

# אלגוריתמי מיון

$O(n^2)$

- מיון הכנסה

• מיון בעזרת עץ חיפוש בינרי موازن:

- בניית עץ AVL על-ידי הכנסת האיברים בזיה אחר זה.

- סרוק את העץ ב- inorder

$O(n \log n)$

• מיון עריםה:

- בניית ערימת מקסימום.

- הוצאת האיבר המקסימלי ושים **בראש הפלט הנוכחי**

$O(n \log n)$

1

ד"ר נעה לויינשטיין ©

עריכה: ד"ר אילת בוטמן, פרופ' אביבית לוי

# משפט

כל אלגוריתם למציאת מbijון מבודד השוואות צריך  
 $\Omega(n \cdot \log n)$  השוואות במקרהorst



# מיין הכנסה

5	2	4	6	1	3
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# מיין הכנסה

5	2	4	6	1	3
---	---	---	---	---	---





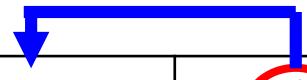
# מיין הכנסה

2	5	4	6	1	3
---	---	---	---	---	---



# מיין הכנסה

2	5	4	6	1	3
---	---	---	---	---	---



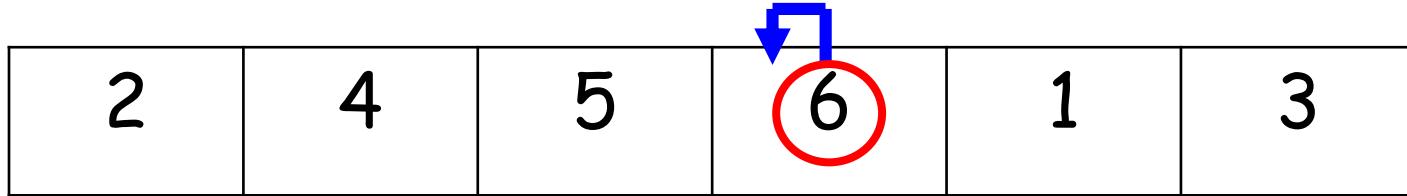


# מיין הכנסה

2	4	5	6	1	3
---	---	---	---	---	---



# מיין הכנסה





# מיין הכנסה

2	4	5	6	1	3
---	---	---	---	---	---



# מיין הכנסה

2	4	5	6	1	3
---	---	---	---	---	---



1

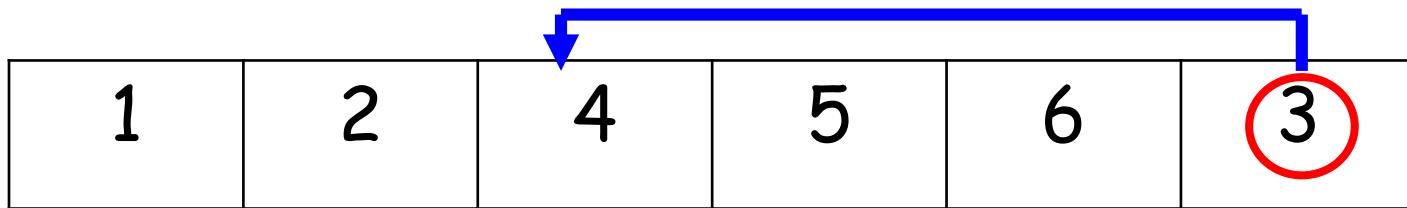


# מיין הכנסה

1	2	4	5	6	3
---	---	---	---	---	---



# מיין הכנסה





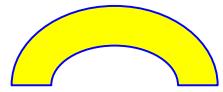
# מיין הכנסה

1	2	3	4	5	6
---	---	---	---	---	---



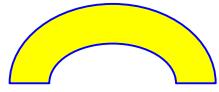
# מיין הכנסה

5	2	4	6	1	3
2	5	4	6	1	3
2	4	5	6	1	3
2	4	5	6	1	3
1	2	4	5	6	3
1	2	3	4	5	6



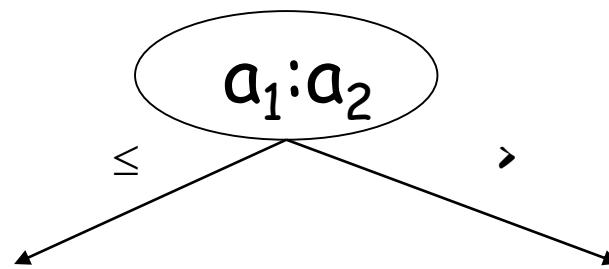
a[1]	a[2]	a[3]
?	?	?

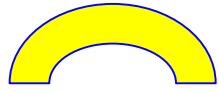
# עַז הַחֲלֹטָה



$a[1]$	$a[2]$	$a[3]$
?	?	?

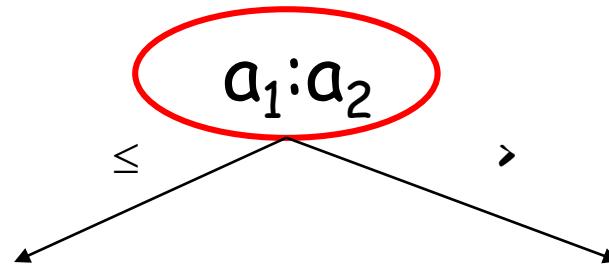
# עַז הַחֲלֹטָה

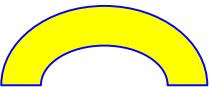




$a[1]$	$a[2]$	$a[3]$
1	2	3

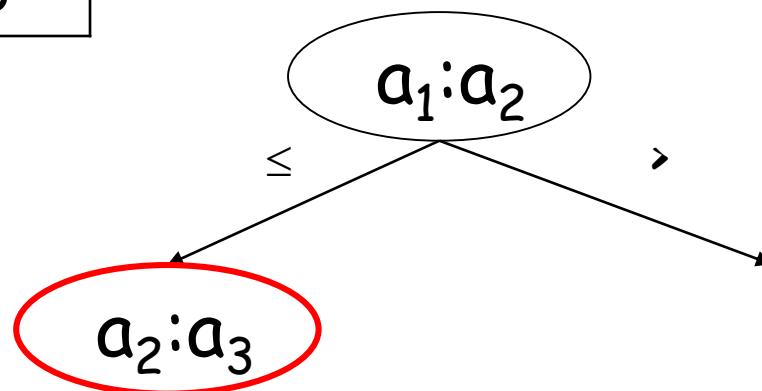
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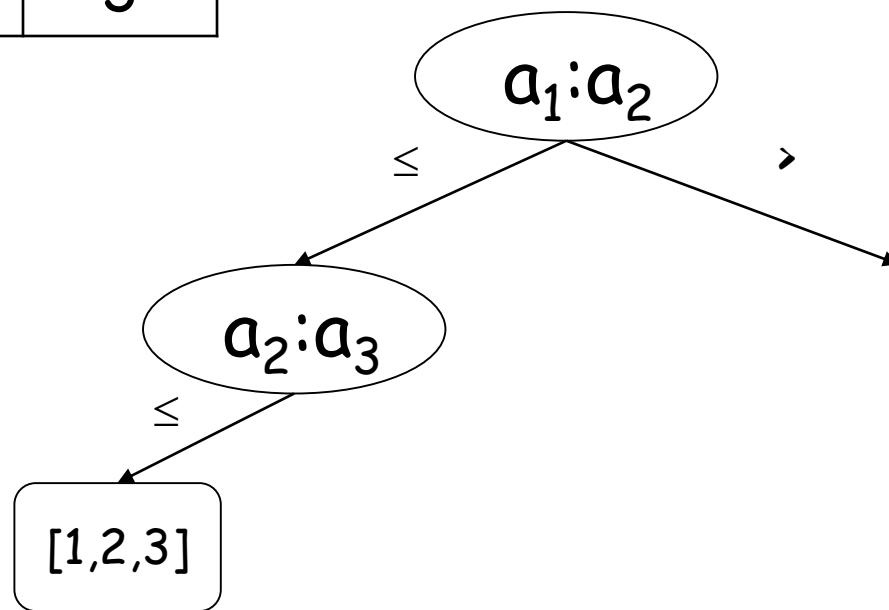
$a[1]$	$a[2]$	$a[3]$
1	2	3

