Homework 5

ECE 345 Algorithms and Data Structures
Winter Semester, 2015

Due: April 14th at 12noon in ECE345 dropbox

- All page numbers are from 2009 3rd edition of Cormen, Leiserson, Rivest and Stein.
- For each algorithm you asked to design you should give a detailed *description* of the idea, proof of algorithm correctness, termination, analysis of time and space complexity. If not, your answer will be incomplete and you will miss credit. You are allowed to refer to pages in the textbook.
- Do not write C code! When asked to describe an algorithm give analytical pseudocode.
- Staple your homework properly. Use a stapler; do not use glue or other weird material to put it together. If you are missing pages, we are not responsible for it but you are!
- Write *clearly*, if we cannot understand what you write you may not get credit for the question. Be as formal as possible in your answers. Don't forget to include your name(s) and student number(s) on the front page !

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1. **[NP-completeness, 20 points]** Set-Cover: Given a finite set U, a collection $C = \{C_1, C_2, \cdots, C_n\}$ of subsets of U, and an integer k, determine whether there is a sub-collection of C with cardinality k that covers U. In other words, determine whether there exists $C' \subset S$ such that $|C'| = k$ and $\bigcup_{C_i \in C'} C_i = U$.

   Prove that Set-Cover is an NP-Complete problem given that Vertex-Cover is an NP Complete problem.

2. **[NP-completeness, 30 points]** There is a list of exams $F = \{f_1, f_2, \cdots, f_k\}$ to be scheduled and a list of students $S = \{s_1, s_2, \cdots, s_m\}$ where each student s_i is required to take a subset of exams F_{s_i}, where $F_{s_i} \subseteq F$. The set M contains the exam sets of all students, i.e. $M = \{F_{s_1}, F_{s_2}, \cdots, F_{s_m}\}$. You are required to schedule these exams into time slots so that no student has to take two exams in the same slot. The problem is to determine if there exists a schedule using only H time slots (available distinct time slots for the final exams). This problem can be formally formulated as:

   H-FINAL $= \{< F, S, MH >$ and there is a valid schedule using no more than H slots $\}$.

   Prove that H-FINAL is NP-Complete assume that H-COLOR is NP-Complete where H-COLOR $= \{<$ $G >$, the nodes of G can be colored with H colors such that no two nodes joined by an edge have the same color $\}$.

3. **[Regular language, regular expression, DFA]** In all cases the alphabet is $\sum = \{a, b\}$.

   (a) Give the regular expression and the state diagram of DFA recognizing the following language $\mathcal{L} = \{$w— w contains at most 3 'a's$\}$.

   (b) Give the regular expression and the state diagram of DFA recognizing the following language $\mathcal{L} = \{$w— w has odd length and has exactly 1 'b'$\}$

   (c) Give the state diagram of DFA recognizing the following language $\mathcal{L} = \{$w— w does not contain the substring 'aab'$\}$