Analysis of Rectangular Coaxial Lines

Sarhan M. Musa and Matthew N. O. Sadiku
College of Engineering
Prairie View A&M University
Prairie View, TX 77446 USA
Email: smmusa, mnsadiku@pvamu.edu

Abstract **– This paper proposes a numerical analysis for calculating the capacitance of rectangular coaxial lines. We illustrate how to model and simulate the capacitance of rectangular coaxial lines using COMSOL, a finite element software. The goal is to use COMSOL to determine the capacitance per unit length of rectangular coaxial lines. We compared the results with those obtained by other methods and found them to be very close.**

I. Introduction

Development in microwave circuits using rectangular coaxial lines as transmission medium has been improving over the past decades [1]. Reid and Webster used rectangular coaxial transmission lines to fabricate a 60 GHz branch line coupler [2]. Xu and Zhou used the finite difference time domain method for analyzing satellite beamforming network consisting of rectangular coaxial lines [3].

Rectangular coaxial lines have been analyzed using different techniques. Such techniques include conformal mapping method [4-5], orthonormal block method [6], quasianalytical method [7], finite difference method [8], and other numerical techniques [9-10]. In this paper, we use COMOSL [11], a finite element package, to model a rectangular coaxial transmission lines. We specifically consider a rectangular line, a square line, and a rectangular line with a diamondwise inner structure.

II. Results and Discussions

The rectangular coaxial line consists of a two-conductor transmission system along which TEM wave propagates. The characteristic impedance of such a lossless line is given by

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{cC} \qquad (1)$$

where

Z_o = characteristic impedance of the line

L = inductance per unit length of the line

C = capacitance per unit length of the line

$c = 3 \times 10^8 m/s$ (the speed of light in vacuum)

As shown in Fig. 1, a rectangular coaxial line consists of inner and outer rectangular conductors with a dielectric material separating them.

Using COMSOL for each type of the rectangular lines involves taking the following steps:

1. Develop the geometry of the inner and outer conductors, such as shown in Fig. 2.
1. Select both conductors/rectangle and take the difference.
2. We select the relative permittivity as 1 for the difference in step 2. For the boundary, we select the outer conductor as ground and inner conductor as port.
3. We generate the finite element mesh as in Fig. 3.
4. We solve the model and obtain the potential shown in Fig.4.
5. As postprocessing, we select Point Evaluation and choose capacitance element 11 to find the capacitance per unit length of the line.

We now consider the three models.

A. *Rectangular cross-section transmission line*

For COMSOL, we use the following values.

Dielectric material:

$\varepsilon_r = 1$, $\sigma = 0$ S/m

Conducting material:

$\varepsilon_r = 1$, $\sigma = 5.8 \times 10^7$ S/m (copper)

where

ε_o = permittivity of free space

$= \frac{1}{36\pi} \times 10^{-9} = 8.854 \times 10^{-12} F/m$.

ε_r = dielectric constant

μ_o = permeability of free space

$= 4\pi \times 10^{-7} = 1.257 \times 10^{-6} H/m$.

σ = conductivity of the conductor

a = width of the inner conductor = 1 mm

b = height of the inner conductor = 0.8 mm

A= width of the outer conductor = 2.2 mm

B = height of the outer conductor = 2 mm

From the COMSOL model, we obtained the capacitance per unit length (based on the dimensions given above) as 72. 94264 pF/m. Using the finite difference (FD) method, we obtained the capacitance per unit length of the line as 71.51 pF/m. Table 1 shows the comparison of the characteristic impedance (using eq. 1) of several methods. It is evident from the table that the results are very close. Zheng used multipole theory method to calculate characteristic impedance values for rectangular transmission line [12], Chen used conformal transformation method [4], Costamagna used Schwarz-Christoffel transformation [18], Lau used approximate derived

1-4244-1280-3/07/$25.00 ©2007 IEEE

formula [13], and we used finite difference method and COMSOL [11] involving finite element analysis.

TABLE 1
COMPARISON OF CHARACTERISTIC IMPEDANCE VALUES OF RECTANGULAR COAXIAL LINE

Name	Z_0
Zheng et al. [12]	45.789
Chen [4]	45.759
Costamagna and Fanni [18]	45.767
Lau [13]	45.778
Finite difference	46.612
COMSOL [11]	45.70

B. Square cross-section transmission line

This is only a special case of the rectangular line. We used the same values for the dielectric and conducting materials. We used the following dimensions for the line.

a = width of the inner conductor = 2 mm
b = height of the inner conductor = 2 mm
A= width of the outer conductor = 4 mm
B = height of the outer conductor = 4 mm

From the COMSOL model, we obtained the capacitance per unit length as 90. 6957 pF/m. Using the finite difference (FD) method, we obtained the capacitance per unit length of the line as 90.714 pF/m. Zheng used Multipole theory method to calculate characteristic impedance values for square transmission lines [12], Lau used approximate derived formula [13], Cockcroft used analysis of right-angle electrode configurations [14], Bowan used interactive analytical conformal transformation [15], Green used finite-difference analysis [16], Ivanov used a rapidly converging interaction method, Costamagna used Schwarz-Christoffel transformation [18], Riblet used series expansion for the elliptic integral [19], and we used finite difference method and finite element analysis involving COMSOL [11]. Table 2 presents the comparison of the characteristic impedance of several methods. It is evident from the table that the results are in agreement.

