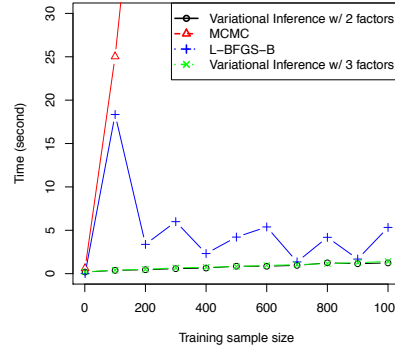


Fig. 8: Time efficiency comparison

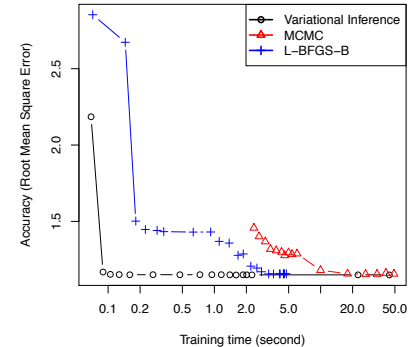


Fig. 9: Accuracy versus training time

quickly and thus improves the recommendation performance. The good performance of Bayes-UCB-CN-V also indicates that the piecewise linear approximation and variational inference is accurate.

5.2.2 Efficiency study. A theoretical efficiency study of MCMC and variational inference algorithms is difficult to analyze due to their iterative nature and deserve future work. Instead, we conducted empirical efficiency study of the training algorithms for Bayes-UCB-CN (MCMC), Bayes-UCB-CN-V (variational inference), Greedy-CN (L-BFGS-B). In addition, the variational inference algorithm for the 3-factor model describe in Section 4.2.4 was also studied. LinUCB and Random were not included because the algorithms are much simpler and thus faster (but also perform much worse). Experiments were conducted on a computer with an Intel Xeon CPU (L5520 @ 2.27GHz) and 32GB main memory. No multi-threading or GP-GPU were used in the comparisons. The programming language R was used to implement all the six algorithms.

From the results in Figure 8, we can see that time consumed by both MCMC and variational inference grows linearly with the training set size. However, variational inference is more than 100 times faster than the MCMC, and significantly faster than the L-BFGS-B algorithm. Comparing the variational inference algorithm with two or three factors, we find that adding another factor to the approximate Bayesian model only slightly slows down the variational inference algorithm. Moreover, when the sample size is less than 1000, the variational inference algorithm can finish in 2 seconds, which makes online updating practical and meets the user requirement well. Implementing the algorithms in more efficient languages like C/C++ can result in even better efficiency.

The training time of all the algorithms is also affected by the accuracy that we want to achieve. To study this, we generated a training set (350 samples) and a test set (150 samples). For each algorithm, we ran it on the training set for multiple times, and every time we used a different number of training iterations and collected both the training time and prediction accuracy on the test set. The whole process was repeated 10 times. For each algorithm and each number of iterations, the 10 training times and prediction accuracies were averaged. From the results shown in Figure 9, we can see that variational inference (VI) converges very fast; L-BFGS-B takes much longer time than VI to converge; MCMC is more than 100 times slower than VI.

Time consumed in the prediction phase of the Bayesian methods is larger than that of Greedy and LinUCB-based methods because of the sampling process. However, for the two factors model Bayes-UCB-CN-V, prediction can be accelerated significantly by the PRODCLIN algorithm without sacrificing the accuracy [MacKinnon et al. 2007]. In addition, since prediction for different songs is trivially parallelizable, scaling variational inference to large music databases should be easy.

We also conducted sample efficiency study of the exact Bayesian model, the approximate Bayesian model, and the minimum mean squared error based frequentist model used for Greedy-CN. We first generated a test set (300 samples), and then tested all the models with different size of training sam-

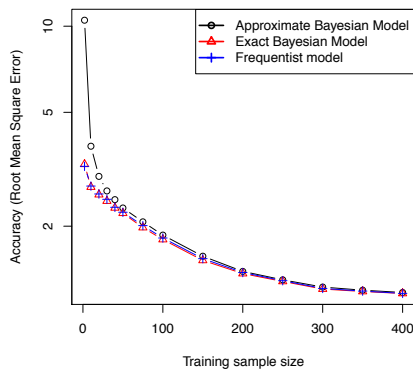


Fig. 10: Sample efficiency comparison

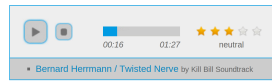


Fig. 11: User evaluation interface

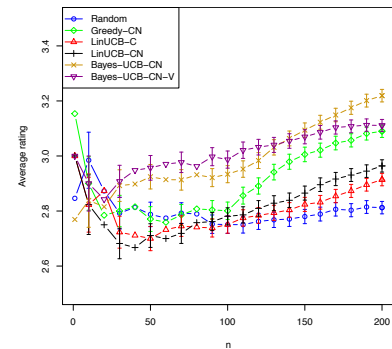


Fig. 12: Performance comparison in user study

ples. The whole process was repeated 10 times, and the average accuracies are shown in Figure 10. We can see that the exact Bayesian model and the frequentist model have almost identical sample efficiency, which confirms that the only difference between Bayes-UCB and Greedy-CN is whether uncertainty is considered or not. The approximate Bayesian model performs slightly worse than the others because of the piecewise linear approximation and the variational inference algorithm.

