



# Multi-Round Influence Maximization

Lichao Sun, Weiran Huang, Philip S. Yu, Wei Chen

University of Illinois at Chicago, Microsoft Research, Tsinghua University

---



SIGKDD 18

# Roadmap



1

**Problem**

2

Methodology

3

Experiments

4

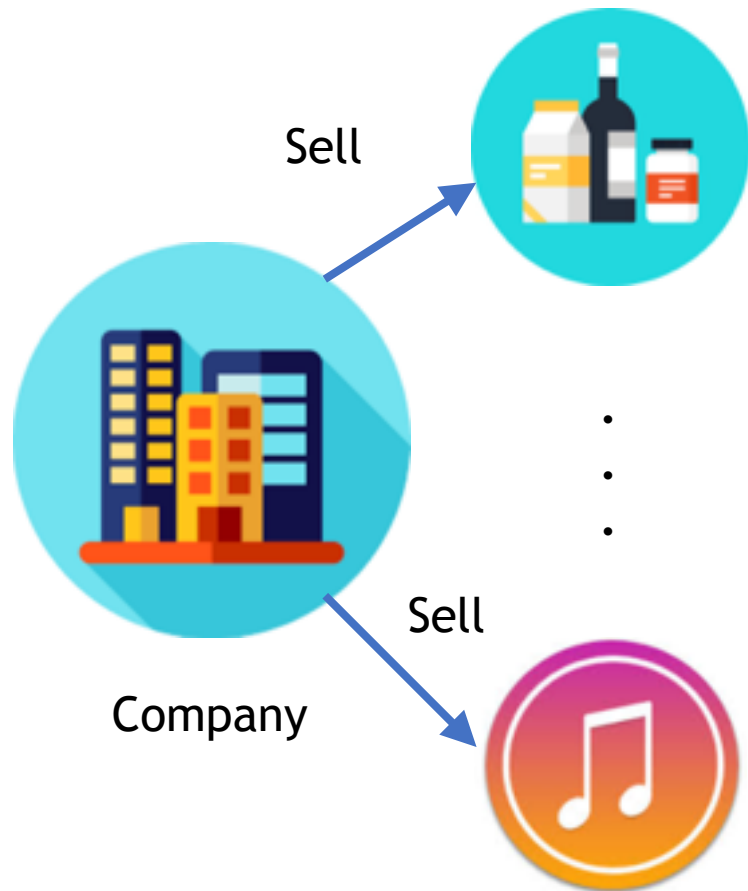
Conclusions

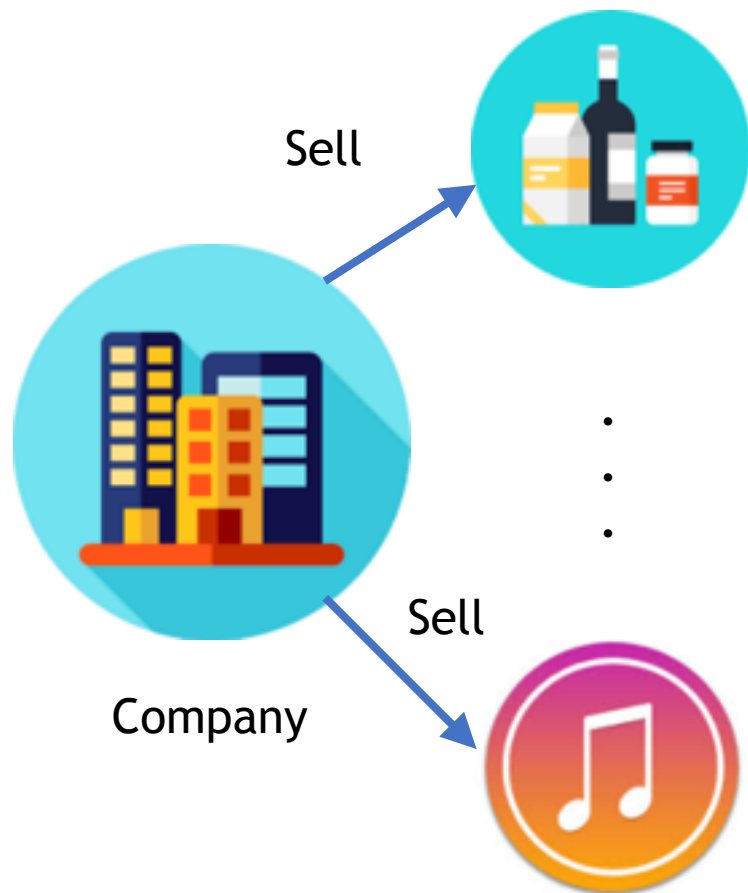
# Problem Statement



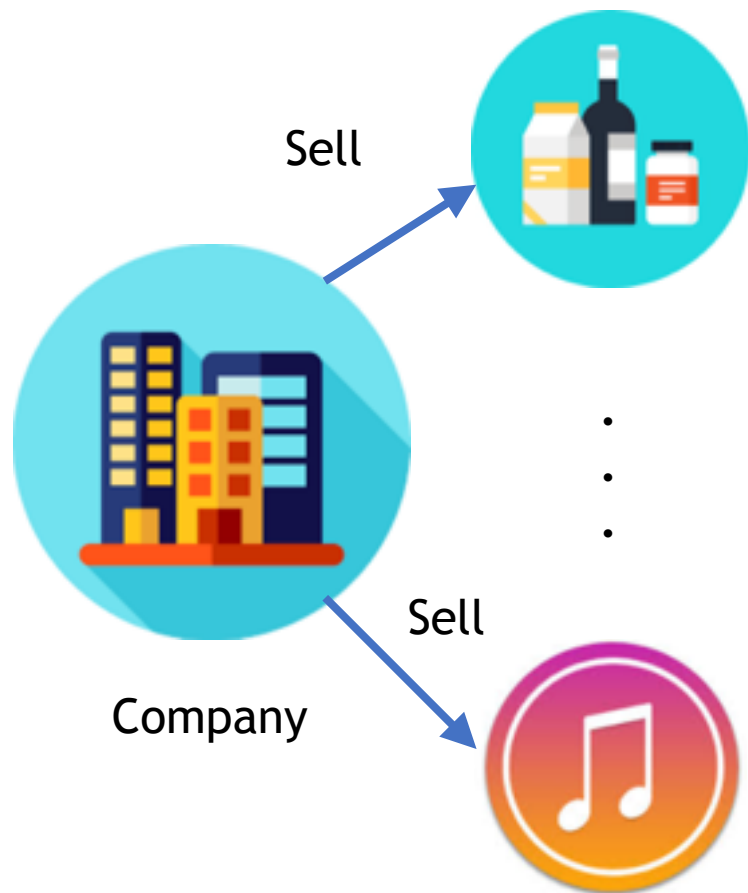


Company





Find Influential member



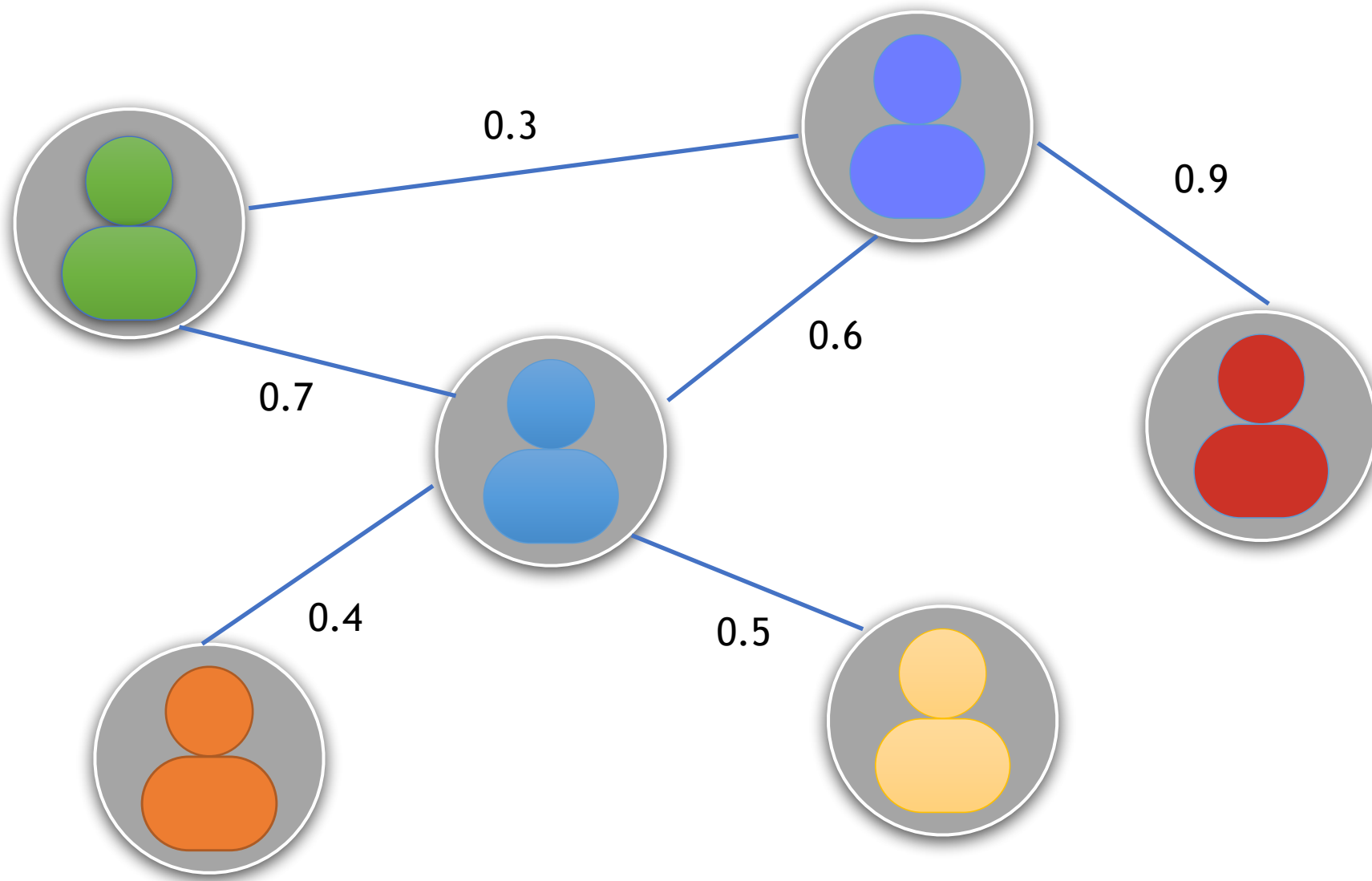
Find a set of Influential members

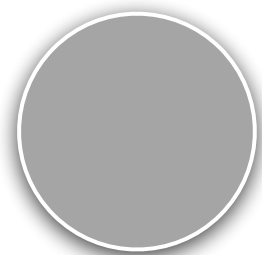
# Single-Round Influence Maximization



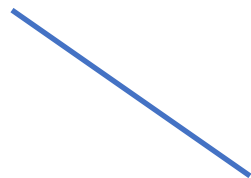


- Influence maximization is the problem of finding a small set of seed nodes in a social network that maximizes the spread of influence under certain influence cascade models. (Kempe, and Kleinberg, 2003)
  - Influence cascade models: (1) Independent cascade model (2) Linear-threshold model (3) Triggering Model
  - Activation probability is known
- Find the optimal solution is NP-hard question (Set cover problem). (Kempe, Kleinberg, and Tardos 2003)
  - Greedy
  - Monto-Carlo simulation





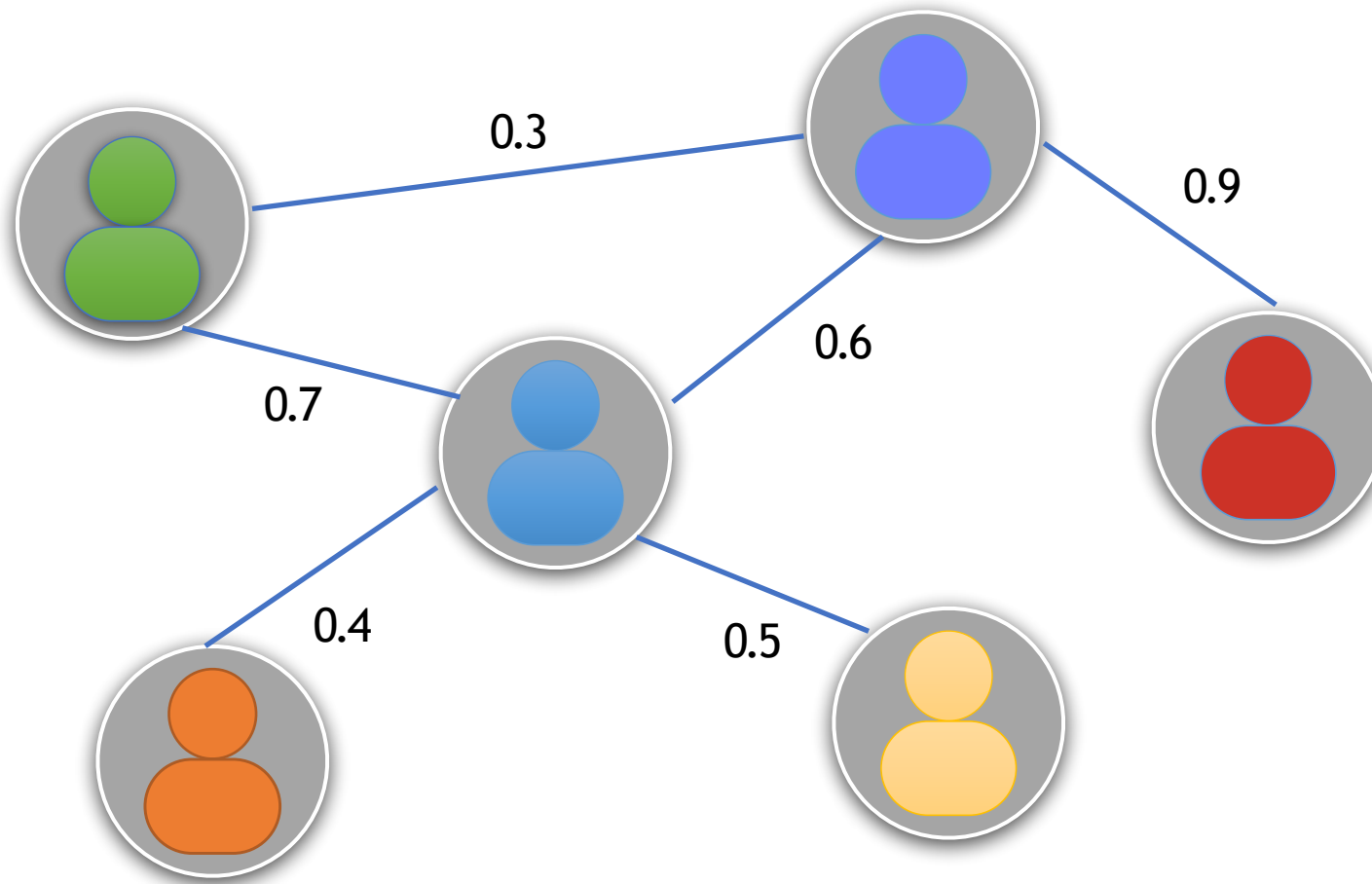
Node

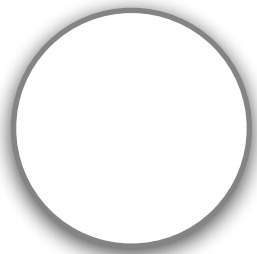


Edge

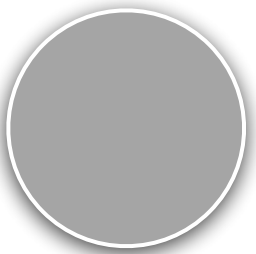
0.9

Influential Probability

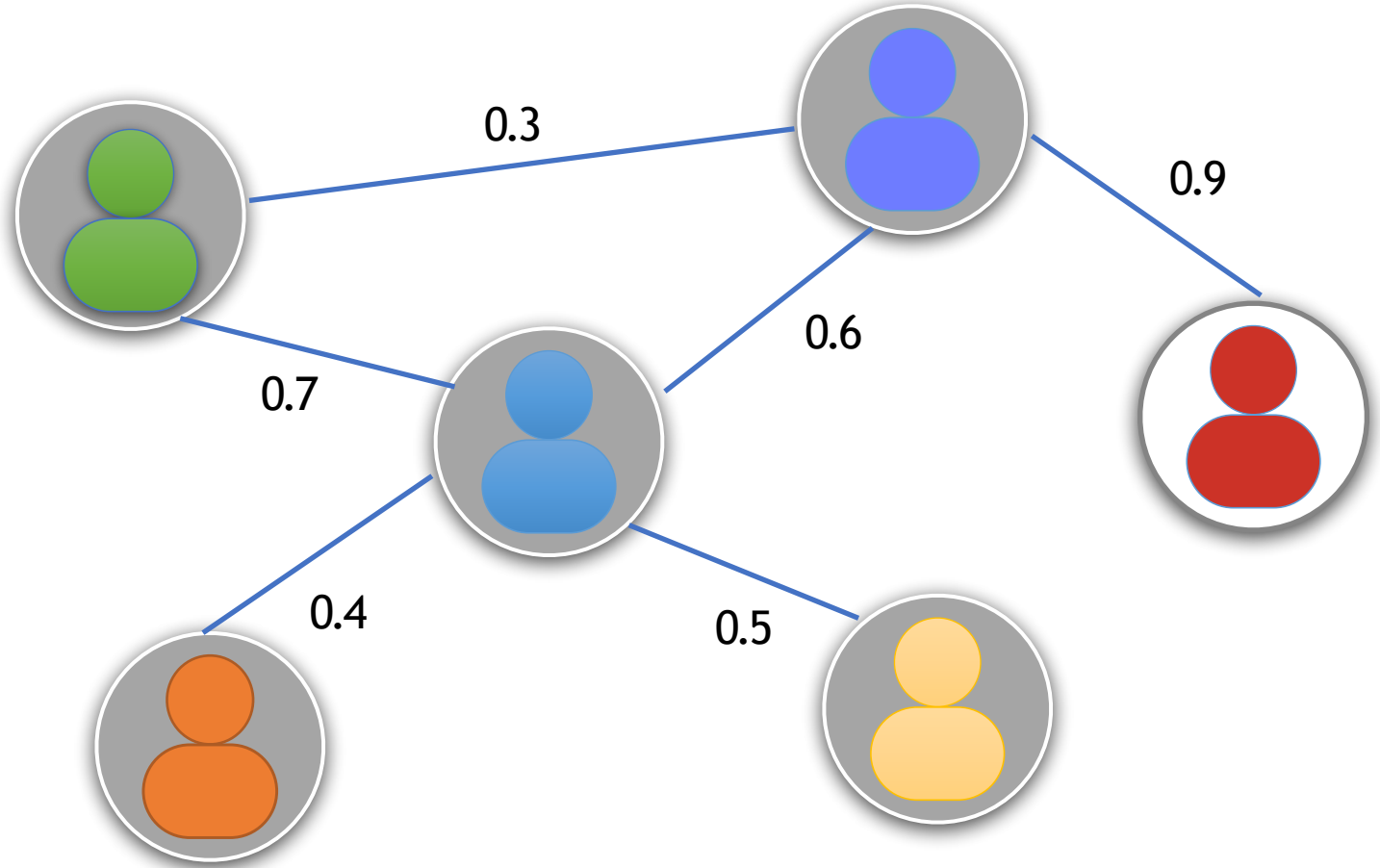
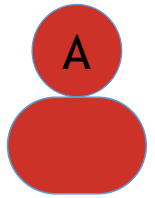


