Two-bit gate in quantum computing and its physical realization
Interim Report

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Abstract

The two-qubit entangling gate is a crucial element in quantum computing. It plays an important role in the preparation of entangled states. Based on the basic theories of quantum computing, this project pays much attention to some theoretical analyses about optical two-qubit gate, in order to emphasize the importance in quantum algorithm. Besides, this project also do some research on some existent designs of two-qubit gate, analysing their advantages as well as drawbacks. Then we will explore some new designs of two-qubit gates, and perform some experiments. This report mainly disscuss some basic theories, imcluding some algorithms of quantum computing and three ways of constructing two qubit-gate.

Key words:optical quantum computing, two-qubit gate, physical realization

1 Introduction

For the recent dozens of years, many quantum information researchers have been looking for proper schemes for universal quantum computing. Because of the robustness of photons, optical scheme has become an important area for quantum computing.

Two-qubit gate is an essencial element in quantum computing. It not only helps generating entangled states, but also plays an indisputable role in almost all the quantum algorithms. The Shor algorithm [1], which works on factoring, is based on the Quantum Fourier Transformation algorithm, needs a large series of Control-R gates. Because of the depth of two-qubit gates, its requirement for the gates' efficiency is extremely high. As a result, constructing a two-qubit gate with high efficiency is of great importance.

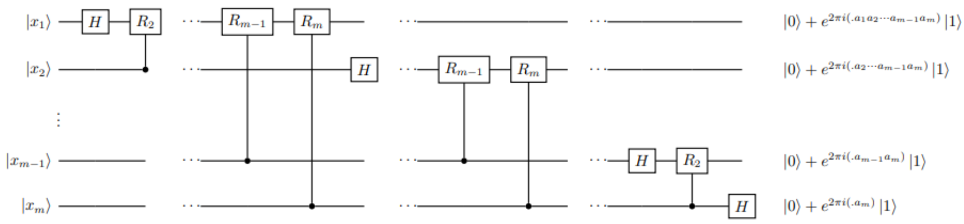


Figure 1: The quantum circuit of Quantum Fourier Transformation [1]

These years, researches on how to construct a universal optical two-qubit gate are paid much attention to, and several breakthroughs have been made in optical quantum computing. In 2001, KLM scheme [2] were put forward by three scientists, laying the foundation of linear optical quantum computing. In 2013, Alberto Peruzzo et al [3] gives us a method of constructing two-qubit gates using linear optics and a optimization algorithm in FPGA. In 2014, Darrick E. Chang et al [4] reveal the quantum nonlinearity optics, and give us a new method to build a deterministic two-qubit gate. In 2022, Thomas Stolz et al in MPQ used a butter-shaped cavity and an atom to construct a Control-Z gate [5], reaching an average efficiency above 40%.

Today, we have been working on constructing deterministic two-qubit gate with cavity-QED. we are also trying to combine the deterministic single-photon source with the gate. If the project be realized, it will become a big leap towards universal quantum computing.

2 Theoretical Analysis

Quantum computing Basis

qubit

The qubit is essencially a kind of vector in two-dimensional Hilbert Space. We don't care about the absolute phase or the magnitude of the qubit, so we often use normalized two-dimensional vector to stand for a qubit. Every qubit can be written as: $|\phi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$.

Qubit gate

Quantum computation is essencially a unitary transfomation of a series of qubits. Qubit gate is the basic element of the unitary transformation. We have concluded that several basic qubit-gates can construct universal qubit gate. They are:

1. Hadamard gate $:H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$
2. $\pi/4$gate : $R = \begin{pmatrix}1 & 0\\ 0 & e^{i\pi/2}\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$
3. $\pi/8$gate : $T = \begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}$
4. CNOT gate : $CNOT = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

In these gates, only CNOT is a entangled two-qubit gate, so our main purpose is to construct a CNOT gate. Since $CNOT = (I \bigotimes H)CZ(I \bigotimes H)$, we can construct a CZ gate as well.

2.1 Random two-qubit gate using linear optics

2.1.1 Linear optical quantum computing

KLM protocol gives that using linear optical elements like beam splitters and wave plate can construct universal quantum computing gates. The hadamard gate can be easily constructed by a 50-50 beam splitter, according to the Hong-Ou-Mandel scheme. And the R and T gates can be realized by wave plates. CNOT gate has a relatively more complex structure. It needs several elements and post-selection to construct. The first CNOT has an efficiency of 1/9. The main loss comes from the post-selection loss.

