

Problem Set 1

Group 2

2026-01-29

1) Prove $\lim_{x \rightarrow -1} 2x + 1 = -1$

Solution:

Determine $\delta > 0$ so that if $|x - (-1)| < \delta$ then $|2x + 1 - (-1)| < \epsilon$.

$$|(2x + 1) - (-1)| < \epsilon$$

$$|2x + 2| < \epsilon$$

$$2|x + 1| < \epsilon$$

$$2|x - (-1)| < \epsilon$$

$$|x - (-1)| < \frac{\epsilon}{2}$$

Given $\epsilon > 0$, Choose $\delta = \frac{\epsilon}{2}$, thus,

$$|x + 1| < \delta$$

$$|x + 1| < \frac{\epsilon}{2}$$

$$2|x + 1| < \epsilon$$

$$|2x + 2| < \epsilon$$

$$|2x + 1 + 1| < \epsilon$$

$$|2x + 1 - (-1)| < \epsilon$$

2) Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function and graph using R with the point/s identified. $f(x) = x^3 - 4x^2 - 2x - 5$ on $[-10, 10]$.

Solution:

$$\begin{aligned}
f'(c) &= \frac{f(b) - f(a)}{b - a} \\
3c^2 - 8c - 2 &= \frac{[(10)^3 - 4(10) - 2(10) - 5] - [(-10)^3 - 4(-10) - 2(-10) - 5]}{10 - (-10)} \\
3c^2 - 8c - 2 &= \frac{575 - (-1385)}{20} \\
3c^2 - 8c - 2 &= 98 \\
3c^2 - 8c - 100 &= 0 \\
c &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-100)}}{6} \\
c &= \frac{8 \pm \sqrt{1264}}{6} \\
c &= \frac{8 \pm 4\sqrt{79}}{6} \\
c &\approx -4.59 \\
c &\approx 7.25
\end{aligned}$$

- 3) Find the point c that satisfies the mean value theorem for integrals on the interval $[-1, 1]$. The function is $f(x) = 2e^x$

Solution:

$$\begin{aligned}
\int_{-1}^1 2e^x dx &= (2e^c)(1 - (-1)) \\
2e^x \Big|_{-1}^1 &= (2e^c)2 \\
(2e - 2e^{-1}) &= 4e^c \\
\frac{2(e - e^{-1})}{4} &= e^c \\
\frac{(e - e^{-1})}{2} &= e^c \\
\ln\left(\frac{(e - e^{-1})}{2}\right) &= c \\
0.16143936157 &\approx c
\end{aligned}$$