Question3

October 17, 2019

0.1 Question 3

Instruction:

[9]: """

```
In this section, you are asked to train a NN with different hyperparameters.
      To start with training, you need to fill in the incomplete code. There are 3
      places that you need to complete:
      a) Backward pass equations for an affine layer (linear transformation + bias).
      b) Backward pass equations for ReLU activation function.
      c) Weight update equations with momentum.
      After correctly fill in the code, modify the hyperparameters in "main()".
      You can then run this file with the command: "python nn.py" in your terminal.
      The program will automatically check your gradient implementation before start.
      The program will print out the training progress, and it will display the
      training curve by the end. You can optionally save the model by uncommenting
      the lines in "main()".
      from __future__ import division
      from __future__ import print_function
      from util import LoadData, Load, Save, DisplayPlot
      import sys
      import numpy as np
[10]: def InitNN(num_inputs, num_hiddens, num_outputs):
          """Initializes NN parameters.
          Arqs:
              num_inputs:
                            Number of input units.
              num hiddens: List of two elements, hidden size for each layer.
              num_outputs: Number of output units.
          Returns:
             model:
                             Randomly initialized network weights.
```

```
W1 = 0.1 * np.random.randn(num_inputs, num_hiddens[0])
W2 = 0.1 * np.random.randn(num_hiddens[0], num_hiddens[1])
W3 = 0.01 * np.random.randn(num_hiddens[1], num_outputs)
b1 = np.zeros((num_hiddens[0]))
b2 = np.zeros((num_hiddens[1]))
b3 = np.zeros((num_outputs))
model = {
    'V_w1': np.zeros((num_inputs, num_hiddens[0])),
    'V_w2': np.zeros(num_hiddens),
    'V_w3': np.zeros((num_hiddens[1], num_outputs)),
    'W1': W1,
    'W2': W2,
    'W3': W3,
    'b1': b1,
    'b2': b2,
    'b3': b3
return model
```

```
[11]: def Affine(x, w, b):
    """Computes the affine transformation.

Args:
    x: Inputs (or hidden layers)
    w: Weights
    b: Bias

Returns:
    y: Outputs
    """

# y = np.dot(w.T, x) + b
y = x.dot(w) + b
return y
```

```
grad_w: Gradients wrt. the weights.
             grad_b: Gradients wrt. the biases.
         # Insert your code here.
         grad_h = np.dot(grad_y, w.T)
         grad_w = np.dot(h.T, grad_y)
         grad_b = np.sum(grad_y, axis=0)
         return grad_h, grad_w, grad_b
[13]: def ReLU(z):
         """Computes the {\it ReLU} activation function.
         Arqs:
             z: Inputs
         Returns:
             h: Activation of z
         return np.maximum(z, 0.0)
[14]: def ReLUBackward(grad_h, z):
         \hookrightarrow inputs.
         Args:
             z: Inputs
         Returns:
             grad_z: Gradients wrt. the hidden state prior to activation.
         grad_z = grad_h * (z > 0)
         return grad_z
[90]: def Softmax(x):
         """Computes the softmax activation function.
         Args:
             x: Inputs
         Returns:
             y: Activation
         return np.exp(x) / np.exp(x).sum(axis=1, keepdims=True)
[16]: def NNForward(model, x):
         """Runs the forward pass.
     var = {
```

```
Arqs:
    model: Dictionary of all the weights.
           Input to the network.
Returns:
           Dictionary of all intermediate variables.
    var:
11 11 11
z1 = Affine(x, model['W1'], model['b1'])
h1 = ReLU(z1)
z2 = Affine(h1, model['W2'], model['b2'])
h2 = ReLU(z2)
y = Affine(h2, model['W3'], model['b3'])
var = {
    'x': x,
    'z1': z1,
    'h1': h1,
    'z2': z2,
    'h2': h2,
    'y': y
}
return var
```

```
[17]: def NNBackward(model, err, var):
          """Runs the backward pass.
          Args:
              model:
                      Dictionary of all the weights.
                        Gradients to the output of the network.
              err:
                        Intermediate variables from the forward pass.
              var:
          11 11 11
          dE_dh2, dE_dW3, dE_db3 = AffineBackward(err, var['h2'], model['W3'])
          dE_dz2 = ReLUBackward(dE_dh2, var['z2'])
          dE_dh1, dE_dW2, dE_db2 = AffineBackward(dE_dz2, var['h1'], model['W2'])
          dE_dz1 = ReLUBackward(dE_dh1, var['z1'])
          _, dE_dW1, dE_db1 = AffineBackward(dE_dz1, var['x'], model['W1'])
          model['dE_dW1'] = dE_dW1
          model['dE_dW2'] = dE_dW2
          model['dE_dW3'] = dE_dW3
          model['dE_db1'] = dE_db1
          model['dE_db2'] = dE_db2
          model['dE_db3'] = dE_db3
          pass
```

```
[18]: def NNUpdate(model, eps, momentum):
    """Update NN weights.

Args:
```

```
Dictionary of all the weights.
    model:
              Learning rate.
    eps:
    momentum: Momentum.
# Insert your code here.
# Update velocities
model['V_w1'] = momentum * model['V_w1'] + (1 - momentum) * model['dE_dW1']
model['V_w2'] = momentum * model['V_w2'] + (1 - momentum) * model['dE_dW2']
model['V_w3'] = momentum * model['V_w3'] + (1 - momentum) * model['dE_dW3']
# Update the weights and biases.
model['W1'] -= eps * model['V_w1']
model['W2'] -= eps * model['V_w2']
model['W3'] -= eps * model['V_w3']
model['b1'] -= eps * model['dE_db1']
model['b2'] -= eps * model['dE_db2']
model['b3'] -= eps * model['dE_db3']
```

```
[53]: def Train(model, forward, backward, update, hypers, verbose=True, diagram=True):
          %matplotlib tk
          """Trains a simple MLP.
          Args:
                               Dictionary of model weights.
              model:
              forward:
                               Forward prop function.
                              Backward prop function.
              backward:
              update:
                              Update weights function.
              eps:
                               Learning rate.