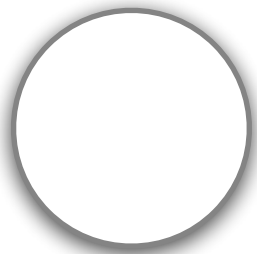


Active

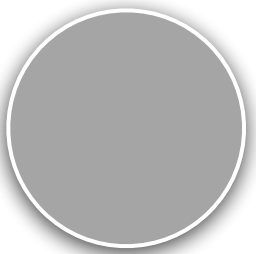


Inactive

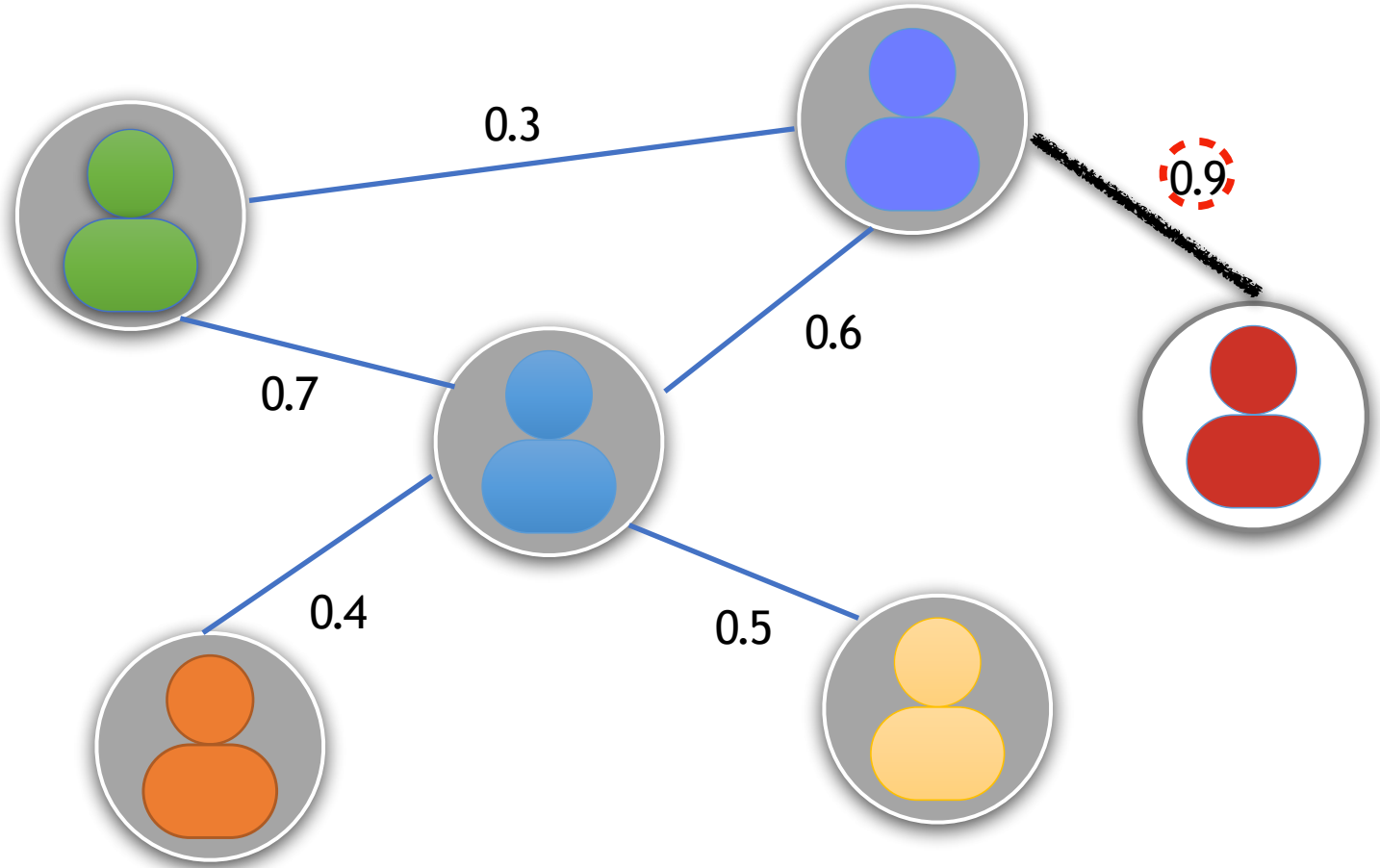
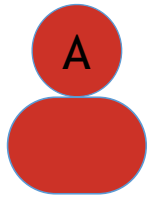


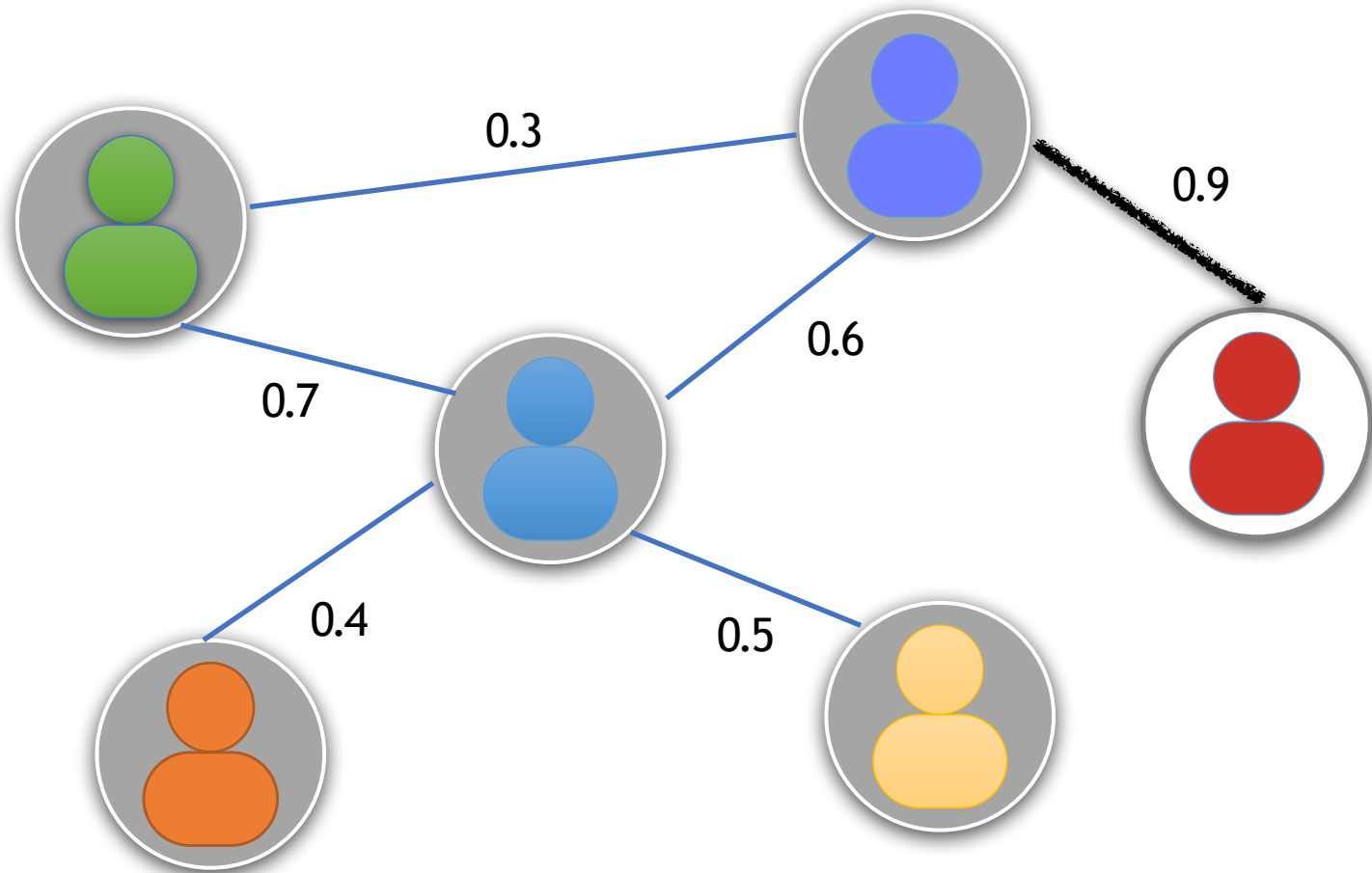
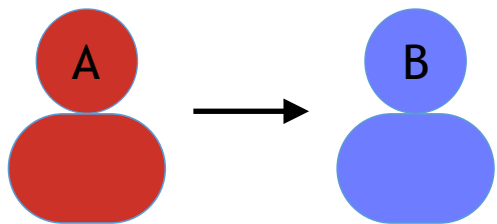


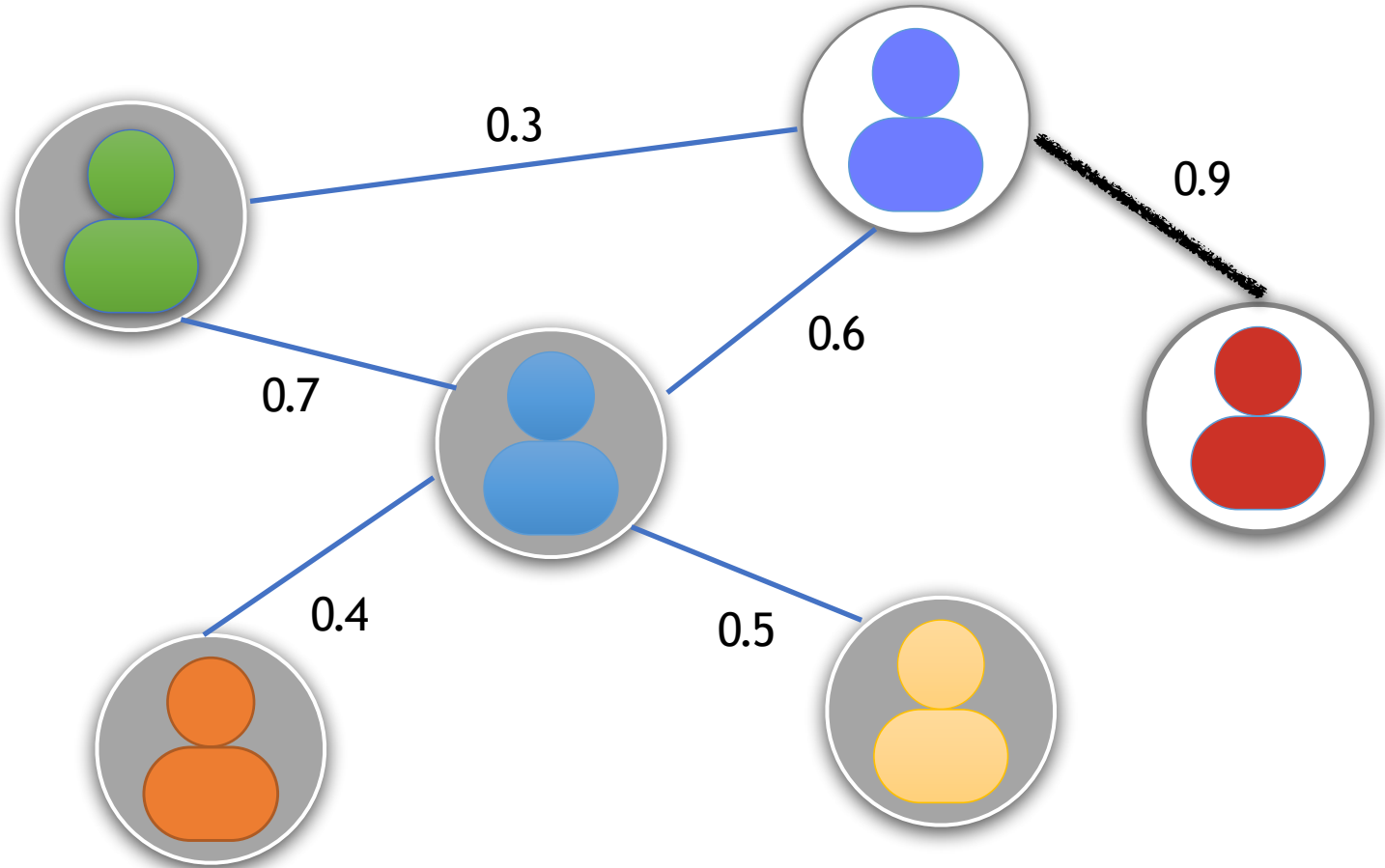
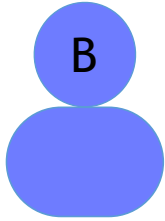
Active

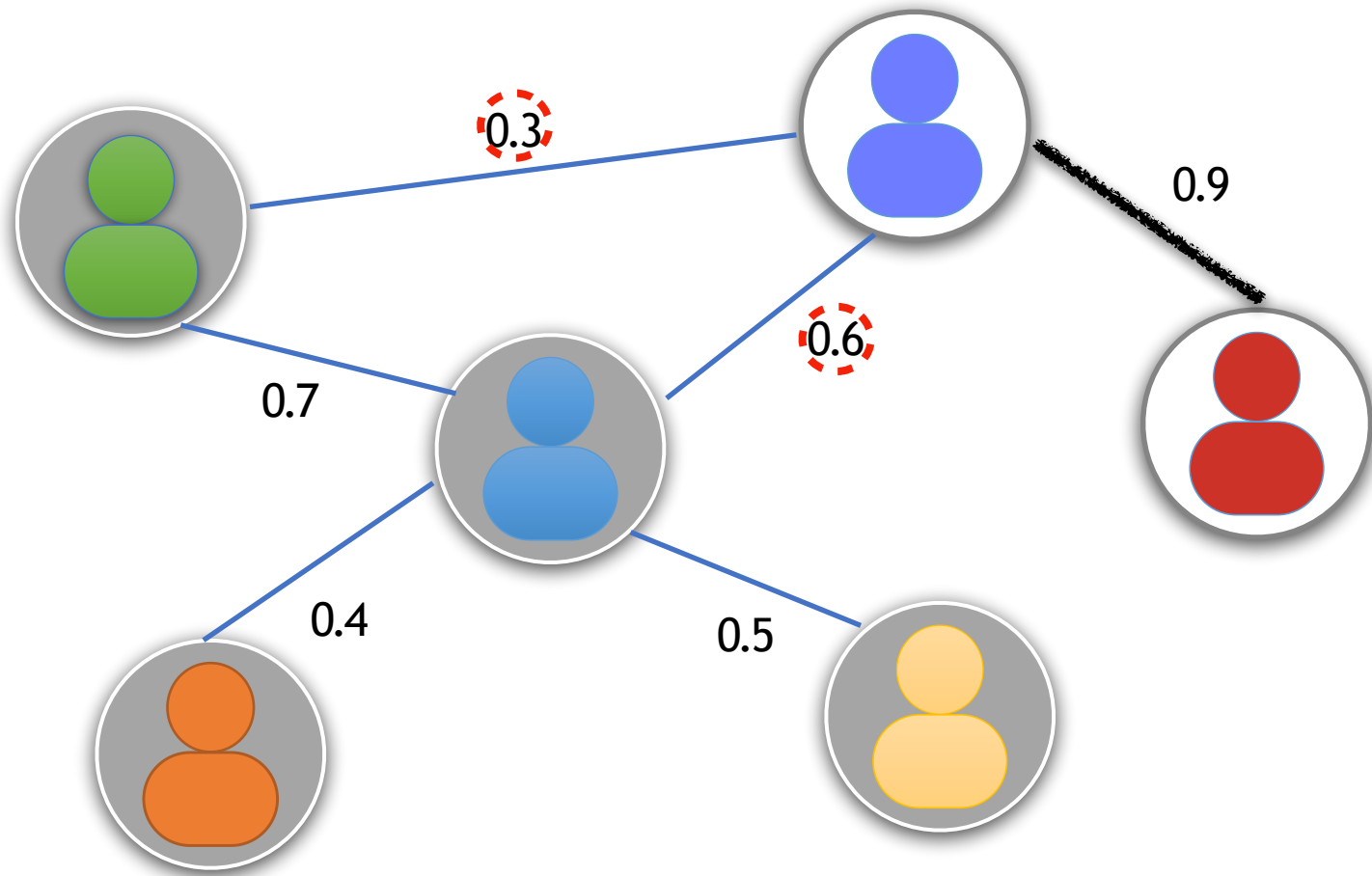
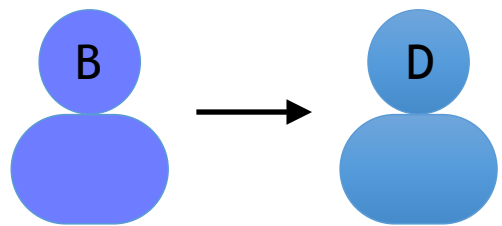
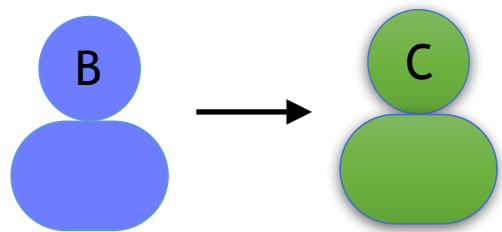


