Q3

November 14, 2019

0.0.1 Load all handwritten digit

[1]:
```
from utils import load_data
```

[2]:
```
X_train, X_valid, X_test, y_train, y_valid, y_test = load_data("digits.npz")
```

0.0.2 Compute covariance matrix

[3]:
```
import numpy as np
```

[4]:
```
S = np.cov(X_train.T, bias=True)
```

[5]:
```
from numpy import linalg as LA
```

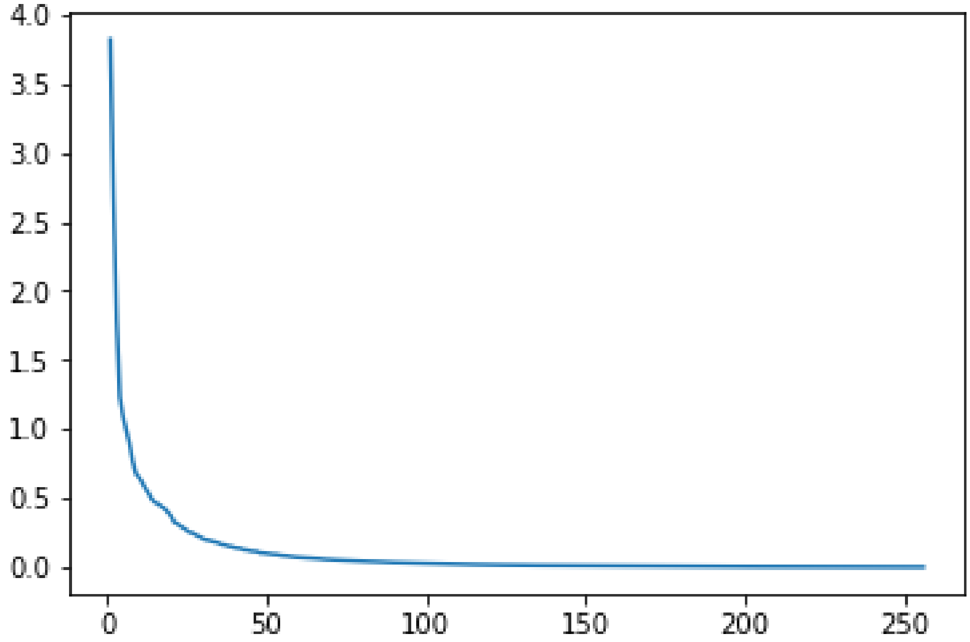
[6]:
```
w, v = LA.eig(S)
arg_w = w.argsort()[::-1]
x = np.arange(1, w.shape[0] + 1)
```

0.0.3 Plot the sorted eigen values

[7]:
```
import matplotlib.pyplot as plt
```

[8]:
```
plt.plot(x, w[arg_w])
```

[8]:
```
[<matplotlib.lines.Line2D at 0x7f635ba71e90>]
```



0.0.4 Plot the first three eigen-images and mean

[9]:
```
def plot_images(images, ax, ims_per_row=5, padding=5, digit_dimensions=(28, 28),
                cmap=plt.cm.binary, vmin=None, vmax=None):
    """Images should be a (N_images x pixels) matrix."""
    N_images = images.shape[0]
    N_rows = np.int32(np.ceil(float(N_images) / ims_per_row))
    pad_value = np.min(images.ravel())
    concat_images = np.full(((digit_dimensions[0] + padding) * N_rows + padding,
                             (digit_dimensions[1] + padding) * ims_per_row +␣
 ↪padding), pad_value)
    for i in range(N_images):
        cur_image = np.reshape(images[i, :], digit_dimensions, order='F')
        row_ix = i // ims_per_row
        col_ix = i % ims_per_row
        row_start = padding + (padding + digit_dimensions[0]) * row_ix
        col_start = padding + (padding + digit_dimensions[1]) * col_ix
        concat_images[row_start: row_start + digit_dimensions[0],
                      col_start: col_start + digit_dimensions[1]] = cur_image
    cax = ax.matshow(concat_images, cmap=cmap, vmin=vmin, vmax=vmax)
    return cax
```

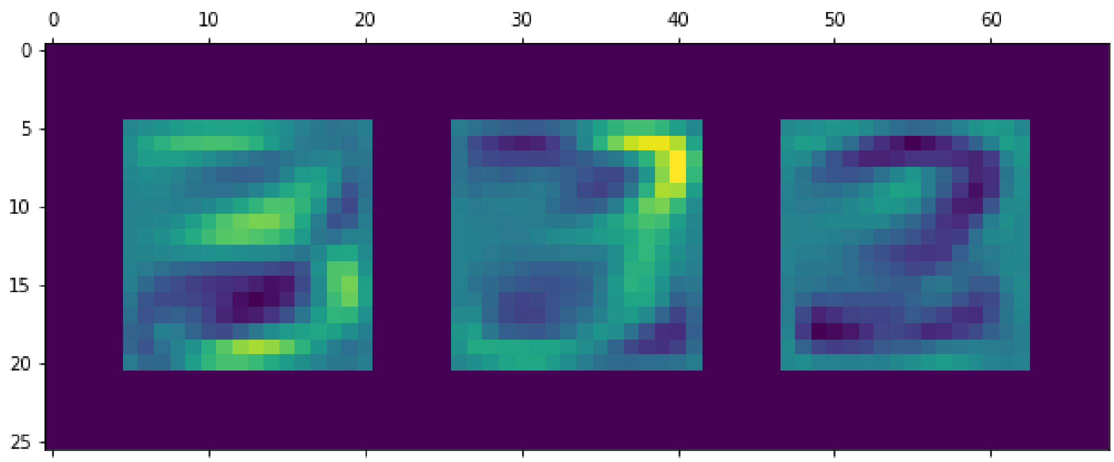
[10]:
```
# Get the first three eigen vectors
first_3 = v[:,arg_w[:3]].T
```