              momentum:
                              Momentum.
              num_epochs:
                              Number of epochs to run training for.
                              Mini-batch size, -1 for full batch.
              batch_size:
          Returns:
              stats:
                              Dictionary of training statistics.
                  - train_ce:
                                    Training cross entropy.
                  - valid_ce:
                                    Validation cross entropy.
                                    Training accuracy.
                  - train_acc:
                  - valid acc:
                                    Validation accuracy.
          11 11 11
          eps, momentum, num_epochs, batch_size = \
              hypers["eps"], hypers["momentum"], hypers["num_epochs"],
       →hypers["batch_size"]
          inputs_train, inputs_valid, inputs_test, target_train, target_valid, \
              target test = LoadData('./toronto face.npz')
          rnd_idx = np.arange(inputs_train.shape[0])
          train_ce_list = []
          valid_ce_list = []
          train_acc_list = []
```

```
valid_acc_list = []
num_train_cases = inputs_train.shape[0]
if batch_size == -1:
    batch_size = num_train_cases
num_steps = int(np.ceil(num_train_cases / batch_size))
for epoch in range(num_epochs):
    np.random.shuffle(rnd_idx)
    inputs_train = inputs_train[rnd_idx]
    target_train = target_train[rnd_idx]
    for step in range(num_steps):
        # Forward prop.
        start = step * batch_size
        end = min(num_train_cases, (step + 1) * batch_size)
        x = inputs_train[start: end]
        t = target_train[start: end]
        var = forward(model, x)
        prediction = Softmax(var['y'])
        train_ce = -np.sum(t * np.log(prediction)) / x.shape[0]
        train_acc = (np.argmax(prediction, axis=1) ==
                     np.argmax(t, axis=1)).astype('float').mean()
        if verbose:
            print(('Epoch {:3d} Step {:2d} Train CE {:.5f} '
                   'Train Acc {:.5f}').format(
                epoch, step, train_ce, train_acc))
        # Compute error.
        error = (prediction - t) / x.shape[0]
        # Backward prop.
        backward(model, error, var)
        # Update weights.
        update(model, eps, momentum)
    valid_ce, valid_acc = Evaluate(
        inputs_valid, target_valid, model, forward, batch_size=batch_size)
    if verbose:
        print(('Epoch {:3d} '
               'Validation CE {:.5f} '
               'Validation Acc {:.5f}\n').format(
            epoch, valid_ce, valid_acc))
    train_ce_list.append((epoch, train_ce))
    train_acc_list.append((epoch, train_acc))
    valid_ce_list.append((epoch, valid_ce))
    valid_acc_list.append((epoch, valid_acc))
```

```
if diagram:
        DisplayPlot(train_ce_list, valid_ce_list, 'Cross Entropy', number=0)
        DisplayPlot(train_acc_list, valid_acc_list, 'Accuracy', number=1)
if verbose:
    print()
train_ce, train_acc = Evaluate(
    inputs_train, target_train, model, forward, batch_size=batch_size)
valid ce, valid acc = Evaluate(
    inputs_valid, target_valid, model, forward, batch_size=batch_size)
test_ce, test_acc = Evaluate(
    inputs_test, target_test, model, forward, batch_size=batch_size)
print('CE: Train %.5f Validation %.5f Test %.5f' %
      (train_ce, valid_ce, test_ce))
print('Acc: Train {:.5f} Validation {:.5f} Test {:.5f}'.format(
    train_acc, valid_acc, test_acc))
stats = {
    'train_ce': train_ce_list,
    'valid_ce': valid_ce_list,
    'train_acc': train_acc_list,
    'valid_acc': valid_acc_list
}
return model, stats
```

```
[20]: def Evaluate(inputs, target, model, forward, batch_size=-1):
          """Evaluates the model on inputs and target.
          Arqs:
              inputs: Inputs to the network.
              target: Target of the inputs.
              model: Dictionary of network weights.
          num_cases = inputs.shape[0]
          if batch_size == -1:
              batch_size = num_cases
          num_steps = int(np.ceil(num_cases / batch_size))
          ce = 0.0
          acc = 0.0
          for step in range(num_steps):
              start = step * batch_size
              end = min(num_cases, (step + 1) * batch_size)
              x = inputs[start: end]
              t = target[start: end]
              prediction = Softmax(forward(model, x)['y'])
              ce += -np.sum(t * np.log(prediction))
```

```
[21]: def CheckGrad(model, forward, backward, name, x):
          """Check the gradients
          Args:
              model: Dictionary of network weights.
              name: Weights name to check.
              x: Fake input.
          11 11 11
          np.random.seed(0)
          var = forward(model, x)
          loss = lambda y: 0.5 * (y ** 2).sum()
          grad_y = var['y']
          backward(model, grad_y, var)
          grad_w = model['dE_d' + name].ravel()
          w_ = model[name].ravel()
          eps = 1e-7
          grad_w_2 = np.zeros(w_.shape)
          check_elem = np.arange(w_.size)
          np.random.shuffle(check_elem)
          # Randomly check 20 elements.
          check_elem = check_elem[:20]
          for ii in check_elem:
              w_{i} = eps
              err_plus = loss(forward(model, x)['y'])
              w [ii] -= 2 * eps
              err minus = loss(forward(model, x)['v'])
              w_[ii] += eps
              grad_w_2[ii] = (err_plus - err_minus) / 2 / eps
          np.testing.assert_almost_equal(grad_w[check_elem], grad_w_2[check_elem],
                                         decimal=3)
```

```
[23]: # Export name
    model_fname = 'nn_model.npz'
    stats_fname = 'nn_stats.npz'
```

```
[141]: # Setup hyperparameters
hyperparameters = {
        "num_hiddens": [16, 32],
        "eps": 0.1,
        "momentum": 0.0,
        "num_epochs": 1000,
```

```
"batch_size": 100
}
```