Inactive

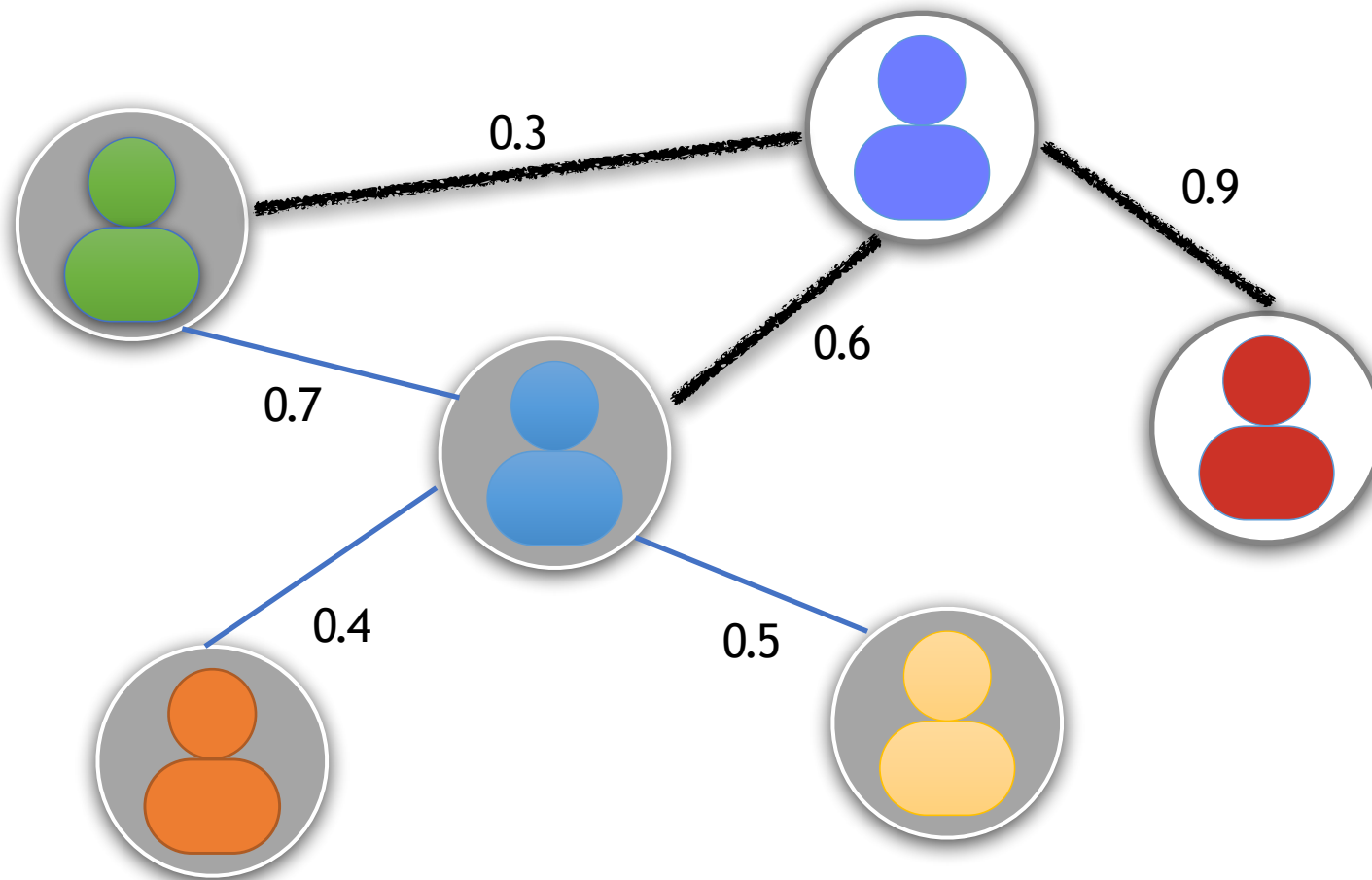
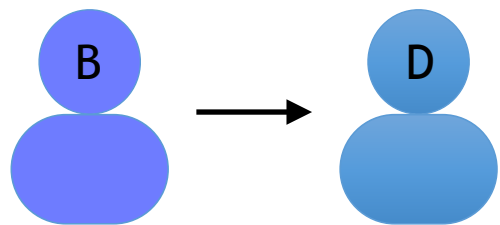
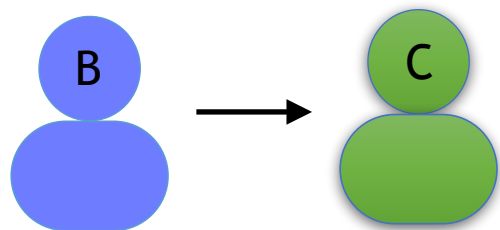


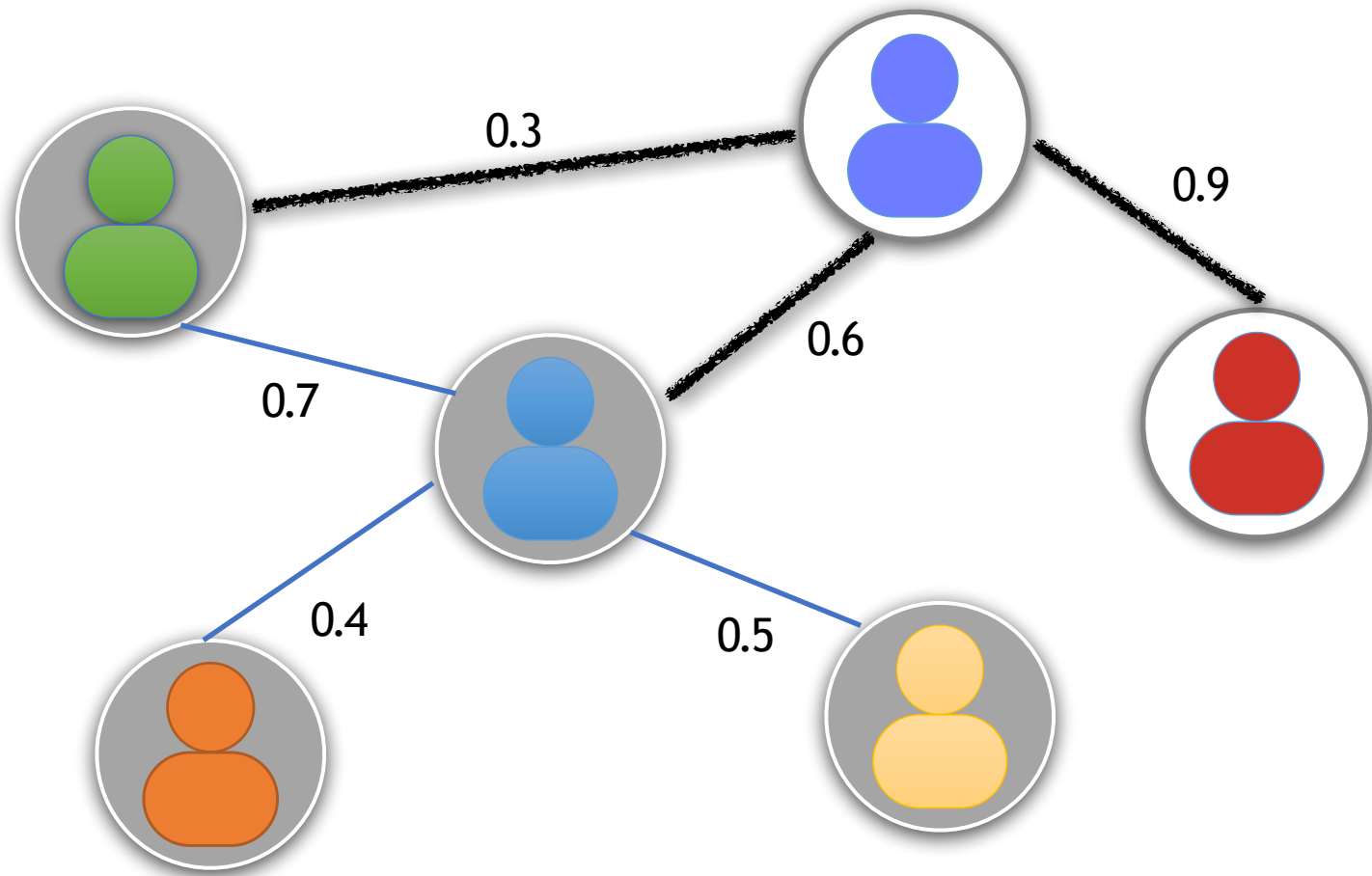
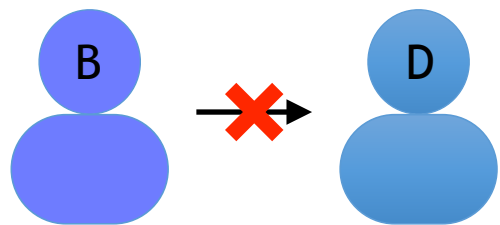
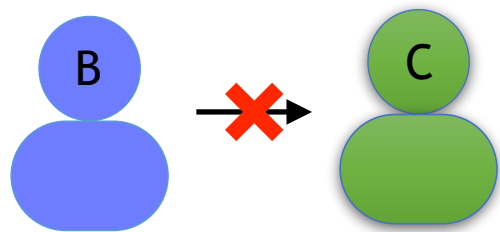


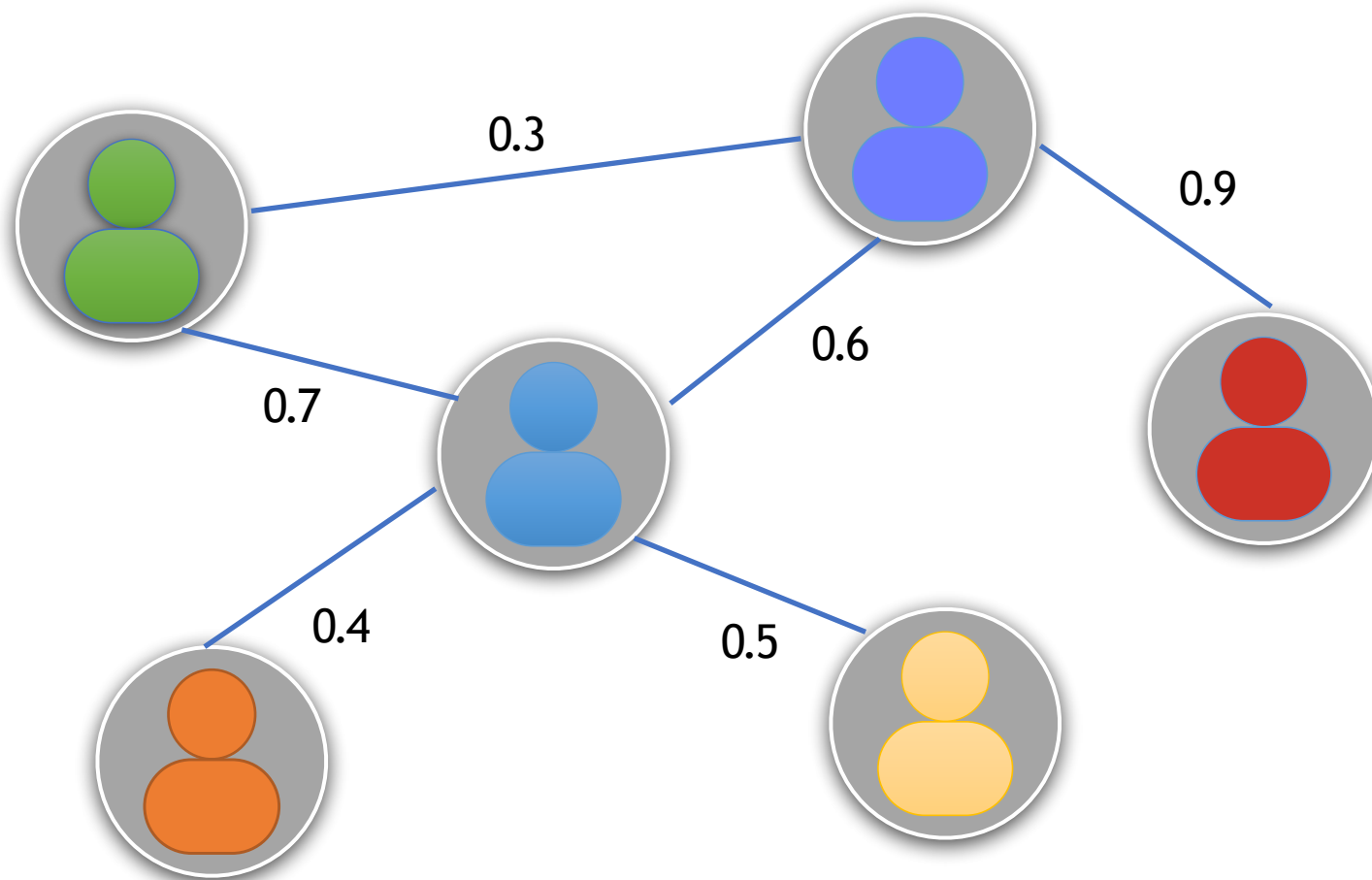
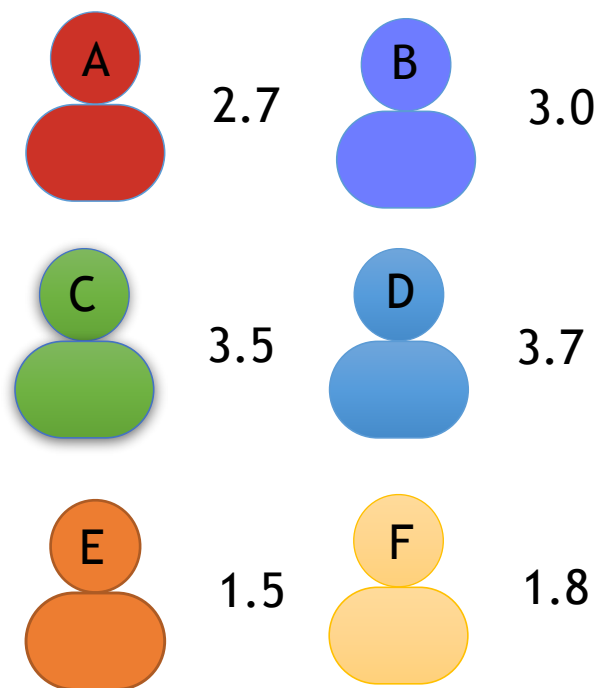