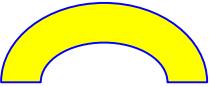
# עַז הַחֲלֹתָה



$a[1]$	$a[2]$	$a[3]$
1	2	3

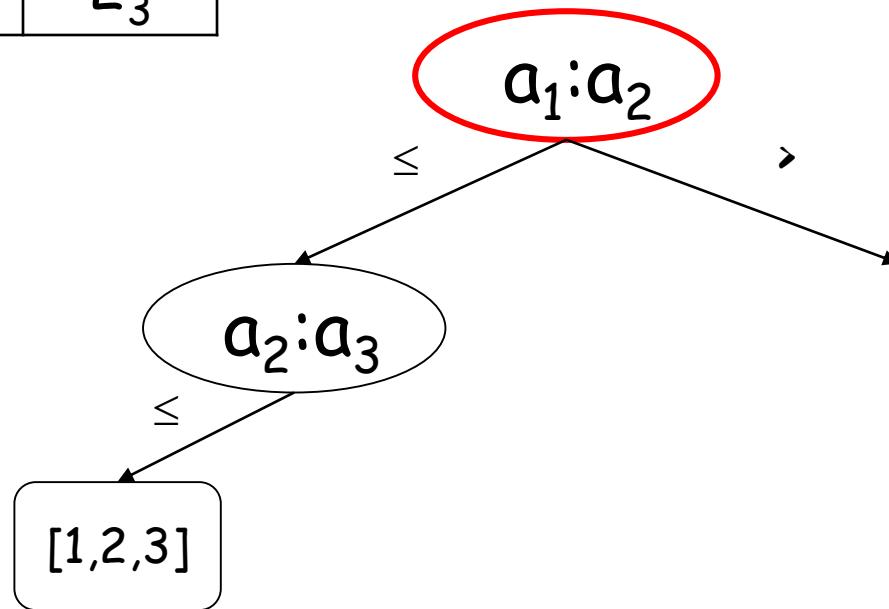
# עץ החלטה





$a[1]$	$a[2]$	$a[3]$
$1_1$	$3_2$	$2_3$

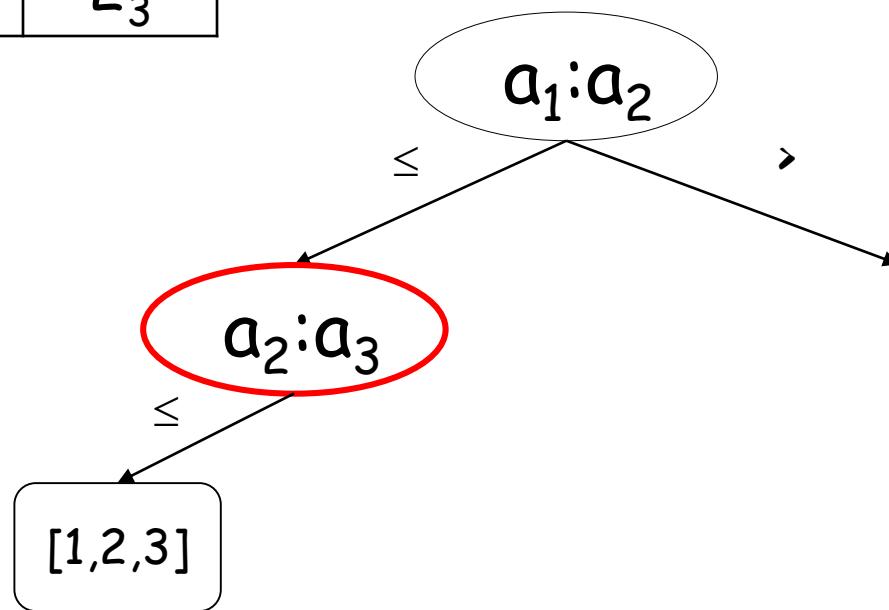
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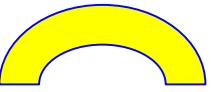