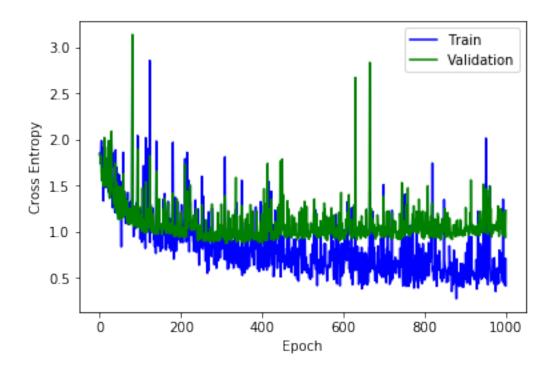
```
[142]: # Input-output dimensions.
       num_inputs = 2304
       num_outputs = 7
       # Initialize model.
       model = InitNN(num_inputs, hyperparameters["num_hiddens"], num_outputs)
       # Uncomment to reload trained model here.
       # model = Load(model_fname)
       # Check gradient implementation.
       print('Checking gradients...')
       x = np.random.rand(10, 48 * 48) * 0.1
       CheckGrad(model, NNForward, NNBackward, 'W3', x)
       CheckGrad(model, NNForward, NNBackward, 'b3', x)
       CheckGrad(model, NNForward, NNBackward, 'W2', x)
       CheckGrad(model, NNForward, NNBackward, 'b2', x)
       CheckGrad(model, NNForward, NNBackward, 'W1', x)
       CheckGrad(model, NNForward, NNBackward, 'b1', x)
```