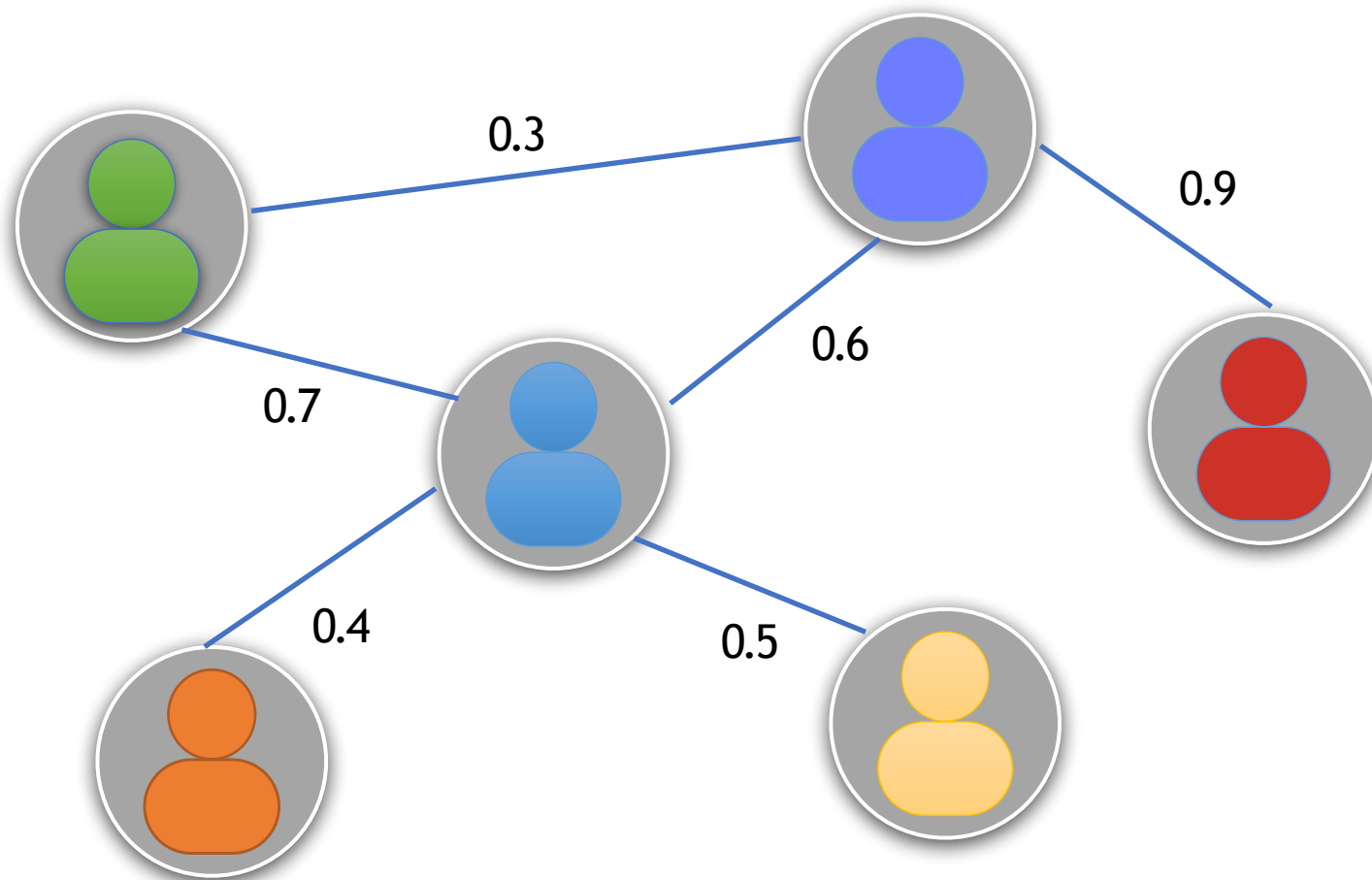
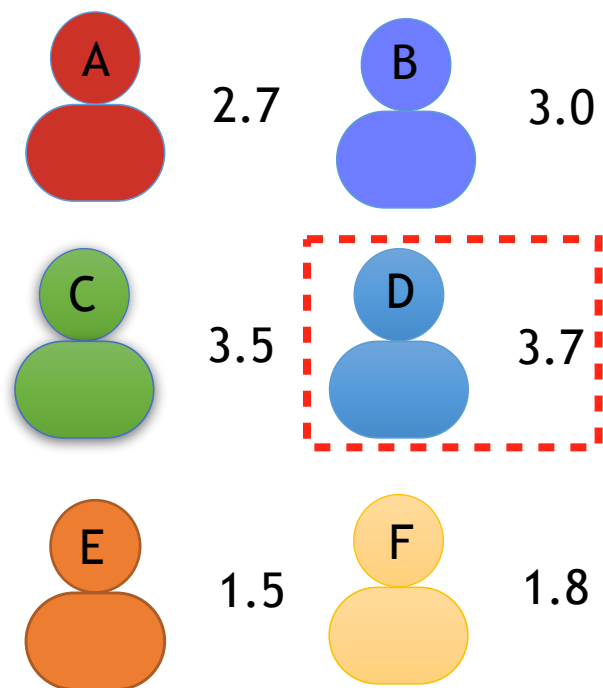


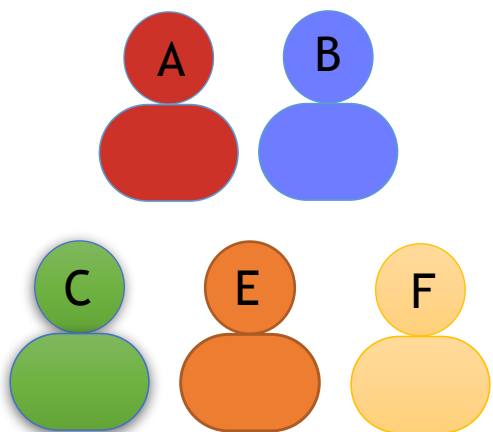




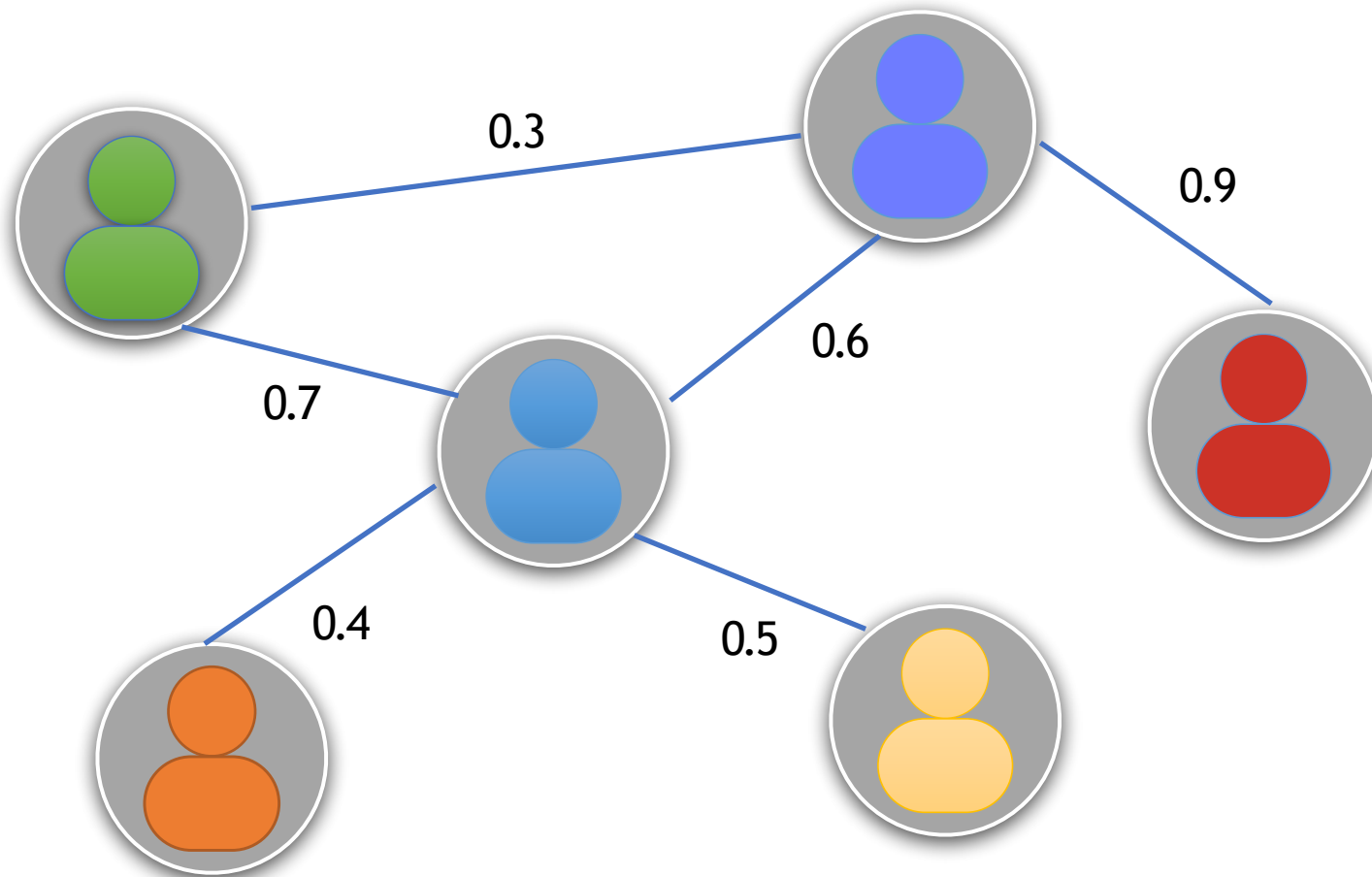








Find the best one which can  
produce largest network  
impact with seed set

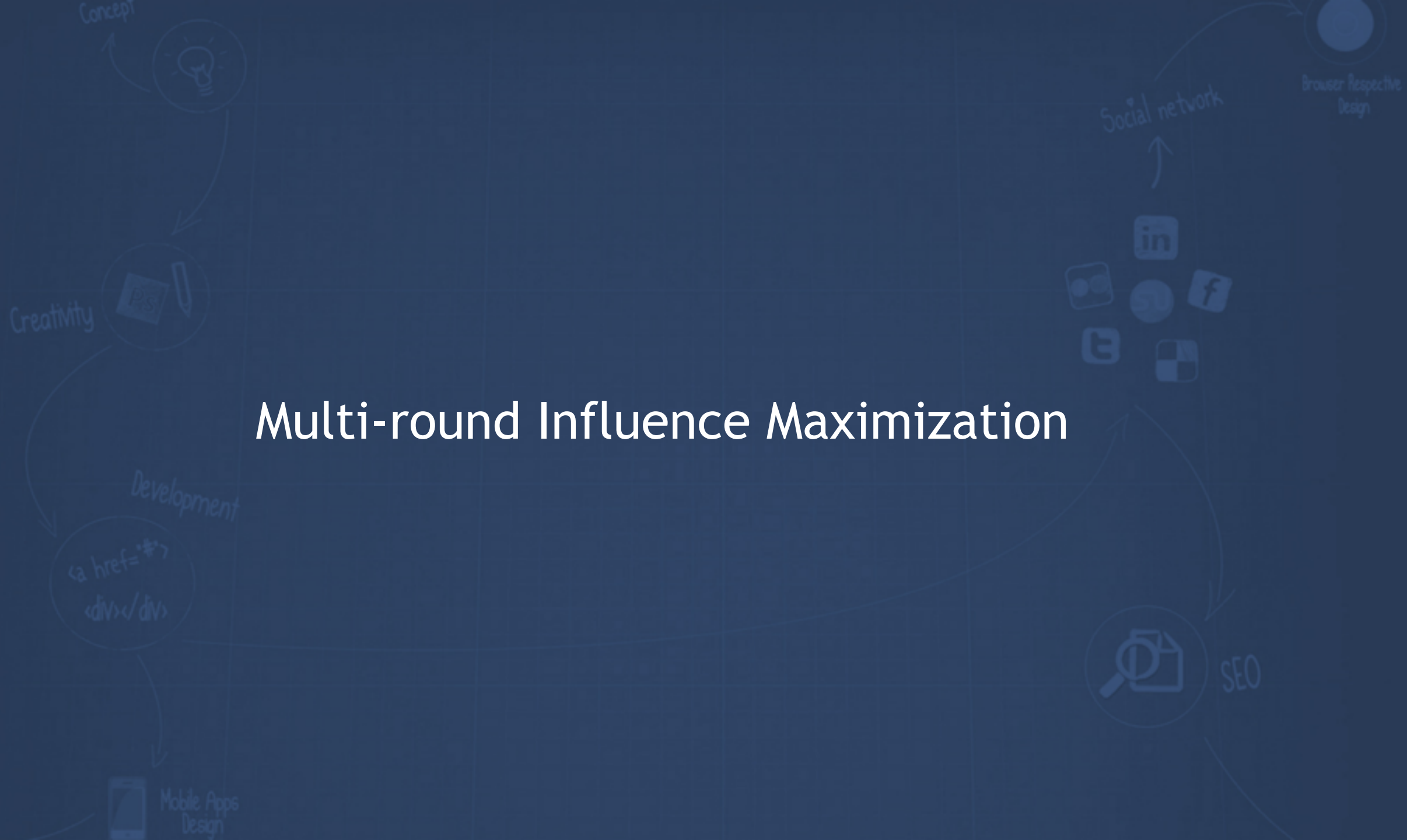


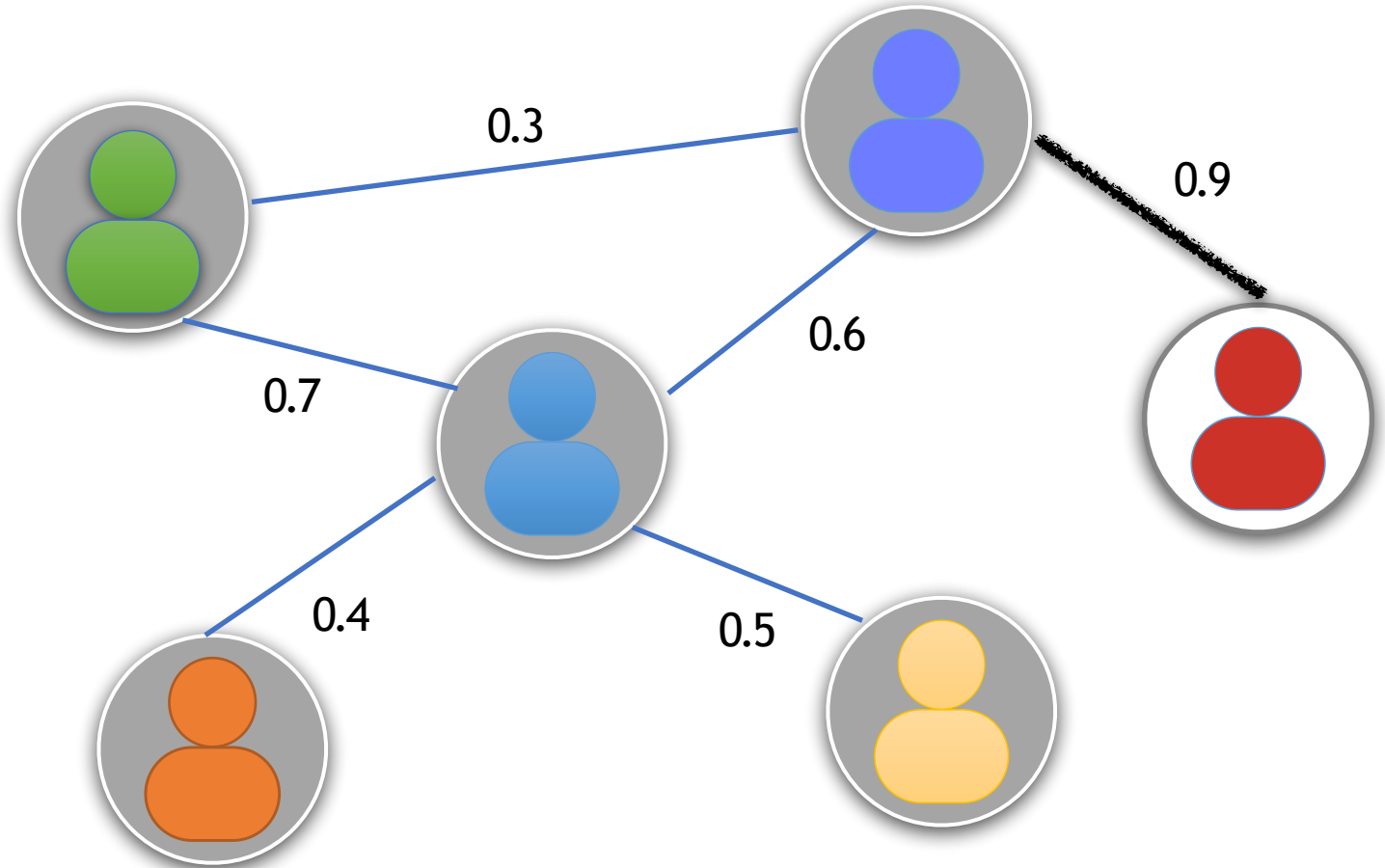
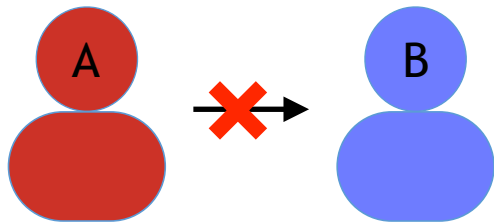
# Summary: Single-Round Influence Maximization

- Given a probability Graph  $G$  and budget  $k$
- Greedy algorithm to pick  $k$  nodes as a seed set
- Every node can be activated at most once
- Every node can not be duplicated in the seed set