5.3 User study

Undergraduate students aged 17-25 years were chosen as our study target. It would be interesting to study the impact of occupations and ages on our method in the future. Most applicants were females, and we selected 15 from them with approximately equal number of males (6) and females (9). Their cultural backgrounds were diversified to include Chinese, Malay, Indian and Indonesian. They all listen to music regularly (at least 3 hours per week). To reduce the number of subjects needed, the within-subject experiment design was used, i.e. every subject evaluated all recommendation algorithms. Every subject was rewarded with a small token payment for their time and effort. For each of the 6 algorithms, a subject evaluated 200 recommendations, a number more than sufficient to cover the cold-start stage. Every recommended song was listened to for at least 30 seconds (except when the subject was very familiar with the song *a priori*) and rated based on a 5-point Likert-scale as before. Subjects were required to rest for at least 4 minutes after listening to 20 songs to ensure the quality of the ratings and simulate recommendation sessions. To minimize the carryover effect of the within-subject design, subjects were not allowed to evaluate more than two algorithms within one day, and there must be a gap of more than 6 hours between two algorithms. The user study lasted one week. Every subject spent more than 14 hours in total. The dataset will be released after the publication of this paper. During the evaluation, the recommendation models were updated immediately whenever a new rating was obtained. The main interface used for evaluation is in Figure 11.

5.3.1 The overall recommendation performance. Because the true model is not known in user study, the regret used in simulations cannot be used here. We thus choose average rating as the evaluation metric, which is also popular in evaluations of RL algorithms. Figure 12 shows the average ratings and standard errors of every algorithm from the beginning to the n -th recommendation.

T-tests at different iterations show Bayes-UCB-CN outperforms Greedy-CN since the 45th iteration with p -values < 0.039 . Bayes-UCB-CN-V outperforms Greedy-CN from the 42th to the 141th iteration with p -values < 0.05 , and afterwards with p -values < 0.1 . Bayes-UCB-CN and Greedy-CN share the same rating model and the only difference between them is that Bayes-UCB-CN balances exploration/exploitation while Greedy-CN only exploits. Therefore, the improvement of Bayes-UCB-CN against Greedy-CN is solely contributed by the exploration/exploitation tradeoff.