TABLE 2
COMPARISON OF CHARACTERISTIC IMPEDANCE VALUES OF SQUARE COAXIAL LINE

Name	Z_0
Zheng et al. [12]	36.79
Lau [13]	36.81
Cockcroft [14]	36.80
Bowan [15]	36.81
Green [16]	36.58
Ivanov and Djankov [17]	36.97
Costamagna and Fanni [18]	36.81
Riblet [19]	36.80
Finite difference	36.75
COMSOL [11]	36.75

C. Rectangular line with Diamondwise structure

The geometry of the cross-section of this line is shown in Fig. 5. The same dielectric and conducting materials used for the rectangular are used for this line. The following values are used for the COMSOL model of the line.

a = width of the inner conductor = 1.414 mm
b = height of the inner conductor = 1.414 mm
d = 1 mm
A= width of the outer conductor = 4 mm
B = height of the outer conductor = 4 mm

For the COMSOL model, we obtained the capacitance per unit line as 57.39278 pF/m.

Fig. 6 illustrates the mesh of the Square cross-section transmission line. Figs. 7 and 8 are the potential distribution of the diamondwise structure with 45° offset angle along y equal to 1 mm and 2 mm respectively. Zheng used multipole theory method to calculate characteristic impedance values for rectangular, square, and diamonwise structure transmission lines [12], Bowan used electric field analysis [15], Riblet used series expansion for the elliptic integral [19], and we used COMSOL involving finite element analysis [11]. Table 3 displays the comparison of the characteristic impedance of several methods. It is evident from the table that the results are in good agreement.

TABLE 3
COMPARISON OF CHARACTERISTIC IMPEDANCE VALUES OF DIAMONDWISE STRUCTURE

Name	Z_0
Zheng et al. [12]	56.742
Bowan [15]	56.745
Riblet [19]	56.745
COMSOL [11]	58.079

III. Conclusion

The capacitance per unit length of rectangular coaxial lines has been calculated using COMSOL, a finite element package. The results of the COMSOL model are compared with those obtained by other methods and found to be in good agreement.

References

[1]. M. L. Crawford, "Generation of standard EM fields using TEM Transmission cells," *IEEE Trans. Electromagn. Compat.*, vol. EMC-16, pp. 189-195, Nov. 1974.

[2]. J.R. Reid and R.T. Webster," A 60 GHz branch line coupler fabricated using integrated rectangular coaxial lines," *Microwave Symposium Digest, 2004 IEEE MTT-S International,* Vol. 2, 6-11 June 2004 PPs.41 - 444 .

[3]. S. Xu and P. Zhou, "FDTD analysis for satellite BFN consisting of rectangular coaxial lines,"*Asia Pacific Microwave Conference,* 1997, pp. 877-880.

[4]. T. S. Chen,"_Determination of the Capacitance, Inductance, and Characteristic Impedance of Rectangular Lines**,"** *IEEE Transactions on Microwave Theory and Techniques*, Volume 8, Issue 5, Sep 1960 Page(s):510 – 519.

[5]. W. Lin nad Y. Xiang," Electrostatic force on the walls of a rectangular coaxial line," *Journal of Electrostatics*, Vol. 43, pp.275-283, 1998.

[6]. O. R. Cruzan and V. G. Garver, "Characteristic impedance of rectangular coaxial transmission lines," *IEEE Transactions on Microwave Theory and Techniques*,.Volume 12, Issue 5, Sep 1964, pp. 488 – 495.

[7]. Q. Zheng, F Xie, Li Ma, and J. Li,"Multiple theory analysis of TE and TM modes in a rectangular coaxial transmission-line family,"*Microwave and optical technology letters*, Vol. 24, No. 6, 2000.

[8]. M. V. Schneider," Computation of impedance and attenuation of TEM-line by finite difference methods,"*IEEE Transactions on Microwave Theory and Techniques,* Vol. MTT-13, No. 6, Nov. 1965, pp. 793-800.

[9]. H. A. Kalhor," Computer aided analysis of characteristic impedance of rectangular coaxial lines,"*Computers & Elec. Engng.,*Vol. 16, No. 2, 1990, pp. 95-98.