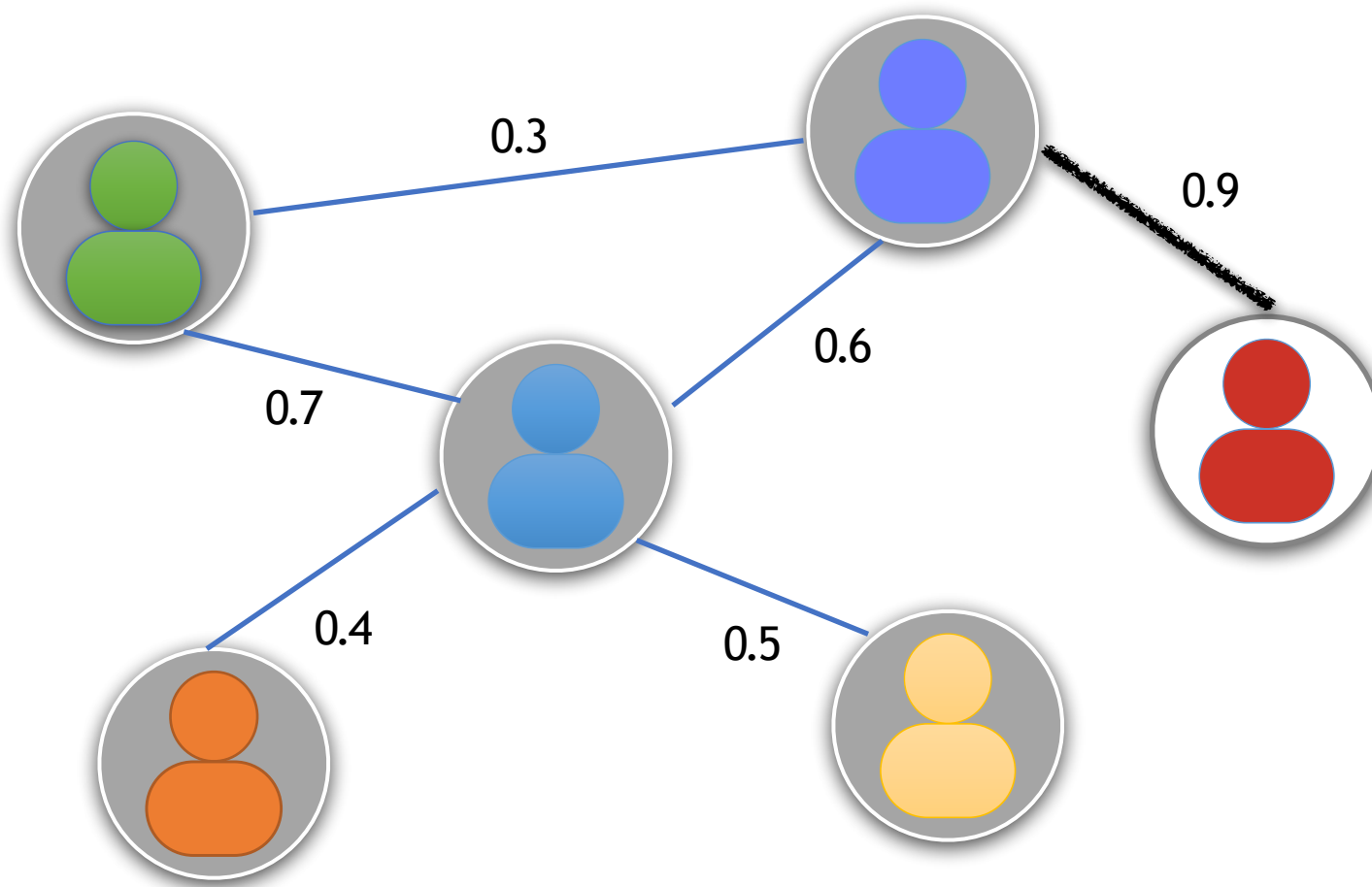
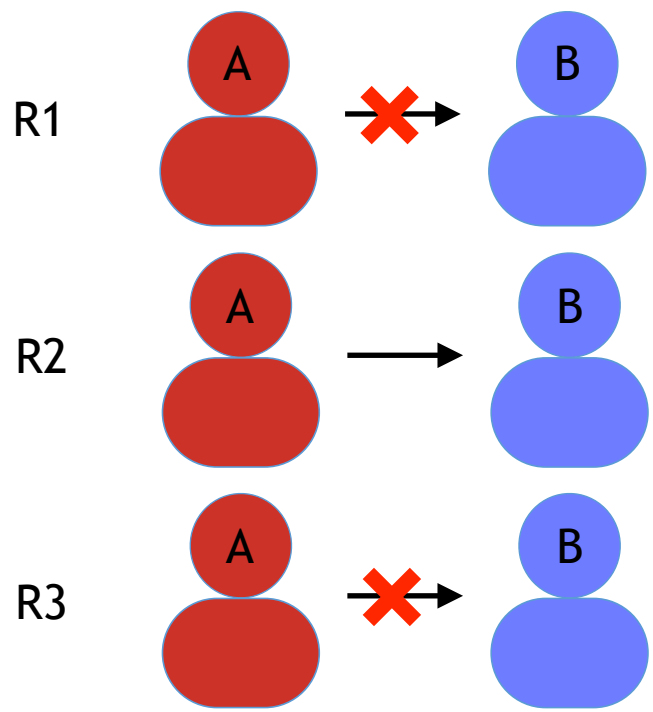
If company wants to do advertisements twice with total  $2k$  budget, how to design the node selection strategy?

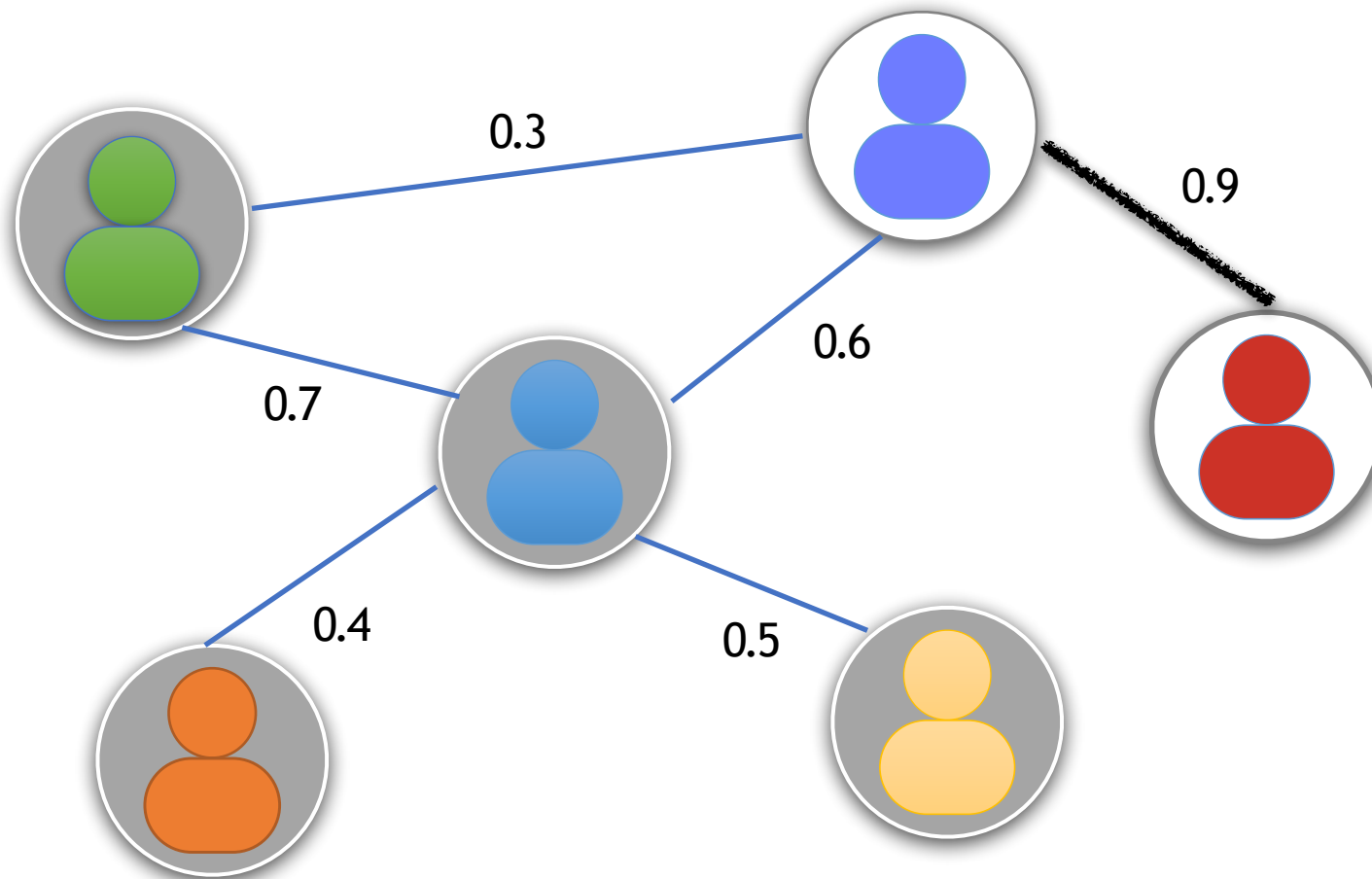
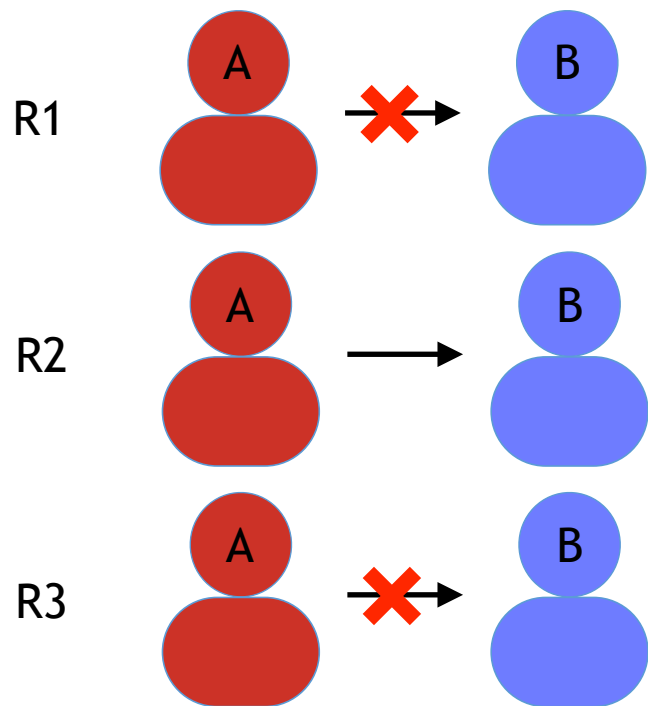
# Multi-round Influence Maximization







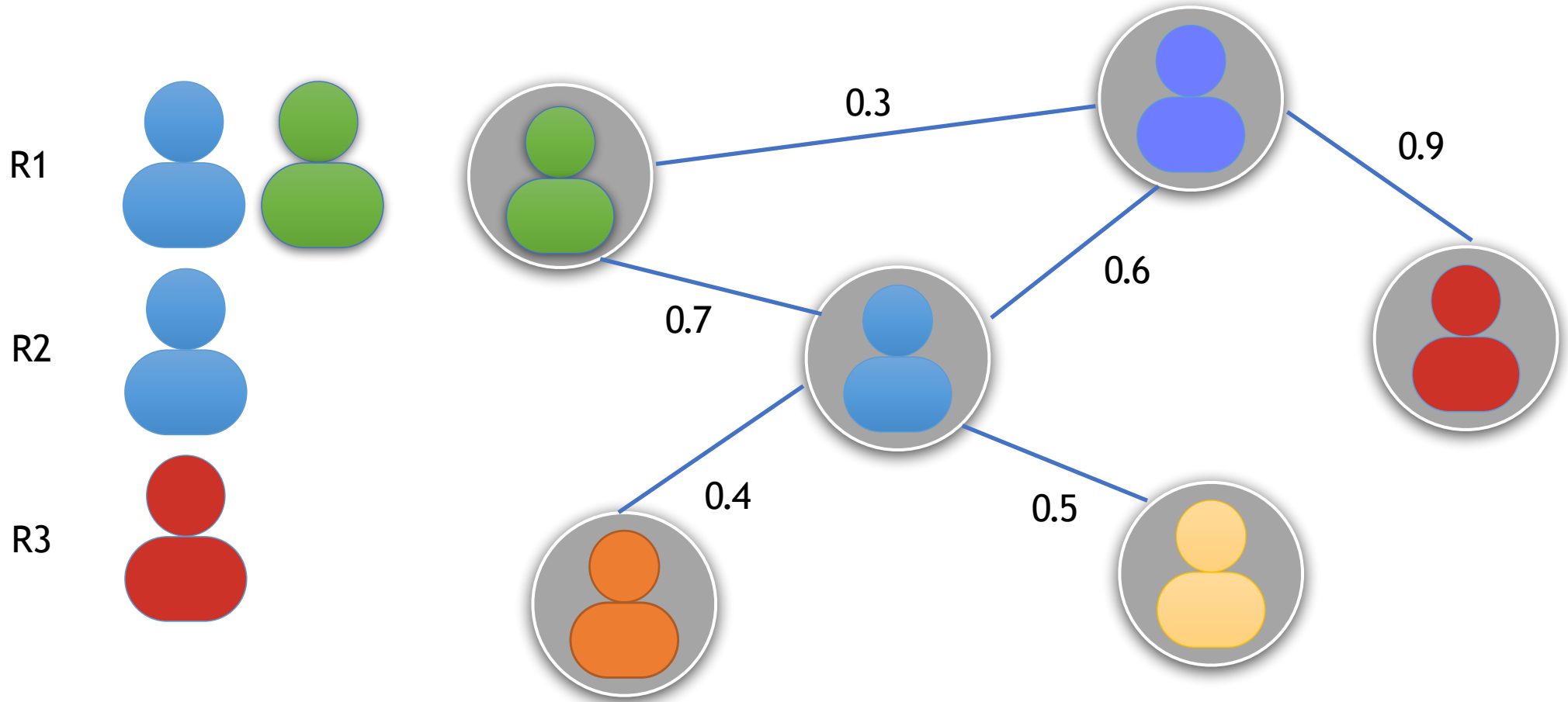




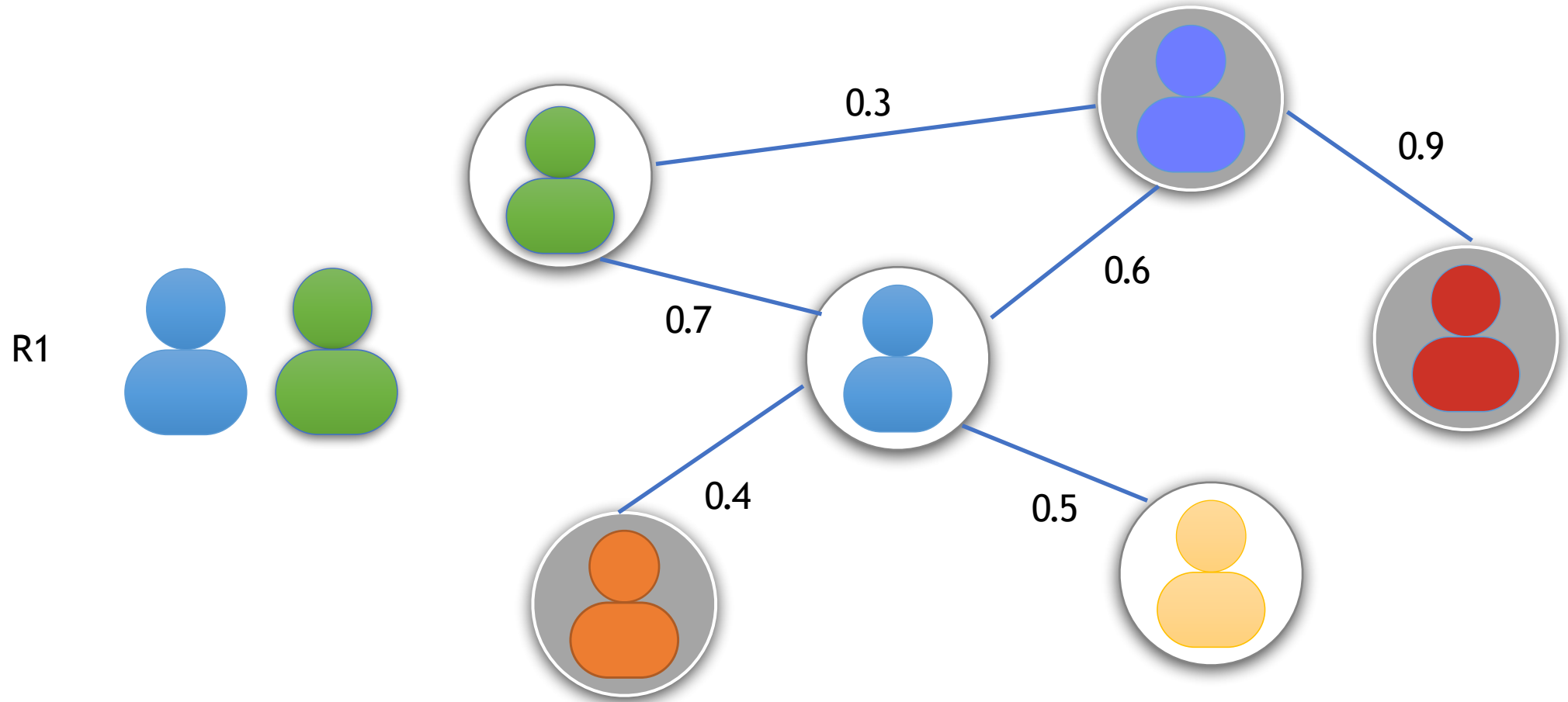
# Summary: Multi-Round Influence Maximization

- Given a probability Graph  $G$ , budget  $k$  and round  $T$
- Greedy pick  $Tk$  nodes as a seed set
- Every node can be activated at most once in each round
- Every node can not be duplicated in the seed set in each round
- Every node can be activated more than once in multiple round (no double count score but can share information)
- Every node can be duplicated in the seed set in multiple round

