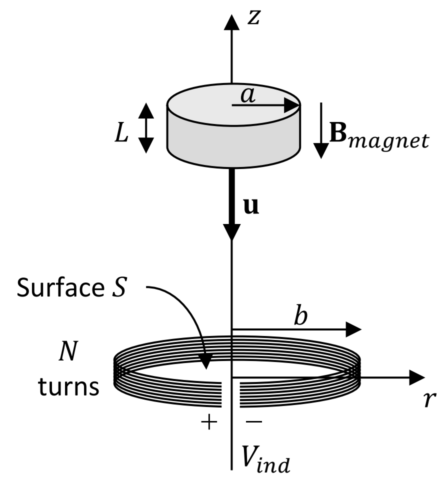
LAB 6

The purpose of Lab #6 is to make use of your current understanding of MATLAB and the functions that you have previously written to solve an engineering problem. This problem is based around an electromagnetic flashlight, which relies on its energy from the movement of a magnet through a coil of wire.

INTRODUCTION



As part of this lab, you will use MATLAB to calculate the induced voltage caused by a magnet which is falling through a coil of wire with a velocity **u** (see the figure to the right). Read through the posted article titled, "*Faraday's Law – Quantitative Experiments*" by R.C. Nicklin which was published in the American Journal of Physics in May 1986. Make sure you read the highlighted sections. This paper discusses a simple experiment which demonstrates how a voltage can be induced by a changing magnetic field.

Recall that this law states:

$$\begin{bmatrix} \textit{Voltage induced} \\ \textit{in a coil of wire} \end{bmatrix} = -\begin{bmatrix} \textit{Number of turns} \\ \textit{the coil has} \end{bmatrix} \begin{bmatrix} \textit{The time rate of change of the} \\ \textit{magnetic flux through the coil} \end{bmatrix}$$

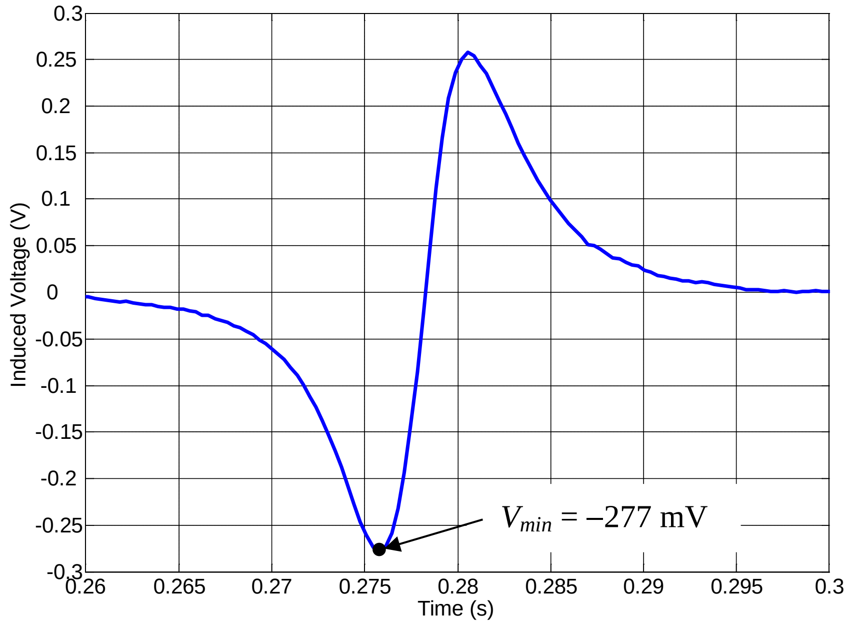
or

$$V_{ind} = -N_{coil}\frac{d\Phi}{dt} = -N_{coil}\frac{d}{dt}\iint_S \mathbf{B}\cdot\mathbf{ds}$$

MATLAB is often used to provide a graphical comparison between results measured in a laboratory experiment and results that are based on a theoretical analysis. This type of comparison is critical in research, as it can provide validation to a new theory which has been developed through the research. Or, if the theory has already been well established, then it can demonstrate that the experimental setup and procedure are correct.