2.1.2 An example

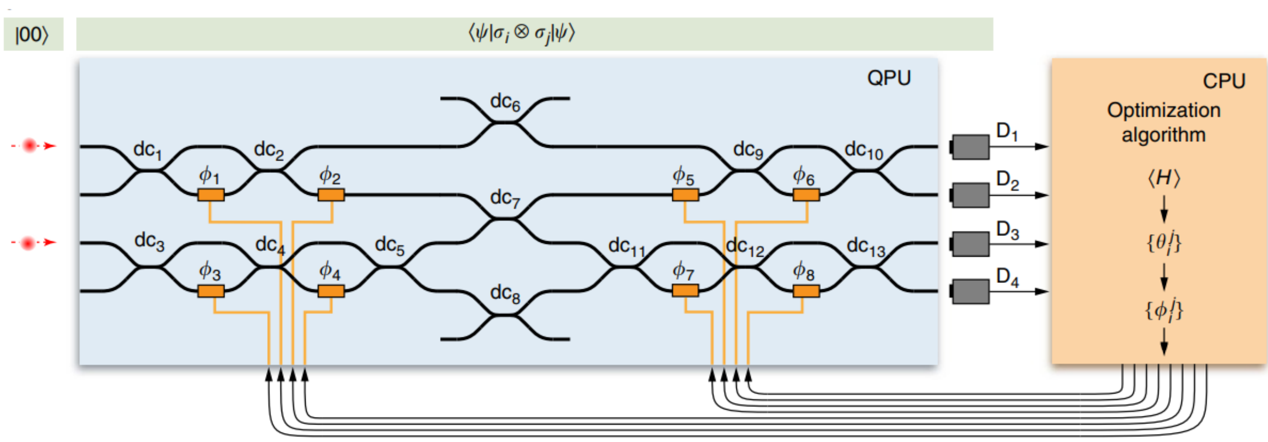


Figure 2: The linear optical CNOT gate using line coding [3]

It has an FPGA optimizer to feed back and adjust the phase shifters, in order to realize a CNOT transformation.

2.2 Deterministic two-qubit gate using optical quantum nonlinearity

2.2.1 Cavity-QED

Since a deterministic two-qubit gate needs a strong interaction between photons, the basic method is to use an atom as a media between the two photons. However, it is not easy to make a single photon interact with an atom, so we try to use a cavity to reflect the photon many times to enhance the interaction between the photon and atom. If the atom can be seen as a two-level system, the interaction satisfies the Jaynes-Cummings Model [6,7]. Anyway, it is not easy to perfectly interact a gaseous single atom with two single photons, because it must be cooled and trapped inside an optical resonator. So, we use a solid state system to realize it. However, although the two-level atom can cause some nonlinearity, the atom can easily dephased. It can not 'store' the first photon for enough time. These limitations can be overcome by employing multilevel atoms with two ground and/or metastable states, in which quantum coherence can be stored for long periods. Such states enable the implementation of a quantum memory, whereby a quantum state of light can be mapped onto atomic states [8,9].

2.2.2 Generation of deterministic photon strings

In order to make the deterministic two-qubit gate work more efficiently, the deterministic single-photon source should be required. According to He Yu Ming [11], we can generate deterministic photon string using quantum dot's P-shell exitation by short pulse string. Since the pulse is much shorter than the spontaneous emission lifetime, we can confirm each time it generates a single photon.

2.2.3 Construction of deterministic two-qubit gate

Assuming that we have a one-sided cavity with a decay rate κ, and a photon with frequency ω_0 interacts with each other, the hamiltonion would be:

$$H = \hbar\omega_0 a^\dagger a + \hbar\int b^\dagger(\omega)b(\omega)d\omega + \hbar\int g(\omega)(a^\dagger b(t) + b^\dagger(t)a)d\omega$$

where g denotes the coupling rate, a denots the module of the cavity, and b denotes the free space mode. According to Walls [14], it can lead to a result as follows: ($H_{sys} = \hbar a^\dagger a$)

$$\dot{a}(t) = \frac{1}{i\hbar}[H_{sys}, a(t)] - \frac{\gamma}{2}a(t) + \sqrt{\gamma}a_{IN}(t)$$

$$\dot{a}(t) = \frac{1}{i\hbar}[H_{sys}, a(t)] + \frac{\gamma}{2}a(t) - \sqrt{\gamma}a_{OUT}(t)$$

