Problem Set 1

Group 2

2026-01-29

1) Prove $\lim_{x \to -1} 2x + 1 = -1$

   Solution:

   Determine $\delta > 0$ so that if $|x - (-1)| < \delta$ then $|2x + 1 - (-1)| < \epsilon$.

$$\begin{aligned} |(2x+1) - (-1)| &< \epsilon \\ |2x+2| &< \epsilon \\ 2|x+1| &< \epsilon \\ 2|x-(-1)| &< \epsilon \\ |x-(-1)| &< \frac{\epsilon}{2} \end{aligned}$$

   Given $\epsilon > 0$, Choose $\delta = \frac{\epsilon}{2}$, thus,

$$\begin{aligned} |x+1| &< \delta \\ |x+1| &< \frac{\epsilon}{2} \\ 2|x+1| &< \epsilon \\ |2x+2| &< \epsilon \\ |2x+1+1| &< \epsilon \\ |2x+1-(-1)| &< \epsilon \end{aligned}$$

2) Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function and graph using R with the point/s identified. $f(x) = x^3 - 4x^2 - 2x - 5$ on $[-10, 10]$.

   Solution:

3) Find the point c that satisfies the mean value theorem for integrals on the interval $[-1, 1]$. The function is $f(x) = 2e^x$

   Solution:

$$
\begin{aligned}
\int_{-1}^{1} 2e^x dx &= (2e^c)(1-(-1)) \\
2e^x\Big|_{-1}^{1} &= (2e^c)2 \\
(2e - 2e^{-1}) &= 4e^c \\
\frac{2(e-e^{-1})}{4} &= e^c \\
\frac{(e-e^{-1})}{2} &= e^c \\
\ln\left(\frac{(e-e^{-1})}{2}\right) &= c \\
0.16143936157 &\approx c
\end{aligned}
$$