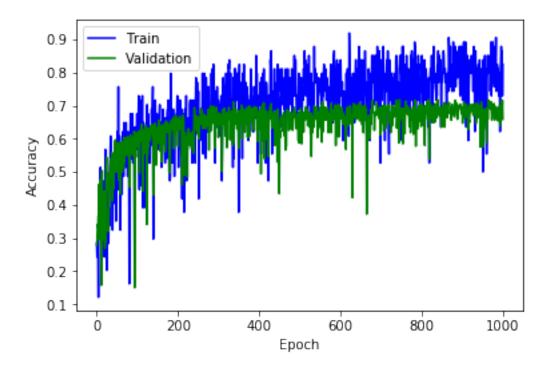
Checking gradients...

0.2 Q 3.1

Train with default parameters

CE: Train 0.72919 Validation 1.23030 Test 1.18131 Acc: Train 0.73681 Validation 0.65871 Test 0.63377





Comment on the stats: The perfomance in terms of accuarcy of validation set is less than the trainning set.

```
[]: #### Plot of cross entropy
! [title] (Figure_0.png)

#### Plot of accuracy
! [title] (Figure_1.png)
```

0.3 Q3.2

```
[26]: import matplotlib.pyplot as plt
[54]: def train_with_hyper_list(hypers, parameters, name):
# Input-output dimensions
```

```
# Input-output dimensions.
  num_inputs = 2304
  num_outputs = 7
  hyper_clone = dict(hypers)
  train_ce = []
  train_acc = []
  valid_ce = []
  valid_acc = []
  for param in parameters:
       # Reinitialize model.
      model = InitNN(num_inputs, hypers["num_hiddens"], num_outputs)
       # Check gradient implementation.
      print('Checking gradients...')
      x = np.random.rand(10, 48 * 48) * 0.1
      CheckGrad(model, NNForward, NNBackward, 'W3', x)
       CheckGrad(model, NNForward, NNBackward, 'b3', x)
       CheckGrad(model, NNForward, NNBackward, 'W2', x)
       CheckGrad(model, NNForward, NNBackward, 'b2', x)
       CheckGrad(model, NNForward, NNBackward, 'W1', x)
       CheckGrad(model, NNForward, NNBackward, 'b1', x)
      print('Gradient check passed...')
      print('Starting trainning...')
      print()
      print(name + " = " + str(param) + ":")
      print()
      hyper_clone[name] = param
       # Train model.
       stats = Train(model, NNForward, NNBackward, NNUpdate, hyper_clone, u
→verbose=False, diagram=False)
       # Only concern the ce of the last iteration
```

```
train_ce.append(stats[1]["train_ce"][-1][1])
    train_acc.append(stats[1]["train_acc"][-1][1])
    valid_ce.append(stats[1]["valid_ce"][-1][1])
    valid_acc.append(stats[1]["valid_acc"][-1][1])
    print()
%matplotlib inline
plt.figure(figsize=(20, 5))
plt.subplot(121)
plt.plot(parameters, train_ce, '-o', label="training set")
plt.plot(parameters, valid_ce, '-o', label="validation set")
plt.xlabel(name)
plt.ylabel("cross entrophy loss")
plt.legend()
plt.subplot(122)
plt.plot(parameters, train_acc, '-o', label="training set")
plt.plot(parameters, valid_acc, '-o', label="validation set")
plt.xlabel(name)
plt.ylabel("accuracy")
plt.legend()
# Report the best hyperparameter setting
argmin = np.argmin(valid_ce)
best = parameters[argmin]
print(name + " choosen: " + str(best))
return best
```

Train with different learning rates

```
[44]: learning_rates = [0.001, 0.1, 0.2, 0.5, 1.0]

[45]: best_eps = train_with_hyper_list(hyperparameters, learning_rates, "eps")

Checking gradients...
Gradient check passed...
Starting trainning...

eps = 0.001:

CE: Train 1.10412 Validation 1.12326 Test 1.15264
Acc: Train 0.60729 Validation 0.57995 Test 0.57662

Checking gradients...
Gradient check passed...
Starting trainning...
```

eps = 0.1:

CE: Train 0.53448 Validation 1.01282 Test 0.96629 Acc: Train 0.80794 Validation 0.69690 Test 0.68312

Checking gradients... Gradient check passed... Starting trainning...

eps = 0.2:

CE: Train 2.45219 Validation 2.53740 Test 2.33438 Acc: Train 0.33817 Validation 0.32936 Test 0.37143

Checking gradients...
Gradient check passed...
Starting trainning...

eps = 0.5:

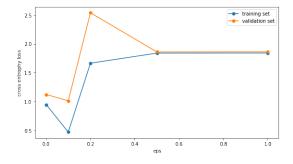
CE: Train 1.86108 Validation 1.85905 Test 1.83904 Acc: Train 0.28542 Validation 0.27924 Test 0.31688

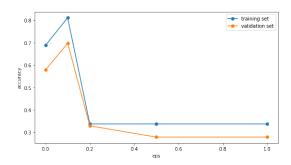
Checking gradients...
Gradient check passed...
Starting trainning...

eps = 1.0:

CE: Train 1.86245 Validation 1.86104 Test 1.84036 Acc: Train 0.28542 Validation 0.27924 Test 0.31688

eps choosen: 0.1



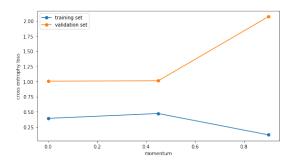


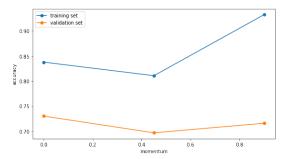
0.3.1 Comment

[84]: hyperparameters["eps"] = best_eps

As the learning rate increase, within 0.1, the larger the learning rate, the cross entropy was more convergent. However, as the learning rate get bigger 0.1, the cross entropy didn't converge as the learning rate increase. It is due to the fact that, the cross entrophy keep escaping from the global minimum.

```
Given the choosen best learning rate, try different momentum.
[47]: momentums = [0.0, 0.45, 0.9]
[48]: best_momentum = train_with_hyper_list(hyperparameters, momentums, "momentum")
     Checking gradients...
     Gradient check passed...
     Starting trainning...
     momentum = 0.0:
     CE: Train 0.38254 Validation 1.00641 Test 0.91246
     Acc: Train 0.86663 Validation 0.73031 Test 0.71169
     Checking gradients...
     Gradient check passed...
     Starting trainning...
     momentum = 0.45:
     CE: Train 0.53448 Validation 1.01282 Test 0.96629
     Acc: Train 0.80794 Validation 0.69690 Test 0.68312
     Checking gradients...
     Gradient check passed...
     Starting trainning...
     momentum = 0.9:
     CE: Train 0.30314 Validation 2.07246 Test 2.10522
     Acc: Train 0.89923 Validation 0.71599 Test 0.66234
     momentum choosen: 0.0
```





0.3.2 Comment

As the momentum increases, the convergence of the training set doesn't change a lot, but the convergence of the validation set increases.

```
[83]: hyperparameters["momentum"] = best_momentum
```

Given the choosen best momentum and learning rate, try different mini-batch sizes.

```
[59]: batches = [200, 500, 800, 900, 1000]
```

Checking gradients...
Gradient check passed...
Starting trainning...

batch_size = 200:

CE: Train 0.66596 Validation 0.97049 Test 0.88406 Acc: Train 0.76378 Validation 0.68019 Test 0.68831

Checking gradients...
Gradient check passed...
Starting trainning...

batch_size = 500:

CE: Train 1.00303 Validation 1.08501 Test 1.08839 Acc: Train 0.63367 Validation 0.62053 Test 0.59221

Checking gradients... Gradient check passed... Starting trainning... batch_size = 800:

CE: Train 1.29053 Validation 1.29646 Test 1.32629 Acc: Train 0.52786 Validation 0.54415 Test 0.52727

Checking gradients...
Gradient check passed...
Starting trainning...

batch_size = 900:

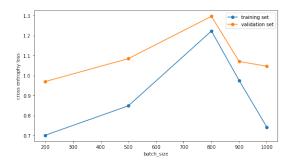
CE: Train 1.01786 Validation 1.07067 Test 1.03939 Acc: Train 0.62596 Validation 0.63246 Test 0.60260