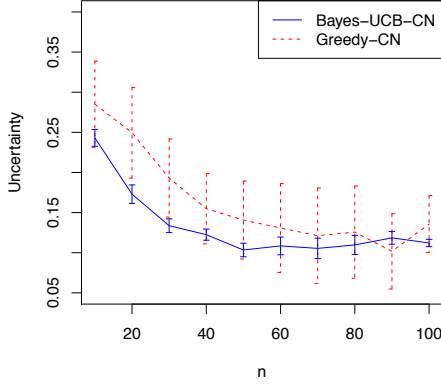


Fig. 13: Uncertainty

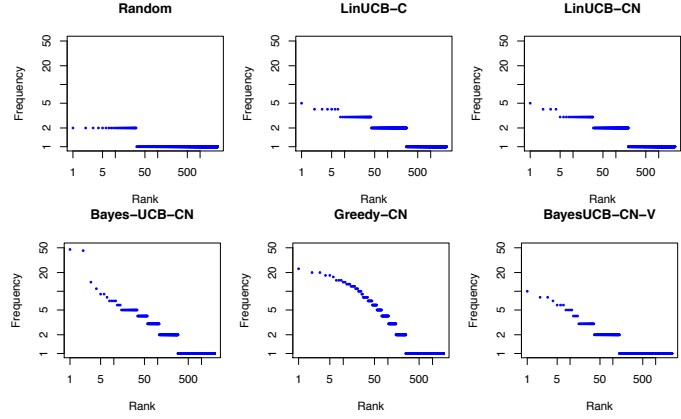


Fig. 14: Distributions of song repetition frequency

More interestingly, when $n \leq 100$ (cold-start stage) the differences between Bayes-UCB-CN and Greedy-CN are even more significant. This is because during the cold-start stage, the uncertainty is very high; Bayes-UCB explores and thus reduces the uncertainty quickly while Greedy-CN always exploits and thus cannot reduce the uncertainty as efficiently as Bayes-UCB-CN. To verify this point, we first define a metric for uncertainty as $\frac{1}{|S|} \sum_{k=1}^{|S|} \text{SD}[p(U_k|\mathcal{D}_n)]$, which is the mean of the standard deviations of all song's posterior distributions $p(U_k|\mathcal{D}_n)$ estimated using the exact Bayesian model. Larger standard deviation means larger uncertainty as illustrated in Figure 1. Given the iteration n , we calculate an uncertainty measure based on each user's recommendation history. The means and standard errors of the uncertainties among all users at different iterations are shown in Figure 13. When the number of training data points increases, the uncertainty decreases. Also as expected, the uncertainty of Bayes-UCB-CN decreases faster than Greedy-CN when n is small, and later the two remain comparable because both have obtained enough training samples to fully train the models. Therefore, this verifies that our bandit approach handles uncertainty better during the initial stage, and thus mitigate the cold-start problem.

Results in Figure 12 also show that all algorithms involving CN outperforms LinUCB-C, indicating that the novelty factor of the rating model improves recommendation performance. In addition, Bayes-UCB-CN outperforms LinUCB-CN significantly, suggesting that multiplying U_c and U_n together works better than linearly combining them.

~~5.3.2 Playlist generation. As discussed in Section 3.1, repeating songs following the Zipf's law is important for playlist generation. Therefore, we evaluated the playlists generated during the recommendation process by examining the distribution of songs repetition frequencies for every user. We generated the plots of the distributions in the same way we generated Figure 3 for the six algorithms. Ideal algorithms should reproduce repetition distributions of Figure 3.~~

~~The results of the six algorithms are shown in Figure 14. As we can see all algorithms with U_c and U_n multiplied together (i.e. Bayes-UCB-CN, Greedy-CN, BayesUCB-CN-V) reproduce the Zipf's law pattern well, while the algorithms without U_c (Random, LinUCB-C) or with U_c and U_n added together (LinUCB-CN) do not. This confirms that our model $U = U_c U_n$ can effectively reproduce the Zipf's law distribution. Thus, we successfully modeled an important part for combining music recommendation and playlist generation.~~

5.3.3 Piecewise linear approximation. In addition to the studies detailed above, the piecewise linear approximation of the novelty model is tested again by randomly selecting four users and showing in Figure 15 their novelty models learnt by Bayes-UCB-CN-V. Specifically, the posterior distributions

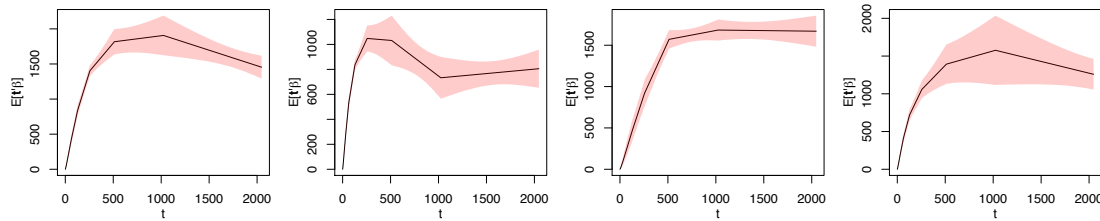


Fig. 15: Four users' diversity factors learnt from the approximate Bayesian model

of $\beta't$ for $t \in (0, 2^{11})$ are presented. The lines represent the mean values of $\beta't$ and the regions around the lines the confidence bands of one standard deviation. The scale of $\beta't$ is not important because $\beta't$ is multiplied together with the content factor, and any constant scaling of one factor can be compensated by the scaling of the other one. Comparing Figure 15 and Figure 4, we can see that the learnt piecewise linear novelty factors match our analytic form U_n well. This again confirms the accuracy of the piecewise linear approximation.

6. DISCUSSION

Exploring user preferences is a central issue for recommendation systems, regardless of the specific media types. Under uncertainty, the greedy approach usually produces suboptimal results, and balancing exploration/exploitation is important. One successful example of exploration/exploitation tradeoff is the news recommender [Li et al. 2012]. Our work in this paper has shown its effectiveness in music recommendation. Given that uncertainty exists universally in all kinds of recommenders, it will be interesting to examine its effectiveness in recommenders for other media types e.g., video and image.

