习题2.

1. (1) 总体：这批机器零件寿命

样本：从总体中随机抽取的8个零件寿命

样本值：$x=(230, 243, 185, 240, 228, 196, 246, 200)$

样本容量：$n=8$

(2) $\bar{x}=221 \qquad s^2=566$

$\bar{\xi}^2 = s^2 + 8(\bar{\xi})^2 = 391294$

2. (1) $\sum(\xi_i-a)^2 = \sum(\xi_i-\bar{\xi}+\bar{\xi}-a)^2$

$= \sum(\xi_i-\bar{\xi})^2 + 2\sum(\xi_i-\bar{\xi})(\bar{\xi}-a) + \sum(\bar{\xi}-a)^2$

$= \sum(\xi_i-\bar{\xi})^2 + n(\bar{\xi}-a)^2$

(2) $\sum(\xi_i-\bar{\xi})^2 = \sum\xi_i^2 - 2\sum\xi_i\bar{\xi} + n\sum\bar{\xi_i}^2$

$= \sum\xi_i^2 - 2\bar{\xi}\sum\xi_i + n\bar{\xi}^2$

$= \sum\xi_i^2 - \bar{\xi}^2$

3. (1) $\bar{\xi} = \frac{1}{n}\sum\xi_i = \frac{1}{n}\sum(\sigma\eta+u) = \sigma\bar{\eta}+u$

(2) $S_\eta^2 = \frac{1}{n}\sum(\eta_i-\bar{\eta})^2 = \frac{1}{\sigma^2}S^2$

4. 由题可知 $\xi_i \sim B(1,p)$

所以有 $E(\xi_i)=p$ $\quad D(\xi_i)=p(1-p)$

(1) $E(\bar{\xi})=E\xi=p$

$D(\bar{\xi})=D(\frac{1}{n}\sum\xi_i)=\frac{1}{n^2}D\sum\xi_i=\frac{1}{n^2}\cdot n(1-p)\cdot p$

$=\frac{1}{n}p(1-p)$

(2) $E(S^{*2})=E(\frac{n}{n-1}S^2)=D\xi=p(1-p)$

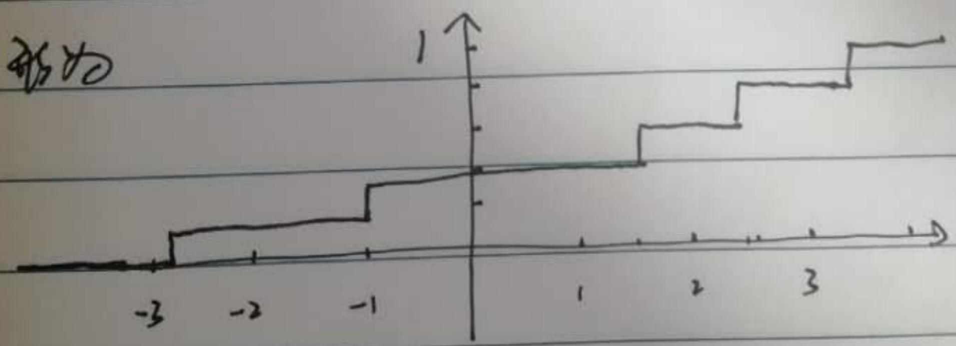
(3) $S^2=\frac{1}{n}\sum(\xi_i-\bar{\xi})^2=\underline{\frac{1}{n}(\sum\xi_i^2}-n\bar{\xi}^2)$

$=\bar{\xi}-\bar{\xi}^2=\bar{\xi}(1-\bar{\xi})$

6. 经验分布函数为

$$F_n(x)=\begin{cases}0, & x\le -2.8\\ 1/5, & -2.8<x\le -1\\ 2/5, & -1<x\le 1.5\\ 3/5, & 1.5<x\le 2.1\\ 4/5, & 2.1<x\le 3.4\\ 1, & 3.4<x\end{cases}$$