$a[1]$	$a[2]$	$a[3]$
$1_1$	$3_2$	$2_3$

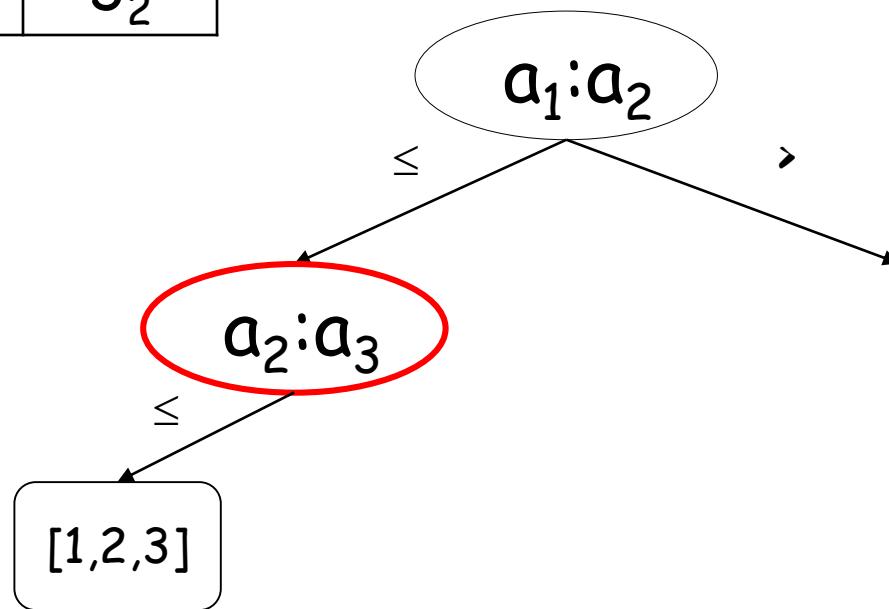
# עַז הַחֲלֹתָה

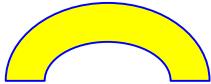




$a[1]$	$a[2]$	$a[3]$
$1_1$	$2_3$	$3_2$

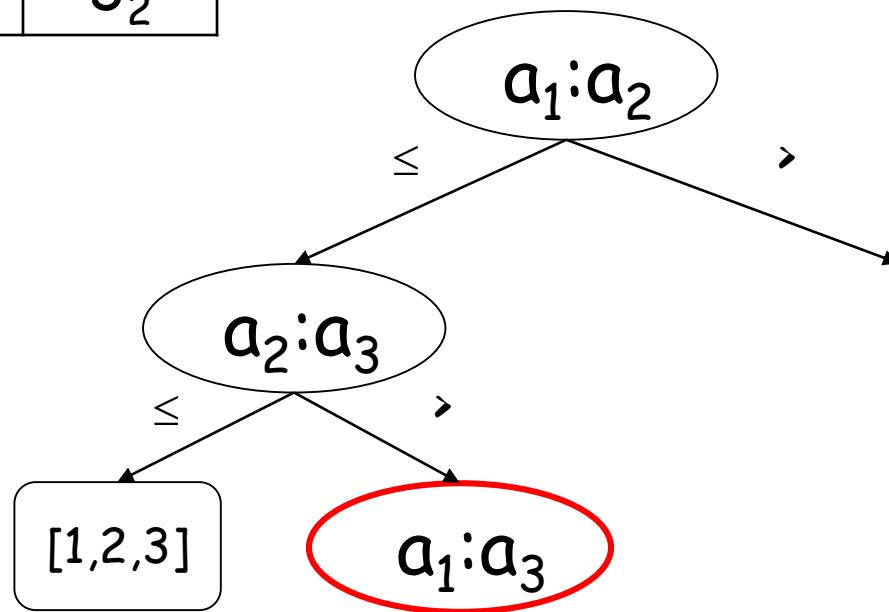
# עז החלטה





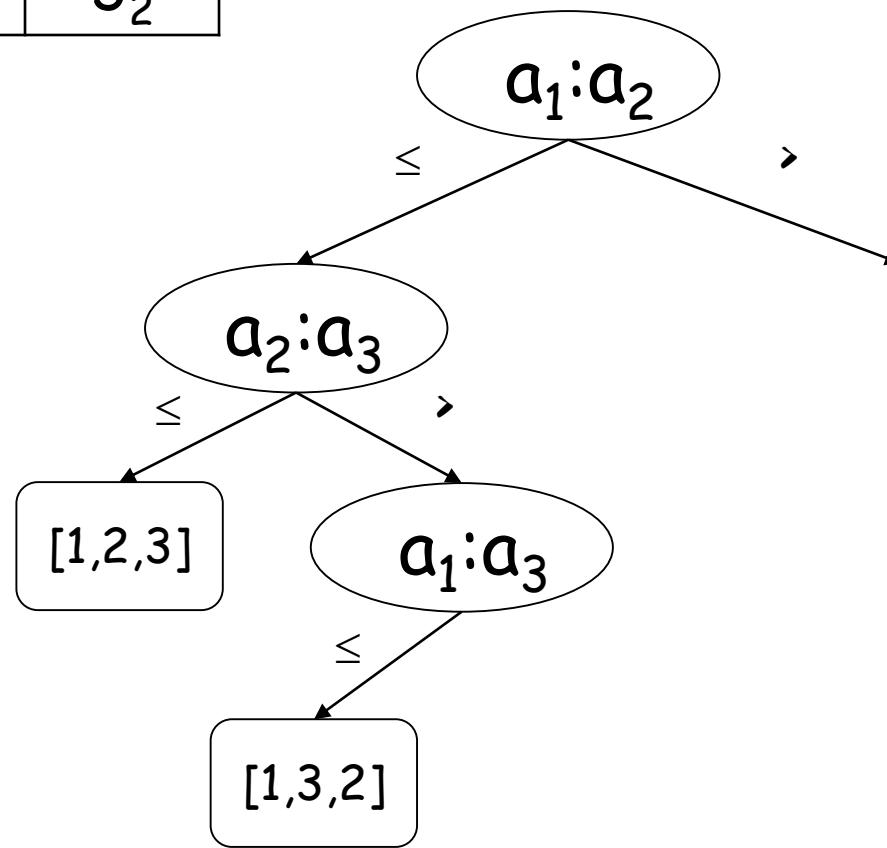
$a[1]$	$a[2]$	$a[3]$
$1_1$	$2_3$	$3_2$

# עץ החלטה



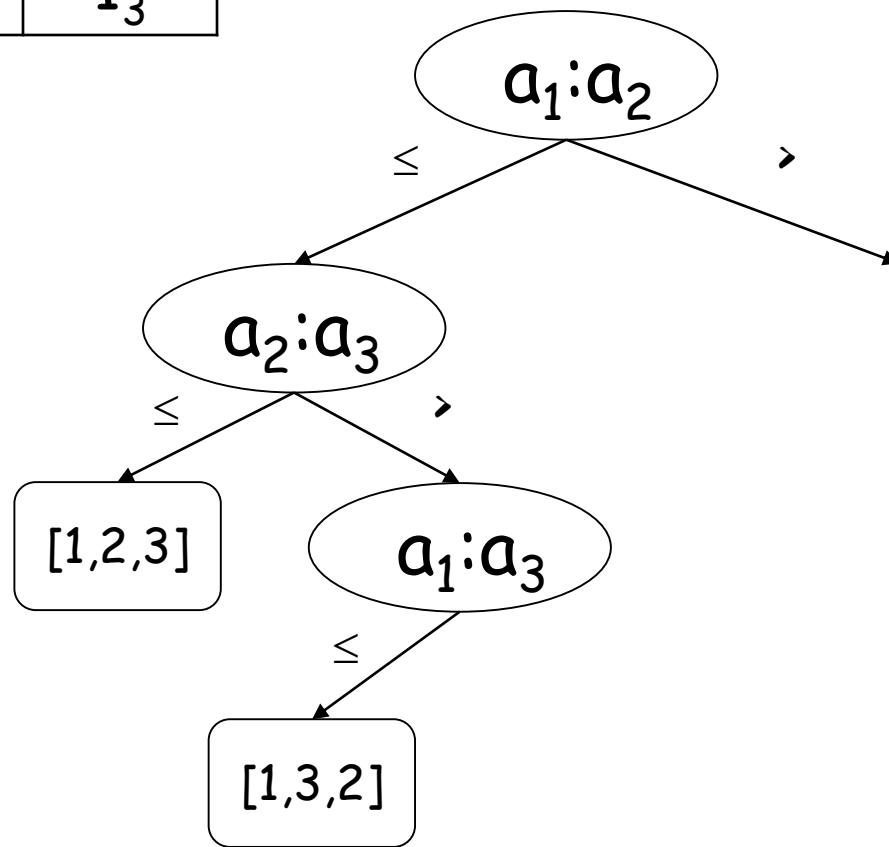
$a[1]$	$a[2]$	$a[3]$
$1_1$	$2_3$	$3_2$

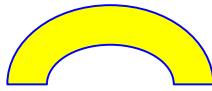
# עץ החלטה



$a[1]$	$a[2]$	$a[3]$
$2_1$	$3_2$	$1_3$

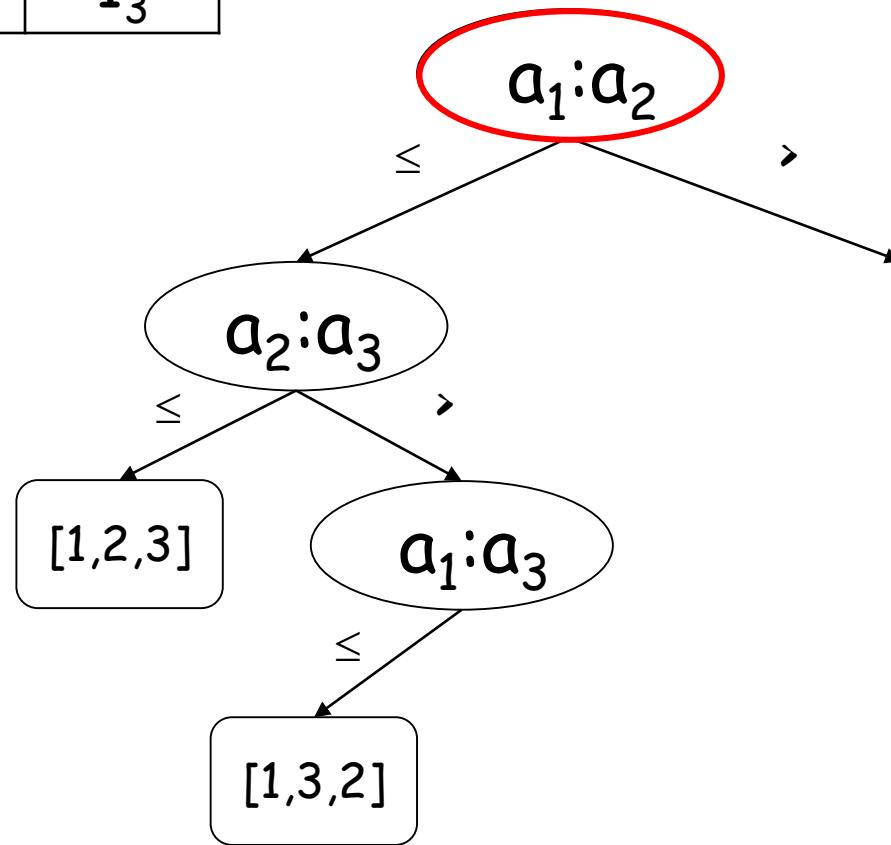
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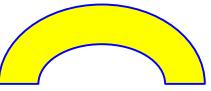