This lab will focus on this type of comparison. Shown below is an example of a measured induced voltage waveform for this setup. In this experiment, the magnet had a radius of a = 0.5 in = 1.27 cm, a length of L = 3/8 in ≈ 1 cm, and fell from a height of h = 15 in = 38 cm before it passed through a 10 turn loop, which had a radius of b = 11/16 in = 1.75 cm. This was measured in the Electromagnetics labs here at the University of Toronto using an oscilloscope attached to a coil.

Part of your lab preparation involves the development of a function which can calculate the induced voltage as a magnet falls through a coil. This will result in the ability to generate a waveform, i.e., a graph of the induced voltage versus time.



As discussed in the paper by R.C. Nicklin, the voltage induced in a coil as a magnet falls through it is given by:

$$V_{ind} = -N_{coil}\frac{d}{dt}\iint_S \mathbf{B}\cdot\mathbf{ds} = -N_{coil}\int_0^b\int_0^{2\pi}\left(\frac{dB_z}{dt}\right)(rd\phi dr) = -N_{coil}(2\pi)\int_0^b\left(\frac{dB_z}{dz}\frac{dz}{dt}\right)(rdr) \qquad \text{Eqn. (1)}$$

Given the expression for B_z of a circular cylindrical magnet, which is provided in the paper, one can evaluate dB_z/dz , and then the induced voltage as a function of time becomes:

$$V_{ind}(t) = -\frac{N_{coil}u(t)\mu_0 M_0 a}{2}\int_0^b\int_0^{2\pi}\left(\frac{a-r\cos\phi}{\alpha^2}\right)\left[\frac{1}{\sqrt{\alpha^2+[z(t)+L/2]^2}}-\frac{1}{\sqrt{\alpha^2+[z(t)-L/2]^2}}-\frac{[z(t)+L/2]^2}{\{\alpha^2+[z(t)+L/2]^2\}^{3/2}}+\frac{[z(t)-L/2]^2}{\{\alpha^2+[z(t)-L/2]^2\}^{3/2}}\right]rd\phi dr$$

with Eqn. (2)

$$\alpha^2 = r^2 + a^2 - 2ra\cos\phi \quad \text{and} \quad z(t) = -\frac{1}{2}gt^2 + h$$

Using this expression, one can write a simple MATLAB function to theoretically calculate the induced voltage waveform of a magnet falling through a coil.

PREPARATION - Individual

1. Some very important things should be observed about Eqn. (1) given in the introduction above. To identify these, answer the following questions:

   a) Why is it that V_{ind} depends only on the *z*-component of the magnetic flux density?

   b) Why is it that the time derivative can be moved inside the integral?

   c) What does the term $\frac{dz}{dt}$ represent? If the magnet starts from rest, i.e., $u(0) = 0$, what would $\frac{dz}{dt}$ be given by?

   d) By replacing the integration with respect to ϕ by 2π what assumption is being made?

   Make sure you understand the answers to these questions, as you will be asked about these during the lab session.

2. Use the final expression for V_{ind} given in the introduction, i.e., Eqn. (2), to write a MATLAB function which calculates the voltage induced in a coil over a range of time values when a magnet falls through it. Your inputs should be the specifications of the problem (i.e., N_{coil}, M_0, a, L, b, h, and t), and you can assume that the magnet starts from rest [i.e, at $z = h$, $t = 0$ s and $u(t = 0\text{ s}) = 0$.] Since the timing of the center of the induced voltage pulse will depend on h, your code should first determine a reasonable time vector so that you are not wasting time calculating the induced voltage when the magnet is far away from the coil (either above or below).

   Since your function will have to evaluate this double integral for each value of *t*, you should make use of MATLAB's built-in integration functions (i.e., `quad2d`). These are based on algorithms that are more efficient then the basic `for` loop structure.

