CSC418/2504 – Winter 2017
Assignment #2

Assignment due date: Friday, March 3, 11:59pm

Hand-in and code to be submitted to the CDF server by the above due date

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I hereby affirm that all the solutions I provide, both in writing and in code, for this assignment are my own. I have properly cited and noted any reference material I used to arrive at my solution, and have not shared my work with anyone else.

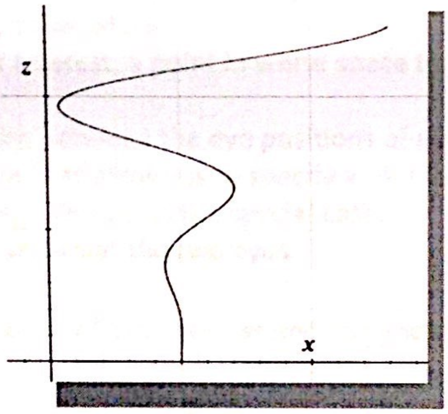
Daiq Li
Signature

(note: -3 marks penalty for not completing properly the above section)

Part 1 – Pen and paper questions

1- Surfaces of revolution

A simple way to create a vase shape is to start with a parametric curve arc as shown below:



Then revolve that arc around the **z** axis. This parametric curve can be derived using an instance of the parametric equation (i.e. for some choice of scalars *a*, *b*):

$$\bar{p}(t) = \left(a + t^2 \cos t, 0, \frac{t}{b}\right), 0 \le t \le 2\pi$$

a) (5 marks) Give a parametric equation and appropriate parameter bounds for the resulting surface of revolution *p(u, v)* in terms of the surface parameters *u* and *v*.

$$\bar{P}(u,v) = ((a+u^2\cos u)\cos v, (a+u^2\cos u)\cdot\sin v, \frac{u}{b})$$

$$u \in [0, 2\pi), \quad v \in [0, 2\pi)$$

b) (5 marks) Give an equation for the tangent plane at a point *p(u, v)* in terms of the surface parameters *u* and *v*.

$$\bar{tg_u} = \frac{\partial \bar{P}(u,v)}{\partial u} = (2u\cos v\cos u - u^2\sin u\cos v, 2u\sin v\cos u - u^2\sin u\sin v, \frac{1}{b})$$

$$\bar{tg_v} = \frac{\partial \bar{P}(u,v)}{\partial v} = (-(a+u^2\cos u)\sin v, (a+a^2\cos u)\cos v, 0)$$

c) (4 marks) Give an equation for the outward-facing unit normal vector at point *p(u, v)* in terms of the surface parameters *u* and *v*.

$$\hat{n} = \frac{\vec{tg_v} \times \vec{tg_u}}{|\bar{tg_v} \times \bar{tg_u}|}, \quad \bar{tg_v} \times \bar{tg_u} = \left[\frac{-\cos v(a+u^2\cos u)}{b}, \frac{\sin(a+u^2\cos}{b}, (2u\cos u - u^2\sin u)(a+u^2\cos u)\right]$$

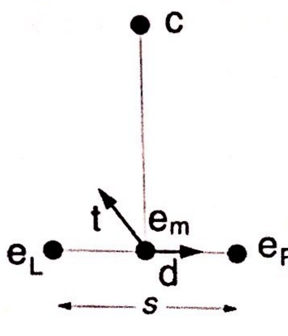
Part 1 – Pen and paper questions

2- Camera Transformations

(16 marks) In stereo rendering, two cameras are needed with slightly different vantage points and view directions. Each is used to render an image for the corresponding eye. This can be specified with the following parameters:

- c: The center of interest, a point in world space that lies along the optical axis of both cameras.
- e_m: The midpoint between the eye positions of each camera.
- t: An "up" vector that allows us to specify a tilt rotation about the axis passing through c and e_m. The z-axis is a special case.
- s: The distance between the two eyes.

All answers should be given in terms of the above stereo parameters, as well as any intermediate quantities you specify.



a) (2 marks) Give an expression for the unit vector d that is perpendicular to both t and $c - e_m$ such that $(c - e_m, t, d)$ is a right–handed coordinate frame.

$\bar d = (c - e_m) \times \bar t \,/\, |(c-e_m)\times \bar t|$

b) (3 marks) Give expressions for e_L and e_R the eye positions of each camera.

$e_L = e_m - \frac{s}{2}\bar d$, $e_R = e_m + \frac{s}{2}\bar d$

c) (3 marks) Give expressions for the basis vectors that make up the two cameras' coordinate frames: (u_L, v_L, w_L), and (u_R, v_R, w_R).

$\bar u_L = \dfrac{t \times (c - e_L)}{|\bar t \times (c - e_L)|}$ $\bar u_R = \dfrac{\bar t \times (c - e_R)}{|\bar t \times (c - e_R)|}$

$\bar v_L = \dfrac{(c - e_L) \times \bar u_L}{|(c - e_L) \times u_L|}$ $\bar v_R = \dfrac{(c - e_R) \times u_R}{|c - e_R| \times \bar u_R}$

$\bar w_L = \dfrac{(c - e_L)}{|c - e_L|}$ $w_R = \dfrac{(c - e_R)}{|c - e_R|}$

d) (5 marks) Give an expression for the 4x4 homogeneous transformation matrix that transforms a point in the camera space of the left eye (p_L) to the camera space of the right eye (p_R). In other words, what is MLR such that $p_R = M_{LR} p_L$.

$$M_{LR} = \begin{bmatrix} 1 & 0 & 0 & S \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e) (3 marks) Give a test that determines whether or not a polygon face with normal n is a backface that can be culled when rendering from both cameras.

Find a point P on the polygon, check if both $(\bar p - \bar e_L)\cdot \vec n > 0$ and $(p - \bar e)\cdot \vec n > 0$. If both > 0, cull.

Part 1 – Pen and paper questions

3- Camera coordinates and coordinate conversion

a) (7 marks) Give the ***world-to-camera*** 3D homogeneous transformation matrix for a camera with:

$$\vec{e}_{wc} = (1,2,5)^T$$

$$\vec{t} = (1,1,0)^T$$

looking at point

$$\vec{p}_{wc} = (-1,-2,0)^T$$

Give the derivation and values for each of the vectors that define the camera's coordinate frame, as well as the final 4x4 transformation matrix.

$$\bar{g} = \frac{\bar{p}_{wc} - \bar{e}_{wc}}{|\bar{p}_{wc} - \bar{e}_{wc}|} = \left(-\frac{2}{\sqrt{45}}, \frac{-4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}\right)^T$$

$$\bar{u} = \frac{\bar{t} \times \bar{g}}{|\bar{t} \times \bar{g}|} = \left(-\frac{5}{\sqrt{86}}, \frac{5}{\sqrt{86}}, -\frac{6}{\sqrt{86}}\right)^T$$

$$\bar{v} = \frac{\bar{g} \times \bar{u}}{|\bar{g} \times \bar{u}|} = \left(\frac{49}{\sqrt{3470}}, \frac{13}{\sqrt{3470}}, \frac{-30}{\sqrt{3470}}\right)^T$$

$$\bar{w} = -\vec{g} = \left(\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{5}{\sqrt{45}}\right)^T$$

$$H = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We also need to rotate the matrix to align coordinates. R

$$M_{wc} = R\,H$$

b) (3 marks) Using the above, show the ***camera-to-world*** 3D homogeneous transformation matrix.

$$M_{cw} = M_{wc}^{-1} = R^{-1} H^{-1}$$

$$= R^{-1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$