[11]:
```
first_3.shape
```

[11]: (3, 256)

[12]:
```
%matplotlib inline
plot_images(first_3, plt, ims_per_row=3, digit_dimensions=(16, 16),␣
 ↪cmap='viridis')
```

[12]: <matplotlib.image.AxesImage at 0x7f635b67d8d0>



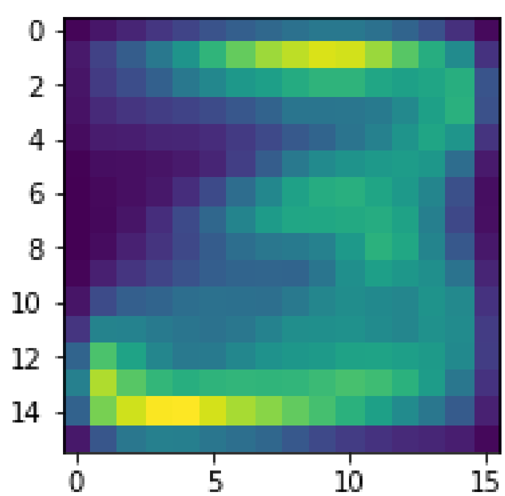
[13]:
```
# Derive the mean
mean = np.mean(X_train, axis=0)
```

[14]:
```
mean.shape
```

[14]: (256,)

[15]:
```
plt.figure(figsize = (10,3))
plt.imshow(mean.reshape((16, 16), order='F'))
```

[15]: <matplotlib.image.AxesImage at 0x7f635b432990>



0.0.5 Q3 (a) Derive the code vectors

[16]:
```python
def predict_1NN(X_train, y_train, X):
    nn = []
    for x in X:
        nn.append(LA.norm(X_train - x, axis=1).argmin())
    return y_train[nn]
```

[17]:
```python
def accuracy(y_predict, y):
    return np.sum(y_predict == y) / float(len(y))
```

[18]:
```python
# Initialize all dimensions
dims = [2, 5, 10, 20, 30]
acc = []
```

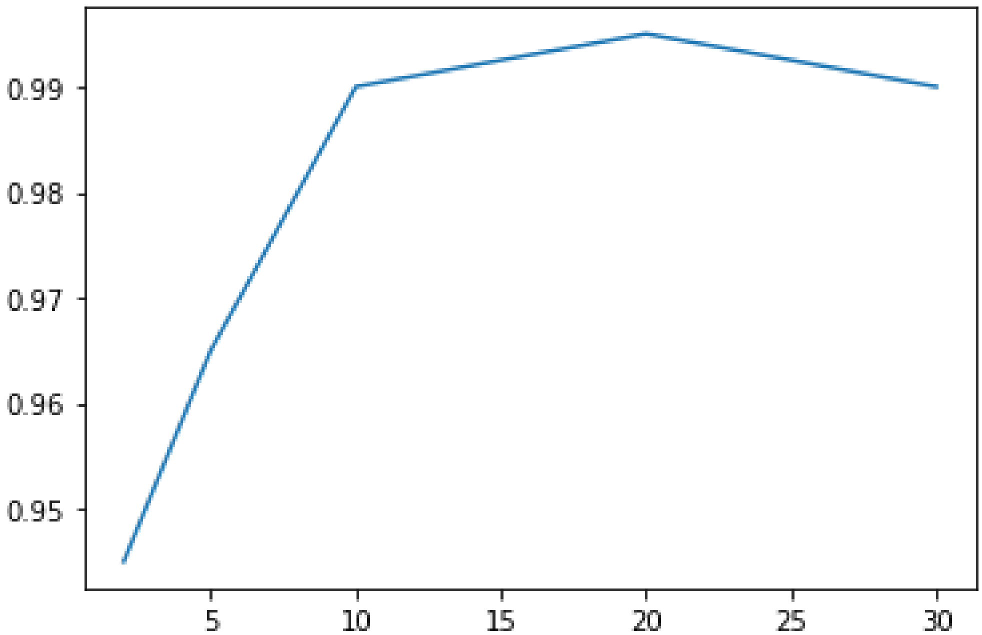
0.0.6 Train with 1NN on the code vectors

[19]:
```python
for dim in dims:
    # Center the data
    Xt_c, Xv_c = X_train - mean, X_valid - mean
    # Project on lower dimensional space
    basis = v[:, arg_w[: dim]]
    Xt_l, Xv_l = Xt_c @ basis, Xv_c @ basis
    # Predict the labels of validation set
    yv_predict = predict_1NN(Xt_l, y_train, Xv_l)
    # Evaluate performance
    acc.append(accuracy(yv_predict, y_valid))
```

0.0.7 Plot the performances with respect to dimensions

[20]:
```python
%matplotlib inline
plt.plot(dims, acc)
```

[20]:
```
[<matplotlib.lines.Line2D at 0x7f635b3a5090>]
```



0.0.8 Q3 (b)

I would choose 20 top eigenvectors, since it reduces the dimensions of the features from 30 to 20. It reduces the cost of the training process and at the same time achieves the highest score on the validation set.

0.0.9 Q3 (c)

[21]:
```
# Center the data
Xtest_c = X_test - mean
# Project on lower dimensional space
basis = v[:, arg_w[: 20]]
Xtest_l, Xt_l = Xtest_c @ basis, Xt_c @ basis
# Predict the labels of validation set
yt_predict = predict_1NN(Xt_l, y_train, Xtest_l)
```

0.0.10 Performance of the final classifier

[22]:
```
accuracy(yt_predict, y_test)
```

[22]:
```
0.99
```