Our models and algorithms could be generalized to other recommenders. First, the mathematical form of the approximate Bayesian model is general enough to cover a family of rating functions that can be factorized as the product of a few linear functions (Section 4.2.4). Moreover, we can often approximate nonlinear functions with linear ones. For instance, we can use a feature mapping function $\phi(\mathbf{x})$ and make $U_c = \theta'\phi(\mathbf{x})$ to capture the non-linearity in our content model. Therefore, it will be interesting to explore our approximate Bayesian model and the variational inference algorithm in other recommendation systems. Second, the proposed novelty model may not be suitable for movie recommendation due to different consumption patterns in music and movie—users may listen to their favorites songs for many times, but repetitions are relatively rare for movies. However, the novelty model may suit recommenders which repeat items (e.g. food or makeup recommenders [Liu et al. 2013]). If their repetition patterns also follow the Zipf's law, both the exact and approximate Bayesian models can be used; otherwise, the approximate Bayesian model can be used at least.

As for extensions of this work, the first interesting direction is to model the correlations between different users to further reduce the amount of exploration. This could be achieved by extending the Bayesian models to hierarchical Bayesian models. Another interesting direction is to consider more factors such as diversity, mood, and genres to generate even better playlists.

For people who prefer collaborative filtering (CF), a future direction could be to use the exploration/exploitation tradeoff idea to boost CF's performance. A simple approach is to use the latent features learnt by existing matrix factorization methods to replace the audio features in our methods, and keep other parts of our methods unchanged.

7. CONCLUSION

In this paper, we described a multi-armed bandit approach to interactive music recommendation that balances exploration and exploitation, mitigates the cold-start problem, and improves recommendation performance. We described a rating model including music audio content and novelty to integrate

music recommendation and playlist generation. To jointly learn the parameters of the rating model, a Bayesian regression model together with a MCMC inference procedure were developed. To make the Bayesian inference efficient enough for online updating and generalize the model for more factors such as diversity, a piecewise linear approximate Bayesian regression model and a variational inference algorithm were built. The results from simulation demonstrate that our models and algorithms are accurate and highly efficient. User study results show that (1) the bandit approach mitigates the cold-start problem and improves recommendation performance, and (2) the novelty model together with the content model capture the Zipf's law of repetitions in recommendations.

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Online Appendix to: Exploration in Interactive Personalized Music Recommendation: A Reinforcement Learning Approach

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A. CONDITIONAL DISTRIBUTIONS FOR THE APPROXIMATE BAYESIAN MODEL

Given N training samples $\mathcal{D} = \{r_i, \mathbf{x}_i, \mathbf{t}_i\}_{i=1}^N$, the conditional distribution $p(\boldsymbol{\theta}|\mathcal{D}, \tau, \boldsymbol{\beta})$ remains a normal distribution:

$$\begin{aligned} p(\boldsymbol{\theta}|\mathcal{D}, \tau, \boldsymbol{\beta}) &\propto p(\tau)p(\boldsymbol{\theta}|\tau)p(\boldsymbol{\beta}|\tau) \prod_{i=1}^N p(r_i|\mathbf{x}_i, \mathbf{t}_i, \boldsymbol{\theta}, \boldsymbol{\beta}, \tau) \\ &\propto p(\boldsymbol{\theta}|\tau) \prod_{i=1}^N p(r_i|\mathbf{x}_i, \mathbf{t}_i, \boldsymbol{\theta}, \boldsymbol{\beta}, \tau) \\ &\propto \exp\left(-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\mu}_{\boldsymbol{\theta}0})'(\sigma^2\mathbf{D}_0)^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_{\boldsymbol{\theta}0})\right) \exp\left(\sum_{i=1}^N -\frac{1}{2}(r_i - \mathbf{x}_i'\boldsymbol{\theta}\mathbf{t}_i'\boldsymbol{\beta})(\sigma^2)^{-1}\right) \\ &\propto \exp\left[-\frac{1}{2}\boldsymbol{\theta}'\left(\tau\mathbf{D}_0^{-1} + \tau\sum_{i=1}^N \mathbf{x}_i\mathbf{t}_i'\boldsymbol{\beta}\boldsymbol{\beta}'\mathbf{t}_i\mathbf{x}_i'\right)\boldsymbol{\theta} + \tau\left(\boldsymbol{\mu}_{\boldsymbol{\theta}0}'\mathbf{D}_0^{-1} + \sum_{i=1}^N r_i\boldsymbol{\beta}'\mathbf{t}_i\mathbf{x}_i'\right)\boldsymbol{\theta}\right] \\ &\propto \exp\left(-\frac{1}{2}\boldsymbol{\theta}'\boldsymbol{\Lambda}_{\boldsymbol{\theta}N}\boldsymbol{\theta} + \boldsymbol{\eta}_{\boldsymbol{\theta}N}'\boldsymbol{\theta}\right) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\Lambda}_{\boldsymbol{\theta}N} &= \tau\left(\mathbf{D}_0^{-1} + \sum_{i=1}^N \mathbf{x}_i\mathbf{t}_i'\boldsymbol{\beta}\boldsymbol{\beta}'\mathbf{t}_i\mathbf{x}_i'\right) \\ \boldsymbol{\eta}_{\boldsymbol{\theta}N}' &= \tau\left(\boldsymbol{\mu}_{\boldsymbol{\theta}0}'\mathbf{D}_0^{-1} + \sum_{i=1}^N r_i\boldsymbol{\beta}'\mathbf{t}_i\mathbf{x}_i'\right) \end{aligned}$$