图形为



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能 力 提 升 内 容

7. 令 $\eta_1=\xi_1+\xi_2+\xi_3$ $\eta_2=\xi_4+\xi_5+\xi_6$

则：$\eta_1 \sim N(0.3)$，$\eta_2 \sim N(0.3)$

且 η_1, η_2 相互独立

则有：$(\frac{\eta_1}{\sqrt{3}})^2 \sim \chi^2(1)$ $(\frac{\eta_2}{\sqrt{3}})^2 \sim \chi^2(1)$

$c\eta = c\eta_1^2 + c\eta_2^2 = 3c((\frac{\eta_1}{\sqrt{3}})^2+(\frac{\eta_2}{\sqrt{3}})^2)$

有：$(\frac{\eta_1}{\sqrt{3}})^2+(\frac{\eta_2}{\sqrt{3}})^2 \sim \chi^2(2)$

即：当 $3c=1$，$c=\frac{1}{3}$ 时，随机变量 $c\eta$ 服从 χ^2 分布

能力提升内容

9. 已知总体 $\xi \in N(u, \sigma^2)$

令 $\eta = \sum\left(\frac{\xi_i - u}{\sigma}\right)^2 \sim \chi^2(n)$

$\sum(\xi_i - u)^2 = \sigma^2 \cdot \eta^{\frac{n}{2}}$ 的特征函数

$$\varphi_{\sigma^2 n}(\sigma^2 t) = (1 - 2\sigma^2 t i)^{\frac{n}{2}}$$

$\sum(\xi_i - u)^2 = \sigma^2 \cdot \eta^{\frac{n}{2}}$ 的分布密度

$$p(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-itx}(1 - 2\sigma^2 t i)^{-\frac{n}{2}} dt$$

$$p(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-x\frac{ui}{\sigma^2}}(1 - 2ui)^{-\frac{n}{2}} \frac{1}{\sigma^2} du.$$

$$= \begin{cases} 0, & x \le 0 \\ \frac{1}{\sigma^2} \cdot \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})}\left(\frac{x}{\sigma^2}\right)^{\frac{n}{2}-1} e^{-\frac{x}{2\sigma^2}}, & x > 0 \end{cases}$$

$$= \begin{cases} 0, & x \le 0 \\ \frac{1}{\sigma^n} \cdot \frac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} x^{\frac{n}{2}-1} e^{-\frac{x}{2\sigma^2}}, & x > 0. \end{cases}$$

12. 因为 $\bar{\xi} \sim N(u, \frac{\sigma^2}{n})$，而 $\xi_{n+1} \sim N(u, \sigma^2)$

且与 $\xi_1, \xi_2, \cdots, \xi_n$ 相互独立. 则

$$(\xi_{n+1} - \bar{\xi}) \sim N\left[u, \left(1 + \frac{1}{n}\right)\sigma^2\right]$$

所以有 $\left(\dfrac{\xi_{n+1}-\bar{\xi}}{\sqrt{1+\frac{1}{n}}\cdot\sigma}\right)\sim N(0,1)$，又 $\because \dfrac{nS^2}{\sigma^2}\sim\chi^2(n-1)$

且 $\dfrac{\xi_{n+1}-\bar{\xi}}{\sqrt{1+\frac{1}{n}}\cdot\sigma}$ 与 $\dfrac{nS^2}{\sigma^2}$ 相互独立

因此，$\eta=\dfrac{\xi_{n+1}-\bar{\xi}}{S}\cdot\sqrt{\dfrac{n-1}{n+1}}=\left(\dfrac{\xi_{n+1}-\bar{\xi}}{\sqrt{1+\frac{1}{n}}\cdot\sigma}\right)\Big/\sqrt{\dfrac{\frac{nS^2}{\sigma^2}}{n-1}}\sim t(n-1)$

