Lab Section (circle): PRA01 PRA02 PRA03 PRA04 PRA05
TA Name: ____________________________ Group #: _______________________
Last Name: _______________ First Name: _____________ Student #: _____________

LAB #4

The purpose of LAB 4 is to allow you to use MATLAB to work through a set of problems based on electric fields. These exercises will make use of your work in previous labs, as well as enabling you to consider how to use MATLAB to solve a design problem based around a resistive sensor.

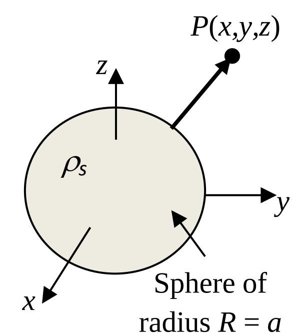
PREPARATION - Individual

1) Read through these Lab #4 notes carefully, and make sure you understand what each part of the lab is asking you to do. This prior review will save you a lot of time during the lab session, which will enable you to complete the lab in the time available.

2) Read through the Technology Brief 7: Resistive Sensors (pgs. 198 – 200) as this will give you a good background for part 2 of this lab.

3) The first part of this lab involves using MATLAB to calculate and plot the electric field intensity, **E**, and the absolute electric scalar potential, V, due to a charged spherical shell, as shown to the right.

   Use Coulomb's law to develop the integrals that you would have to evaluate to calculate the three components of **E** and the value of V at a general point $P(x,y,z)$ due to this charged shell. You do not have to solve these integrals. Assume that the shell has a radius a, and non-uniform charge density, ρ_S. Write your handwritten solution in the box below and make sure to show all your work.



4) With our work from part 3, you can now adapt your *ring_of_charge* function from Lab #3 to calculate the components of **E** and the value of *V* at any point *P*(*x*,*y*,*z*) for the case of an infinitely-thin spherical shell charged with the non-uniform charge density ρ_S. Since this is a surface integration it can be approximately calculated by two nested *for* loops. Using this function as a foundation, create a new function *sphere_of_charge* that carries out these calculations and write it out in the box on the next page below (this must be written by hand). Your new function *must* be based on the integrations that you would need to evaluate if you solved this problem using Coulomb's law. Remember, each integration requires a summation, and so you will have to be careful how you do the two summations required for the charged sphere calculation. *Hint*: To implement the double integral shown below one option is to use the `for` loop structure given:

$$Int = \int_0^{2\pi} \int_0^{\pi} a^2 \sin\theta \, d\theta d\varphi$$

```
N = 500;
dtheta = pi/N;
dphi = 2*pi/N;
phi = linspace(dphi,2*pi,N);
theta = linspace(0,pi,N+1);
for e=1:length(phi)
    for m=1:length(theta)
        dInt1(m)=(a^2)*sin(theta(m))*dtheta*dphi;
    end
    dInt2(e)=sum(dInt1);
end
Int=sum(dInt2);
```

Observe that the result of this integration provides the surface area of a sphere of radius a. Notice how the `phi` and `theta` vectors must be defined carefully. For the $1 \times N$ vector `phi`, it must start at `dphi` since if it started at zero, it would count that point twice due to the fact that `phi` = 2π specifies the same position as `phi` = 0. For the $1 \times (N+1)$ vector `theta`, it must have $N+1$ points so that the spacing between the subsequent entries of `theta` correspond to `dtheta`, i.e., `theta(2) - theta(1) = dtheta`. You can verify that the spacing between the subsequent entries of `phi` correspond to `dphi`, as expected.

You must ensure your function works *before* you come to the lab. To do so, compare the results from your function to those determined from a theoretical analysis (i.e., Gauss's law), for a sphere of radius a = 1 cm and uniformly charged with $\rho_s = -5$ nC/m^2:

Point $P(x, y, z)$	**E** – Theory (in terms of E_x, E_y, and E_z)	**E** - Matlab Function (in terms of E_x, E_y, and E_z)
(0, 0, 0) cm		
(0, 0, 2) cm		
(−1, 2, −1) cm	(No need to provide theoretical values for this point)	

IN-LAB WORK - Group

1. Electric-Field of a Spherical Shell

1.1. Uniformly Charged Spherical Shell

For this section of the lab consider a uniformly charged spherical shell in free space ($\varepsilon = \varepsilon_0$), centered about the origin with radius a = 0.5 m and a charge density of ρ_s = 2 μC/m^2.

Write a new function that will make use of your *sphere_of_charge* function to calculate and plot the electric field magnitude and the absolute electric scalar potential for this shell along the y – axis from y = –3 m to y = 3 m. You should have around 100 points for your y–vector, and use an N = 200 for each integration.

For these two figures, plot the theoretical values of E_{tot} and V using a series of red o's (i.e., `'ro'`) and compare this to the calculated values for the field and the potential both inside and outside the shell using a blue solid line (i.e., `'b-'`). Recall that these theoretical values are given by:

$$E_{theory} = \begin{cases} 0 & |y| < a \\ \dfrac{\rho_S a^2}{\varepsilon_0 y^2} & |y| \geq a \end{cases} \qquad V_{theory} = \begin{cases} \dfrac{\rho_S a}{\varepsilon_0} & |y| < a \\ \dfrac{\rho_S a^2}{\varepsilon_0 |y|} & |y| \geq a \end{cases}$$

Since E_{theory} and V_{theory} are given by two different expressions depending on the value of y, you must create these vectors carefully. One way to do this is to assign the appropriate values to your E_{theory} and V_{theory} vectors using an `if` statement as you cycle through the y-vector in the `for` loop of your plotting function.