Due to the symmetry between $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$, we can easily obtain

$$p(\boldsymbol{\beta}|\mathcal{D}, \tau, \boldsymbol{\theta}) \propto \exp\left(-\frac{1}{2}\boldsymbol{\beta}'\boldsymbol{\Lambda}_{\boldsymbol{\beta}N}\boldsymbol{\beta} + \boldsymbol{\eta}_{\boldsymbol{\beta}N}'\boldsymbol{\beta}\right),$$

where

$$\begin{aligned}\mathbf{\Lambda}_{\beta N} &= \tau \left(\mathbf{E}_0^{-1} + \sum_{i=1}^N \mathbf{t}_i \mathbf{x}_i' \boldsymbol{\theta} \boldsymbol{\theta}' \mathbf{x}_i \mathbf{t}_i' \right) \\ \boldsymbol{\eta}_{\beta N}' &= \tau \left(\boldsymbol{\mu}_{\beta 0}' \mathbf{E}_0^{-1} + \sum_{i=1}^N r_i \boldsymbol{\theta}' \mathbf{x}_i \mathbf{t}_i' \right).\end{aligned}$$

The conditional distribution $p(\tau|\mathcal{D}, \boldsymbol{\theta}, \boldsymbol{\beta})$ also remains a Gamma distribution:

$$\begin{aligned}p(\tau|\mathcal{D}, \boldsymbol{\theta}, \boldsymbol{\beta}) &\propto \tau^{a_N-1} \exp(-b_N \tau) \\ p(\tau|\mathcal{D}, \boldsymbol{\theta}, \boldsymbol{\beta}) &\propto p(\tau) p(\boldsymbol{\theta}|\tau) p(\boldsymbol{\beta}|\tau) \prod_{i=1}^N p(r_i|\mathbf{x}_i, \mathbf{t}_i, \boldsymbol{\theta}, \boldsymbol{\beta}, \tau) \\ &= b_0^{a_0} \frac{1}{\Gamma(a_0)} \tau^{a_0-1} \exp(-b_0 \tau) \times \\ &\quad \frac{1}{(2\pi)^{p/2} |\sigma^2 \mathbf{D}_0|^{1/2}} \exp\left(-\frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\mu}_{\theta 0})' (\sigma^2 \mathbf{D}_0)^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu}_{\theta 0})\right) \\ &\quad \times \frac{1}{(2\pi)^{K/2} |\sigma^2 \mathbf{E}_0|^{1/2}} \exp\left(-\frac{1}{2} (\boldsymbol{\beta} - \boldsymbol{\mu}_{\beta 0})' (\sigma^2 \mathbf{E}_0)^{-1} (\boldsymbol{\beta} - \boldsymbol{\mu}_{\beta 0})\right) \\ &\quad \times \left(\frac{1}{(2\pi)^{1/2} |\sigma^2|^{1/2}} \right)^N \exp\left(\sum_{i=1}^N -\frac{1}{2} (r_i - \mathbf{x}_i' \boldsymbol{\theta} \mathbf{t}_i' \boldsymbol{\beta})^2 (\sigma^2)^{-1}\right) \\ &\propto \tau^{a_N-1} \exp(-b_N \tau)\end{aligned}$$

where a_N and b_N are the parameters of the Gamma distribution, and they are

$$\begin{aligned}a_N &= \frac{p + K + N}{2} + a_0 \\ b_N &= b_0 + \frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\mu}_{\theta 0})' \mathbf{D}_0^{-1} (\boldsymbol{\theta} - \boldsymbol{\mu}_{\theta 0}) + \frac{1}{2} (\boldsymbol{\beta} - \boldsymbol{\mu}_{\beta 0})' \mathbf{E}_0^{-1} (\boldsymbol{\beta} - \boldsymbol{\mu}_{\beta 0}) \\ &\quad + \frac{1}{2} \sum_{i=1}^N (r_i - \mathbf{x}_i' \boldsymbol{\theta} \mathbf{t}_i' \boldsymbol{\beta})^2\end{aligned}$$