   As an example, consider the function `quadl`[1]. Using this function we can implement the integral:

$$B_z = \frac{\mu_0 M_0 a}{4\pi} \int_0^{2\pi} \left(\frac{a - r\cos\phi'}{\alpha^2} \right) \left[\frac{(L/2 - z)}{\sqrt{\alpha^2 + (z - L/2)^2}} + \frac{(L/2 + z)}{\sqrt{\alpha^2 + (z + L/2)^2}} \right] d\phi'$$

   where

$$\alpha^2 = r^2 + a^2 - 2ra\cos\phi'$$

   with the MATLAB code:

```
alphasq=@(phi,r,a) r^2+a^2-2*r*a*cos(phi);
integrand=@(phi) (uo*Mo*a/(4*pi))*((a-r*cos(phi))./(alphasq(phi,r,a))).*((L/2-z)./
((alphasq(phi,r,a)+(z-L/2)^2).^0.5)+(L/2+z)./((alphasq(phi,r,a)+(z+L/2)^2).^0.5));

Bz=quadl(integrand,0,2*pi);
```

[1] see http://www.mathworks.com/access/helpdesk/help/techdoc/ref/quadl.html

This code also makes use of *anonymous functions*[2], which have the basic syntax of:

```
functionname = @(argumentlist) expression
```

These are also called "in-line" functions since they allow you to define a function within another function without having to create a completely separate m-file. In the above example, when the built-in function `quadl` is called it passes the vector `phi` to the anonymous function `integrand`. Due to the fact that within the "function" `integrand`, `phi` is a vector, we are dividing, multiplying, and squaring a vector so we must use dotted operations (i.e., `./`, `.*`, and `.^0.5`).

The "function" `integrand` then calls the anonymous function `alphasq` and passes the vector `phi` and the two numbers `r` and `a`. Since we are now evaluating an expression that does not depend on two vectors being multiplied or divided, for example, we do not need the dot operators in this function.

For the evaluation of the double integral which is required to find V_{ind}, you should base your code around the structure given above, but make use of the function `quad2d`[3].

To verify that your function is working correctly try to test the following cases for specific values of time (i.e., set your time vector to a single value), most of which are taken from the measured V_{ind} shown above:

Experimental Variables:	$N_{coil} = 10$, $M_0 = 7.675\times10^5$ A/m, $a = 1.27$ cm, $L = 1$ cm, $b = 1.75$ cm, $h = 38$ cm ($z = h$ at $t = 0$)	
Time	**Measured V_{ind}**	**Calculated V_{ind}**
270 ms	–60.2 mV	
283 ms	+172 mV	
288 ms	+43 mV	
250 ms	--	
300 ms	--	

You must come to your Lab #6 Lab Session with a final version of your *working* code.

[2] see http://www.mathworks.com/access/helpdesk/help/techdoc/MATLAB_prog/f4-70115.html

[3] See http://www.mathworks.com/access/helpdesk/help/techdoc/ref/quad2d.html

IN-LAB WORK - Group

Design of an Electromagnetic Flashlight

1. Induced Voltage Due to a Falling Magnet: Theoretical Comparison

The function that you have written during your preparation for this lab can now be used to compare the *measured* waveform given in the preparation to the waveform determined from your numerical MATLAB calculations, which is based on fundamental electromagnetic *theory*. The measured data is available to you in a text file, *Vindmeas.txt*, in the Lab Notes section of the course website. If you imported this data into MATLAB, your workspace should then contain a variable called `Vindmeas`, which is a matrix of size 2942 x 2. The first column is the time in seconds, and the second column is the measured induced voltage in Volts.

Once this measured data is imported into your workspace, you can generate the *theoretically-based* waveform by running your own MATLAB function. Since the waveform is centered about 278.3 ms, you should calculate V_{ind} over the minimum range of 260 ms to 300 ms.

Now, generate a plot that compares these two waveforms using a solid blue line for the measured results and a series of red o's for the calculated results.

Answer the following questions about this induced voltage waveform:

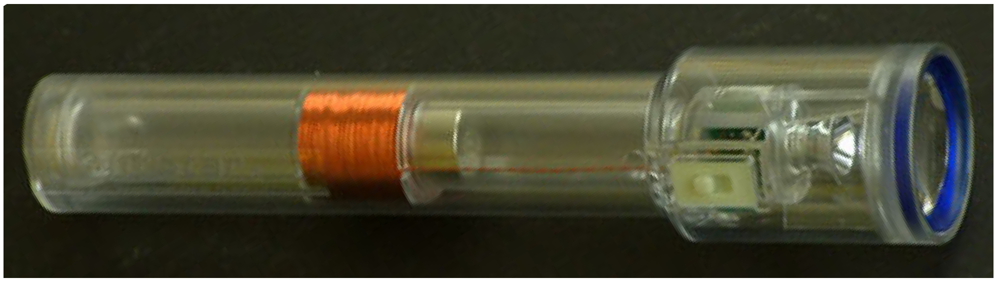
(a) Why does the waveform have an "odd" function characteristic about its midpoint?

