Modeling of Arctic Ice: Redux

David Li
Dr. McKelvy
18 Sep 2013

Importance of the Field

- Weather patterns in NA and Europe
- Interrelationships between ocean, atmosphere, Arctic systems
- Meteorologists & general public

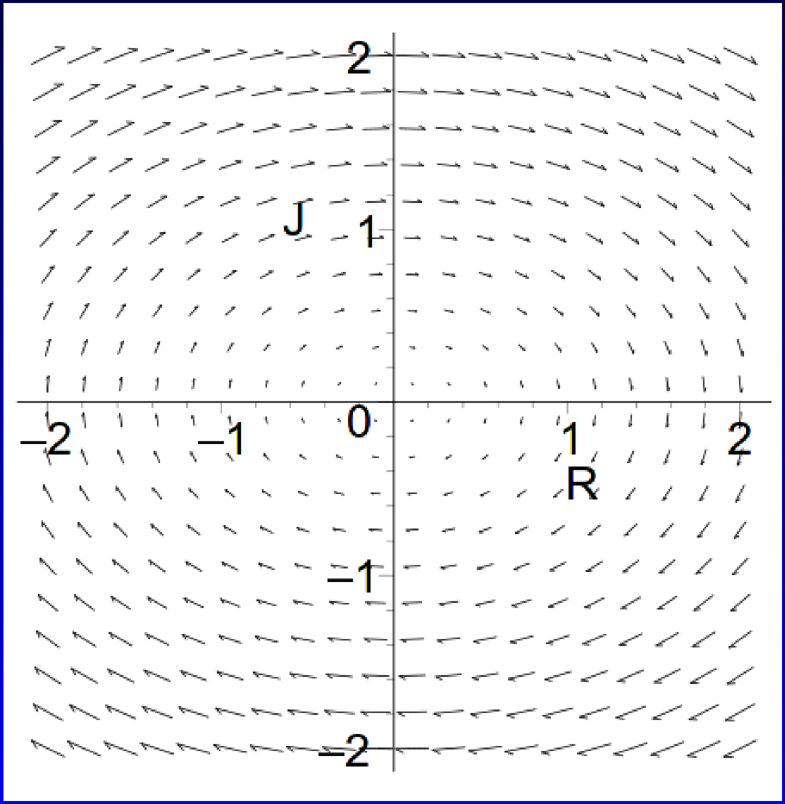
Review of Literature

Eigenvalues, Eigenvectors, and Differential Equations

Cherry, William (2013). Eigenvalues, Eigenvectors, and Differential Equations. *University of North Texas*. Retrieved on September 14, 2013 from http://wcherry.math.unt.edu/math2700/diffeq.pdf.

- Eigenvalues tell us about the solutions to a system of ODEs
- Real: eigenvectors point in asymptotic/repellent directions
- Imaginary: rotations

$$\frac{dR}{dt} = aJ, \ \frac{dJ}{dt} = -bR, \ \lambda = \pm i\sqrt{ab}$$



Introduction to bifurcation theory

Crawford, J.D. & Kueny, C. & Saphir, B. & Shadwick, B. (1989). *Introduction to bifurcation theory*. Retrieved from http://www.osti.gov/scitech/biblio/5396551.