B. VARIATIONAL INFERENCE

To calculate the joint posterior distribution $p(\boldsymbol{\theta}, \tau, \boldsymbol{\beta}|\mathcal{D})$, we can use Gibbs sampling based on the conditional distributions. However, this is slow too, and therefore, we resort to variational inference (mean field approximation specifically). We assume that $p(\boldsymbol{\theta}, \tau, \boldsymbol{\beta}|\mathcal{D}) \approx q(\boldsymbol{\theta}, \boldsymbol{\beta}, \tau) = q(\boldsymbol{\theta})q(\boldsymbol{\beta})q(\tau)$. In the restricted distribution $q(\boldsymbol{\theta}, \boldsymbol{\beta}, \tau)$, every variable is assumed independent from the other variables. Because all the conditional distributions $p(\boldsymbol{\theta}|\mathcal{D}, \tau, \boldsymbol{\beta})$, $p(\tau|\mathcal{D}, \boldsymbol{\theta}, \boldsymbol{\beta})$, and $p(\boldsymbol{\beta}|\mathcal{D}, \boldsymbol{\theta}, \tau)$ are in the exponential families, their restricted distributions $q(\boldsymbol{\theta})$, $q(\boldsymbol{\beta})$, $q(\tau)$ lie in the same exponential families as their conditional distributions. We then obtain the restricted distributions and update rules as in Section 4.2.2.

The expectation of b_N with respect to $q(\theta)$ and $q(\beta)$ might be a bit tricky to derive. We thus show it as the following:

$$\begin{aligned} b_N &= b_0 + \frac{1}{2} \mathbb{E} [(\theta - \mu_{\theta_0})' \mathbf{D}_0^{-1} (\theta - \mu_{\theta_0})] + \frac{1}{2} \mathbb{E} [(\beta - \mu_{\beta_0})' \mathbf{E}_0^{-1} (\beta - \mu_{\beta_0})] + \frac{1}{2} \mathbb{E} \left[\sum_{i=1}^N (r_i - \mathbf{x}_i' \theta \mathbf{t}_i' \beta)^2 \right] \\ &= b_0 + \frac{1}{2} [\text{tr} [\mathbf{D}_0^{-1} (\mathbb{E}[\theta\theta'])]] + (\mu'_{\theta_0} - 2\mathbb{E}[\theta']) \mathbf{D}_0^{-1} \mu_{\theta_0} \\ &\quad + \frac{1}{2} [\text{tr} [\mathbf{E}_0^{-1} (\mathbb{E}[\beta\beta'])]] + (\mu'_{\beta_0} - 2\mathbb{E}[\beta']) \mathbf{E}_0^{-1} \mu_{\beta_0} + \frac{1}{2} \sum_{i=1}^N \mathbb{E} [(r_i - \mathbf{x}_i' \theta \mathbf{t}_i' \beta)^2]. \end{aligned}$$

Since θ and β are assumed independent, we have

$$\begin{aligned} \mathbb{E} [(r_i - \mathbf{x}_i' \theta \mathbf{t}_i' \beta)^2] &= \mathbb{E} [r_i^2 - 2r_i \mathbf{x}_i' \theta \mathbf{t}_i' \beta + \mathbf{x}_i' \theta \mathbf{t}_i' \beta \mathbf{x}_i' \theta \mathbf{t}_i' \beta] \\ &= r_i^2 - 2r_i \mathbf{x}_i' \mathbb{E}[\theta] \mathbf{t}_i' \mathbb{E}[\beta] + \mathbf{x}_i' \mathbb{E}[\theta\theta'] \mathbf{x}_i \mathbf{t}_i' \mathbb{E}[\beta\beta'] \mathbf{t}_i. \end{aligned}$$

Therefore b_N can be calculated as

$$\begin{aligned} b_N &= b_0 + \frac{1}{2} [\text{tr} [\mathbf{D}_0^{-1} (\mathbb{E}[\theta\theta'])]] + (\mu'_{\theta_0} - 2\mathbb{E}[\theta']) \mathbf{D}_0^{-1} \mu_{\theta_0} \\ &\quad + \frac{1}{2} [\text{tr} [\mathbf{E}_0^{-1} (\mathbb{E}[\beta\beta'])]] + (\mu'_{\beta_0} - 2\mathbb{E}[\beta']) \mathbf{E}_0^{-1} \mu_{\beta_0} \\ &\quad + \frac{1}{2} \left[\sum_{i=1}^N r_i^2 - 2r_i \mathbf{x}_i' \mathbb{E}[\theta] \mathbf{t}_i' \mathbb{E}[\beta] + \mathbf{x}_i' \mathbb{E}[\theta\theta'] \mathbf{x}_i \mathbf{t}_i' \mathbb{E}[\beta\beta'] \mathbf{t}_i \right]. \end{aligned}$$