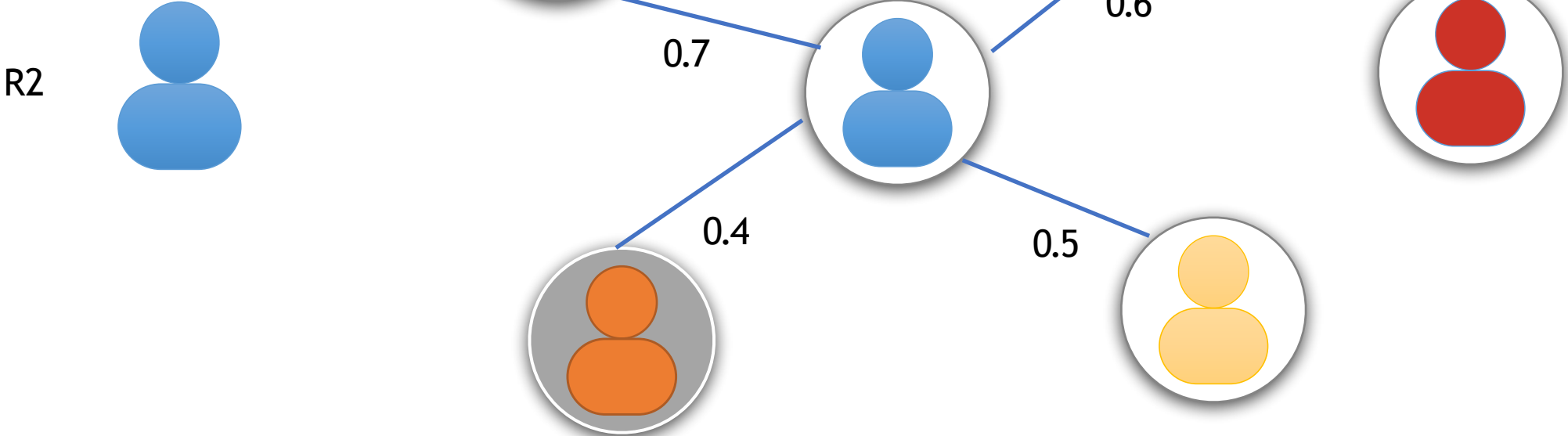
# Non-Adaptive Multi-Round Influence Maximization



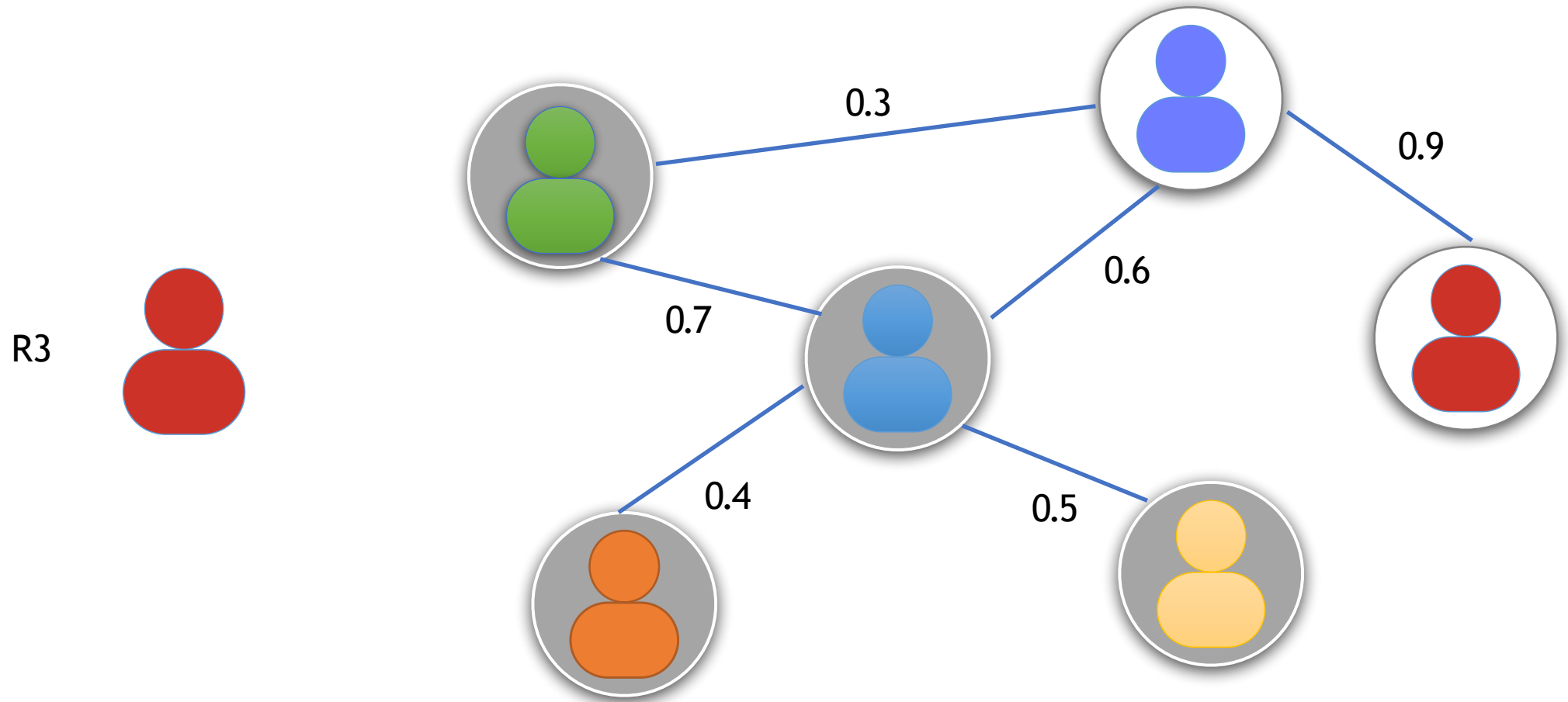
# Non-Adaptive Multi-Round Influence Maximization



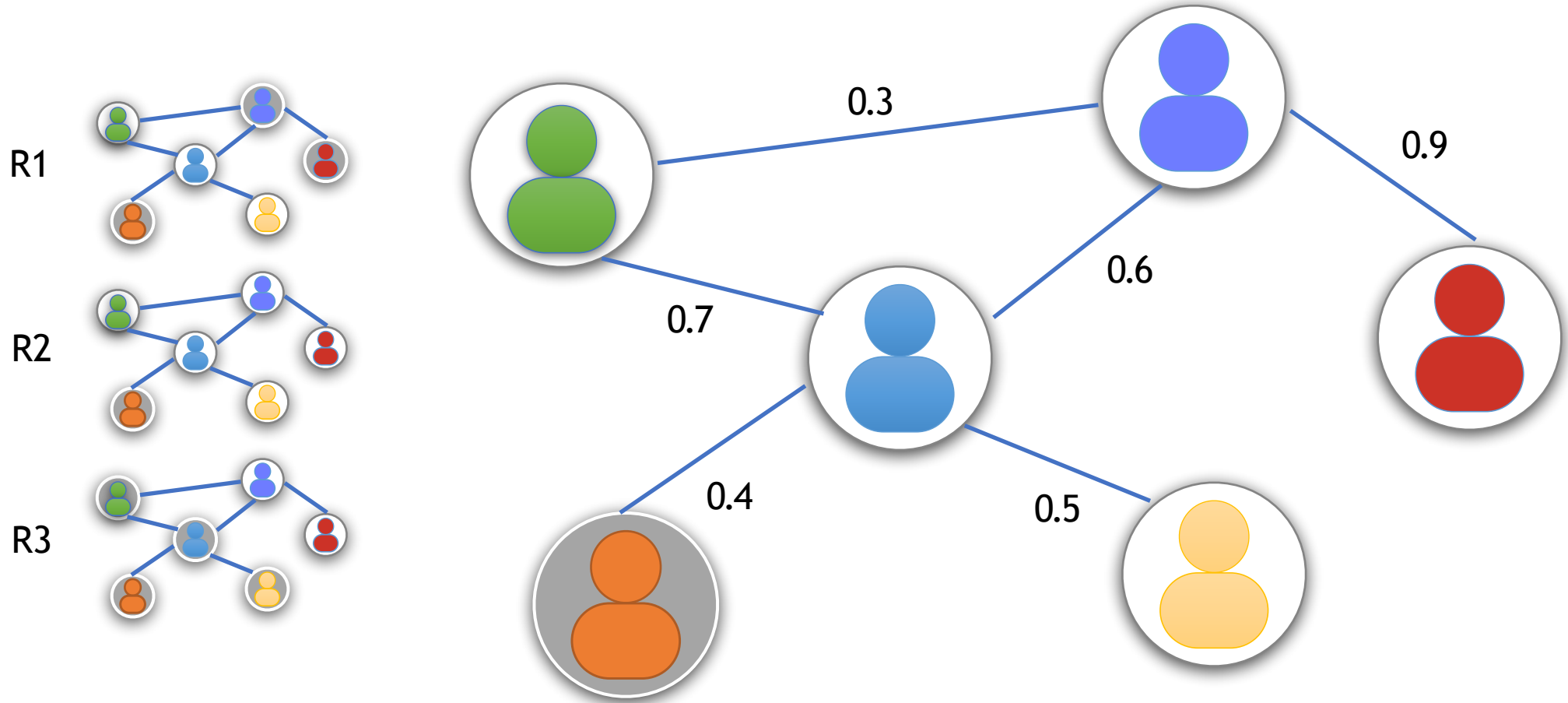
# Non-Adaptive Multi-Round Influence Maximization



# Non-Adaptive Multi-Round Influence Maximization

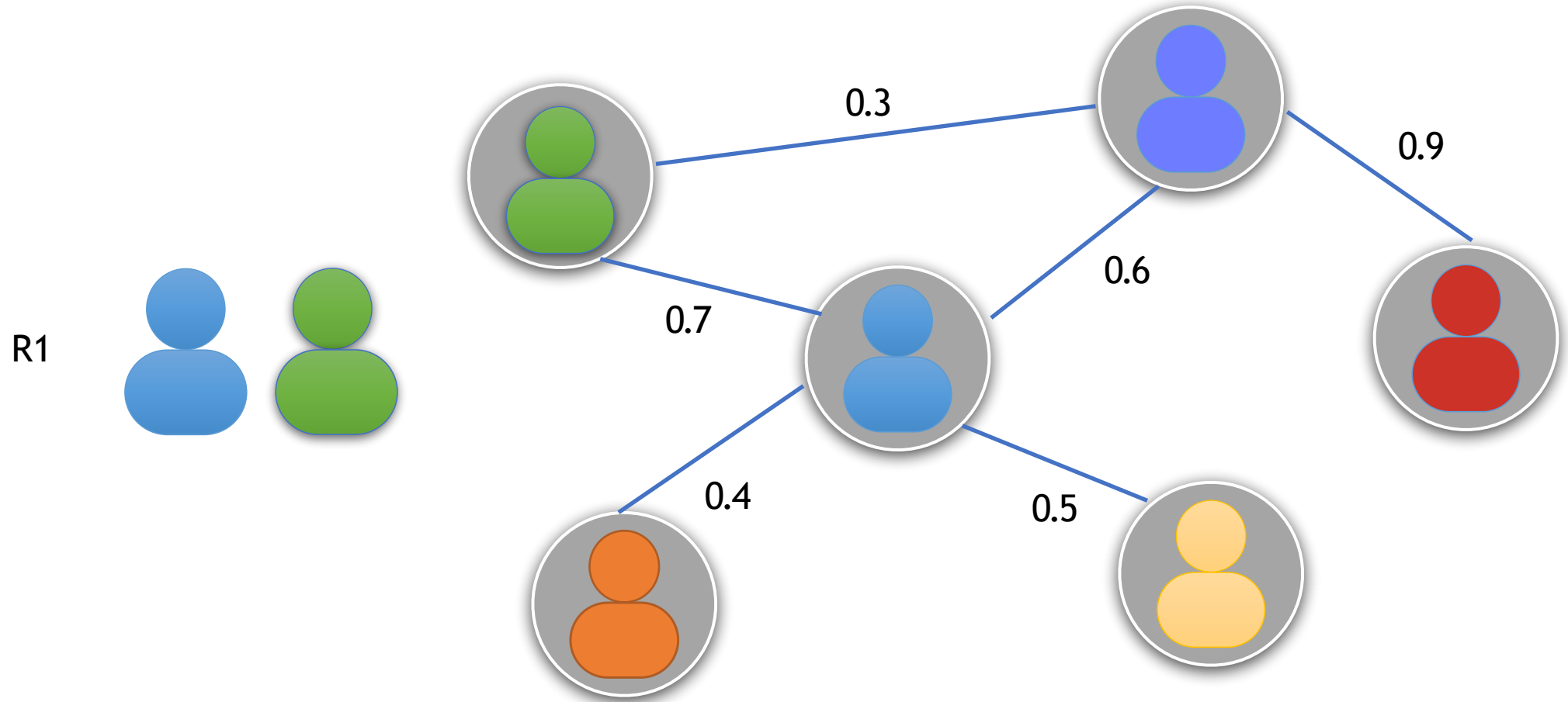


# Non-Adaptive Multi-Round Influence Maximization

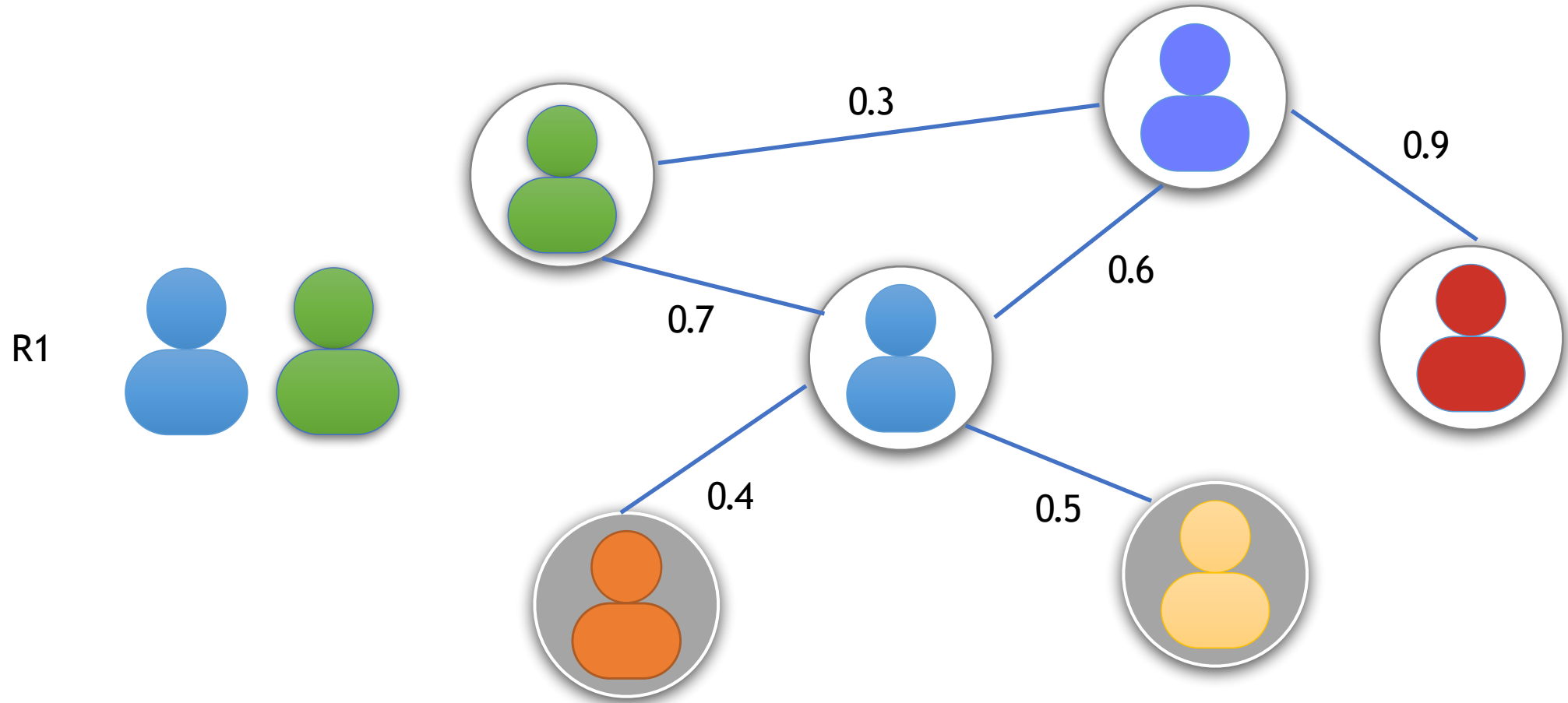




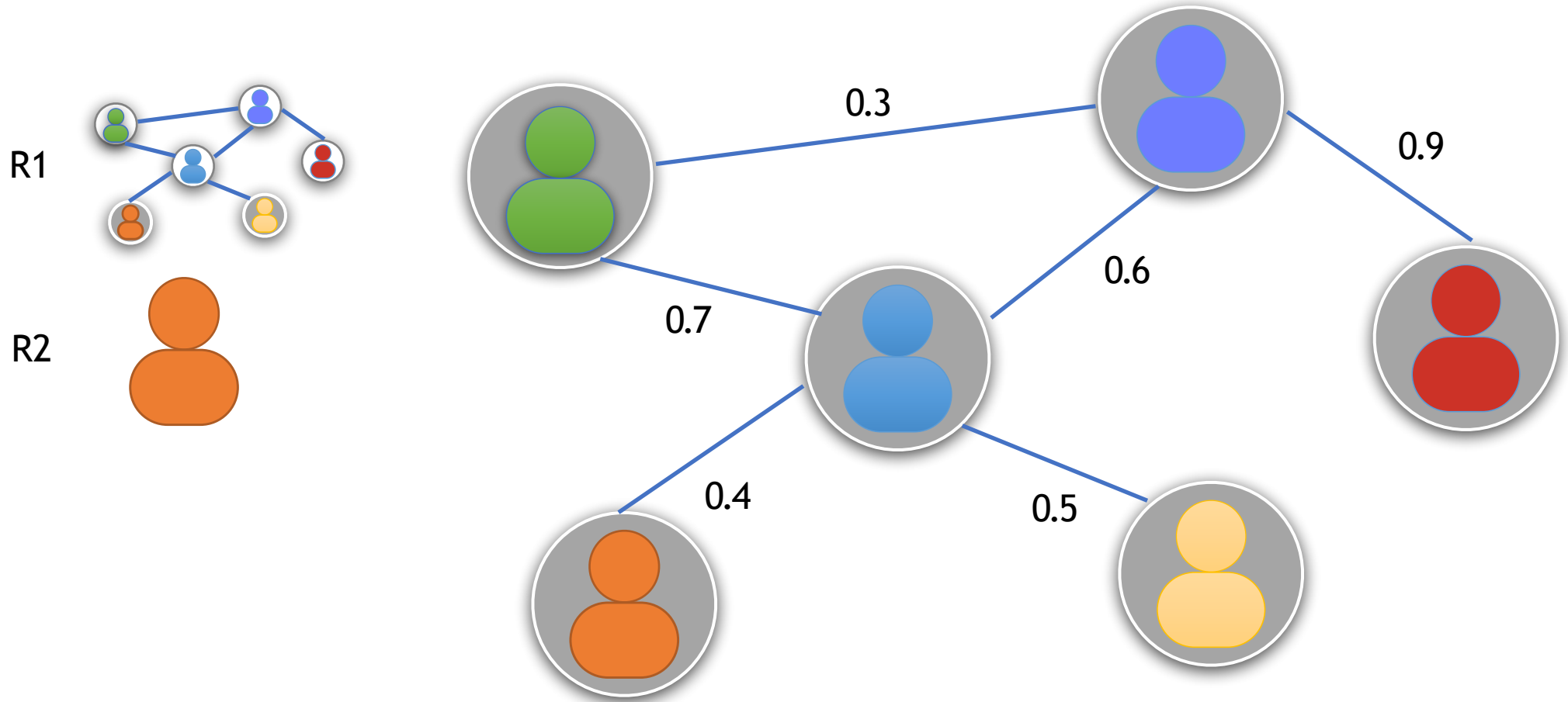
# Adaptive Multi-Round Influence Maximization



