

习题2.

1. (1) 总体: 这批机器零件毛坯

样本: 从总体中随机抽取的 8 个零件毛坯

样本值: $x = (230, 243, 185, 240, 228, 196, 246, 200)$

样本容量: $n = 8$

$$(2) \quad \bar{x} = 221 \quad s^2 = 566$$

$$\bar{s}^2 = s^2 + 8(\bar{x})^2 = 391294$$

$$\begin{aligned} 2. (1) \quad \sum (\xi_i - a)^2 &= \sum (\xi_i - \bar{\xi} + \bar{\xi} - a)^2 \\ &= \sum (\xi_i - \bar{\xi})^2 + 2 \sum (\xi_i - \bar{\xi})(\bar{\xi} - a) + \sum (\bar{\xi} - a)^2 \\ &= \sum (\xi_i - \bar{\xi})^2 + n(\bar{\xi} - a)^2 \end{aligned}$$

$$\begin{aligned} (2) \quad \sum (\xi_i - \bar{\xi})^2 &= \sum \xi_i^2 - 2 \sum \xi_i \bar{\xi} + n \bar{\xi}^2 \\ &= \sum \xi_i^2 - 2 \bar{\xi} \sum \xi_i + n \bar{\xi}^2 \\ &= \sum \xi_i^2 - \bar{\xi}^2 \end{aligned}$$

$$3. (1) \quad \bar{\xi} = \frac{1}{n} \sum \xi_i = \frac{1}{n} \sum (\sigma \eta + u) = \sigma \bar{\eta} + u$$

$$(2) \quad S_{\eta}^2 = \frac{1}{n} \sum (\eta_i - \bar{\eta})^2 = \frac{1}{\sigma^2} S^2$$

4. 由题可知 $\xi_i \sim B(1, p)$

所以有 $E(\xi_i) = p$ $D(\xi_i) = p(1-p)$

(1) $E(\bar{\xi}) = E\{\xi\} = p$

$$D(\bar{\xi}) = D\left(\frac{1}{n} \sum \xi_i\right) = \frac{1}{n^2} D \sum \xi_i = \frac{1}{n^2} \cdot n(1-p) \cdot p$$

$$= \frac{1}{n} p(1-p)$$

(2) $E(S^2) = E\left(\frac{n}{n-1} S^2\right) = D\xi = p(1-p)$

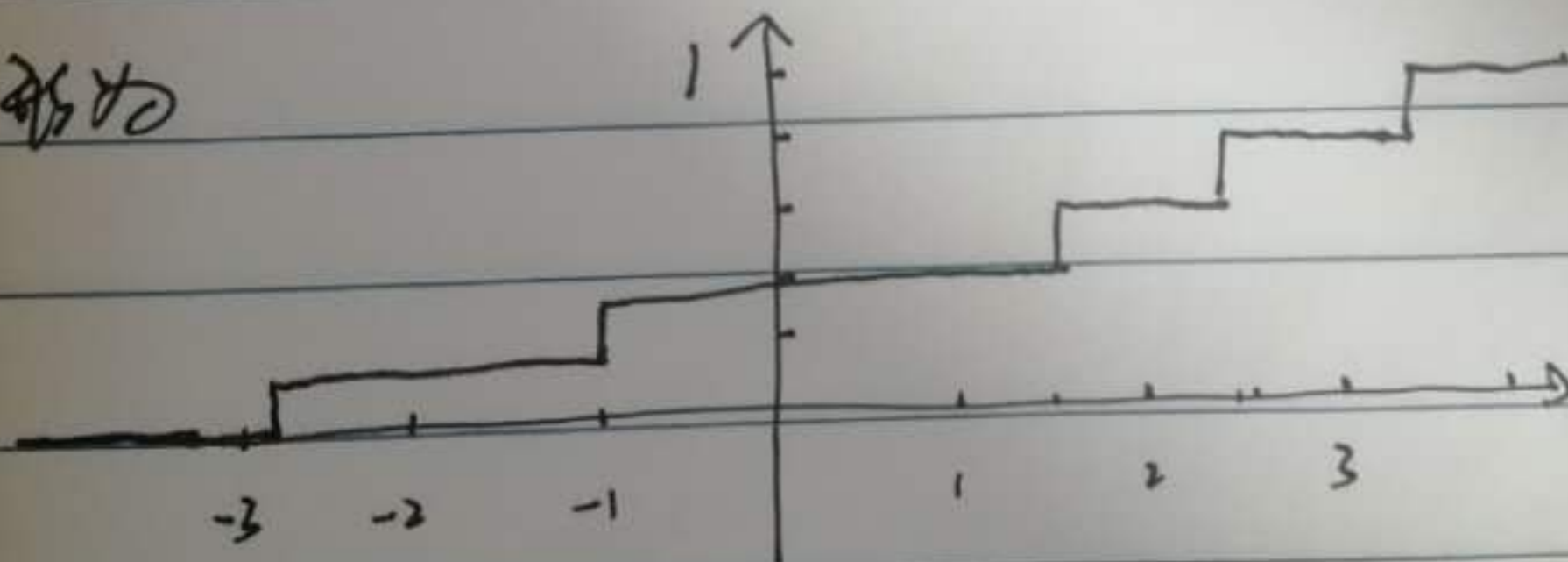
(3) $S^2 = \frac{1}{n} \sum (\xi_i - \bar{\xi})^2 = \frac{1}{n} (\sum \xi_i^2 - n\bar{\xi}^2)$

$$= \bar{\xi} - \bar{\xi}^2 = \bar{\xi}(1-\bar{\xi})$$

6. 经验分布函数为

$$F_n(x) = \begin{cases} 0 & , x \leq -2.8 \\ 1/5 & , -2.8 < x \leq -1 \\ 2/5 & , -1 < x \leq 1.5 \\ 3/5 & , 1.5 < x \leq 2.4 \\ 4/5 & , 2.4 < x \leq 3.4 \\ 1 & , 3.4 < x \end{cases}$$

图形为



遇顺境处之淡然，逢逆境处之泰然。

能力提升内容

7. 令 $\eta_1 = \xi_1 + \xi_2 + \xi_3$ $\eta_2 = \xi_4 + \xi_5 + \xi_6$

则 $\eta_1 \sim N(0, 3)$, $\eta_2 \sim N(0, 3)$

且 η_1, η_2 相互独立

则有: $(\frac{\eta_1}{\sqrt{3}})^2 \sim \chi^2(1)$ $(\frac{\eta_2}{\sqrt{3}})^2 \sim \chi^2(1)$

$c\eta = c\eta_1^2 + c\eta_2^2 = 3c[(\frac{\eta_1}{\sqrt{3}})^2 + (\frac{\eta_2}{\sqrt{3}})^2]$

有: $(\frac{\eta_1}{\sqrt{3}})^2 + (\frac{\eta_2}{\sqrt{3}})^2 \sim \chi^2(2)$

即: 当 $3c=1$, $c=\frac{1}{3}$ 时, 随机变量 $c\eta$ 服从 χ^2 分布