The moments of θ , β , and τ :

$$\mathbb{E} [\beta\beta'] = \mathbf{\Lambda}_{\beta N}^{-1} + \mathbb{E}[\beta]\mathbb{E}[\beta'], \quad \mathbb{E}[\beta] = \mathbf{\Lambda}_{\beta N}^{-1} \boldsymbol{\eta}_{\beta N}, \quad \mathbb{E} [\theta\theta'] = \mathbf{\Lambda}_{\theta N}^{-1} + \mathbb{E}[\theta]\mathbb{E}[\theta'], \quad \mathbb{E}[\theta] = \mathbf{\Lambda}_{\theta N}^{-1} \boldsymbol{\eta}_{\theta N}, \quad \mathbb{E}[\tau] = \frac{a_N}{b_N}$$

C. VARIATIONAL LOWER BOUND

The following is the variational lower bound, where $\psi(\cdot)$ is the digamma function.

$$\begin{aligned} \mathcal{L} &= \mathbb{E}[\ln(\mathcal{D}, \tau, \theta, \beta)] - \mathbb{E}[\ln q(\theta, \tau, \beta)] \\ &= \mathbb{E}[\ln p(\tau)] + \mathbb{E}[\ln p(\theta|\tau)] + \mathbb{E}[\ln p(\beta|\tau)] + \sum_{i=1}^N \mathbb{E}[\ln p(r_i | \mathbf{x}_i, \mathbf{t}_i, \theta, \beta, \tau)] - \mathbb{E}[\ln q(\theta)] - \mathbb{E}[\ln q(\beta)] - \mathbb{E}[\ln q(\tau)] \\ &= a_0 \ln b_0 + (a_0 - 1) [\psi(a_N) - \ln b_N] - b_0 \frac{a_N}{b_N} - \frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_0| + \frac{p}{2} (\psi(a_N) - \ln(b_N)) \\ &\quad - \frac{a_N}{2b_N} [\text{tr}(\mathbf{D}_0 \mathbf{\Lambda}_{\beta N}^{-1}) + (\mu_{\theta_0} - \mathbb{E}[\theta])' \mathbf{D}_0^{-1} (\mu_{\theta_0} - \mathbb{E}[\theta])] - \frac{K}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{E}_0| + \frac{K}{2} (\psi(a_N) - \ln(b_N)) \\ &\quad - \frac{a_N}{2b_N} [\text{tr}(\mathbf{E}_0 \mathbf{\Lambda}_{\beta N}^{-1}) + (\mu_{\beta_0} - \mathbb{E}[\beta])' \mathbf{E}_0^{-1} (\mu_{\beta_0} - \mathbb{E}[\beta])] - \frac{1}{2} \ln(2\pi) + \frac{1}{2} (\psi(a_N) - \ln b_N) \\ &\quad - \frac{a_N}{2b_N} \sum_{i=1}^N (r_i^2 + \mathbf{x}_i' \mathbb{E}[\theta\theta'] \mathbf{x}_i \mathbf{t}_i' \mathbb{E}[\beta\beta'] \mathbf{t}_i) + \frac{a_N}{b_N} \sum_{i=1}^N r_i \mathbf{x}_i' \mathbb{E}[\theta] \mathbf{t}_i' \mathbb{E}[\beta] + \frac{K}{2} [1 + \ln(2\pi)] + \frac{1}{2} \ln |\mathbf{\Lambda}_{\beta N}^{-1}| \\ &\quad + \frac{p}{2} [1 + \ln(2\pi)] + \frac{1}{2} \ln |\mathbf{\Lambda}_{\theta N}^{-1}| - (a_N - 1) \psi(a_N) - \ln b_N + a_N \end{aligned}$$

It might be a bit tricky to derive

$$\mathbb{E}[\ln p(\boldsymbol{\theta}|\tau)] = \iint p(\boldsymbol{\theta}|\tau)q(\boldsymbol{\theta})d\boldsymbol{\theta}q(\tau)d\tau$$

which is part of the lower bound \mathcal{L} . We assume that $P = p(\boldsymbol{\theta}|\tau)$, and $Q = q(\boldsymbol{\theta})$, and we have $\int p(\boldsymbol{\theta}|\tau)q(\boldsymbol{\theta})d\boldsymbol{\theta} = -H(Q, P)$, where $H(Q, P)$ is the cross entropy between Q and P . Given Q and P are multivariate normal distributions, the KL-divergence between Q and P and the entropy of Q are

