Homework 1

ECE 345 Algorithms and Data Structures
Winter Semester, 2015

Due: 1PM, Jan 27, in ECE345 dropbox (next to SFB560)

This homework is designed so you practice your background in introductory discrete mathematics and combinatorics. You need have this background material practiced well because future homeworks will use it extensively!

- All page numbers are from 2009 edition of Cormen, Leiserson, Rivest and Stein.
- For each algorithm you asked to design you should give a detailed *description* of the idea, proof of algorithm correctness, termination, analysis of time and space complexity. If not, your answer will be incomplete and you will miss credit. You are allowed to refer to pages in the textbook.
- Do not write C code! When asked to describe an algorithm give analytical pseudocode.
- Staple your homework properly. Use a stapler; do not use glue or other weird material to put it together. If you are missing pages, we are not responsible for it but you are!
- Write *clearly*, if we cannot understand what you write you may not get credit for the question. Be as formal as possible in your answers. Don't forget to include your name(s) and student number(s) on the front page !

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1. [**Permutations and Combinations, 6+6+6 points**]
   (a) In how many ways can ten engineers and four models be seated at a round table if all the models do not sit together?
   (b) In how many ways can you divide 28 persons into three groups having 4, 12 and 12 persons?
   (c) There are 6 boxes numbered 1,2,...,6. Each box needs to be filled up either with a red or a blue ball in such a way that at least 1 box contains a blue ball and the boxes containing blue balls are consecutively numbered. What is the total number of ways in which this can be done ?
2. [**Recurrences, 20 points**] Solve the following recurrences by giving tight Θ-notation bounds.
   (a) $T(n) = 3T(n/4) + n \log n$
   (b) $T(n) = 4T(n/2) + cn$
   (c) $T(n) = 3T(n/3) + n/2$
   (d) $T(n) = 16T(n/4) + n!$
   (e) $T(n) = \sqrt{n}T(\sqrt{n}) + 100n$
   (f) $T(n) = T(n/5) + T(4n/5) + \Theta(n)$

3. **[Asymptotics, 24 points]** Sort the following 24 functions from asymptotically smallest to asymptotically largest, indicating ties if there are any. Do as much as you can, you will receive partial credit if you do not sort all of them. You will also need to turn in proofs that *justify your answers* to receive full credit! All logarithms are base 2 unless otherwise stated.

$n^{4.5} - (n-2)^{4.5}$	n	$n^{1+1/\log n}$	$\log^*(n/2)$	$\sum_{i=1}^{n} \log i$
$\sum_{i=1}^{n} \left(\frac{1}{i-1} - \frac{1}{i+1}\right) + 2$	$\log^*(\log^* n)$	$\log n!$	$(\log n)^{\log^* n}$	n^5
$\log^* 2^n$	$2^{\log^* n}$	e^n	$\lfloor \log\log(n!) \rfloor$	
$\left(1 - \log \frac{1}{1-1/n}\right)^n$	$n^{\log\log n/\log n}$	$(\log n)^{(n/2)}$	$(\log n)^{\log n}$	$\left(1 + \frac{1}{200n}\right)^{200n}$
$n^{1/\log\log n}$	$n^{\log\log n}$	$\log^{(200)} n$	$l0g^2 n$	$n(\log n)^2$

To simplify notation, write $f(n) \ll g(n)$ to mean $f(n) = o(g(n))$ and $f(n) \equiv g(n)$ to mean $f(n) = \Theta(g(n))$. For example, the functions n^2, n, $\binom{n}{2}$, n^3 could be sorted either as $n \ll n^2 \equiv \binom{n}{2} \ll n^3$ or as $n \ll \binom{n}{2} \equiv n^2 \ll n^3$.

4. **[Induction, 10+10 points]** Use mathematical induction to prove the following statements. Make sure to show clearly all three steps of induction (inductive basis, inductive hypothesis and inductive step) or you will miss credit.

   (a)
$$\left(1 - \frac{1}{1+2}\right)\left(1 - \frac{1}{1+2+3}\right)\cdots\left(1 - \frac{1}{1+2+3+\cdots+n}\right) = \frac{n+2}{3n}$$

   (b)
$$6n + 12 < 3^n$$

   for all $n \geq 4$.

5. **[Graphs, Proof by contradiction, 15 points]**

   A simple graph is a undirected graph with no multiple edges or loops. In a simple graph with at least two vertices, show that there are at least two vertices with the same degree. Use proof by contradiction. Make sure to show clearly all the steps to receive full credit.

6. **[Induction, 10+10 points]**

$$a_1 = 2$$
$$a_2 = 4$$
$$a_n = a_{n-1} + 2a_{n-2}, \forall n \geq 3$$

   Prove that:

$$a_n \geq 2^n, \forall n \geq 1$$