a[1]	a[2]	a[3]
2 <sub>1</sub>	3 <sub>2</sub>	1 <sub>3</sub>

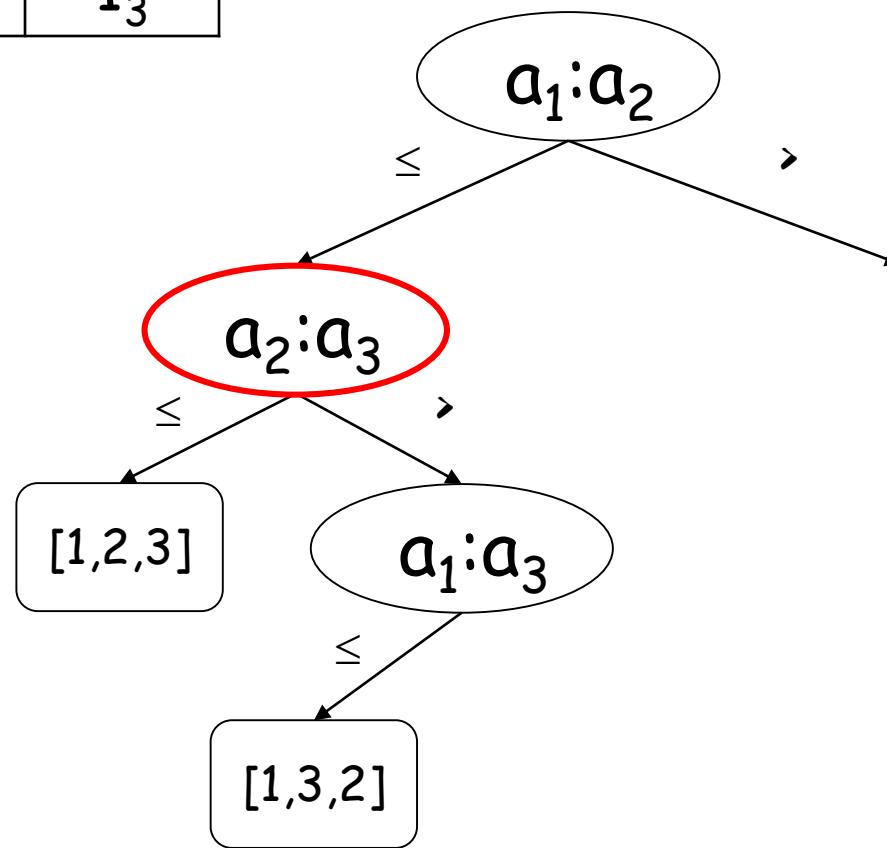
## עץ החלטה

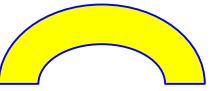




$a[1]$	$a[2]$	$a[3]$
$2_1$	$3_2$	$1_3$

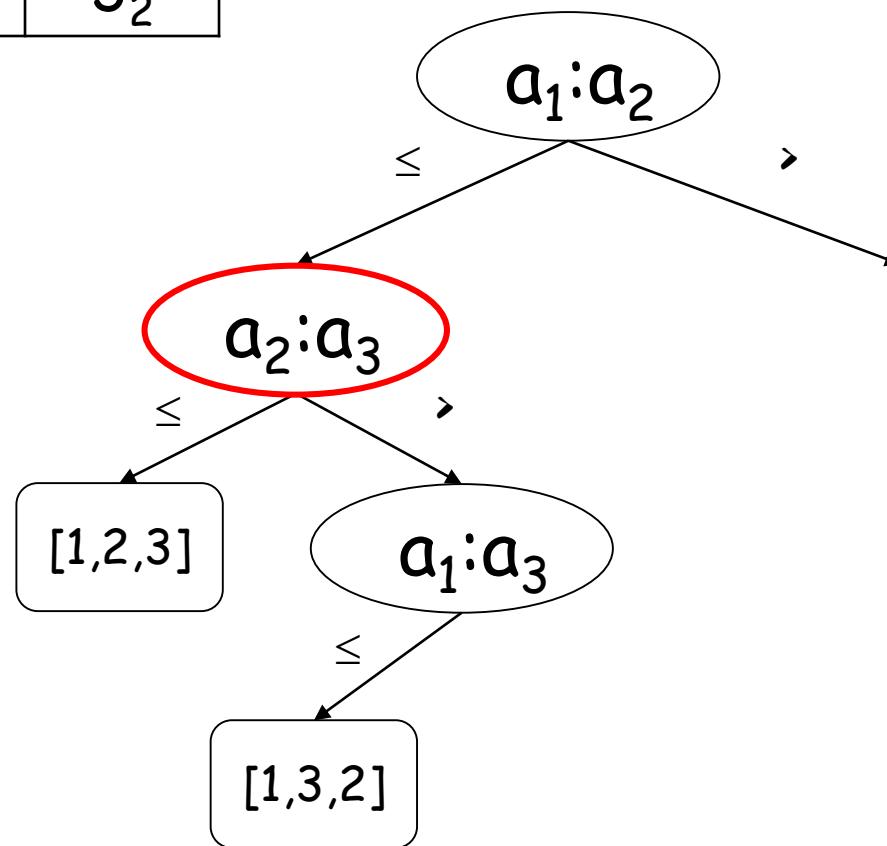
# עץ החלטה

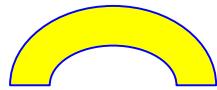




$a[1]$	$a[2]$	$a[3]$
$2_1$	$1_3$	$3_2$

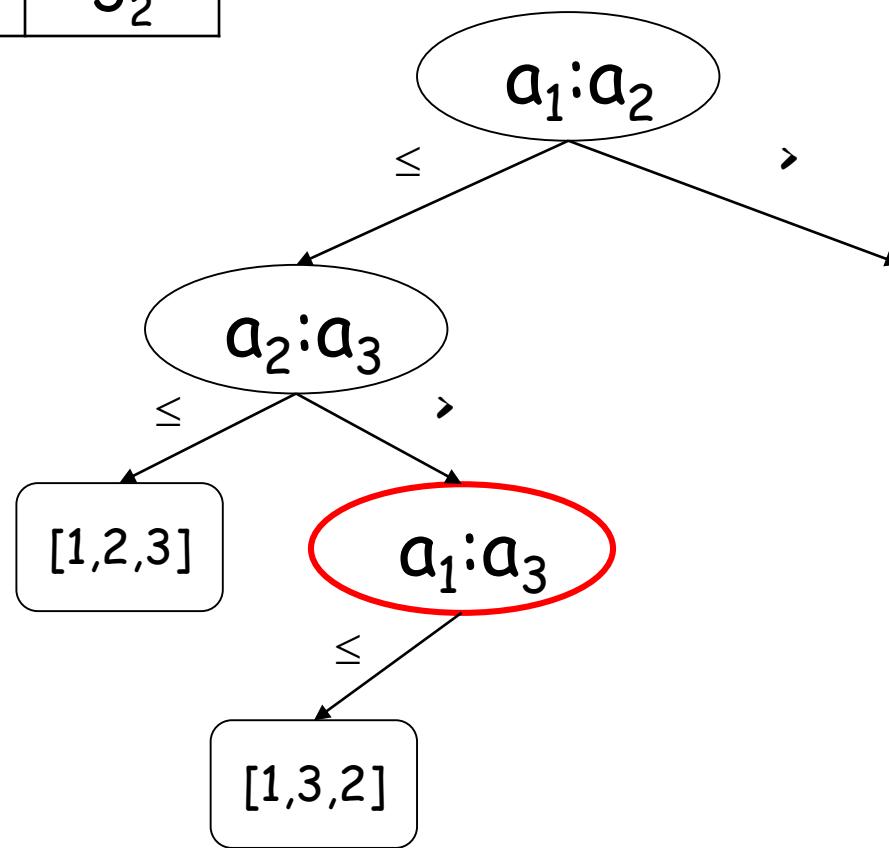
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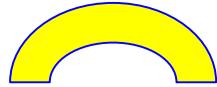




a[1]	a[2]	a[3]
2 <sub>1</sub>	1 <sub>3</sub>	3 <sub>2</sub>

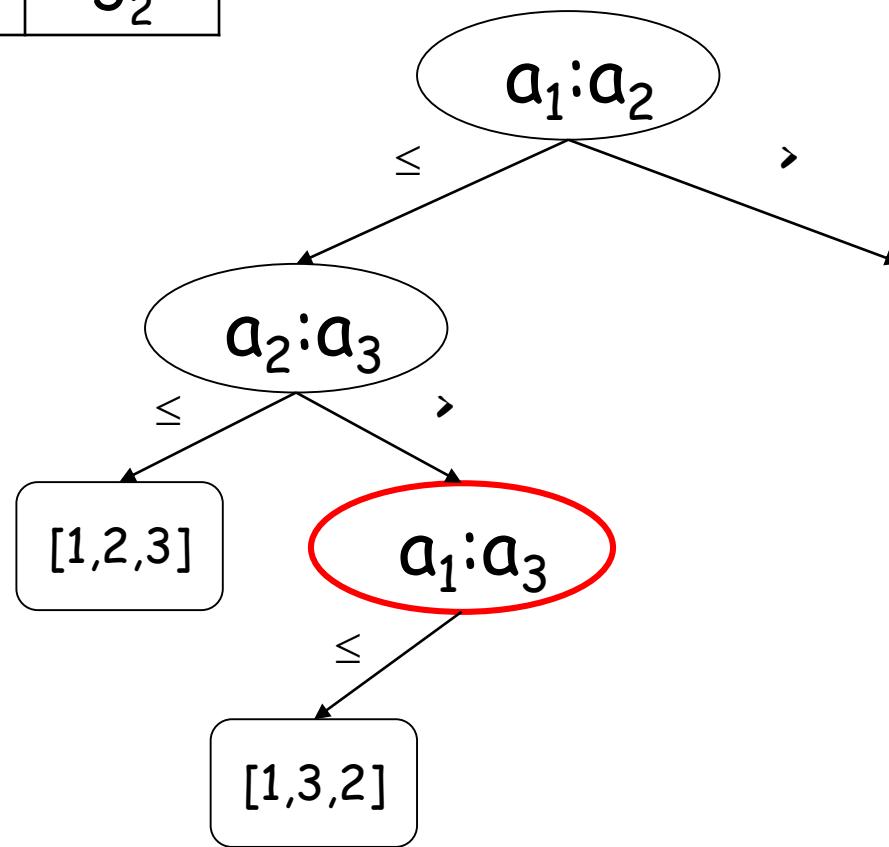
# עץ החלטה





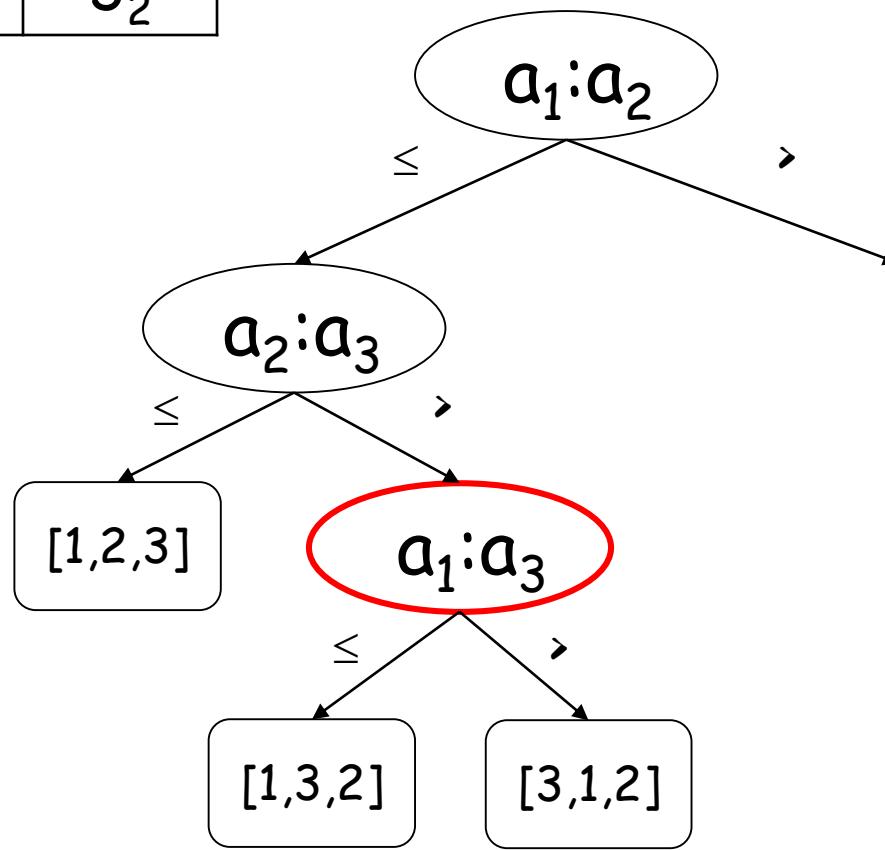
a[1]	a[2]	a[3]
1 <sub>3</sub>	2 <sub>1</sub>	3 <sub>2</sub>

