

Problem Set 1

Group 2

2026-01-29

- 1) Prove $\lim_{x \rightarrow -1} 2x + 1 = -1$

Solution:

Determine $\delta > 0$ so that if $|x - (-1)| < \delta$ then $|2x + 1 - (-1)| < \epsilon$.

$$\begin{aligned}|(2x + 1) - (-1)| &< \epsilon \\ |2x + 2| &< \epsilon \\ 2|x + 1| &< \epsilon \\ 2|x - (-1)| &< \epsilon \\ |x - (-1)| &< \frac{\epsilon}{2}\end{aligned}$$

Given $\epsilon > 0$, Choose $\delta = \frac{\epsilon}{2}$, thus,

$$\begin{aligned}|x + 1| &< \delta \\ |x + 1| &< \frac{\epsilon}{2} \\ 2|x + 1| &< \epsilon \\ |2x + 2| &< \epsilon \\ |2x + 1 + 1| &< \epsilon \\ |2x + 1 - (-1)| &< \epsilon\end{aligned}$$

- 2) Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function and graph using R with the point/s identified. $f(x) = x^3 - 4x^2 - 2x - 5$ on $[-10, 10]$.

Solution:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 - 8c - 2 = \frac{[(10)^3 - 4(10) - 2(10) - 5] - [(-10)^3 - 4(-10) - 2(-10) - 5]}{10 - (-10)}$$

$$3c^2 - 8c - 2 = \frac{575 - (-1385)}{20}$$

$$3c^2 - 8c - 2 = 98$$

$$3c^2 - 8c - 100 = 0$$

$$c = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-100)}}{6}$$

$$c = \frac{8 \pm \sqrt{1264}}{6}$$

$$c = \frac{8 \pm 4\sqrt{79}}{6}$$

$$c \approx -4.59$$

$$c \approx 7.25$$

- 3) Find the point c that satisfies the mean value theorem for integrals on the interval $[-1, 1]$. The function is $f(x) = 2e^x$

Solution:

$$\int_{-1}^1 2e^x dx = (2e^c)(1 - (-1))$$

$$2e^x \Big|_{-1}^1 = (2e^c)2$$

$$(2e - 2e^{-1}) = 4e^c$$

$$\frac{2(e - e^{-1})}{4} = e^c$$

$$\frac{(e - e^{-1})}{2} = e^c$$

$$\ln\left(\frac{(e - e^{-1})}{2}\right) = c$$

$$0.16143936157 \approx c$$