(b) If the magnet was dropped from a height of h = 10 cm, rather than h = 38 cm, how would the induced waveform change? List at least two changes. Recall, that we have defined t = 0 s as the time at which the magnet begins to fall.

2. Induced Voltage Due to a Falling Magnet: Electromagnetic Flashlight Design

The main purpose for developing a theoretical model of a physical system is so that we can use that model to design or engineer a "new" system that solves a specific problem. In this part you will use your theoretically-based model that you have coded into a MATLAB function to "solve" a simple problem. You can have some faith that this is a good model, since you have just verified that it matched up quite well with a measured result. Generally, the verification against a single measurement is not sufficient to prove the validity of a model, but it is enough for our purposes.

Consider the design of an electromagnetic flashlight, which consists of a magnet which can be moved through a coil, as illustrated below. As the flashlight is shaken, the energy generated can be stored in either a capacitor or a rechargeable battery. With the use of low-power LEDs, it is possible to generate enough energy to light the LEDs for a considerable time (~30 minutes) with a reasonable number of "shakes" (30 – 50).



The purpose of this part of the lab is to make use of your code which calculates the induced voltage waveform to assess a collection of possible flashlight designs. Listed below are a number of flashlight designs about which your "supervisor" asked for your expert opinion. These designs have been developed based on a number of factors including cost, ergonomics, and manufacturability. It is up to you to use your engineering abilities to determine how well these designs will work.

In order to determine if these designs will "work" or not, you need to decide upon a reasonable "metric", about which to gauge the effectiveness of that design. In this case, the metric will be the *peak value of the induced voltage for each "shake"*.

Hint: To quickly find the peak voltage of an induced waveform you can using the MATLAB function `max` (type `help max` for more information). For example:

```
>>[Vmaxmeas,Index_maxmeas]=max(abs(Vindmeas(:,2)));
Vmaxmeas = 0.2773   and   Index_maxmeas = 812
```

This example demonstrates that the measured voltage waveform has a peak absolute voltage of 277.3 mV, and this occurs at the 812th element of the vector, or at a time of `Vindmeas(812,1)=0.2758` seconds.

It can be shown that the larger the peak value of the voltage, the larger the energy generated for that "shake". Therefore, ask your TA to provide you with a set of possible design options and from these select the one that seems to provide the best electrical performance.

Design Option	**Values Common to all Designs** $N_{coil} = 500$, $M_0 = 7.675\times10^5$ A/m, $b = 2$ cm		
	Design Specifics ($z = h$ at $t = 0$)	**Calculated Peak V_{ind}**	**Design Choice**
A	$a_A =$, $L_A =$, $h_A =$		
B	$a_B =$, $L_B =$, $h_B =$		
C	$a_C =$, $L_C =$, $h_C =$		
D	$a_D =$, $L_D =$, $h_D =$		

Note: When comparing these designs you need to make sure that you consider the fact that as the drop height, h, changes the center of the induced pulse will also change. To support your design choice, you must plot all four of the induced voltage waveforms on a single plot.

Make sure you discuss this section of the lab with your TA before you leave and be prepared to support your choice by explaining the reasons why this design outperforms the others.

Bonus Assumptions of the Model, Practical Implementation, and Estimate of Energy Generation

In the above analysis we have used the expression for V_{ind} due to a falling magnet to assess the relative performance of a number of flashlight designs. Describe one assumption that we have made by modeling the operation of this flashlight with the "falling magnet" analysis.

If we connect this coil to a capacitor, we can store the energy generated by each shake. However, due to the "odd" function characteristics of the induced voltage the net energy transferred will be zero. What electrical component must be add to the system to make sure that net energy stored with each shake is positive.

Suppose that we are able to make use of the total energy generated by each shake, and this energy was stored in a 1 mF capacitor. If we would like to power a white LED light for 30 minutes, how many shakes would be needed if you used your flashlight design choice from above? Assume that the LED light consumes 70 mW of power. *Hint*: To approximately calculate the derivative of a vector, such as `Vind`, you can use the function `diff`. Also, to approximately integrate a vector you can use the function `trapz`.