13. 已知 $\xi\sim t(n)$，则有 $\xi=\dfrac{\alpha}{\sqrt{\beta/n}}$，

其中 $\alpha\sim N(0,1)$，$\beta\sim\chi^2(n)$，且 α,β 相互独立

又因为 $\alpha^2\sim\chi^2(1)$

$\therefore\ \xi^2=\dfrac{\frac{\alpha^2}{1}}{\frac{\beta}{n}}\sim F(1,n)$

15. (1) $u_{0.99}=2.3263$. (2) $u_{0.04}=-1.7507$. (3) $\chi^2_{0.975}(15)=27.4884$

(4) $\chi^2_{0.025}(15)=6.2621$. (5) $\chi^2_{0.95}(50)=67.5048$ (6) $\chi^2_{0.95}(100)=124.3421$

(7) $t_{0.975}(19)=2.0930$. (8) $t_{0.975}(99)=1.9842$. (9) $F_{0.95}(2,6)=5.1433$

(10) $F_{0.99}(3,40)=4.3126$. (11) $F_{0.05}(2,6)=0.0517$.

(12) $F_{0.01}(3,40)=0.0379$.

能力提升内容

23. (1). 因为 $E\bar{\xi} = E\xi = Np$. $D\bar{\xi} = \frac{D\xi}{n} = \frac{Np(1-p)}{n}$

由中心极限定理知，样本均值 $\bar{\xi}$ 的极限分布为

$\bar{\xi} \sim N\left(Np, \frac{Np(1-p)}{n}\right)$

(2) 因为 $E\bar{\xi} = E\xi = \lambda$. $D\bar{\xi} = \frac{D\xi}{n} = \frac{\lambda^2}{n}$

由中心极限定理知，样本均值 $\bar{\xi}$ 的极限分布为

$\bar{\xi} \sim N\left(\lambda, \frac{\lambda^2}{n}\right)$

26. (1) 样本极差 $R^2 = \xi_{(2)} - \xi_{(1)}$

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}}$$

$$F(x) = \int_{-\infty}^{x} p(x)dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}} dx$$

$$F(x+z) - F(x) = \int_{x}^{x+z} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-u)^2}{2\sigma^2}} dx$$

$$F_{R_2}(z) = \begin{cases} 0, & z \le 0 \\ 2\int_{-\infty}^{0} [F(x+z) - F(x)]\, p(x)\, dx, & z > 0. \end{cases}$$

$$= \begin{cases} 0, & z \le 0 \\ 2\int_{-\infty}^{0} [F(x+z) - F(x)]\, d\, F(x), & z > 0 \end{cases}$$

$$= \begin{cases} 0, & z \le 0 \\ 2\int_{-\infty}^{0} F(x+z)\, d(F(x)) - 1, & z > 0 \end{cases}$$

$$= \begin{cases} 0, & z \le 0 \\ 2\int_{-\infty}^{0} \cdots, & z > 0 \end{cases}$$

(2) $F_{\xi_{(1)}}(x) = 1-[1-F(x)]^2$

(3) $F_{\xi_{(2)}}(x) = [F(x)]^2$

27.(1) $p(x) = \begin{cases} 4, & \theta-\frac{1}{2}<x<\theta+\frac{1}{2} \\ 0, & 其它. \end{cases}$

$F(x) = \int_{\theta-\frac{1}{2}}^{x} 4\,dx = 4(x-\theta+\frac{1}{2})$

$\xi_{(1)}$ 的分布密度

$p_{\xi_{(1)}}(x) = n[1-F(x)]^{n-1}p(x) = \begin{cases} n(4\theta-4x-1)^{n-1}4. & (\theta-\frac{1}{2},\theta+\frac{1}{2}) \\ 0 & 其它 \end{cases}$

(2)

$p_{\xi_{(n)}}(x) = n[F(x)]^{n-1}p(x) = \begin{cases} n(4x-4\theta+2)^{n-1}\cdot 4, & \theta-\frac{1}{2}<x<\theta+\frac{1}{2} \\ 0, & 其它 \end{cases}$

(3)

$p_{\xi_{(1)},\xi_{(2)}}(x,y) = \begin{cases} n(n-1)[F(y)-F(x)]^{n-2}p(x)\cdot p(y)., & x<y \\ 0., & x\geq y \end{cases}$