1. (a) $x^3 = 9$

$$f(x) = x^3 - 9$$
$$f(2.05) = 2.05^3 - 9 = -0.3849$$
$$f(3.05) = 3.05^3 - 9 = 19.3726$$

the interval is $[2.05, 3.05]$

(b) $3x^3 + x^2 = x + 5$

$$f(x) = 3x^3 + x^2 - x - 5$$
$$f(1.1) = 3 \times 1.1^3 + 1.1^2 - 1.1 - 5 = -0.897$$
$$f(2.1) = 3 \times 2.1^3 + 2.1^2 - 2.1 - 5 = 25.093$$

the interval is $[1.1, 2.1]$

(c) $\cos^2 x + 6 = x$

$$f(x) = \cos^2 x - x + 6$$
$$f(2\pi) = \cos^2(2\pi) - 2\pi + 6 = 7 - 2\pi > 0$$
$$f(2\pi + 1) = \cos^2(2\pi + 1) - (2\pi + 1) + 6 = \cos^2(2\pi + 1) - 1.28 < 0$$

the interval is $[2\pi, 2\pi + 1]$

2. (a) $x^5 + x = 1$

$$f(x) = x^5 + x - 1$$
$$f(0) = -1 < 0$$
$$f(1) = 1 > 0$$

the interval is$[0, 1]$

(b) $\sin x = 6x + 5$

$$f(x) = \sin x - 6x - 5$$
$$f(0) = \sin 0 - 5 = -5 < 0$$
$$f(-1) = \sin(-1) + 6 - 5 = \sin(-1) + 1 > 0$$

the interval is$[-1, 0]$

(c) $\ln x + x^2 = 3$

$$f(x) = \ln x + x^2 - 3$$

$$f(\sqrt{3}-1) = \ln(\sqrt{3}-1) + (\sqrt{3}-1)^2 - 3 = \ln(\sqrt{3}-1) + 3 - 2\sqrt{3} + 1 - 3 < 0$$

$$f(\sqrt{3}) = \ln\sqrt{3} + \sqrt{3}^2 - 3 = \ln\sqrt{3} > 0$$

the interval is $[\sqrt{3}-1, \sqrt{3}]$

3. (a)

k	a_k	$f(a_k)$	c	$f(c_k)$	b	$f(b_k)$
0	2.05	$-$	2.55	$+$	3.05	$+$
1	2.05	$-$	2.3	$+$	2.55	$+$
2	2.05	$-$	2.175	$+$	2.3	$+$

(b)

k	a_k	$f(a_k)$	c	$f(c_k)$	b	$f(b_k)$
0	1.1	$-$	1.6	$+$	2.1	$+$
1	1.1	$-$	1.35	$+$	1.6	$+$
2	1.1	$-$	1.225	$+$	1.35	$+$

(c)

k	a_k	$f(a_k)$	c	$f(c_k)$	b	$f(b_k)$
0	2π	$-$	6.7832	$-$	$2\pi+1$	$+$
1	6.7832	$-$	7.0332	$-$	$2\pi+1$	$+$
2	7.0332	$-$	7.1582	$-$	$2\pi+1$	$+$

4. (a)

k	a_k	$f(a_k)$	c	$f(c_k)$	b	$f(b_k)$
0	0	$-$	0.5	$-$	1	$+$
1	0.5	$-$	0.75	$-$	1	$+$
2	0.75	$-$	0.875	$+$	1	$+$

(b)

k	a_k	$f(a_k)$	c	$f(c_k)$	b	$f(b_k)$
0	-1	$-$	-0.5	$-$	0	$+$
1	-0.5	$-$	-0.25	$-$	0	$+$
2	-0.25	$-$	-0.125	$-$	0	$+$

(c)

k	a_k	$f(a_k)$	c	$f(c_k)$	b	$f(b_k)$
0	$\sqrt{3}-1$	$-$	1.2321	$-$	$\sqrt{3}$	$+$
1	1.2321	$-$	1.4821	$-$	$\sqrt{3}$	$+$
2	1.4821	$-$	1.6071	$+$	$\sqrt{3}$	$+$

5. $x^4 = x^3 + 10$

(a)

$$f(x) = x^4 - x^3 - 10$$
$$f(2) = 2^4 - 2^3 - 10 = -2 < 0$$
$$f(3) = 3^4 - 3^3 - 10 = 44 > 0$$

the interval is $[2, 3]$.

(b)

$$\frac{1}{2^{n+1}} < 10^{-10}$$
$$2^{n+1} > \frac{1}{10^{-10}}$$
$$2^{n+1} > 10^{-10}$$
$$n + 1 > \frac{10}{log_{10}2} = 33.2193$$

33 steps are required.

6. The Bisection Method will converge to a real number, but which is not the root.