# עץ החלטה



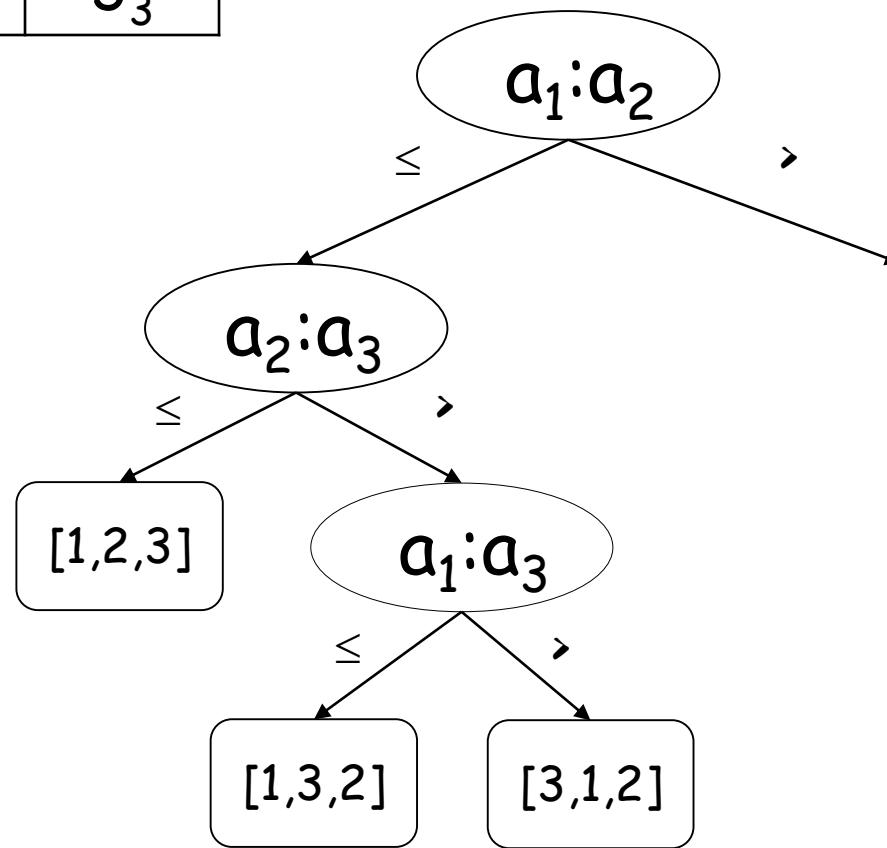
$a[1]$	$a[2]$	$a[3]$
$1_3$	$2_1$	$3_2$

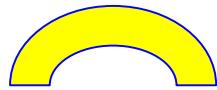
# עץ החלטה



$a[1]$	$a[2]$	$a[3]$
$2_1$	$1_2$	$3_3$

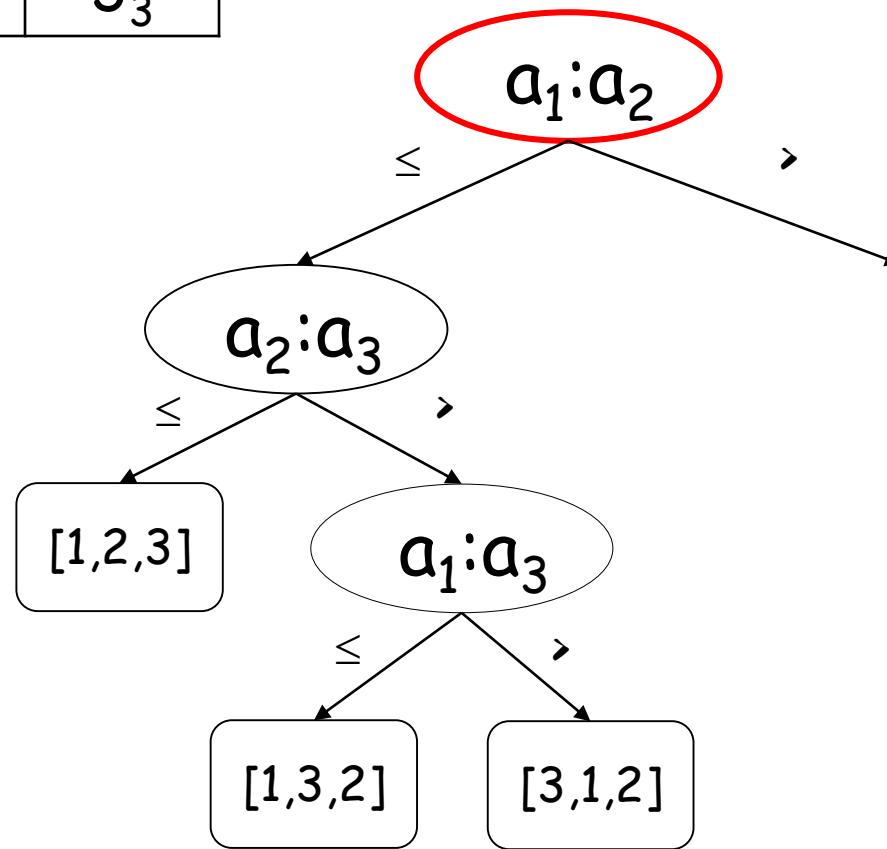
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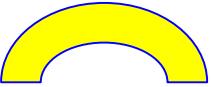




