Part 2, Question 1

R: ABCDEFGHI

$S = \{ A \to BC,\quad AD \to E,\quad BD \to FG,\quad BDH \to I \}$

a)

$A \to BC \qquad A^+ = ABC$

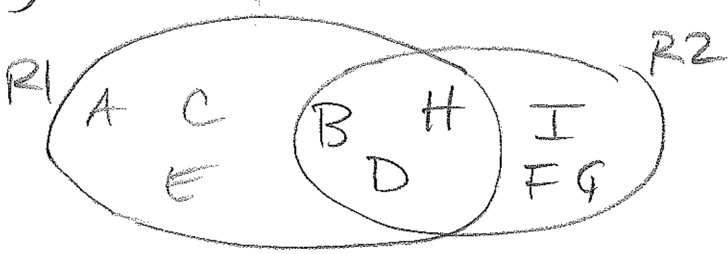
$AD \to E \qquad AD^+ = ADEBCFG$

$BD \to FG \qquad BD^+ = BDFG$

$BDH \to I \qquad BDH^+ = BDHIFG$

} all violate BCNF

b) Choosing to decompose based on $BDH \to I$



R1: ABCDEH

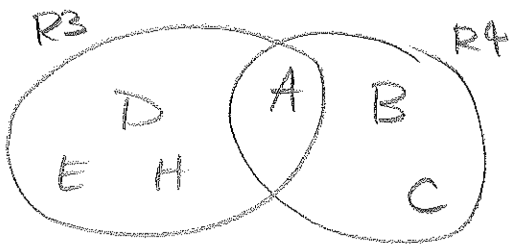
$A^+ = ABC \therefore A \to BC$

Not a superkey

$\therefore$ split again

R2: BDFGHI

see next page



R3: ADEH

$AD^+ = ADEBCFG$

$\therefore AD \to E$

Not a superkey

$\therefore$ split again

R4: ABC

$A^+ = ABC \qquad \therefore A \to BC$

superkey for this relation

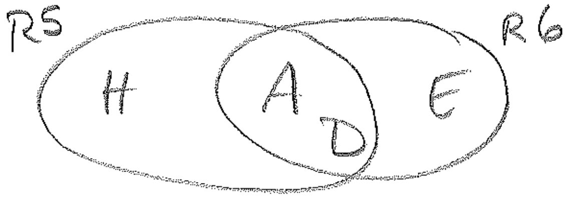
$B^+ = B$

$C^+ = C$

$\cancel{AB^+ =}$ superset of A

$AC^+ = AC$

$BC^+ = BC$



R5: ADH

$A^+ = ABC$

$D^+ = D$

$H^+ = H$

$AD^+ = ADEBCFG$

$AH^+ = AHBC$

$DH^+ = DH$

∴ no FDs!

R6: ADE

$A^+ = ABC$

$D^+ = D$

$E^+ = E$

$AD^+ = ADEBCFG$ ∴ $AD \to E$

a superkey for this relation ∴ satisfies BCNF

$AE^+ = AEBC$

$DE^+ = DE$

(all 2-attribute relations satisfy BCNF!)

R2: BDFGHI

$BD^+ = BDFG$ ∴ $BD \to FG$

not a superkey ∴ split

R7: BDHI

B, D, H and I alone yield nothing

$BD^+ = BDFG$

$BH^+ = BH$

$BI^+ = BI$

$DH^+ = DH$

$DI^+ = DI$

$BDH^+ = BDHIFG$ ∴ $BDH \to I$

a superkey for this relation

$BDI^+ = BDIFG$

$BHI^+ = BHI$

$DHI^+ = DHI$

R8 : BDFG

B, D, F and G alone yield nothing

$BD^+ = BDFG$

$BF^+ = BF$

$BG^+ = BG$

$DF^+ = DF$

$DG^+ = DG$

$FG^+ = FG$

$\cancel{BDF^+} =$ superset of BD

$\cancel{BDG^+} =$ " " "