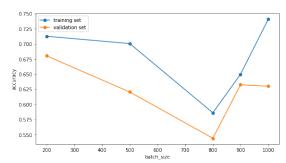
Checking gradients... Gradient check passed... Starting trainning...

batch_size = 1000:

CE: Train 0.98664 Validation 1.04708 Test 1.07973 Acc: Train 0.63041 Validation 0.63007 Test 0.62857

batch_size choosen: 200





0.3.3 Comment

The cross entrophy losses of both training and test set first increase until the batch size reach 800. After 800, the cross entrophy losses start to decrease. However, the accuracy of predicting the label on the validation set is never as high as the when the batch size is 200. It is obvious that, when the batch is 200, the difference of the cross entrophy losses between the validation set and the training set is the greatest. It indicates that the smaller batch size helps to reduce overfit of the model.

[85]: hyperparameters["batch_size"] = best_batch_size

When tuning the eps, momentum, and batch size of the network, I first tried different values of learning rate and select the one which yeilded the lowest cross validation error, then fixed the momentum and batch size one by one by fixing the others and trying different values and select the one yielded the lowest cross validation error.

0.4 Q3.3

Try 3 different values fo the number of hiddent units

```
[82]: # Fix momentum
     hyperparameters ["momentum"] = 0.9
      # Change learning rate and number of epochs
      hyperparameters["eps"] = 0.1
      hyperparameters["num_epochs"] = 400
      index = 0
      hidden_units = [[1, 2], [30, 60], [50, 100]]
      for hidden_unit in hidden_units:
          # Reinitialize model.
          model = InitNN(num_inputs, hidden_unit, num_outputs)
          # Uncomment to reload trained model here.
          # model = Load(model fname)
          # Check gradient implementation.
          print('Checking gradients...')
          x = np.random.rand(10, 48 * 48) * 0.1
          CheckGrad(model, NNForward, NNBackward, 'W3', x)
          CheckGrad(model, NNForward, NNBackward, 'b3', x)
          CheckGrad(model, NNForward, NNBackward, 'W2', x)
          CheckGrad(model, NNForward, NNBackward, 'b2', x)
          CheckGrad(model, NNForward, NNBackward, 'W1', x)
          CheckGrad(model, NNForward, NNBackward, 'b1', x)
          print('Gradient check passed...')
          print()
          print('Starting trainning...')
          print("hiddent_unit" + " = " + str(hidden_unit) + ":")
          # Train model.
          stats = Train(model, NNForward, NNBackward, NNUpdate, hyperparameters,
       →verbose=False, diagram=False)
          %matplotlib inline
          plt.figure(figsize=(20, 5))
          plt.subplot(121)
          train = np.array(stats[1]["train_ce"])
          valid = np.array(stats[1]["valid_ce"])
```

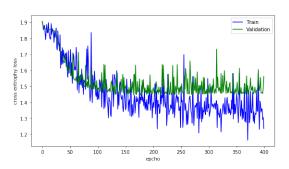
```
plt.plot(train[:, 0], train[:, 1], 'b', label='Train')
plt.plot(valid[:, 0], valid[:, 1], 'g', label='Validation')
plt.xlabel("epcho")
plt.ylabel("cross entrophy loss")
plt.legend()
plt.subplot(122)
train = np.array(stats[1]["train_acc"])
valid = np.array(stats[1]["valid_acc"])
plt.plot(train[:, 0], train[:, 1], 'b', label='Train')
plt.plot(valid[:, 0], valid[:, 1], 'g', label='Validation')
plt.xlabel("epcho")
plt.ylabel("accuracy")
plt.legend()
plt.draw()
plt.pause(0.0001)
print()
```

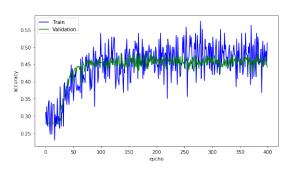
Checking gradients...
Gradient check passed...

Starting trainning...

hiddent_unit = [1, 2]:

CE: Train 1.39660 Validation 1.56225 Test 1.50709 Acc: Train 0.46680 Validation 0.44153 Test 0.44935



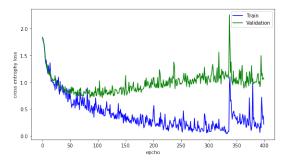


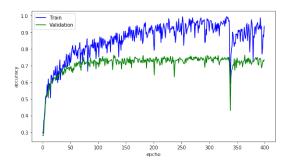
Checking gradients...
Gradient check passed...