$$\begin{aligned} D_{KL}(Q\|P) &= \frac{1}{2} \left[\text{tr}(\Sigma_P^{-1}\Sigma_Q) + (\boldsymbol{\mu}_P - \boldsymbol{\mu}_Q)' \Sigma_P^{-1} (\boldsymbol{\mu}_P - \boldsymbol{\mu}_Q) - \ln \frac{|\Sigma_Q|}{|\Sigma_P|} - p \right] \\ &= \frac{1}{2} \left[\text{tr}(\tau \mathbf{D}_0 \boldsymbol{\Lambda}_{\boldsymbol{\theta}_N}^{-1}) + (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})' \tau \mathbf{D}_0^{-1} (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N}) - \ln |\boldsymbol{\Lambda}_{\boldsymbol{\theta}_N}^{-1}| + \ln \left| \frac{1}{\tau} \mathbf{D}_0 \right| - p \right] \\ H(Q) &= \frac{1}{2} (p + p \ln(2\pi) + \ln |\boldsymbol{\Lambda}_{\boldsymbol{\theta}_N}^{-1}|). \end{aligned}$$

Therefore

$$\begin{aligned} \int p(\boldsymbol{\theta}|\tau)q(\boldsymbol{\theta})d\boldsymbol{\theta} &= -H(Q, P) \\ &= -H(Q) - D_{KL}(Q\|P) \\ &= -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_0| + \frac{p}{2} \ln \tau - \frac{1}{2} [\text{tr}(\tau \mathbf{D}_0 \boldsymbol{\Lambda}_{\boldsymbol{\theta}_N}^{-1}) + (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})' \tau \mathbf{D}_0^{-1} (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})], \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}[\ln p(\boldsymbol{\theta}|\tau)] &= \iint p(\boldsymbol{\theta}|\tau)q(\boldsymbol{\theta})d\boldsymbol{\theta}q(\tau)d\tau \\ &= -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_0| + \frac{p}{2} \mathbb{E}[\ln \tau] - \frac{1}{2} [\text{tr}(\mathbf{D}_0 \boldsymbol{\Lambda}_{\boldsymbol{\theta}_N}^{-1}) + (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})' \mathbf{D}_0^{-1} (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})] \mathbb{E}[\tau] \\ &= -\frac{p}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_0| + \frac{p}{2} (\psi(a_N) - \ln(b_N)) \\ &\quad - \frac{a_N}{2b_N} [\text{tr}(\mathbf{D}_0 \boldsymbol{\Lambda}_{\boldsymbol{\theta}_N}^{-1}) + (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})' \mathbf{D}_0^{-1} (\boldsymbol{\mu}_{\boldsymbol{\theta}_0} - \boldsymbol{\mu}_{\boldsymbol{\theta}_N})] \end{aligned}$$

Table I. : Table of symbols

Symbol	Explanation
\mathcal{D}	Training data set. The first l training samples: $\mathcal{D}_l = \{(\mathbf{x}_i, t_i, r_i)\}_{i=1}^l$
\mathcal{S}	Song set
U_c, U_n	User preference of music content and novelty, respectively
U_k	Mean rating for song k
Ω	Parameter set
θ	User preference of different music features
$\beta't$	Piecewise linear approximation of novelty $U_n(t)$
\mathbf{x}	Music audio feature vector
s	Novelty recovery speed
t	Time elapsed since the last listening of the song
$\sigma^2, 1/\tau$	Variance of the residuals in user ratings
$\mu_{\theta 0}, \sigma^2 \mathbf{D}_0$	Mean and covariance matrix for the prior of θ in the approximate model, respectively
$\mu_{\beta 0}, \sigma^2 \mathbf{E}_0$	Mean and covariance matrix for the prior of β in the approximate model, respectively
$\Lambda_{\theta N}, \Lambda_{\beta N}$	Precision matrices for the posterior distributions of θ and β in the approximate model, respectively
c_0, d_0, e_0, f_0, g_0	Parameters of the prior distributions for the exact Bayesian model

Table II. : Music Content Features

Feature name	Description
Zero Crossings	The rate of sign-changes along a signal.
Spectral Centroid	The “center of mass” of the spectrum. Measures the brightness of the sound.
Spectral Flux	The squared change in normalized amplitude between two consecutive time frames. It measures how much the sound changes between frames.
Spectral Rolloff	Measures the amount of the right-skewedness of the power spectrum.
MFCC	Mel-Frequency Cepstral Coefficients. It models the auditory perception system and is widely used in speech and music domain.
Spectral Flatness Measure	Measures how much the audio signal sounds like a tone instead of noise.
Spectral Crest Factor	Another measure of noisiness. Similar to Spectral Flatness Measure.
Chroma	Pitch based feature. It projects the spectrum into 12 bins, representing the 12 distinct pitches of the chromatic musical scale.