a[1]	a[2]	a[3]
2 <sub>1</sub>	1 <sub>2</sub>	3 <sub>3</sub>

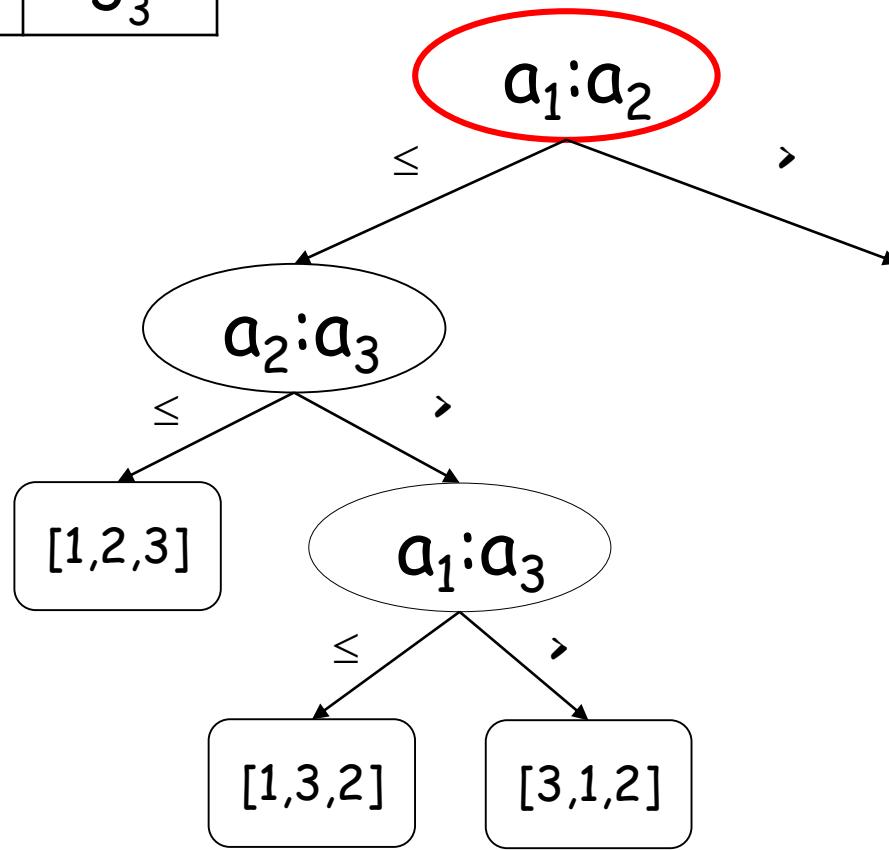
# עץ החלטה





$a[1]$	$a[2]$	$a[3]$
$1_2$	$2_1$	$3_3$

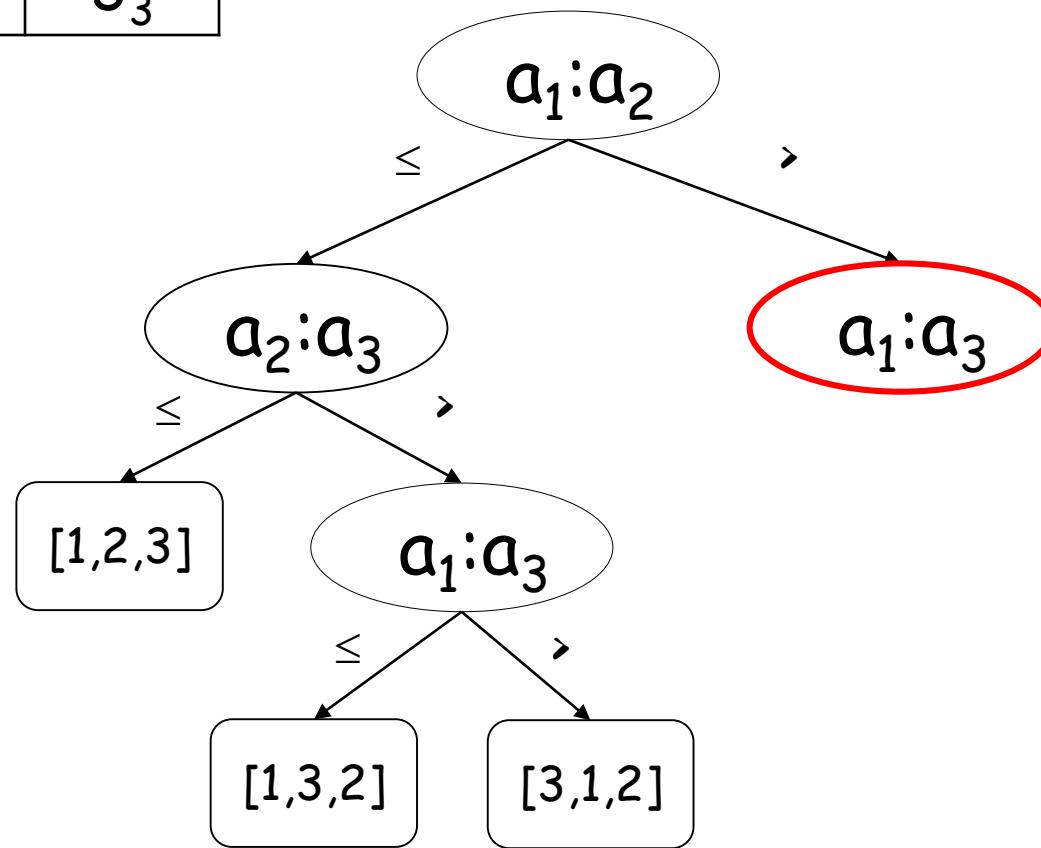
# עץ החלטה

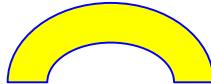




$a[1]$	$a[2]$	$a[3]$
$1_2$	$2_1$	$3_3$

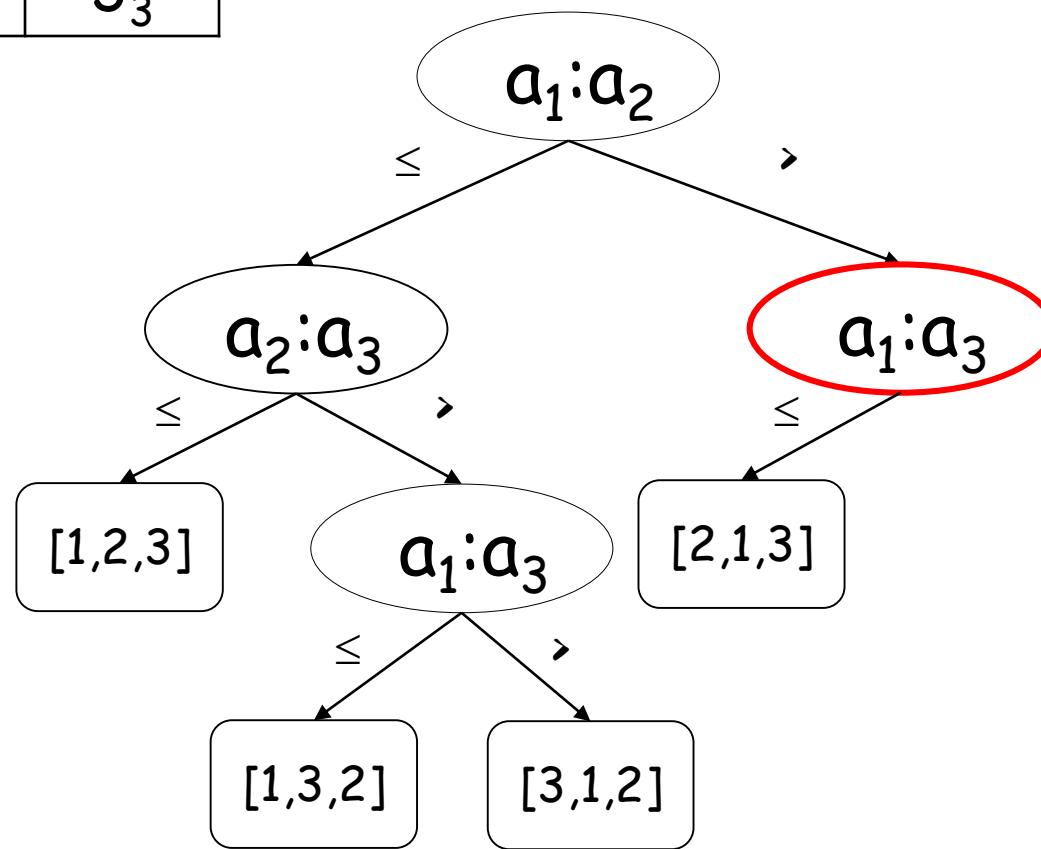
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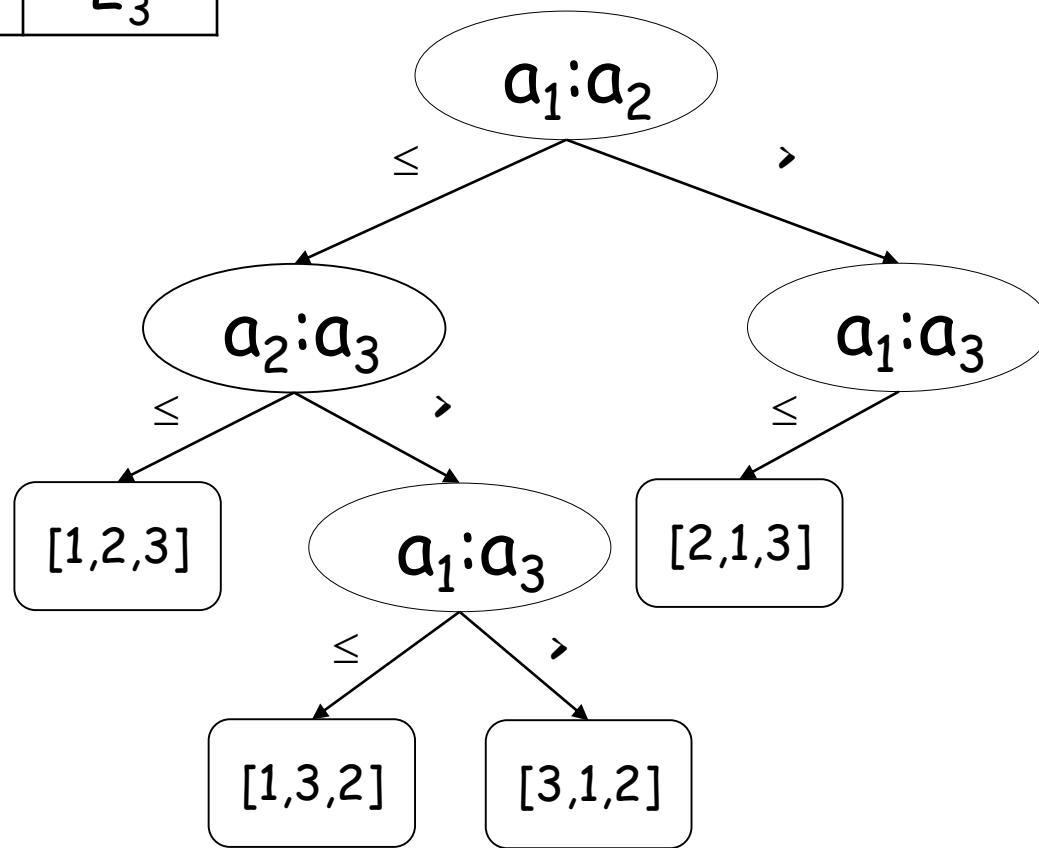
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$1_2$	$2_1$	$3_3$

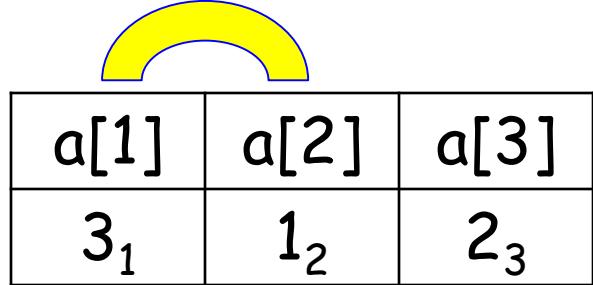
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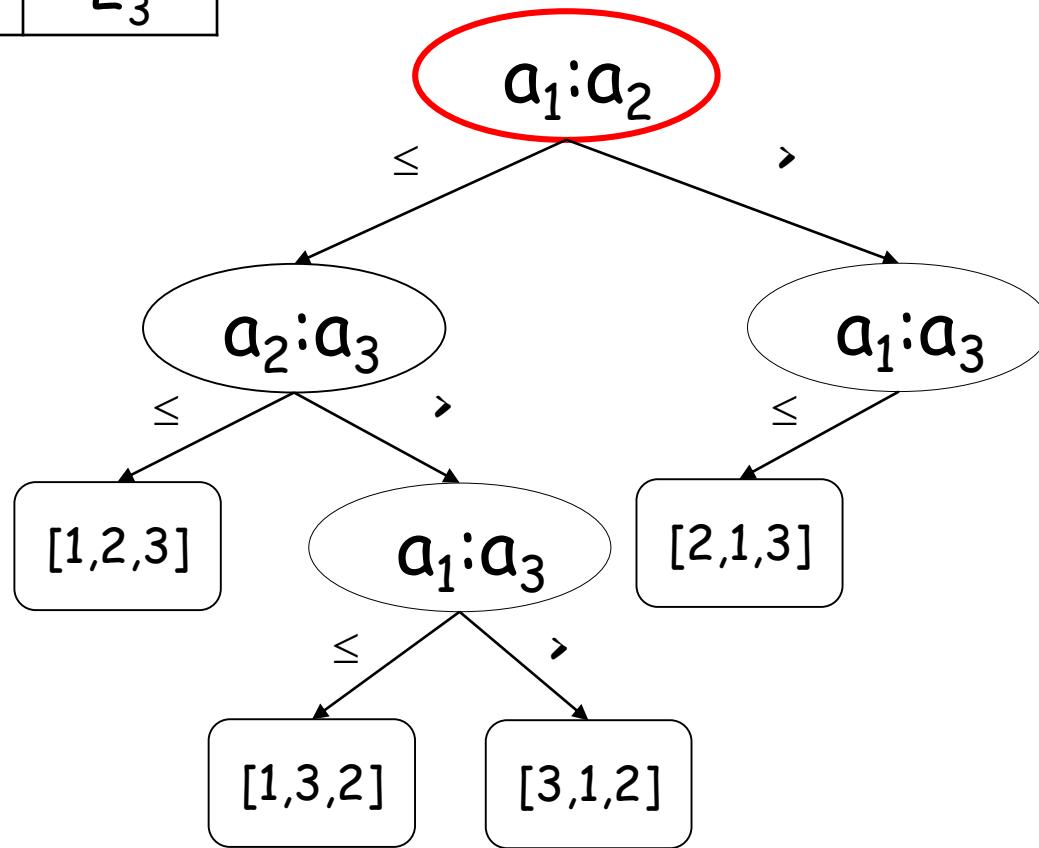
$a[1]$	$a[2]$	$a[3]$
$3_1$	$1_2$	$2_3$