After a Fourier Transform, the equations can be simplified to:

$$a_{OUT}(\omega) = \frac{\gamma/2 + i(\omega - \omega_0)}{\gamma/2 - i(\omega - \omega_0)}a_{IN}(\omega)$$

If the *omega* is around Ω:

$$a_{OUT}(\omega) = \frac{\gamma/2 + i(\Omega - \omega_0)}{\gamma/2 - i(\Omega - \omega_0)}a_{IN}(\omega)$$

After a fourier transform:

$$a_{OUT}(t) = \frac{\gamma/2 + i(\Omega - \omega_0)}{\gamma/2 - i(\Omega - \omega_0)}a_{IN}(t)$$

$$\Rightarrow a_{OUT}(t) = \frac{\gamma/2 + i\Delta}{\gamma/2 - i\Delta}a_{IN}(t)$$

If $Delta >> \gamma$, $a_{OUT}(t) = -a_{IN}(t) = e^{i\pi}a_{IN}$

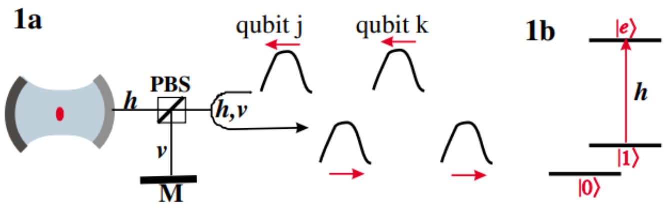


Figure 3: The CNOT gate using CQED [15]

This kind of interaction can seen as a Control-Z gate between the atom in the cavity and the 'h'mode of input-output photon. We denote this as $U_{p,a}^{CPF} = e^{i\pi|0\rangle_a\langle 0|\otimes|h\rangle_p\langle h|}$. Since $U_{j,k}^{CPF} = e^{i\pi|h\rangle_j\langle h|\otimes|h\rangle_k\langle h|}$, Duan [15] find out that

$$U_{j,k}^{CPF}|\Psi_{j,k}\rangle \otimes |\Phi_{a,i}\rangle = U_{a,j}^{CPF}R_a(-\pi/2)U_{a,k}^{CPF}R_a(\pi/2)U_{a,j}^{CPF}|\Psi_{j,k}\rangle \otimes |\Phi_{a,i}\rangle$$

That means we can construct a CZ gate between two photons

2.3 Construcing two-qubit gates in one-way quantum computing

2.3.1 Basic knowledge about graph state

In quantum computing, a graph state is a special type of multi-qubit state that can be represented by a graph. Each qubit is represented by a vertex of the graph, and there is an edge between every interacting pair of qubits. In particular, they are a convenient way of representing certain types of entangled states, like:

$$|G\rangle = \prod_{(a,b)\in E} CZ_{a,b}|+\rangle^{\otimes|V|}$$

in which G=(V,E). Graph states are useful in quantum error-correcting codes, entanglement measurement and purification and for characterization of computational resources in measurement based quantum computing models.

2.3.2 Generation of cluster state

The cluster state is a special graph state whose vertexes are linked as a string by edges. We can a use a voigt magnitude to drive the spin of quantum dot, and there will be a Lamor precession [13]. Every $pi/2$ it rotates, we use a pump pulse to deterministically generate a single photon. Finally, the photon strings will form a cluster state.

2.3.3 Constructing two-qubit gate by measuring

Cluster state is such a special state that many quantum computing process can be easily realized on it.

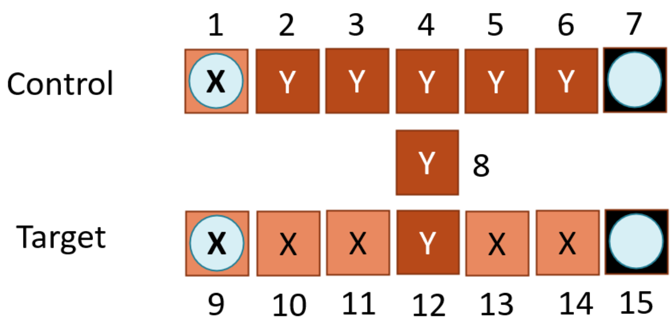


Figure 4: The construction of CNOT by measuring

In order to construct a two-qubit gate, the qubit series need to be measured as fig4. The qubit-1 is the input control qubit, and the qubit-9 is the input target qubit. The qubit-7 is the output control qubit, and the qubit-15 is the output target qubit. All the others are auxilliary qubit. According to one-way computing theory, after the measurements, the CNOT will be realized.

