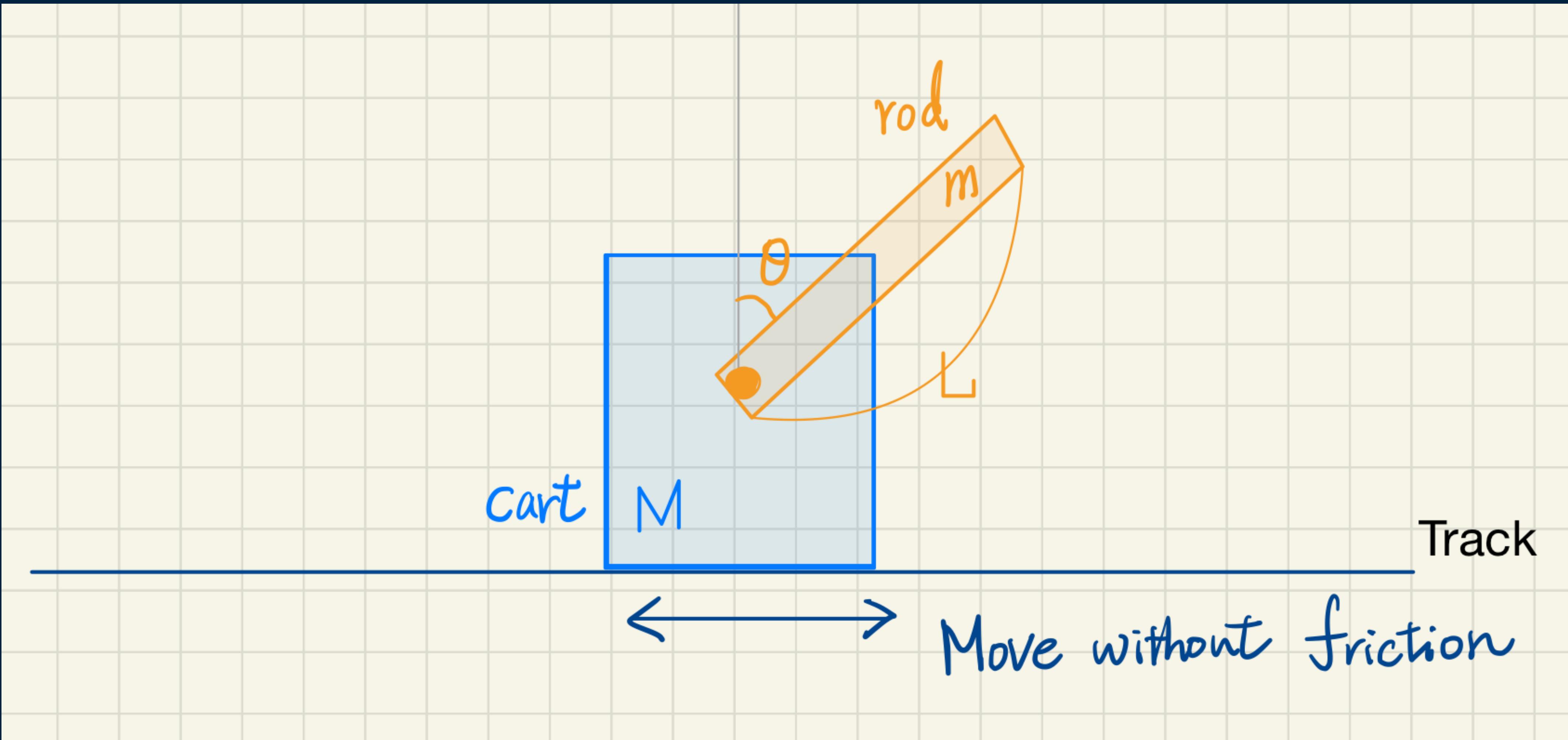


FINAL PROJECT 廖苡鈞 | 呂俐君 | 夏良語

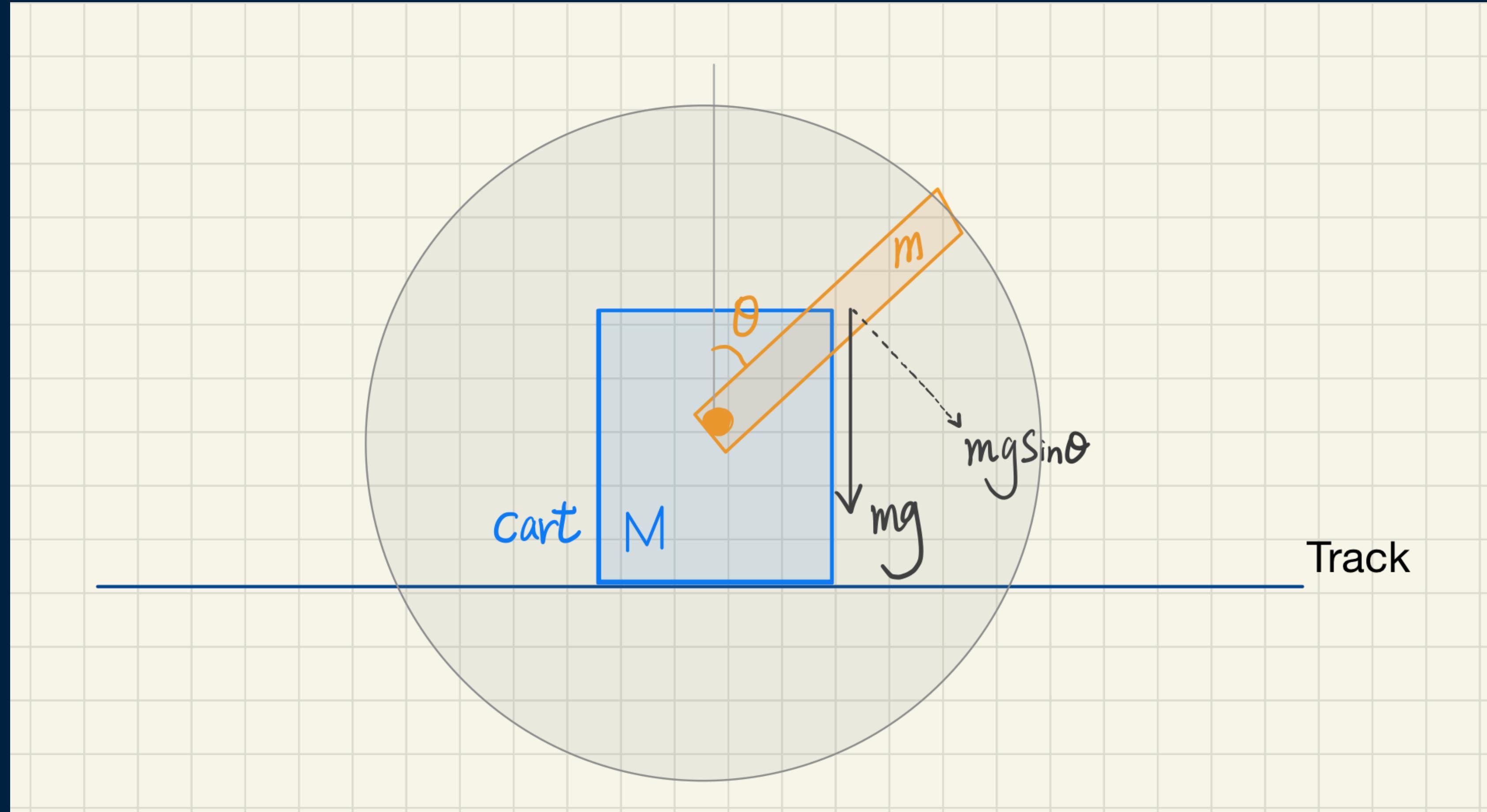
B09901014 B08901207 B09901049

INVERTED PENDULUM

裝置介紹



力學



(I) 看棒子與臺車的相對運動

$$\vec{\tau} = I\alpha = \left(\frac{1}{3}mL^2\right) \cdot \alpha$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \frac{L}{2} \cdot mg \sin\theta$$