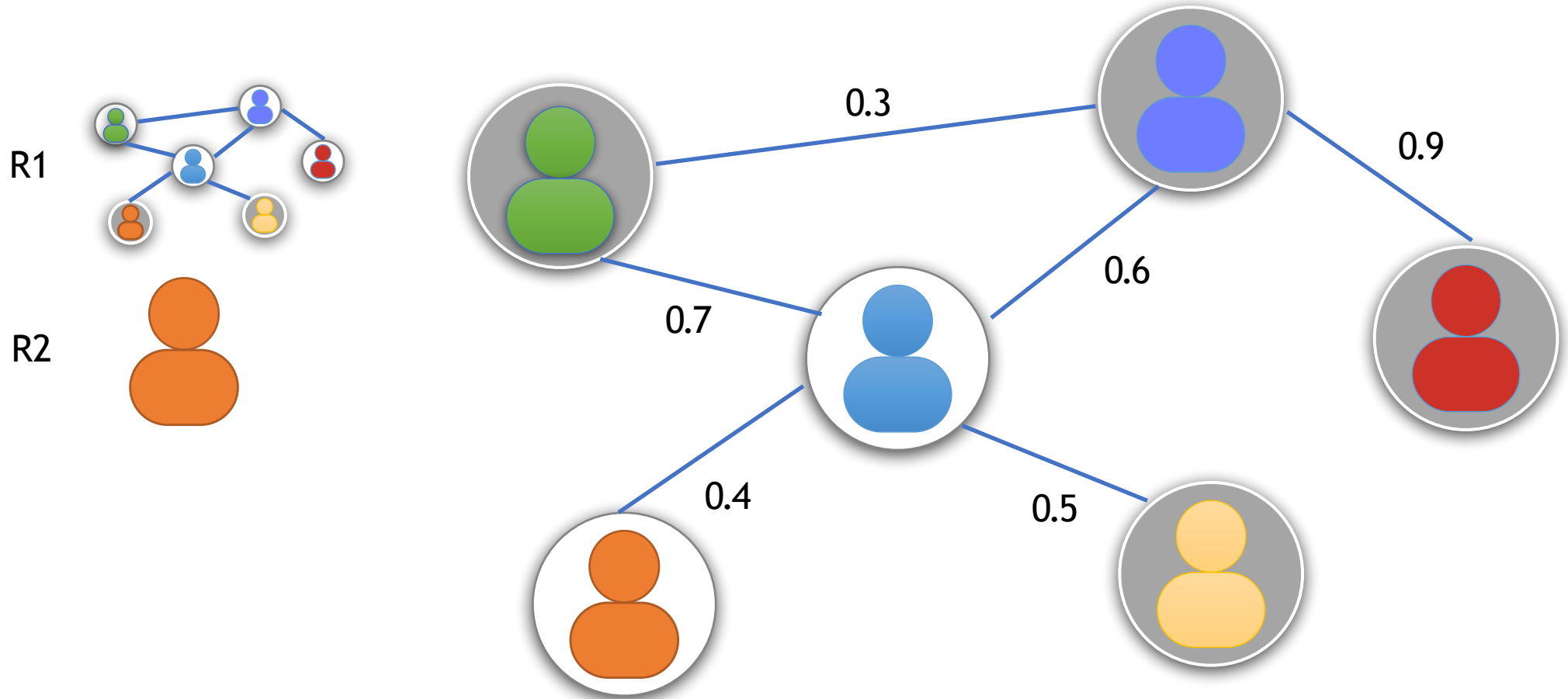
# Adaptive Multi-Round Influence Maximization



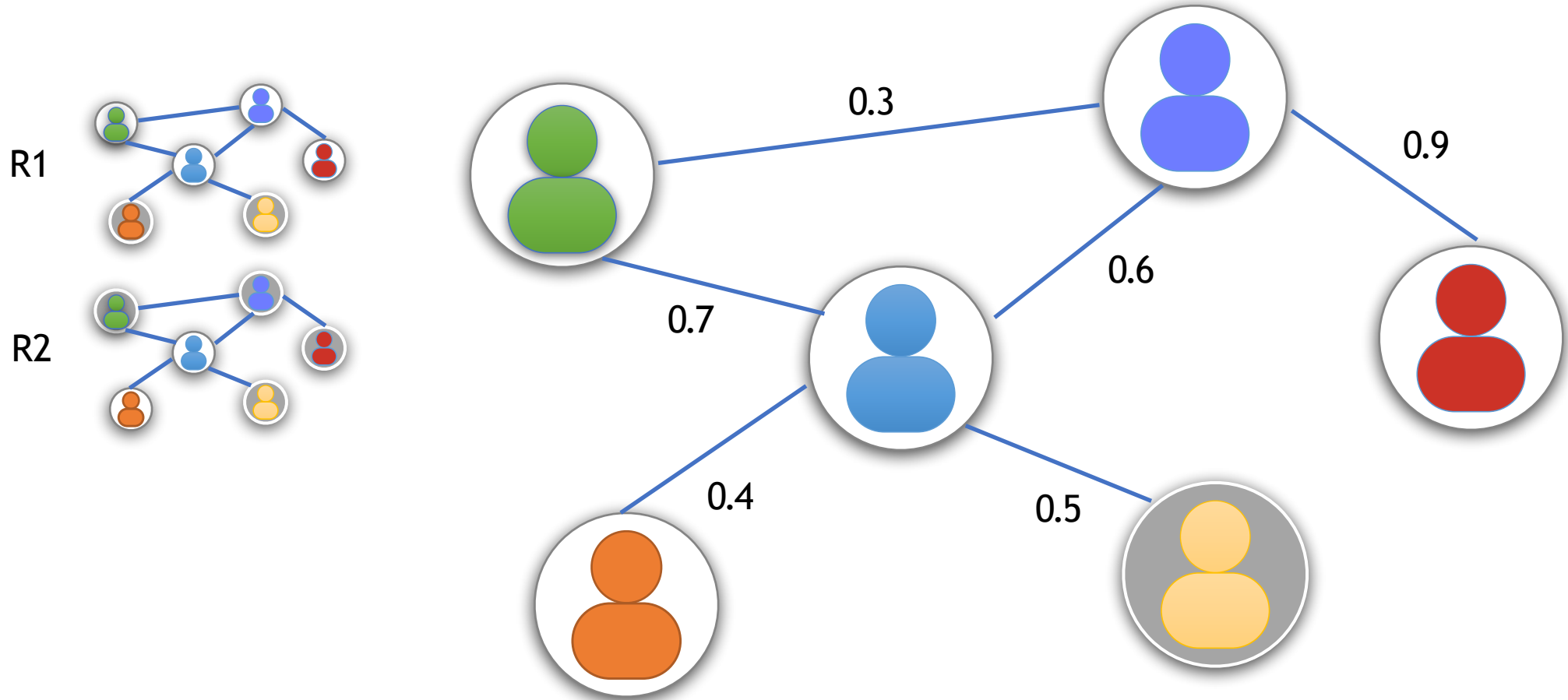
# Adaptive Multi-Round Influence Maximization



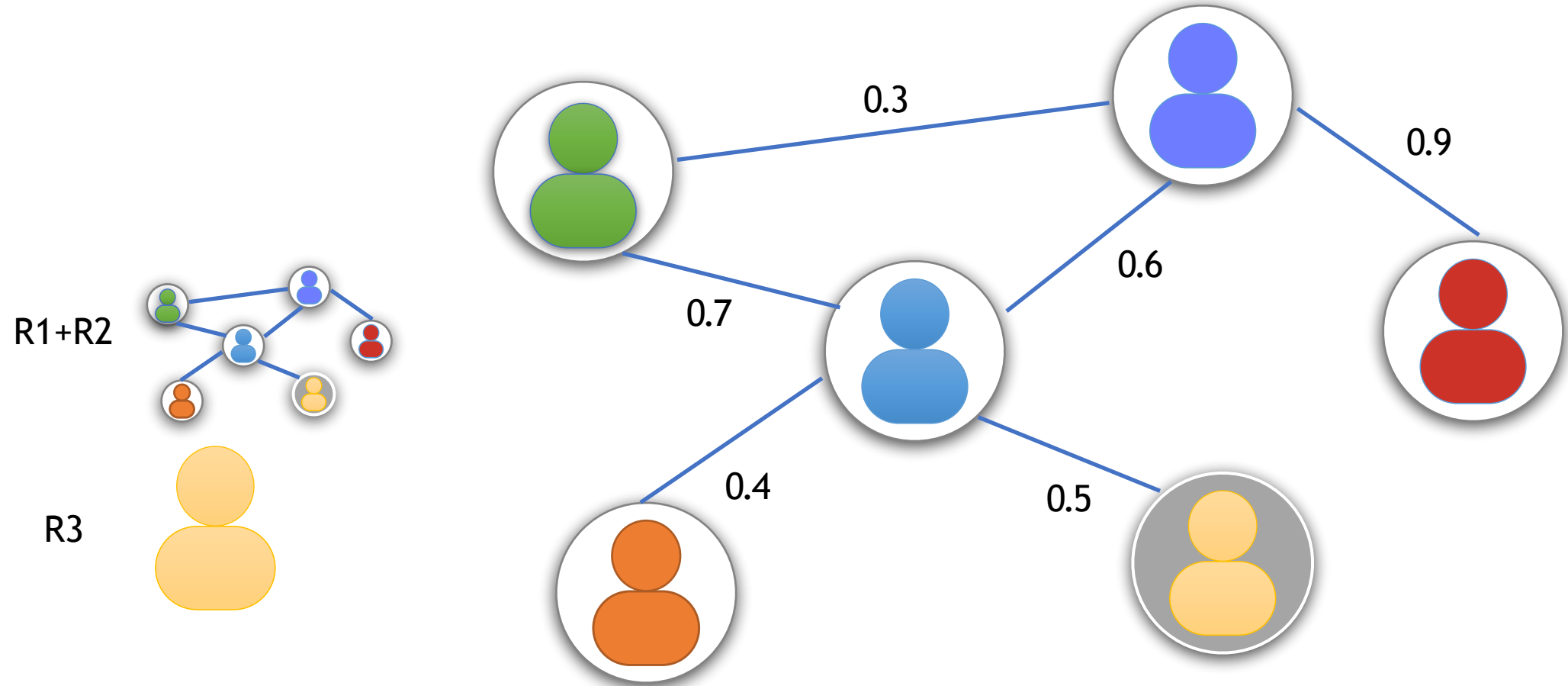
# Adaptive Multi-Round Influence Maximization



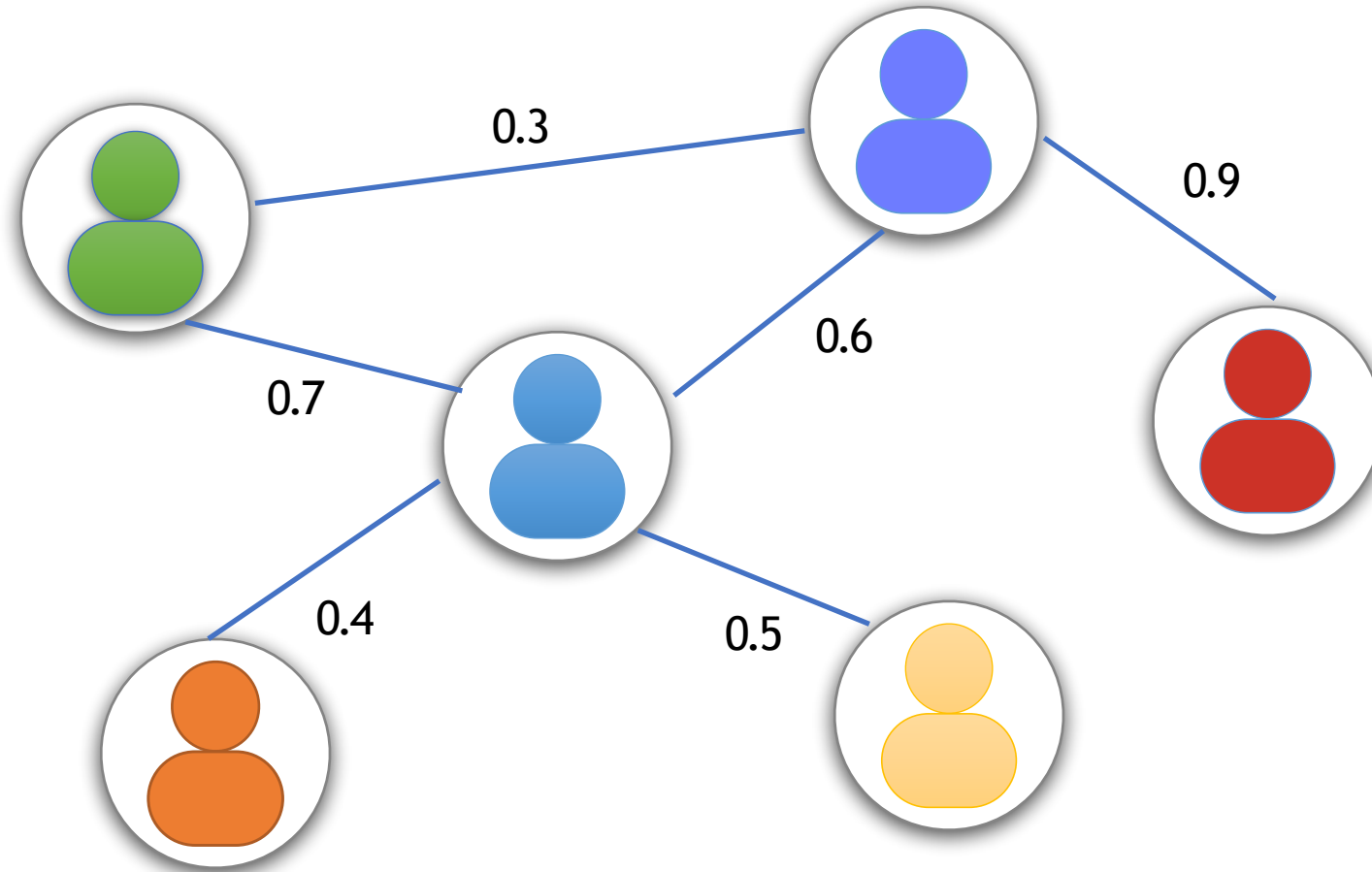
# Adaptive Multi-Round Influence Maximization