3 Experiment and Data Analysis

Because of the limitness of the equipments in the laboratory, I have only complete the two-qubit gate using linear optics.

3.1 Linear optical two-qubit gate

Based on KLM scheme, we choose line coding as my coding method. In order to reduce the cost of experiment as well as simplify the operating difficulty, we used the 12-mode photonic processor produced by QuiX Quantum company [16].

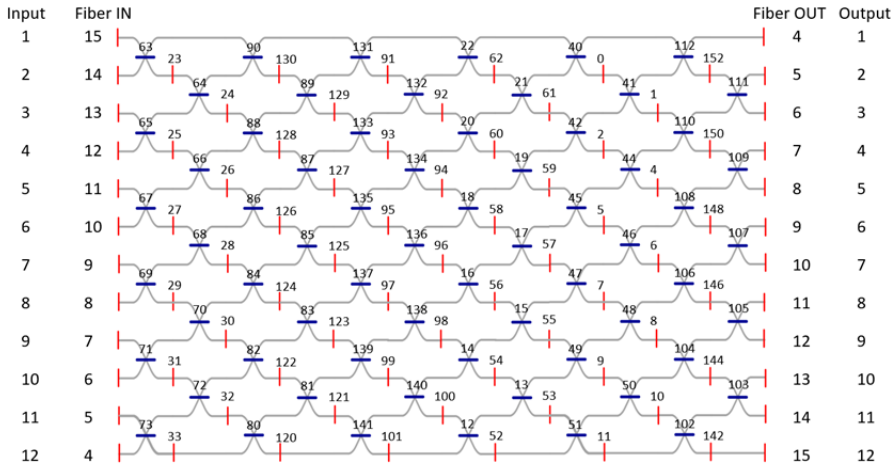


Figure 5: The phonic processor. The horizontal blue lines represent tuneable beam splitters, the vertical red lines phase shifters. [16]

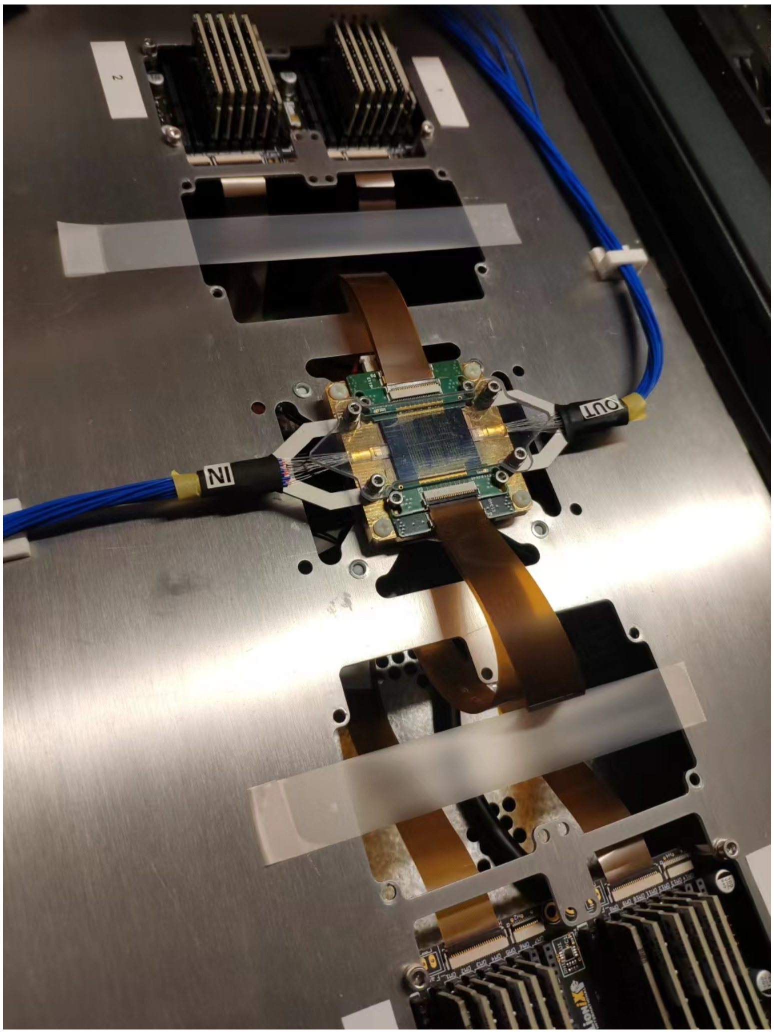
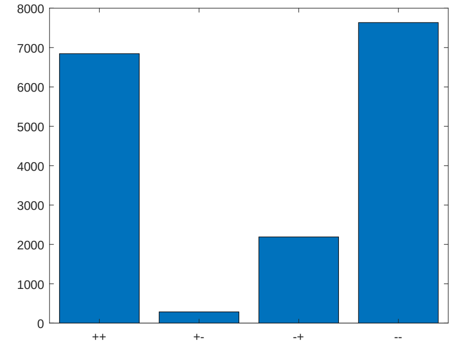