[10]. H. Shibata, S. Minakawa, and R. Terakado," A numerical calculation of the capacitance for the rectangular coaxial line with offset inner conductor having an anisotropic dielectric," *IEEE Transactions on Microwave Theory and Techniques,* Vol. MTT-31, No. 5, May 1983, pp. 385-391.

[11]. http://www.comsol.com/

[12]. Q. Zheng, W. Lin, F. Xie, and M. Li, "Multipole theory analysis of a rectangular transmission line family," *Microwave and Optical Technology Letters,* Vol. 18, No. 6, pp. 382-384, Aug. 1998.

[13]. K. H. Lau," Loss calculation for rectangular coaxial lines," *IEE Proceedings,* Vol. 135, Pt. H. No. 3, pp. 207-209, June 1988.

[14]. J. D. Cockcroft, "The effect of curved boundaries on the distribution of electrical stress round conductors," *J. IEE,* Vol. 66, pp. 385-409, Apr. 1926.

[15]. F. Bowan," Notes on two dimensional electric field problems," *Proc. Lond. Math. Soc.,* Vol. 39, No. 211, pp. 205-215, 1935.

[16]. H. E. Green," The characteristic impedance of square coaxial line," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-11, pp. 554-55, Nov.1963.

[17]. S. A. Ivanov and G. L. Djankov," Determination of the characteristic impedance by a step current density approximation," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-32, pp. 450-452, Apr. 1984.

[18]. E. Costamagna and A. Fanni,"Analysis of rectangular coaxial structures by numerical inversion of the Schwarz-Christoffel transformation," *IEEE Trans. Magn.,* Vol. 28, pp. 1454-1457, Mar. 1992.

[19]. H. J. Riblet, "Expansion for the capacitance of a square in a square with a comparison," *IEEE Trans. Microwave Theory Tech.,* Vol. 44, pp. 338-340, Feb. 1996.

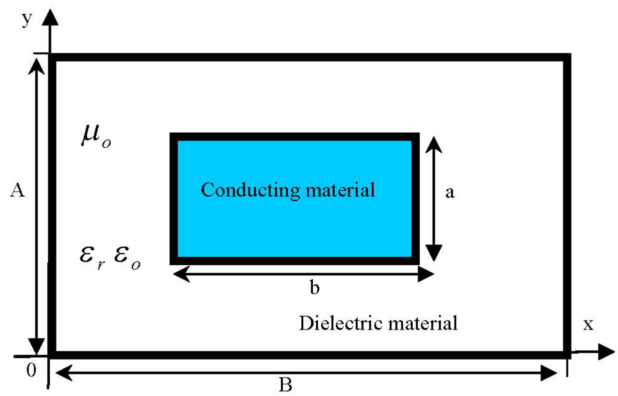

Fig. 1. Cross-section of the rectangular coaxial line.

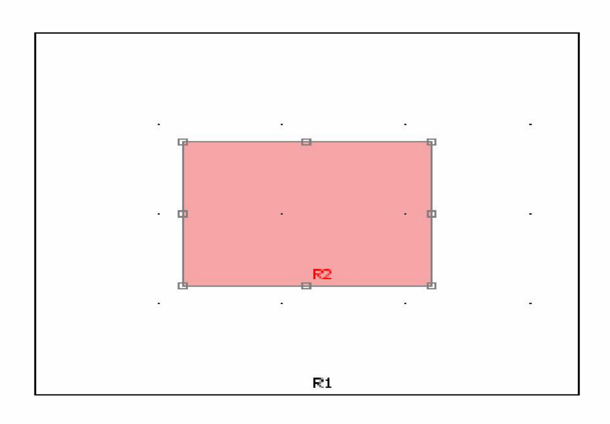

Fig. 2. Geometry of the rectangular coaxial line model.

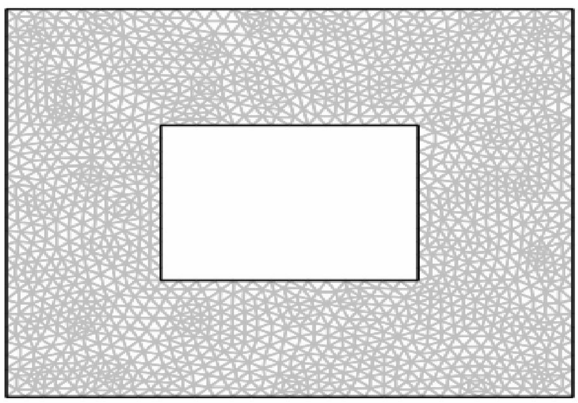

Fig. 3. Mesh of the rectangular coaxial line.