— Bifurcation: qualitative change due to a parameter change

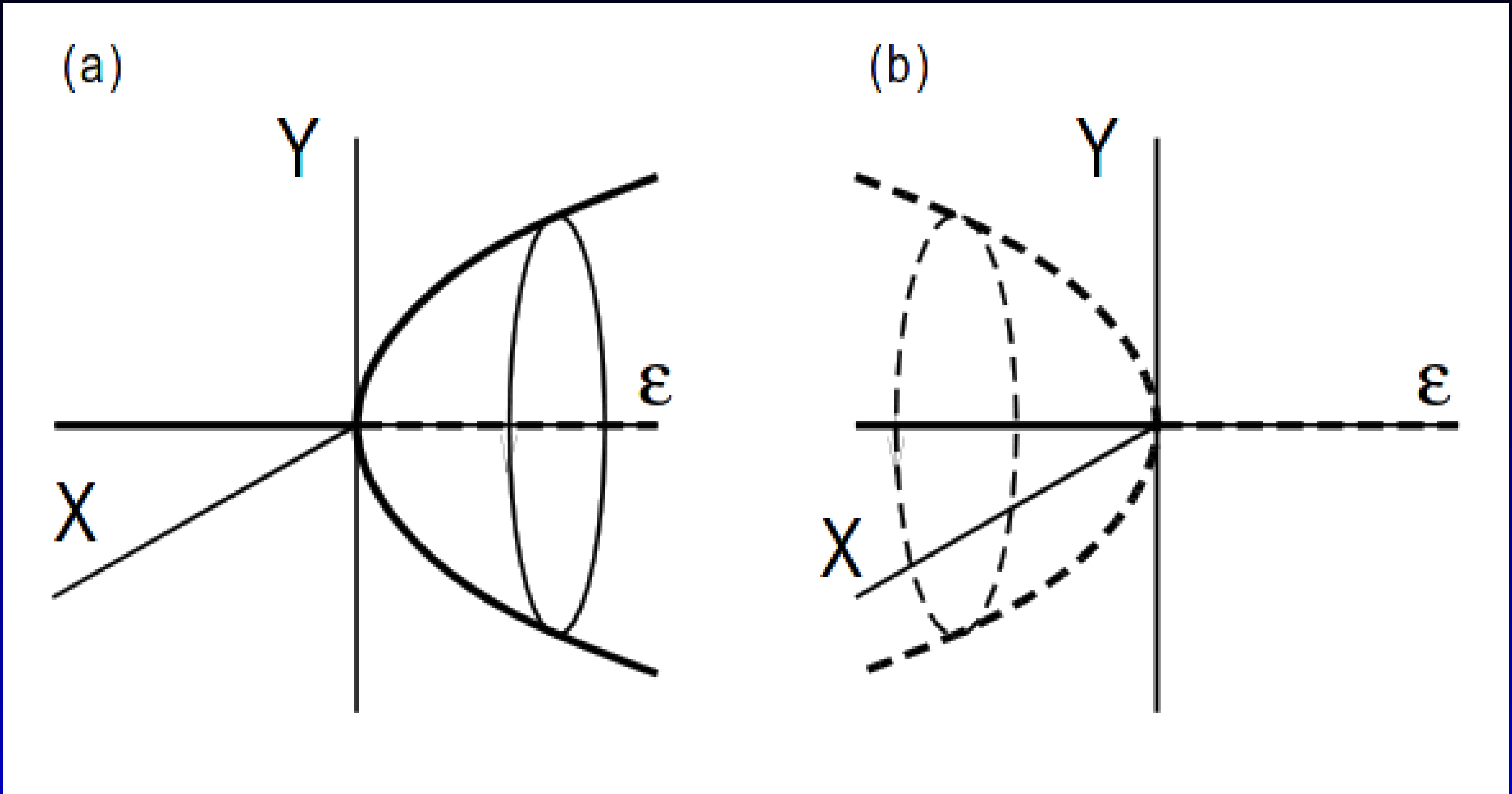
— Dynamical system: set of differential equations, often used to model a physical system

— Changes: (dis)appearance of an equilibrium, periodic orbit

Physics 161: Introduction to Chaos

Cross, Michael (2000). *Physics 161: Introduction to Chaos*. Retrieved from http://crossgroup.caltech.edu/Chaos_Course/.

- Stationary bifurcation: single real eigenvalue passes through 0
- Hopf bifurcation: complex conjugate pair of eigenvalues passes through imaginary axis



(a)
Y
ε
X
(b)
Y
ε
X

Precision and representation issues

Johansson, Fredrik (2011). Precision and representation issues. *mpmath v0.17 documentation*. Retrieved on September 14, 2013 from http://mpmath.googlecode.com/svn/trunk/doc/build/technical.html.

- Numerical error causes:
  - Rounding/cancellation (finite precision)
  - Truncation (approximations to infinite sequences/continuous functions)
- `mpmath`: library for floating-point arithmetic
- "In general, mpmath only guarantees that it will use at least the user-set precision to perform a given calculation"

— “The user may have to manually set the working precision higher than the desired accuracy for the result, possibly much higher.”