# Adaptive Multi-Round Influence Maximization



# Adaptive Multi-Round Influence Maximization

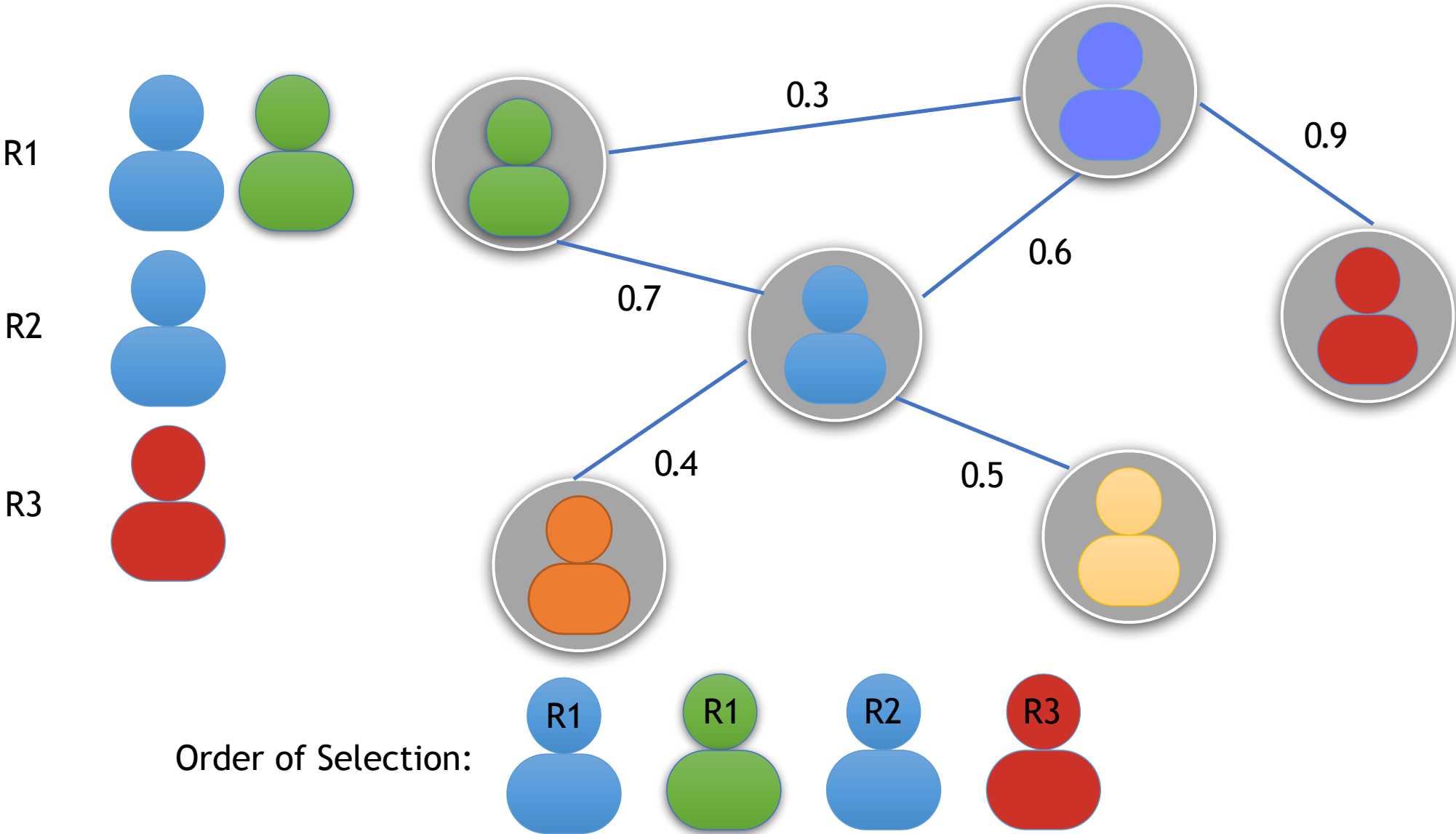


# Summary: Adaptive versus Non-Adaptive

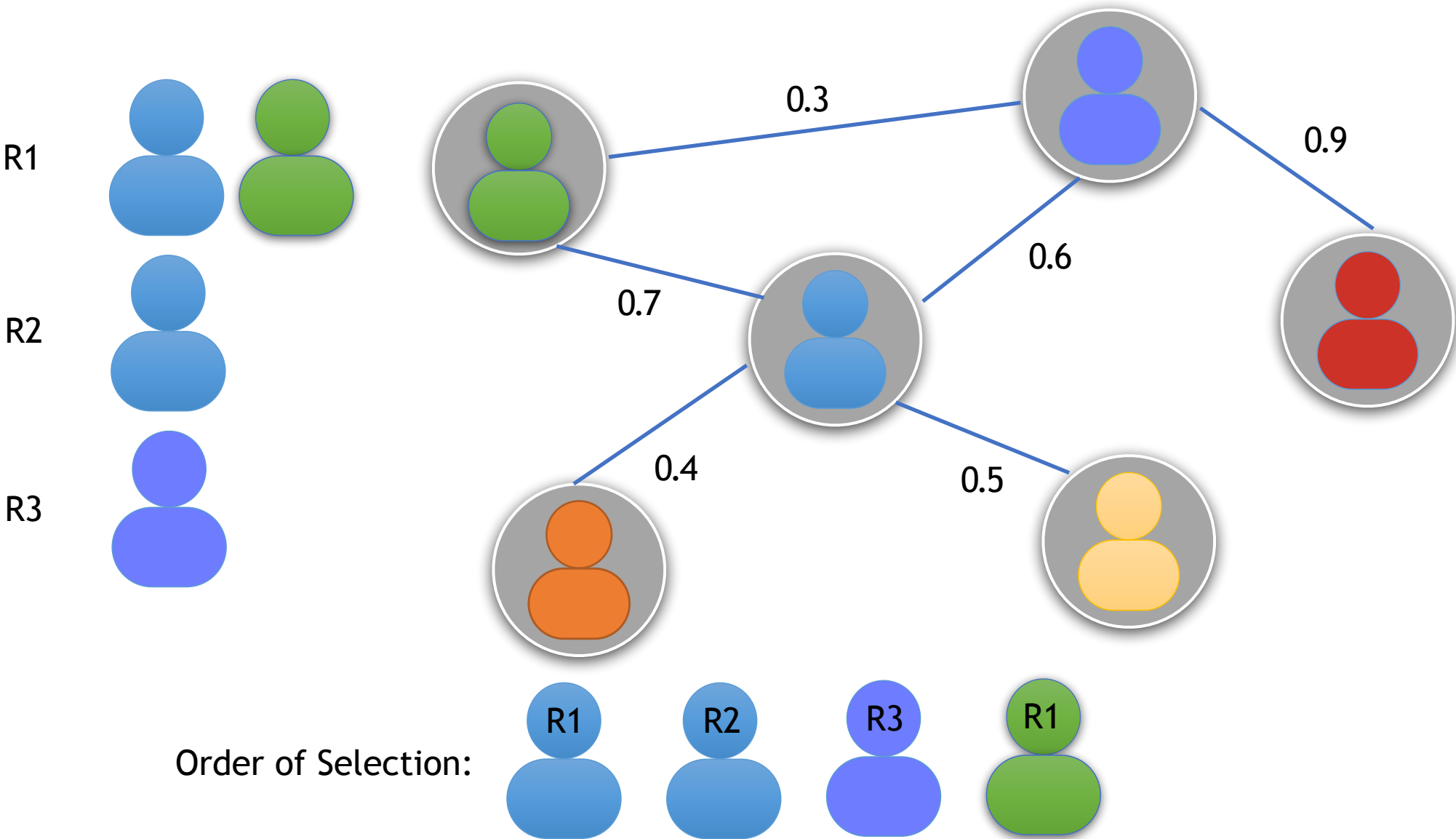
- Non-adaptive algorithm:
  - select all seeds at the beginning
  - do not need to wait for the information feedbacks from previous rounds
- Adaptive algorithm
  - use all influence information in previous rounds for next node selection
  - generally better performance than non-adaptive algorithm



# Non-Adaptive Cross-Round Node Selection



# Non-Adaptive Cross-Round Node Selection



# Summary: Within-round versus Cross-round

- Within-round setting
  - adaptive and non-adaptive
  - faster due to small search space
- Cross-round setting
  - non-adaptive only
  - better performance due to global greedy (large search space)

# Summary: Multi-Round Influence Maximization

- Non-adaptive model
  - cross-round seed selection (good performance)
  - within-round seed selection (fast)
- Adaptive model
  - within-round seed selection (fast, good performance, feedbacks)

**THEOREM 3.2.** Let  $S^*$  be the optimal solution of the non-adaptive MRIM under cross-round setting. For every  $\varepsilon > 0$  and  $\ell > 0$ , with probability at least  $1 - \frac{1}{n^\ell}$ , the output  $S^o$  of CR-Greedy satisfies

$$\rho(S^o) \geq \left( \frac{1}{2} - \varepsilon \right) \rho(S^*),$$

if CR-Greedy uses  $R = \lceil 31k^2T^2n \log(2kn^{\ell+1})/\varepsilon^2 \rceil$  as input. In this case, the total running time is  $O(k^3\ell T^4n^2m \log(n)/\varepsilon^2)$ , assuming each simulation finishes in  $O(m)$  time.

**THEOREM 3.3.** Let  $S^*$  be the optimal solution of the non-adaptive MRIM under within-round setting. For every  $\varepsilon > 0$  and  $\ell > 0$ , with probability at least  $1 - \frac{1}{n^\ell}$ , the output  $S^o$  of WR-Greedy satisfies

$$\rho(S^o) \geq \left( 1 - e^{-(1-\frac{1}{e})} - \varepsilon \right) \rho(S^*),$$

if WR-Greedy uses  $R = \lceil 31k^2n \log(2kn^{\ell+1}T)/\varepsilon^2 \rceil$  as input. In this case, the total running time is  $O(k^3\ell Tn^2m \log(nT)/\varepsilon^2)$ , assuming each simulation finishes in  $O(m)$  time.

**THEOREM 4.3.** Let  $\pi^{\text{ag}}$  represents the policy corresponding to the AdaGreedy algorithm. For any  $\varepsilon > 0$  and  $\ell > 0$ , if we use  $R = \lceil 31k^2n \log(2kn^{\ell+1}T)/\varepsilon^2 \rceil$  simulations for each influence spread estimation, then with probability at least  $1 - \frac{1}{n^\ell}$ ,

$$f_{\text{avg}}(\pi^{\text{ag}}) \geq \left( 1 - e^{-(1-\frac{1}{e})} - \varepsilon \right) f_{\text{avg}}(\pi^*).$$

In this case, the total running time for  $T$ -round AdaGreedy is  $O(k^3\ell Tn^2m \log(nT)/\varepsilon^2)$ .

Time: Non-adaptive CR > Adaptive WR = Non-adaptive WR

Performance: Non-adaptive CR  $\geq$  Non-adaptive WR

Adaptive WR  $\geq$  Non-adaptive WR

**THEOREM 3.2.** Let  $\mathcal{S}^*$  be the optimal solution of the non-adaptive MRIM under cross-round setting. For every  $\varepsilon > 0$  and  $\ell > 0$ , with probability at least  $1 - \frac{1}{n^\ell}$ , the output  $\mathcal{S}^o$  of CR-Greedy satisfies

$$\rho(\mathcal{S}^o) \geq \left( \frac{1}{2} - \varepsilon \right) \rho(\mathcal{S}^*),$$

if CR-Greedy uses  $R = \lceil 31k^2T^2n \log(2kn^{\ell+1})/\varepsilon^2 \rceil$  as input. In this case, the total running time is  $O(k^3\ell T^4n^2m \log(n)/\varepsilon^2)$ , assuming each simulation finishes in  $O(m)$  time.

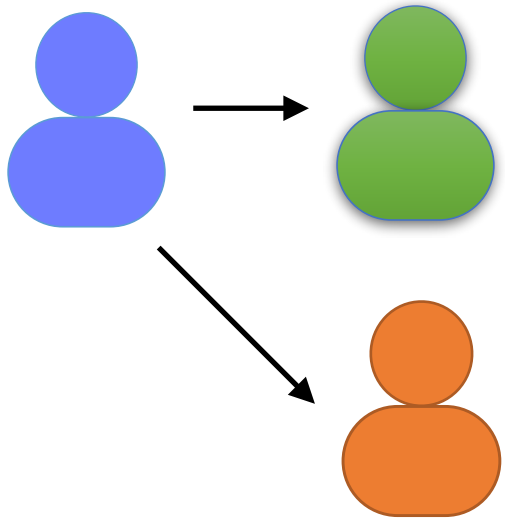
**THEOREM 4.3.** Let  $\pi^{\text{ag}}$  represents the policy corresponding to the AdaGreedy algorithm. For any  $\varepsilon > 0$  and  $\ell > 0$ , if we use  $R = \lceil 31k^2n \log(2kn^{\ell+1}T)/\varepsilon^2 \rceil$  simulations for each influence spread estimation, then with probability at least  $1 - \frac{1}{n^\ell}$ ,

$$f_{\text{avg}}(\pi^{\text{ag}}) \geq \left( 1 - e^{-(1-\frac{1}{\varepsilon})} - \varepsilon \right) f_{\text{avg}}(\pi^*).$$

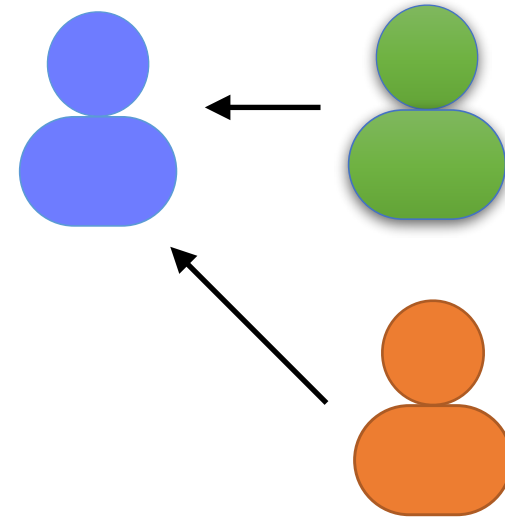
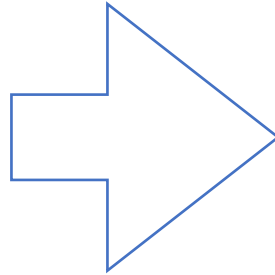
In this case, the total running time for  $T$ -round AdaGreedy is  $O(k^3\ell Tn^2m \log(nT)/\varepsilon^2)$ .

Which is better??? Non-adaptive CR, Adaptive WR ???

# Optimization: Reverse Reachable Set



Monto-Carlo



Reverse-Reachable

# Optimization: Reverse Reachable Set

root 1: <2, 3, 4, 5>
<del>root 2: &lt;1&gt;</del>
<del>root 3: &lt;1, 2, 4, 5&gt;</del>
root 3: <2>
<del>root 4: &lt;1&gt;</del>
<del>root 5: &lt;1, 3&gt;</del>

count
root 1: 4
root 2: 3
root 3: 2
root 4: 2
root 5: 2

root 1: <2, 3, 4, 5>
root 3: <2>

count
root 2: 2
root 3: 1
root 4: 1
root 5: 1

We implement all multi-round model with reverse reachable set algorithm.

**Speed up 1000 times!!!**



# Experiment



Dataset	Description	node	edge
Flixster	movie social	29357	212614
NetHEPT	author/co-author	15233	62774

**Table 1: The performance of influence spread on NetHEPT.**

Method/Simulations	Round				
	1	2	3	4	5
SG (R = 10000)	290.1 [288.8, 291.4]	505.7 [504.0, 507.3]	688.6 [686.6, 690.4]	868.2 [866.2, 870.2]	1027.3 [1025.2, 1029.4]
SG-R (R = 10000)	289.5 [288.2, 290.8]	516.3 [514.6, 518.0]	714.0 [712.0, 716.0]	884.9 [882.7, 887.1]	1042.0 [1039.7, 1044.2]
E-WR-Greedy (R = 10000)	290.7 [289.4, 292.0]	528.9 [527.2, 530.6]	738.8 [736.9, 740.8]	930.2 [928.0, 932.3]	1097.6.9 [1095.3, 1099.8]
WR-IMM (R = 10000)	290.9 [289.7, 292.3]	532.8 [531.1, 534.5]	745.3 [743.2, 747.3]	930.1 [928.0, 932.2]	1093.1 [1090.8, 1095.3]
CR-Greedy (R = 10000)	267.8 [266.5, 269.1]	528.7 [527.2, 530.4]	730.4 [728.5, 732.4]	938.5 [933.7, 937.8]	1121.3 [1119.0, 1123.5]
CR-IMM (R = 10000)	283.0 [281.7, 284.2]	517.4 [515.7, 519.2]	721.9 [720.0, 723.9]	931.6 [929.4, 933.7]	1129.7 [1127.7, 1131.9]
AdaGreedy (R = 150)	288.3 [276.7, 299.7]	533.4 [519.4, 547.3]	758.1 [743.6, 772.7]	960.1 [943.9, 976.3]	1141.5 [1123.7, 1160.0]
AdalMM (R = 150)	<b>291.8</b> [281.3, 302.4]	<b>544.4</b> [531.6, 557.2]	<b>761.8</b> [746.6, 776.9]	<b>965.8</b> [949.7, 982.0]	<b>1146.3</b> [1129.1, 1163.5]

**Table 3: Influence spread with different adaptive degree.**

Num. of Rounds	1	2	5	10
Num. of Seeds	50	25	10	5
AdalMM (R = 100)	883.0 [856.0, 910.1]	1040.3 [1022.6, 1058.1]	1141.0 [1119.3, 1162.6]	1204.7 [1178.2, 1231.3]

**Table 4: Running time of the algorithms, in seconds.**

	SG	SG-R	E-WR-Greedy	WR-IMM
NetHEPT (R = 5)	439.2 [407, 470.94]	87.8 [81.5, 94.2]	551.2 [527.9, 574.4]	1.97 [1.91, 2.03]
Flixster (R = 5)	4862.3 [4773.3, 4951.3]	972.5 [990.3, 954.7]	2478.9 [2422.4, 2535.5]	3.16 [3.14, 3.18]

	CR-Greedy	CR-IMM	AdaGreedy	AdalMM
NetHEPT (R = 5)	2105.6 [2036.2, 2175.0]	2.13 [2.05, 2.21]	465.4 [473.8, 457.0]	2.01 [1.93, 2.09]
Flixster (R = 5)	9587.6 [9145.3, 10029.9]	3.61 [3.59, 3.63]	2305.5 [2161.0, 2450.0]	3.23 [3.16, 3.30]

# Conclusion



# Summary and Future Plan

[lsun29@uic.edu](mailto:lsun29@uic.edu)

- Non-adaptive model
  - cross-round seed selection (good performance)
  - within-round seed selection (fast)
- Adaptive model
  - within-round seed selection (fast, good performance, feedbacks)
- Multi-round is not only a new model, but an idea brings more research opportunities.

*Thank  
you*