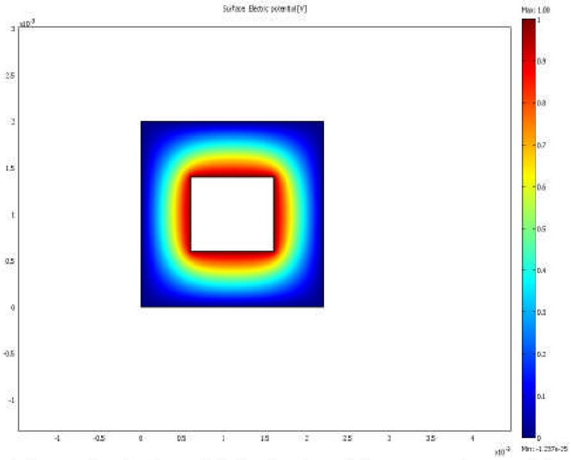

Fig. 4. Image for the Potential distribution of the rectangular coaxial line.

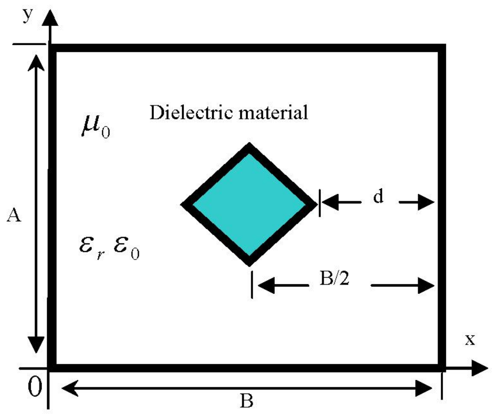

Fig. 5. Cross-section of the diamondwise structure with 45° offset angle.

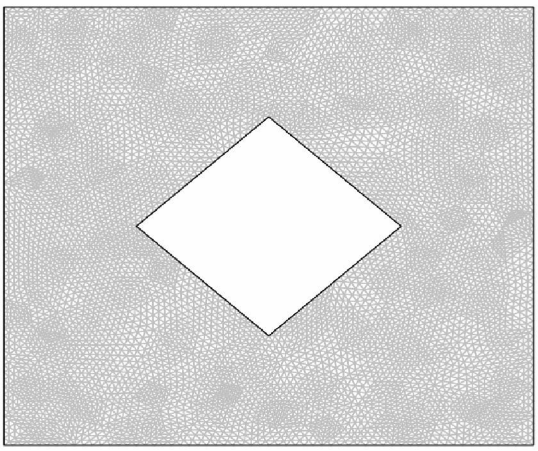

Fig. 6. Mesh of the diamondwise structure with 45° offset angle.

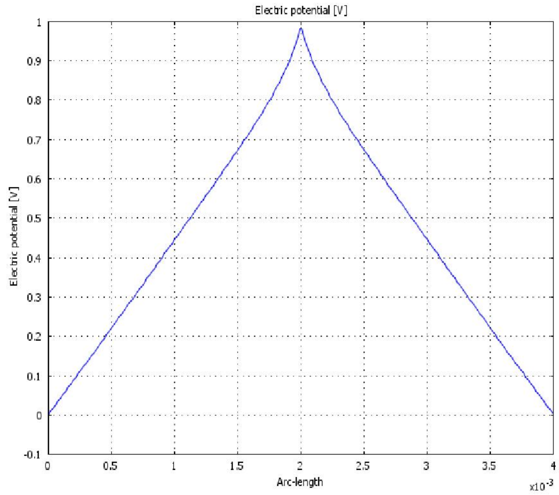

Fig. 7. The potential distribution of the diamondwise structure with 45° offset angle along line y = 1 mm.

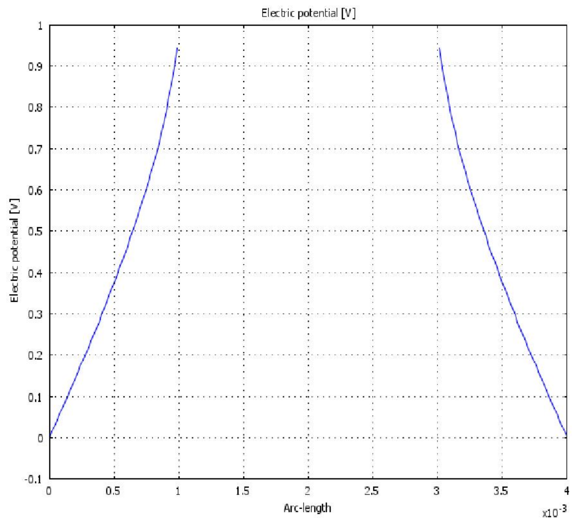

Fig. 8. The potential distribution of the diamondwise structure with 45° offset angle along line y = 2 mm.