$$\Rightarrow \frac{L}{2} \cdot mg \sin\theta = \frac{1}{3}mL^2\alpha$$

(I) 看棒子與臺車的相對運動

$$\alpha = \frac{3g \sin \theta}{2L}$$

$$\omega + = \alpha \cdot dt$$

$$\theta + = \omega \cdot dt$$

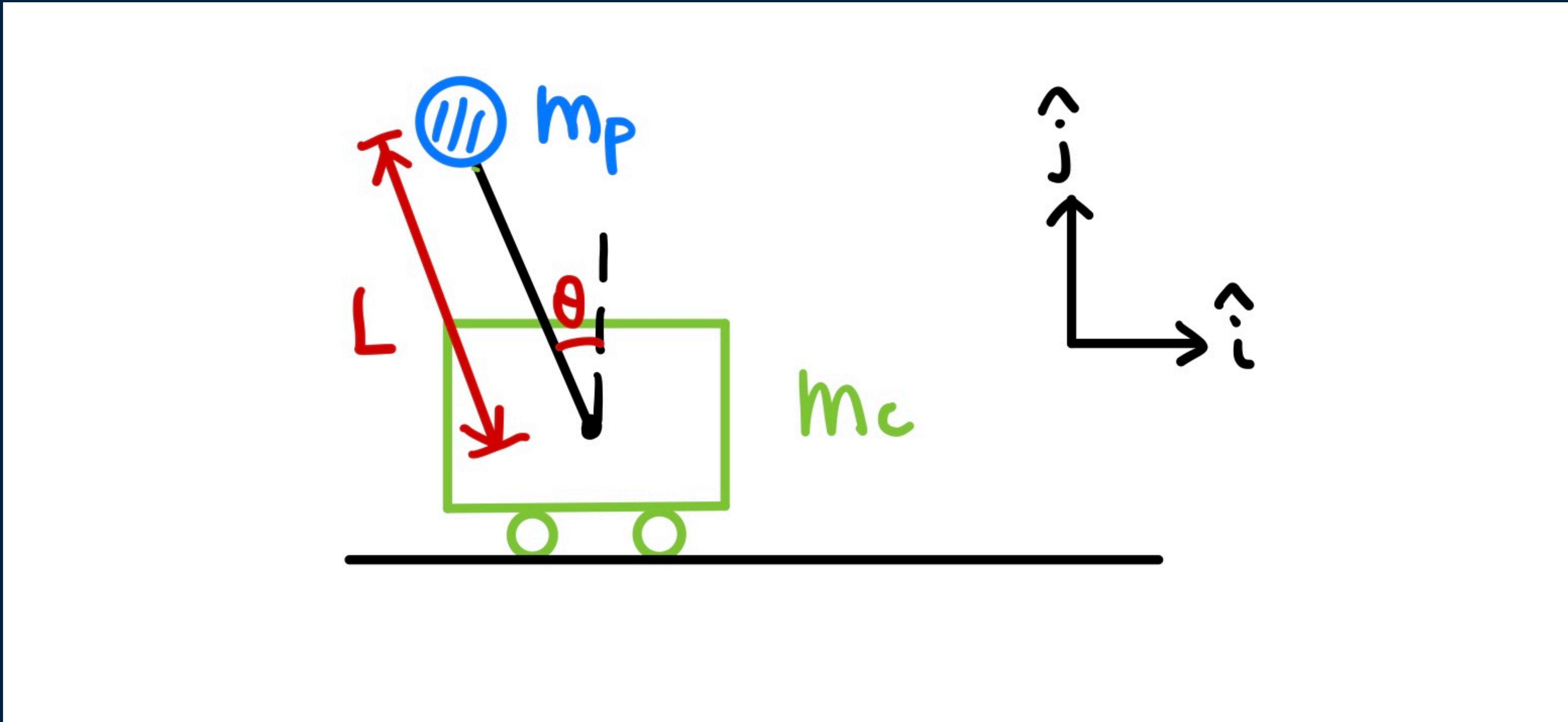
(II) 整體系統 計算質心位置

$$\sum F_x = 0 \Rightarrow C_{Mx} = \frac{M\chi_M + m\chi_m}{M+m} = \text{Fixed}$$