Representation:

$$mantissa \times 2^{exponent}$$

mantissa & exponent are arbitrary-precision integers

Current State of Knowledge

— Differential equations (dynamical systems) are used to model real-world phenomena
— Various mathematical tools exist to help analyze these models
— Bifurcations are interesting as they represent changes in behavior ("tipping points")
— Numerical error can be a problem when computing

Outstanding Questions

- How do these mathematical concepts apply to real-world research? (particularly Hopf bifurcations)

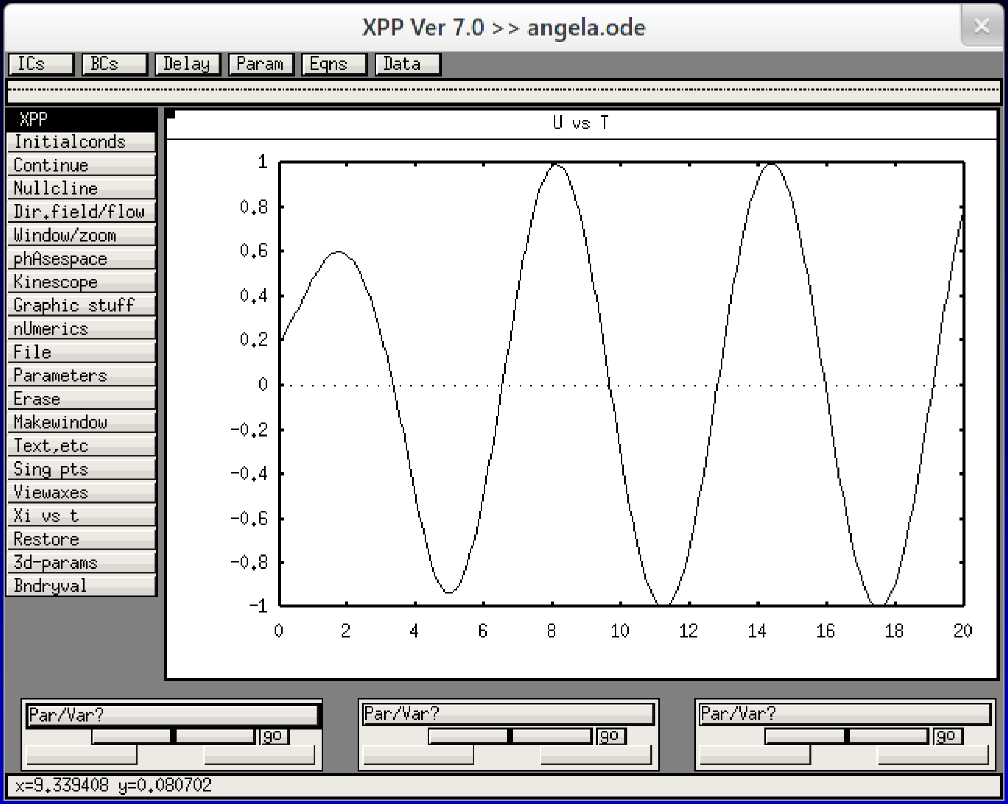
Research Question

How does the level of greenhouse gas as a slow parameter (one that continuously varies at a slow rate) affect the behavior of an Arctic sea ice model, and how can those conclusions be applied to a general climate model?

Proposed Procedure

1. Implement model (Python)
   - Requires RK4 solver
   - Uses `mpmath` for arbitrary-precision calculations
2. Run model with varying parameter
   - Model described in Abbot, 2011
   - Some parameters updated
   - Bifurcation parameter (ε) will be greenhouse gas level

3. Generate diagrams
   — Software: XPP AUTO
   — (Research needed: can AUTO use data from an external program?)
4. Analyze
   — The hard step


XPP Ver 7.0 >> angela.ode
ICs
BCs
Delay
Param
Eqns
Data
XPP
Initialconds
Continue
Nullcline
Dir.field/flow
Window/zoom
phAsespace
Kinescope
Graphic stuff
nUmerics
File
Parameters
Erase
Makewindow
Text,etc
Sing pts
Viewaxes
Xi vs t
Restore
3d-params
Bndryval
U vs T
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10
12
14
16
18
20
Par/Var?
go
Par/Var?
go
Par/Var?
go
x=9.339408 y=0.080702

Resources Needed

- No facilities needed
- No equipment needed
- Qualified scientist: Dr. Baer may be able to assist in verifying calculations
- Critical component: understanding of climate models