Note: By using Coulomb's law to calculate the fields inside and outside this spherical shell, we have made no assumptions about the symmetry of the field. Therefore, your results should prove that the field inside a uniformly charged spherical shell, which is fully closed, is zero!

1.2. Non-Uniformly Charged Spherical Shell

Is the electric field always zero within a charged spherical shell? To find this out, consider the example of a spherical shell in free space ($\varepsilon = \varepsilon_0$), centered about the origin with radius a = 0.5 m and a non-uniform charge density ρ_S. Each group will have a different charge density, so ask your TA to tell you what the charge density function is for your group.

For this non-uniformly charged shell, create a 2D plot of the variation of E_x, E_y, E_z, E_{tot} and V along the z-axis, over the range -3 m $\leq z \leq 3$ m.

Is the field as you would expect it to be? To answer this you could consider this shell as a collection of stacked rings which have different charge densities (according to how ρ_S changes) and different radii.

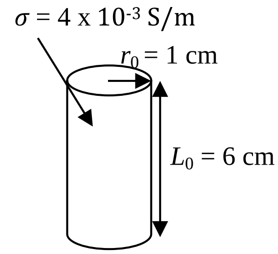
Make sure you discuss this section of the lab with your TA before you leave.

2. Practical Application: Resistive Sensors

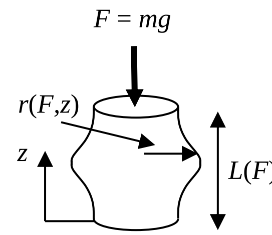
In this part of the lab, you will have the opportunity to apply your knowledge of MATLAB to assist in the design of a precision weight scale based on a resistive sensor.

Consider the following scenario:

As part of your summer internship with a research group within the ECE department, your supervisor asks you to assist a graduate student in the design of a precision weight scale based on a resistive sensor. The sensor makes use of a piezoresistive material which exhibits a change in its resistance as a force is applied. This is illustrated in the figures below.



No Compressive Force
Resistance = R_0



Compressive Force $F = mg$
Resistance = $R(F)$

The initial uncompressed resistance is given by:

$$R_0 = \frac{L_0}{\sigma \pi r_0^2} \qquad \text{Eqn. (1)}$$

As a compressive force is applied, the piece of material becomes shorter, with length given by $L(F)$, and the soft material becomes distorted so that the radius of the material now changes along the length of the material, expressed by the function $r(F,z)$. This means that the compressed resistance as a function of force is given by:

$$R(F) = \int_0^{L(F)} \frac{dz}{\sigma \pi [r(F,z)]^2} \qquad \text{Eqn. (2)}$$

Through careful measurements the graduate student had established the fact that the length of the piezoresistive material depends on the compressive force through the expression:

$$L(F) = L_0 e^{-0.0012F} \qquad \text{Eqn. (3)}$$

At the time, the graduate student did not know how to deal with the fact that the radius of the material changed as a function of both F and z. So, to simplify the analysis the student estimated that the radius only depended on the applied force F, through the expression $r \approx (0.003F + r_0)$, so that the resistance was thus

$$R_{approx}(F) \approx \frac{L_0 e^{-0.0012F}}{\sigma \pi (0.003F + r_0)^2} \qquad \text{Eqn. (4)}$$

The problem is that this expression did not accurately represent the measured resistances of this material for various loading conditions. So, the student has asked for your help to improve the accuracy of this model, so that the weight scale will be more precise.

The student has provided you with their measured data in the form of a text file (*meas_resistance1.txt* that is available on the course website). This data has two columns, the first one being mass (in kilograms) and the second being the measured resistance of this piezoresistive material under the load of that mass.

The student also tells you that through their most recent measurements they have discovered that a more accurate representation of the radius of the compressed material can be modeled by the expression:

$$r(F,z) = 0.0005\sqrt{10F}\, e^{-\left\{5\left[z - \frac{L(F)}{2}\right]F\right\}^2} + r_0 \qquad \text{Eqn. (5)}$$

Use your knowledge of MATLAB to calculate a more accurate value for the resistance of this material as a function of the compressive force. This should be based around the evaluation of the integral given in Eqn. (2) using the equations (3) and (5). As part of your "report", you should include:

a) A plot of resistance versus mass for the range of 10^{-4} kg to 100 kg. It is suggested that you use a log scale for your *x*-axis (mass axis) (you can use the `semilogx` command to do this).

b) In this figure include a plot of R_{approx}, Eqn. (4), and a plot of the resistance of the material if it is assumed that the radius does not change at all, i.e., $R = \frac{L(F)}{\sigma \pi r_0^2}$.

c) Also include in this plot the measured data saved in *meas_resistance1.txt*. This can be loaded into the MATLAB workspace by saving it in your working directory, selecting it, and then choosing "Import Data…" from the File menu.

d) How does your more accurate calculation of *R* compare to the three other plots? Does it seem that this is a better representation of how this material actually responds to a compressive force?

e) Can you explain why your graph of *R* behaves as it does?

f) Your supervisor has also asked you to give your assessment of what range of masses this scale would be most accurate. Are there any parts of the range 10^{-4} kg < *m* < 100 kg over which this scale would not accurately measure the mass?

g) Are there any assumptions that have been made in the development of this more accurate calculation of *R*? From your results, are these valid assumptions?

Make sure you discuss this section of the lab with your TA before you leave.