$$\Rightarrow \chi_M = \frac{C_{Mx_0}(M+m) - m\chi_m}{M}$$

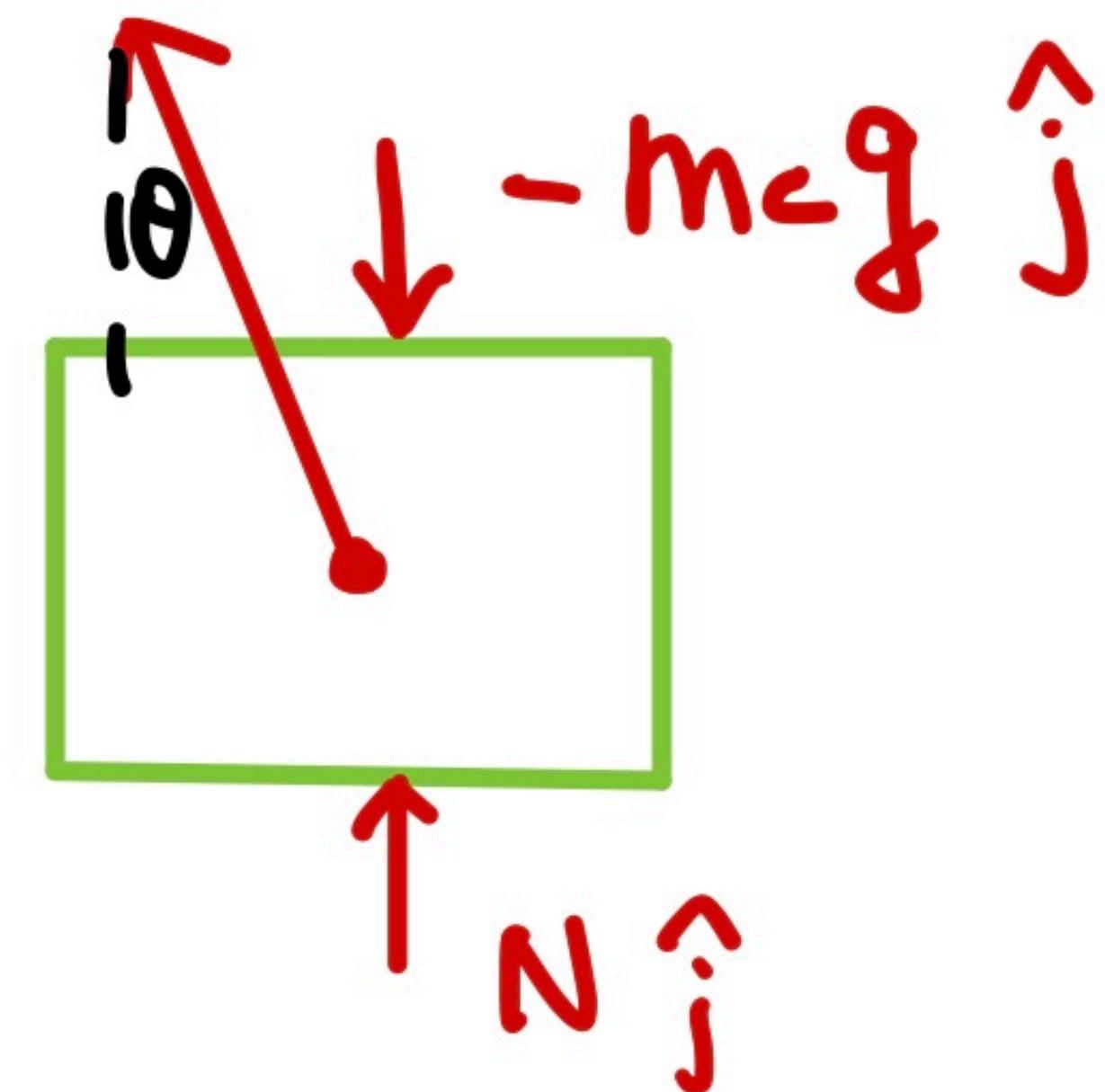
where $C_{Mx_0} = \frac{M\chi_{M_0} + m\chi_{m_0}}{M+m}$

APPLICATIONS

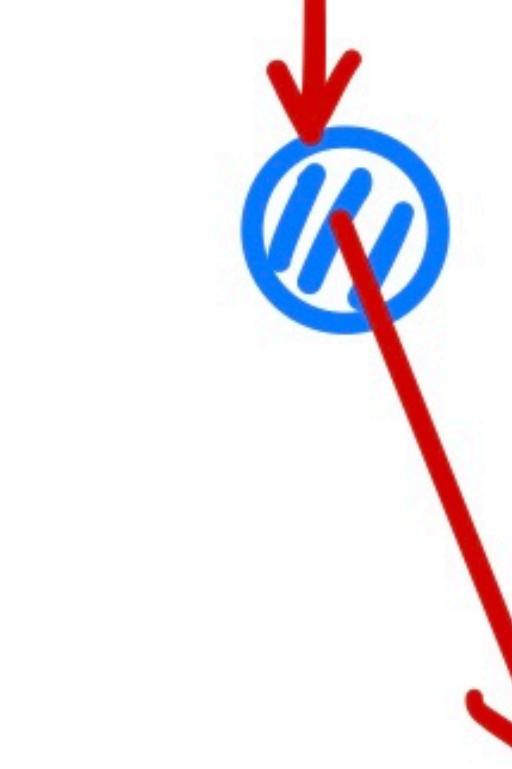


APPLICATIONS

$$-T \sin\theta \hat{i} + T \cos\theta \hat{j}$$

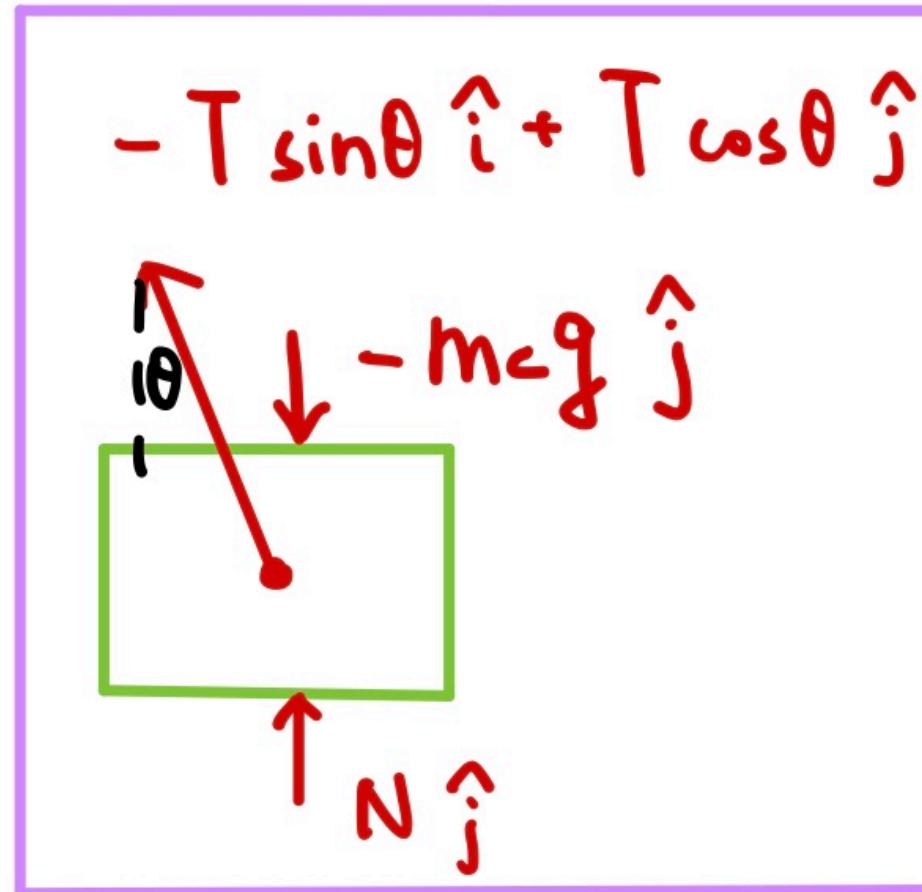


$$-m_P g \hat{j}$$



$$T \sin\theta \hat{i} - T \cos\theta \hat{j}$$

APPLICATIONS



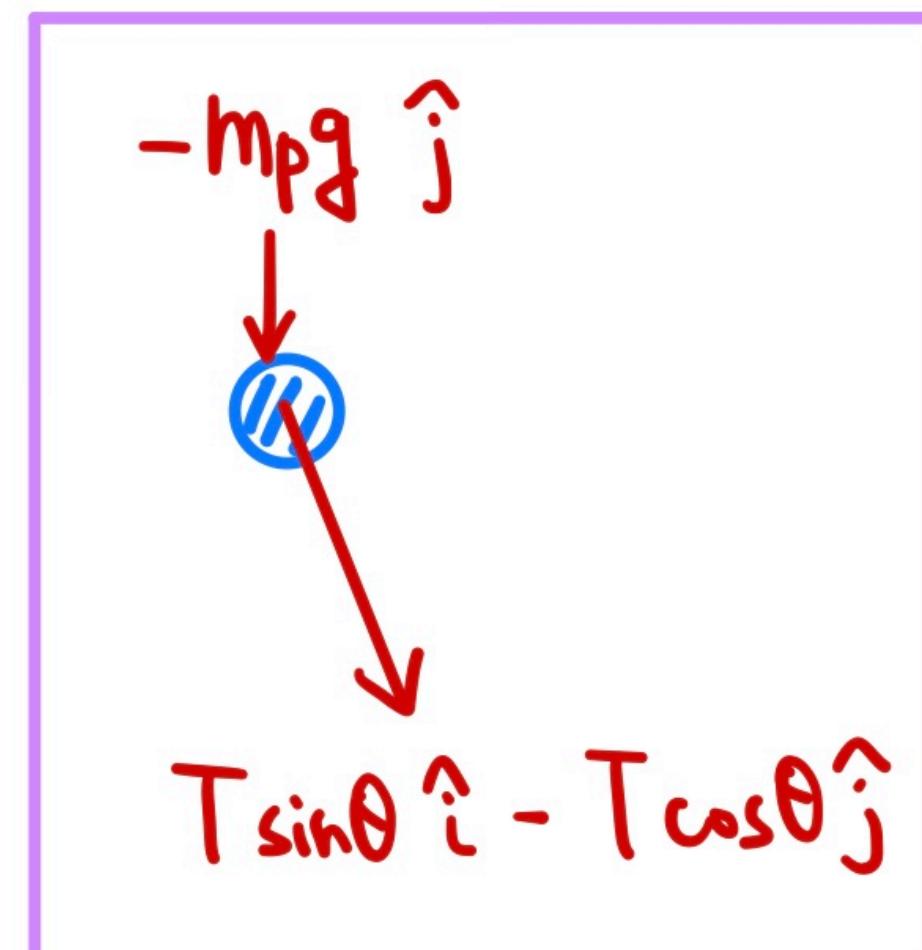
$$-T \sin\theta \hat{i} + T \cos\theta \hat{j}$$

$$-m_c g \hat{j}$$

$$N \hat{j}$$

$\hat{i}:$

$$-T \sin\theta = m_c a_{cx}$$



$$-m_p g \hat{j}$$

$$T \sin\theta \hat{i} - T \cos\theta \hat{j}$$

$\hat{i}:$

$$T \sin\theta = m_p a_{px}$$

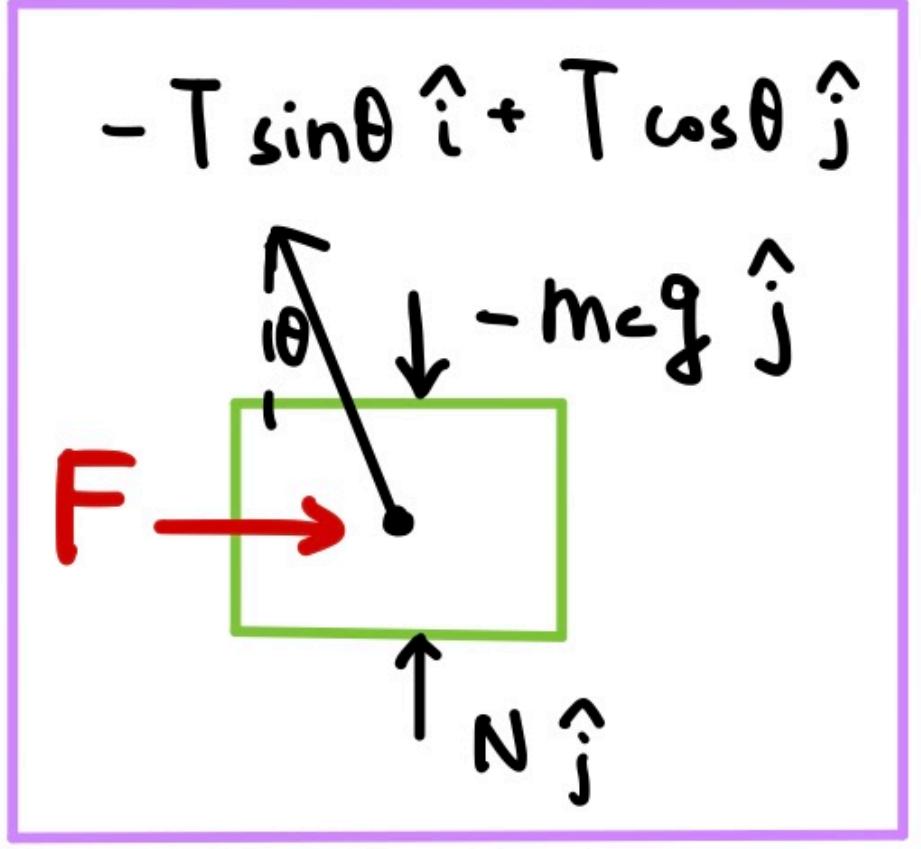
 $\hat{j}:$

$$-T \cos\theta - m_p g = m_p a_{py}$$

APPLICATIONS

```
T = -k*(rod.axis.mag-L) * rod.axis.norm()
cart.a.x = -T.x/cart.m
pendulum.a = vec(0,-g,0) + T/pendulum.m
```

APPLICATIONS



$$-\mathbf{T} \sin\theta \hat{i} + \mathbf{T} \cos\theta \hat{j}$$

$$\mathbf{F}$$

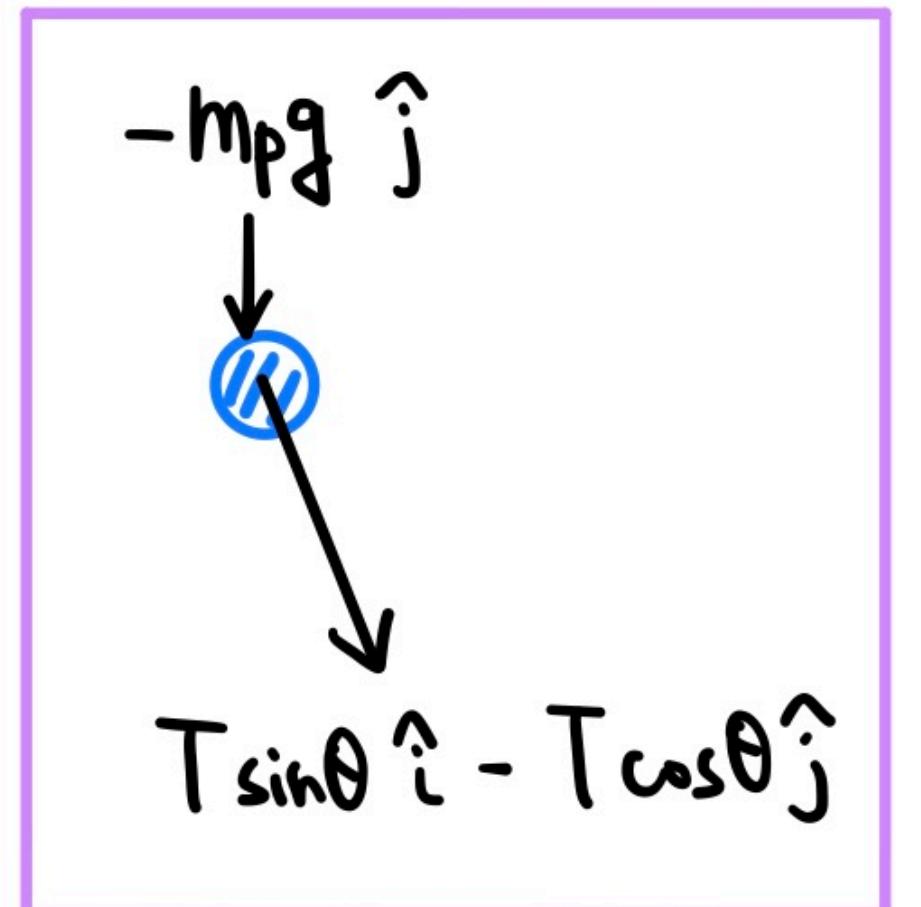
$$\mathbf{N}$$

$$-m_c g \hat{j}$$

$$N \hat{j}$$

$\hat{i}:$

$$F - T \sin\theta = m_c a_{cx}$$



$$-m_p g \hat{j}$$

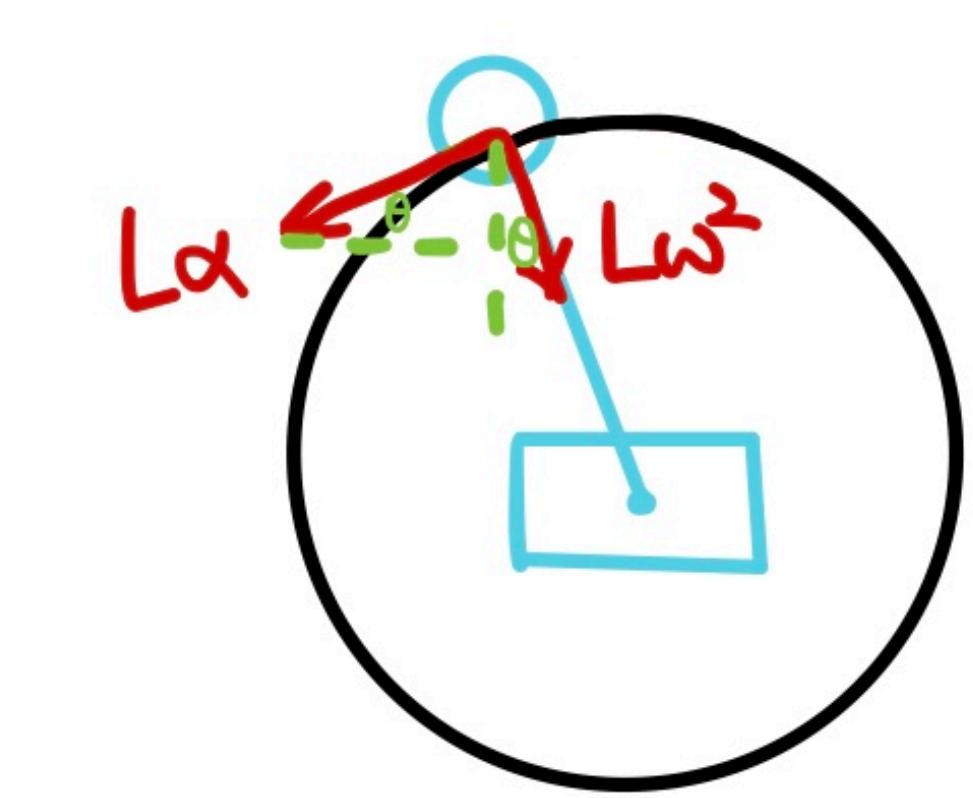
$$T \sin\theta \hat{i} - T \cos\theta \hat{j}$$

$\hat{i}:$

$$T \sin\theta = m_p a_{px}$$
 $\hat{j}:$

$$-T \cos\theta - m_p g = m_p a_{py}$$

APPLICATIONS



The diagram shows a circular platform with a horizontal axis of rotation. A vertical line segment representing a pendulum is attached to the center of the platform. A red arrow labeled $L\alpha$ points left, perpendicular to the line segment. A green arrow labeled $L\omega^2$ points down along the line segment. The angle between the line segment and the vertical is labeled θ .

$$\begin{aligned} \mathbf{a}_p &= \mathbf{a}_c + \mathbf{a}_{pc} = a_{cx}\hat{i} + (L\alpha - L\omega^2) \\ &= a_{cx}\hat{i} + L\alpha[-\cos\theta\hat{i} - \sin\theta\hat{j}] \\ &\quad - L\omega^2[-\sin\theta\hat{i} + \cos\theta\hat{j}] \end{aligned}$$

APPLICATIONS

1. $F - T \sin\theta = m_c a_{cx}$
2. $T \sin\theta = m_p a_{px}$
3. $-T \cos\theta - m_p g = m_p a_{py}$
4. $a_p = a_{cx} \hat{i} + L \alpha [-\cos\theta \hat{i} - \sin\theta \hat{j}]$
 $- L \omega^2 [-\sin\theta \hat{i} + \cos\theta \hat{j}]$

$$\Rightarrow T \sin\theta = m_p a_{px}$$

$$= m_p a_{cx} - m_p L \alpha \cos\theta + m_p L \omega^2 \sin\theta \text{ (1)}$$

$$-T \cos\theta - m_p g = -m_p L \alpha \sin\theta - m_p L \omega^2 \cos\theta \text{ (2)}$$

APPLICATIONS

(1). $\cos\theta + (2) \cdot \sin\theta :$

$$-m_p g \sin\theta = m_p a_{cx} \cos\theta - m_p L \alpha$$

$F - T \sin\theta = m_c a_{cx} :$

$$F + m_p L \alpha \cos\theta - m_p L \dot{\omega}^2 \sin\theta = (m_p + m_c) a_{cx}$$

APPLICATIONS

ex:



自平衡滑行車

FINAL PROJECT 廖苡鈞 | 呂俐君 | 夏良語

B09901014 B08901207 B09901049

THANK YOU