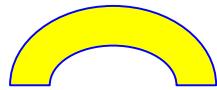
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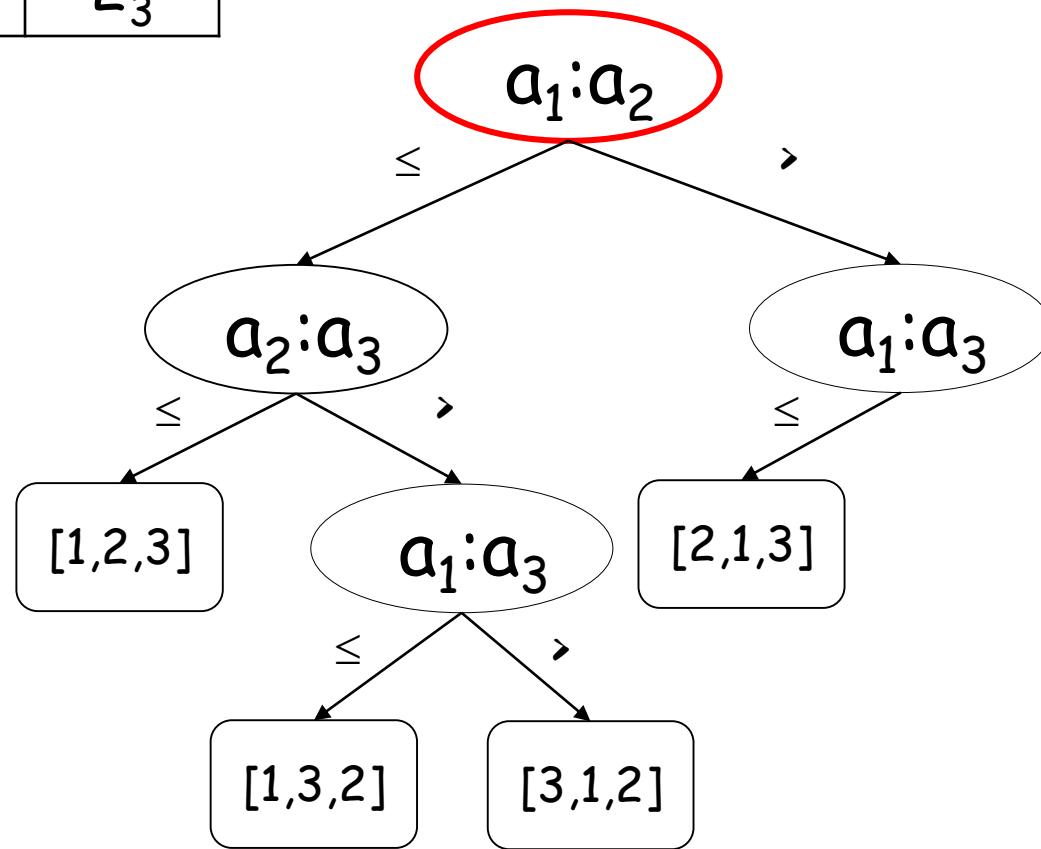
# עַז הַחֲלֹתָה

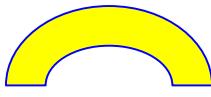




a[1]	a[2]	a[3]
1 <sub>2</sub>	3 <sub>1</sub>	2 <sub>3</sub>

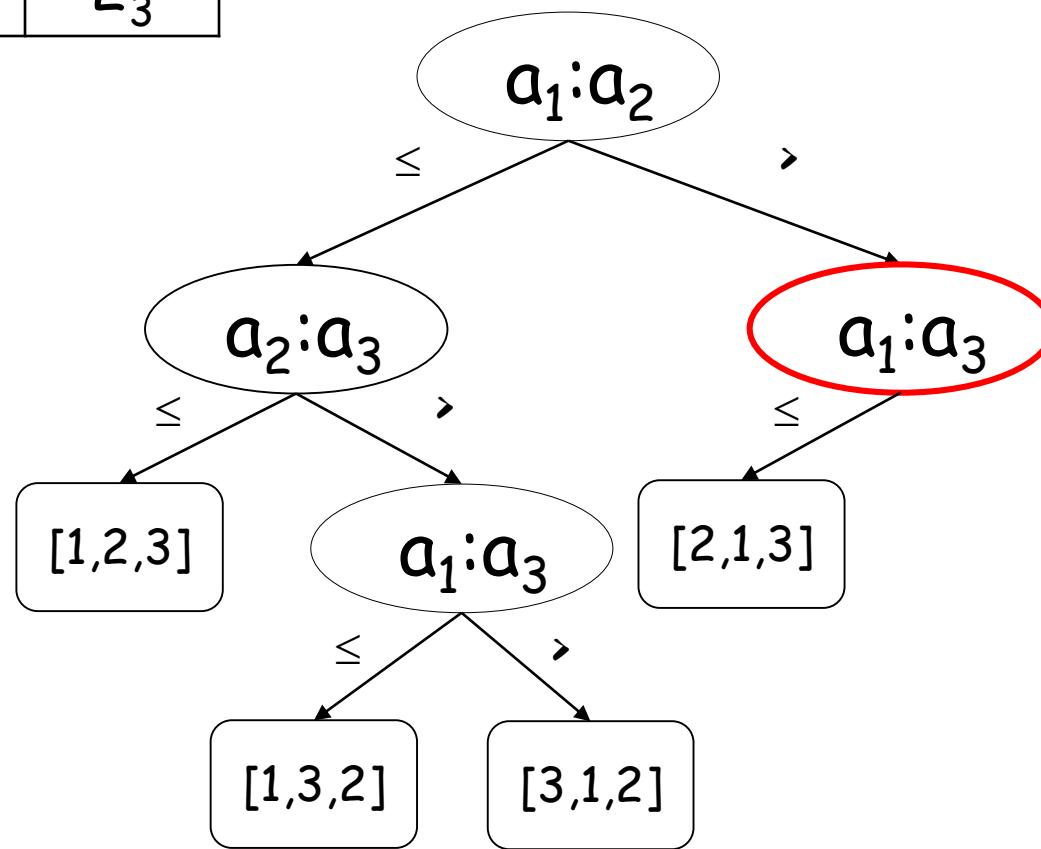
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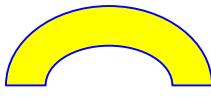




$a[1]$	$a[2]$	$a[3]$
$1_2$	$3_1$	$2_3$

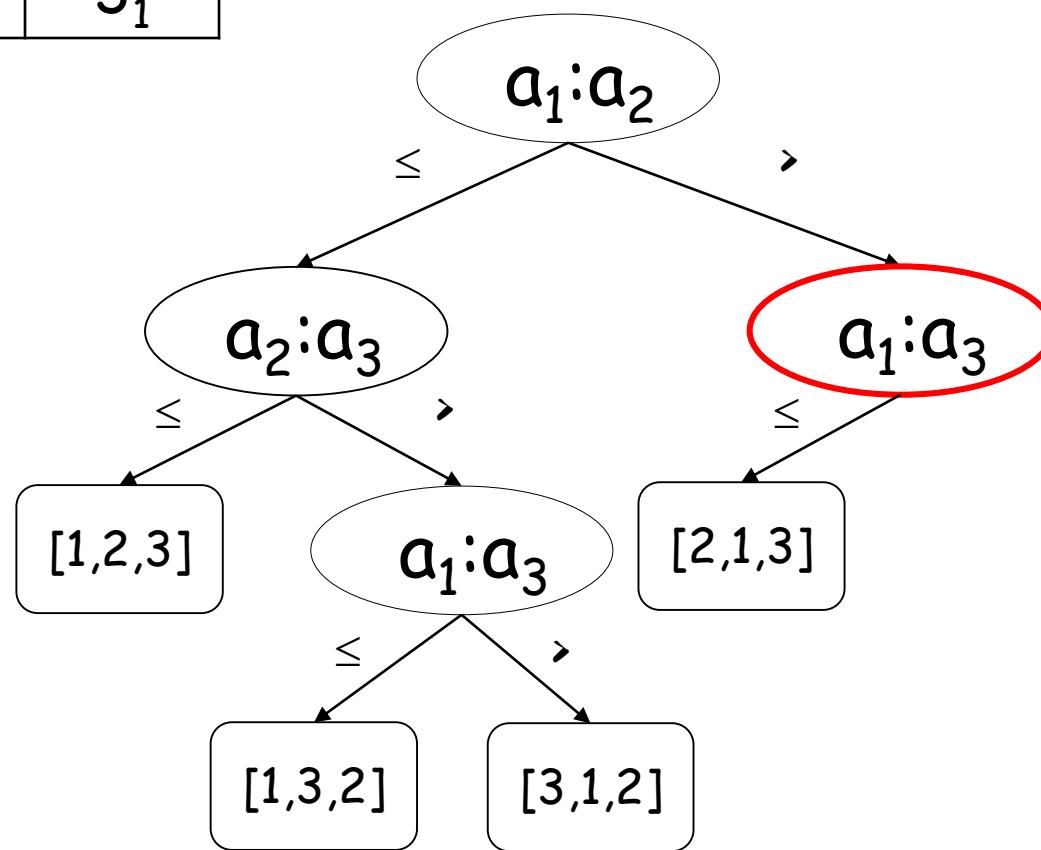
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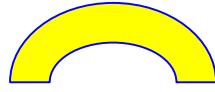