Starting trainning...

hiddent_unit = [30, 60]:

CE: Train 0.25548 Validation 1.06624 Test 0.96192 Acc: Train 0.90427 Validation 0.73508 Test 0.70130



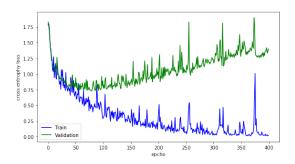


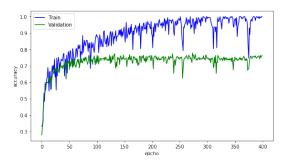
Checking gradients...
Gradient check passed...

Starting trainning...

hiddent_unit = [50, 100]:

CE: Train 0.01521 Validation 1.39853 Test 1.02426 Acc: Train 0.99911 Validation 0.76372 Test 0.77403





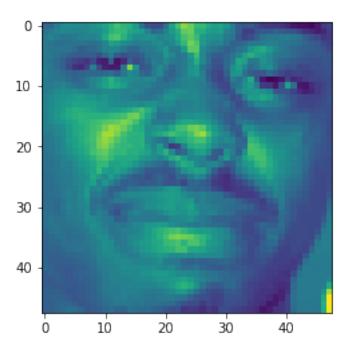
0.4.1 Comment:

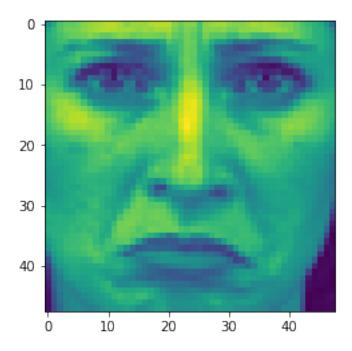
The networks with more hidden units have more generality. It can be seen from the fact that the accuracy on the validation and test set is higher than the network with less hidden units. However, The networks with more hidden unit are more likely to overfit, since the accuracy on the training set is much higher than the accuracy on the validation set on those networks. The networks with only 3 hidden units is underfitting, and the accuracy difference is very small between the training set and validation set.

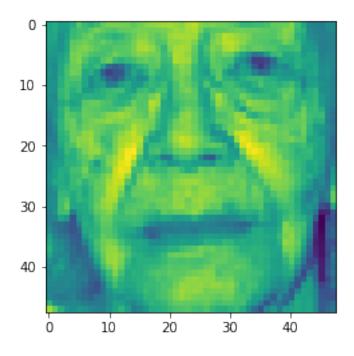
$0.5 \quad Q3.4$

```
[88]: # Setup hyperparameters
       hyperparameters = {
           "num_hiddens": [16, 32],
           "eps": 0.1,
           "momentum": 0.0,
           "num_epochs": 1000,
           "batch_size": 100
       }
       # Set everything to the best configuration
       hyperparameters["eps"] = best_eps
       hyperparameters["momentum"] = best_momentum
       hyperparameters["batch_size"] = best_batch_size
[91]: # Train the final model
       model = InitNN(num_inputs, hyperparameters["num_hiddens"], num_outputs)
       stats = Train(model, NNForward, NNBackward, NNUpdate, hyperparameters,
       →verbose=False, diagram=False)
       # Load the test data
       inputs_train, inputs_valid, inputs_test, target_train, target_valid, \
           target_test = LoadData('./toronto_face.npz')
      CE: Train 0.63639 Validation 0.96965 Test 1.16427
      Acc: Train 0.76615 Validation 0.71360 Test 0.70130
[136]: # Predict label with the given model
       y = NNForward(model, inputs_test)['y']
       # Transfer the prediction to probabilities
       prob = Softmax(y)
       # Calculate the score (confidence level) for each sample
       scores = np.amax(prob, axis=1)
       # Find the unconfident prediction
       unconfident = []
       for i in range(len(scores)):
           if scores[i] < 0.3:</pre>
               unconfident.append(i)
[137]: unconfident
[137]: [87, 276, 339]
```

```
[138]: %matplotlib inline
for image_index in unconfident:
    plt.imshow(np.reshape(inputs_test[image_index], newshape=(48,48)))
    plt.pause(0.0001)
```







0.5.1 Comment

It is hard even human being to define what emotion the person is in the image. For example, in the first image, It is hard to tell if the person is happy or sad. And for the second image, it hard to tell if the person is angry or sad. Therefore, it is not likely that the network will predict it correctly.

[]: