Modeling of Arctic Ice

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Importance of the Field

- Weather patterns in NA and Europe
- Interrelationships between ocean, atmosphere, Arctic systems
- Meteorologists & general public

Review of Literature

History of Sea Ice in the Arctic

Polyak, Leonid & Alley, R. B. & Andrews, J. T. & Brigham-Grette, J. & Cronin, T. M. & Darby, D. A. & Dyke, A. S. & Fitzpatrick, J. J. & Funder, S. & Holland, M. (2010). History of Sea Ice in the Arctic. *Quaternary Science Reviews, 29*.

- Seasonal levels
- Shrinking
  - Warming
  - Extended summer melt season

- Changing atmospheric patterns (clouds & heating)

An Arctic Wild Card in the Weather

Greene, Charles H. & Monger, B. C. (2012). An Arctic Wild Card in the Weather. *Oceanography, 25*.

- Arctic Oscillation
- Exposed ocean absorbs heat—“ice-albedo feedback mechanism”
- Heat released via evaporation…
- Increased pressure, moisture = weakened polar vortex/jet stream

- Polar vortex contains cold air
- Jet stream controls persistence in middle latitudes

Bifurcations leading to summer Arctic sea ice loss

Abbot, Dorian S. & Silber, M. & Pierrehumbert, R. T. (2011). Bifurcations leading to summer Arctic sea ice loss. *Journal of Geophysical Research: Atmospheres, 116*.

- Bifurcation = qualitative change in system
- Mathematical model can't make *specific* predictions…
- No saddle node bifurcations found, but

Hopf unstudied…

$$\frac{dE}{dt} = (1 - \alpha(E))F_s(t) - A(E) - BT(E, t)$$

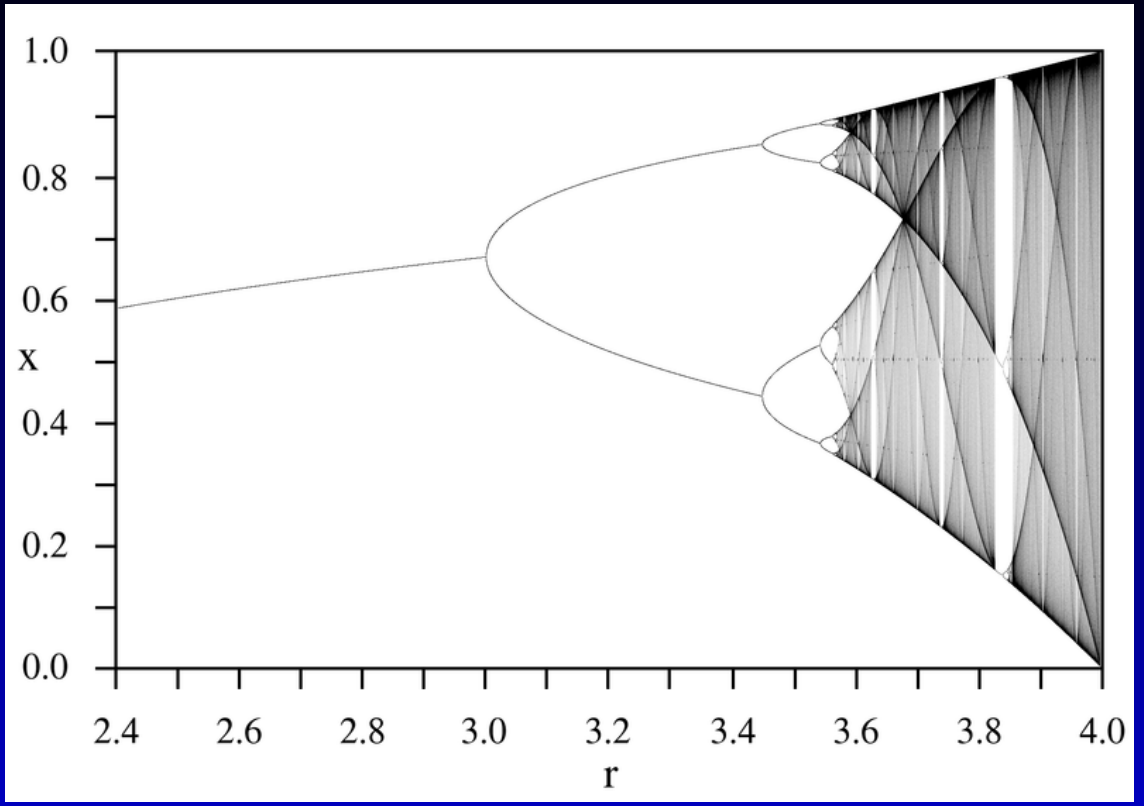
$$+ \Delta A_{ghg} + (-Ev\mathcal{R}(-E))$$

The Slow Passage Through a Hopf Bifurcation

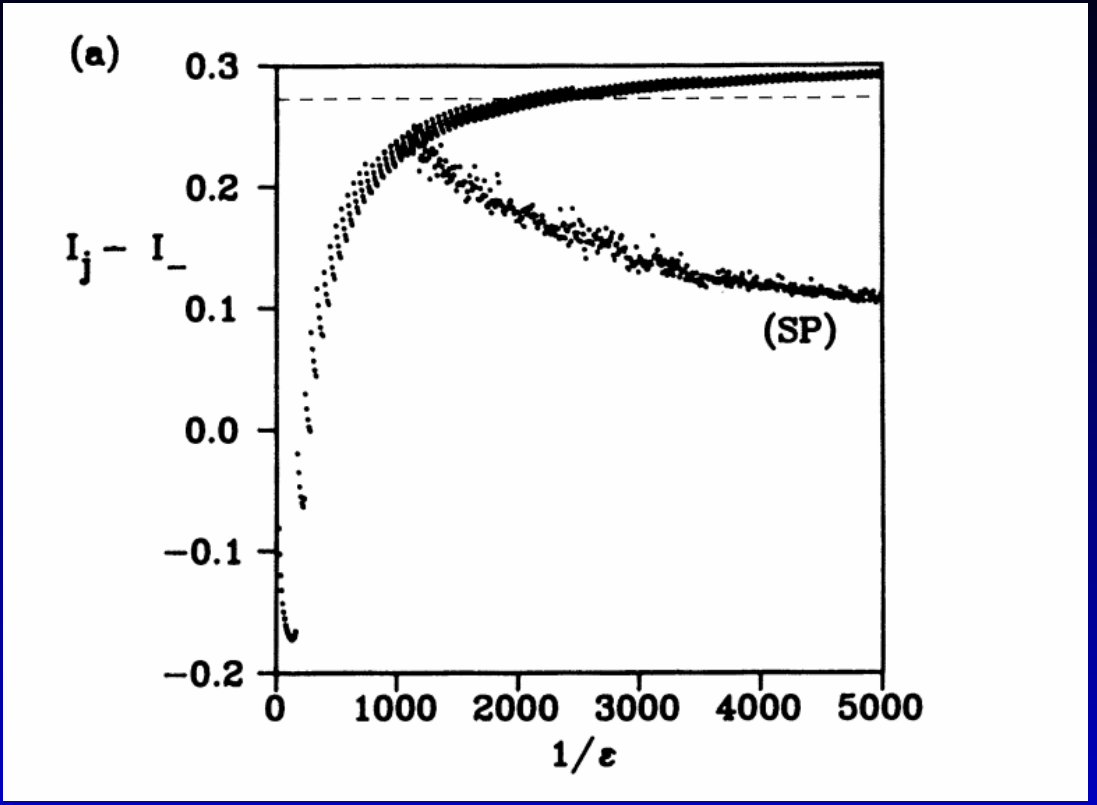
Baer, Stephen & Erneux, T. & Rinzel, J. (1989). The Slow Passage Through a Hopf Bifurcation: Delay, Memory Effects, and Resonance. *SIAM Journal on Applied Mathematics, 49*.

- Slow passage = continuous parameter that "slowly" changes
- Can lead to problems in computer calculations

- Solution: more precision & **awareness** of this problem



$$x_{n+1} = r\,x_n(1 - x_n)$$


(a)
0.3
0.2
0.1
0.0
−0.1
−0.2
$I_j - I_-$
(SP)
0
1000
2000
3000
4000
5000
$1/\varepsilon$

Current State of Knowledge

- Arctic ice melting = changing weather patterns…
- …and the ice is melting
- Already have had many severe winter storms
- Models don’t predict a specific type of qualitative change in the future

Shadow Area

- Jargon of two fields: climate modeling and bifurcation theory
- Details of these weather systems
- How to apply conclusions from a naïve model
- Better understand the mathematics

Outstanding Questions

- Are other types of bifurcations possible?
- Bifurcation generally means a sudden change
- How accurate can we make these models?
- How well do the models' predictions relate to real weather?

Research Question

How does the level of greenhouse gas as a slow parameter (one that continuously varies at a slow rate) affect the behavior of an Arctic sea ice model, and how can those conclusions be applied to a general climate model?

Proposed Procedure

1. Implement model
2. Run model with varying parameter
3. Generate diagrams
4. Analyze

```
def runge_kutta(f, t0, y0, h, steps):
    t = t0
    y = y0
    for n in range(steps):
        k1 = f(t, y)
        k2 = f(t + h / 2, y + k1 * (h / 2))
        k3 = f(t + h / 2, y + k2 * (h / 2))
        k4 = f(t + h, y + k3 * h)

        t += h
        y += (h / 6) * (k1 + 2 * k2 + 2 * k3 +

    return t, y
```

Materials needed:

- Computer
- Time

Assistance: Dr. Baer, to help verify calculations

Polyak, Leonid & Alley, R. B. & Andrews, J. T. & Brigham-Grette, J. & Cronin, T. M. & Darby, D. A. & Dyke, A. S. & Fitzpatrick, J. J. & Funder, S. & Holland, M. (2010). History of Sea Ice in the Arctic. *Quaternary Science Reviews, 29*.
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