Figure 6: The photo of the photonic processor [16]

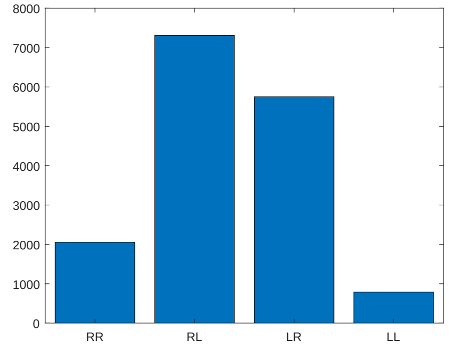
It is a 12-mode M-Z interferometer, in which we can adjust the phase shifters and the tunable beam spliters to modulate the transfer matrix to meet our needs.

In this experiment, we only used the 1,2,3,4 input ports and 2,3,6,8,as output ports. We define that input port 1 and 2 respectly stand for H and V of the control qubit, and 3 and 4 stand for the target qubit. Similarly, output port 2 and 3 stand for the control qubit, and 6,8 stand for target qubit. We wanted to pour the photons into the input port 1 and port 3, then we transform the control qubit by a hadamard gate before entering the CNOT. So while the input state is $|HH\rangle$, after the hadamard it will become $\frac{1}{\sqrt{2}}(|HH\rangle + |VH\rangle)$. Then it will become an entangled state $\frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$ after the CNOT. Finally, we need to measure it by pauli X,Y,and Z, since the final state can be written as $\phi = \frac{1}{\sqrt{2}}(|{+}{+}\rangle + |{-}{-}\rangle) = \frac{1}{\sqrt{2}}(|LR\rangle + |RL\rangle) = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$,

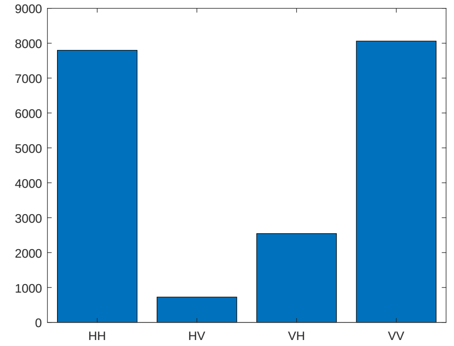
We calculated the matrix and modulate the phase shifters and the tunable beam spliters to construct the gates. The CNOT needed to work by post-selection. Finally, we used the random single photon source with a brightness of 50MHZ, purity of 0.97 and identity of 0.9 to perform the experiment and got the data.



(a) Measuring by X



(b) Measuring by Y



(c) Measuring by Z

Figure 7: The output state measured by pauli X,Y,Z

The average efficiency of the gate is 17.33%, which is ralatively a little higher than the first CNOT gates, whose efficiency is 1/9, but still far from meeting the needs of most algorithms. The low efficiency comes from the high loss of randomness of the gate as well as randomness of the photon source. As can be seen from the graphs, the accuracy of the gate is not so high as well. That's because the equipment cannot be perfectly debuged. Sometimes the vibration and the curved optical fiber can cause an effect.

4 For the future

The low efficiency of linear optical two quibit-gate comes from the randomness and the post-selection process. So in order to construct two-qubit gate with higher efficiency, we have to explore the methods to generate deterministic single photons and construct deterministic gates. These needs some nonlinearity, which will be our main focus in the near future.

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