$BFG^+ = BFG$

$DFG^+ = DFG$

$\therefore BD \rightarrow FG$

a superkey for this relation

$\boxed{\text{Final decomposition}}$

R4 : ABC $\quad A \rightarrow BC$

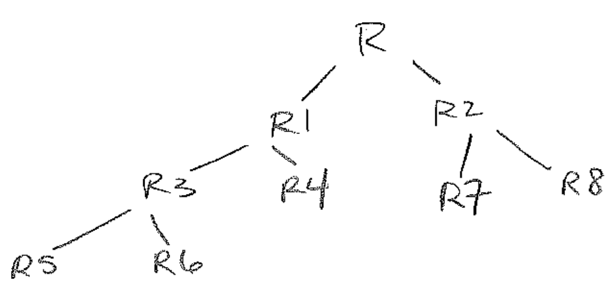
R5 : ADH $\quad \phi$

R6 : ADE $\quad AD \rightarrow E$

R7 : BDHI $\quad BDH \rightarrow I$

R8 : BDFG $\quad BD \rightarrow FG$

Aside: Summary of splits :



Part 2, Question 2

a) All keys

Observation: B is not on the RHS of any FD
∴ it is part of every key.

$B^+ = BDFCEA$

∴ B is a key

∴ No superset of B is a key.

∴ the <u>only</u> key is B.

Phew!
That spared me 2^6 closures!

b) Remove redundant FDs

S =	1. $ACDE \rightarrow F$	$ACDE^+_{S-\{\#1\}} = ACDE$
	2. $B \rightarrow D$	$B^+_{S-\{\#2\}} = BF$
	~~3. $B \rightarrow F$~~	$B^+_{S-\{\#3\}} = BDCEF$
	4. $BCDF \rightarrow A$	$BCDF^+_{S-\{\#3,4\}} = BCDFE$
	~~5. $BD \rightarrow C$~~	$BD^+_{S-\{\#3,5\}} = BDEFC$
	6. $BD \rightarrow E$	$BD^+_{S-\{\#3,5,6\}} = BDF$
	7. $BD \rightarrow F$	$BD^+_{S-\{\#3,5,7\}} = BDE$
	8. $BEF \rightarrow C$	$BEF^+_{S-\{\#3,5,8\}} = BEFD$
	~~9. $BEF \rightarrow D$~~	$BEF^+_{S-\{\#3,5,9\}} = BEFD$

Simplify LHSs

$S' =$ 1. $ACDE \to F$ $\quad ACD^+ = ACD \quad ACE^+ = ACE$
$\quad CDE^+ = CDE \quad ADE^+ = ADE$ } ∴ can't even remove one attribute from LHS

2. $B \to D$ can't simplify

4. $B\cancel{C}\cancel{D}\cancel{F} \to A$ $\quad BCD^+ = BCDFA$ ∴ remove F from LHS
$Bc^+ = BCDEFCA$ ∴ remove D also.
$B^+ = BDEFCA$ ∴ remove C also.

6. $B\cancel{D} \to E$ $\quad B^+ = BDE$ ∴ remove D

7. $B\cancel{D} \to F$ $\quad B^+ = BDEF$ ∴ remove D

8. $B\cancel{E}\cancel{F} \to C$ $\quad B^+ = BDEFC$
∴ simplify LHS to B

Remove redundant FDs

$S'' =$ 1. $ACDE \to F$ $\quad ACDE^+_{S'' - \{\#1\}} = ACDE$

2. $B \to D$

4. $B \to A$

6. $B \to E$

} Not redundant since, for each, there is no other way to get the RHS.

~~7. $B \to F$~~ $\quad B^+_{S'' - \{\#7\}} = BDAECF$

8 $B \to C$ } No other way to get C

No other simplifications are possible.

Minimal basis:

$ACDE \rightarrow F$

$B \rightarrow ACDE$

c) R1: ACDEF

R2: ABCDE

$\left.\begin{array}{l} R1: ACDEF \\ R2: ABCDE \end{array}\right\}$ 3NF schema.

d) Must project the FDs onto R1 + R2 to check for potential redundancy.

R1: ACDEF

clearly, nothing in R1 < ACDE yields any other attributes

So the only FD is $ACDE \rightarrow F$.

This admits no redundancy, since the LHS is a superkey for R1.

R2: ABCDE

Clearly, no subset of ABCDE, other than B alone, yields any other attributes in R2.

$B \rightarrow ACDE$ is ∴ the only FD.

Again, it admits no redundancy: The LHS is a superkey for R2.

There, our 3NF schema does not allow redundancy.