Problem Set 1

Group 2

2026-01-29

1. Prove $\lim_{x \to -1} 2x + 1 = -1$

Solution:

Determine $\delta > 0$ so that if $|x - (-1)| < \delta$ then $|2x + 1 - (-1)| < \epsilon$.

$$\begin{aligned} |(2x+1) - (-1)| &< \epsilon \\ |2x + 2| &< \epsilon \\ 2|x + 1| &< \epsilon \\ 2|x - (-1)| &< \epsilon \\ |x - (-1)| &< \frac{\epsilon}{2} \end{aligned}$$

Given $\epsilon > 0$, Choose $\delta = \frac{\epsilon}{2}$, thus,

$$\begin{aligned} |x + 1| &< \delta \\ |x + 1| &< \frac{\epsilon}{2} \\ 2|x + 1| &< \epsilon \\ |2x + 2| &< \epsilon \\ |2x + 1 + 1| &< \epsilon \\ |2x + 1 - (-1)| &< \epsilon \end{aligned}$$

2. Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function and graph using R with the point/s identified. $f(x) = x^3 - 4x^2 - 2x - 5$ on $[-10, 10]$.

Solution:

$$
\begin{aligned}
f'(c) &= \frac{f(b)-f(a)}{b-a} \\
3c^2-8c-2 &= \frac{[(10)^3-4(10)-2(10)-5]-[(-10)^3-4(-10)-2(-10)-5]}{10-(-10)} \\
3c^2-8c-2 &= \frac{575-(-1385)}{20} \\
3c^2-8c-2 &= 98 \\
3c^2-8c-100 &= 0 \\
c &= \frac{-(-8)\pm\sqrt{(-8)^2-4(3)(-100)}}{6} \\
c &= \frac{8\pm\sqrt{1264}}{6} \\
c &= \frac{8\pm4\sqrt{79}}{6} \\
c &\approx -4.59 \\
c &\approx 7.25
\end{aligned}
$$

3. Find the point c that satisfies the mean value theorem for integrals on the interval $[-1,1]$. The function is $f(x)=2e^x$

Solution:

$$
\begin{aligned}
\int_{-1}^{1} 2e^x dx &= (2e^c)(1-(-1)) \\
2e^x\Big|_{-1}^{1} &= (2e^c)2 \\
(2e-2e^{-1}) &= 4e^c \\
\frac{2(e-e^{-1})}{4} &= e^c \\
\frac{(e-e^{-1})}{2} &= e^c \\
\ln\left(\frac{(e-e^{-1})}{2}\right) &= c \\
0.16143936157 &\approx c
\end{aligned}
$$