$a[1]$	$a[2]$	$a[3]$
$1_2$	$2_3$	$3_1$

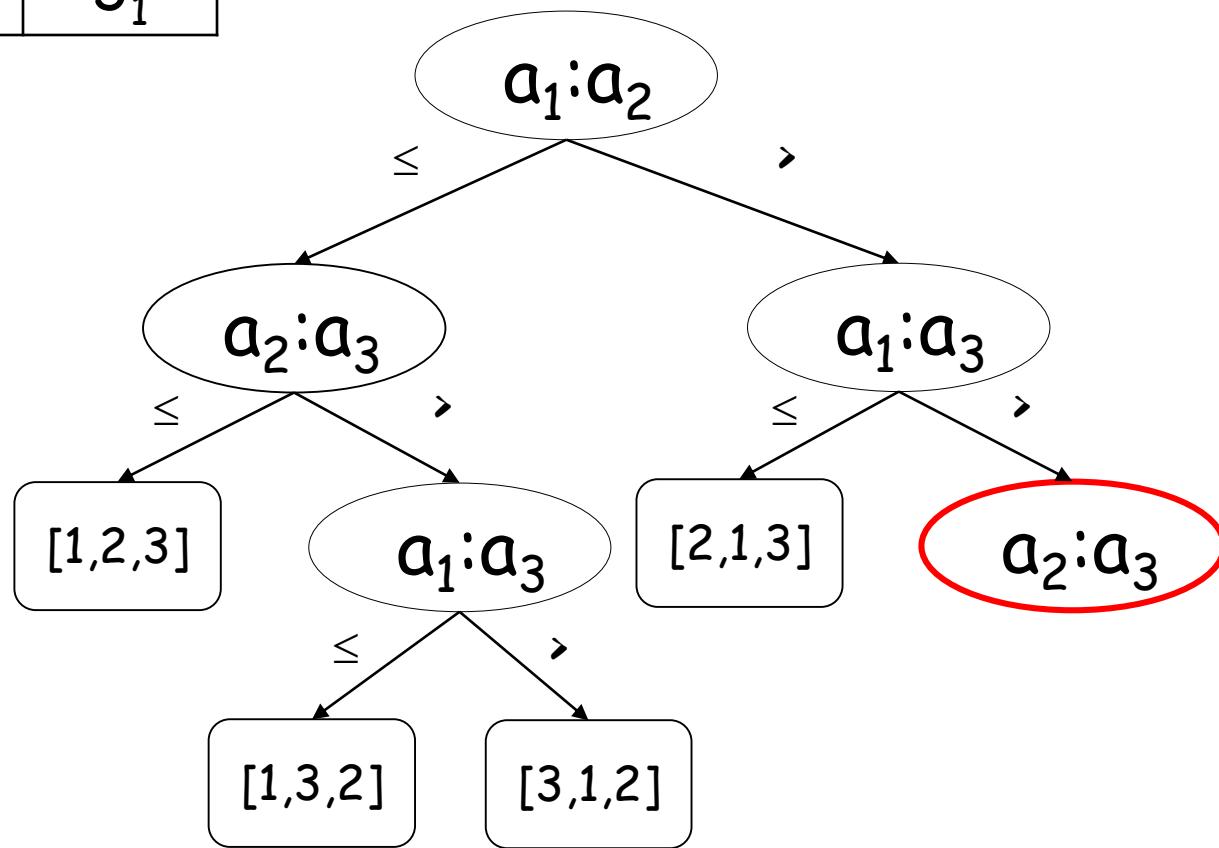
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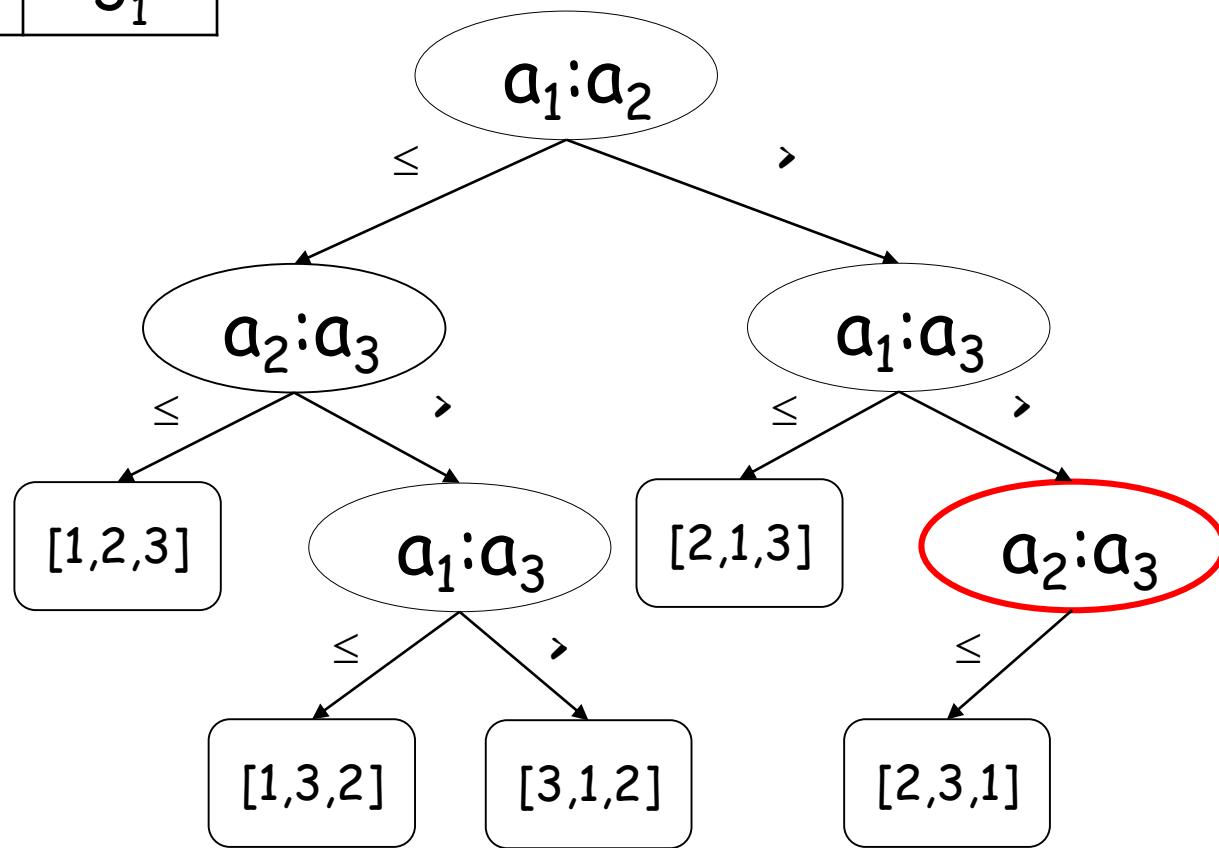
a[1]	a[2]	a[3]
1 <sub>2</sub>	2 <sub>3</sub>	3 <sub>1</sub>

# עץ החלטה



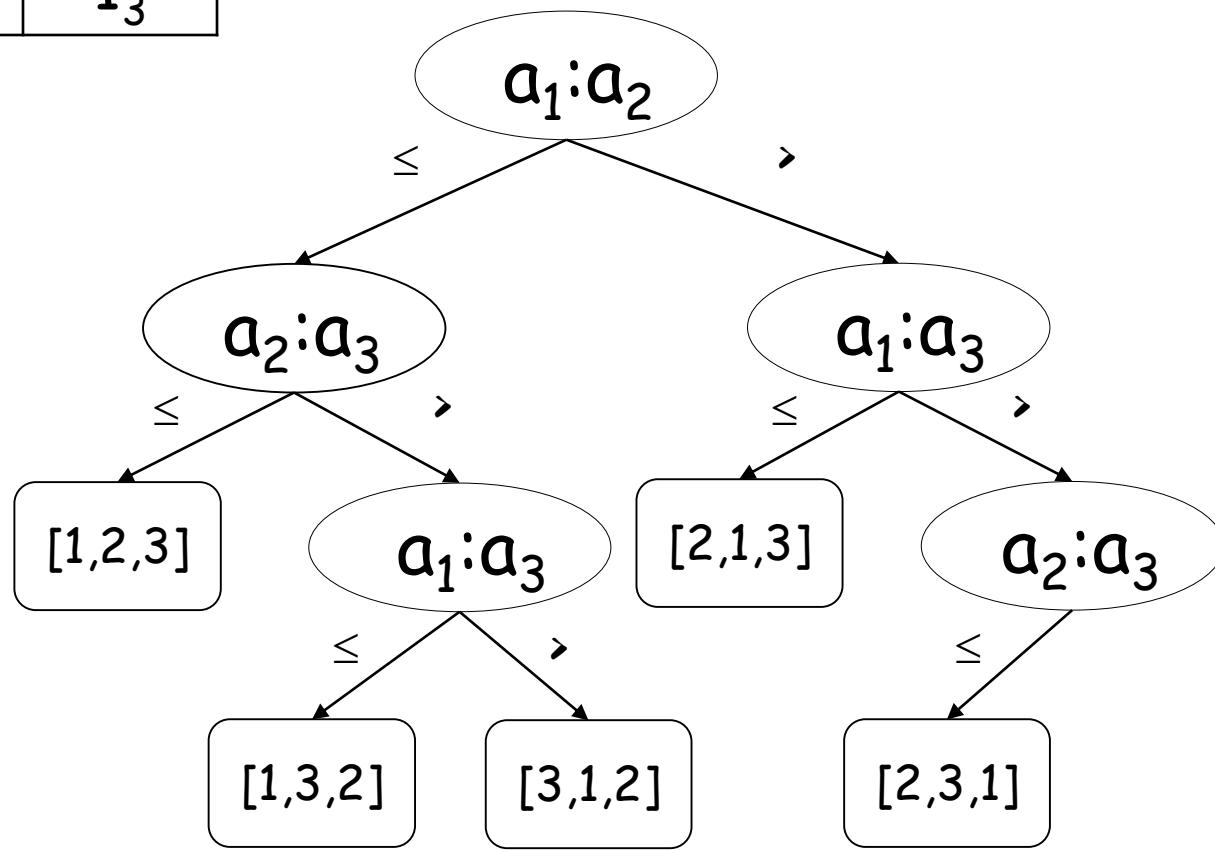
$a[1]$	$a[2]$	$a[3]$
$1_2$	$2_3$	$3_1$

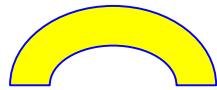
# עץ החלטה



$a[1]$	$a[2]$	$a[3]$
$3_1$	$2_2$	$1_3$

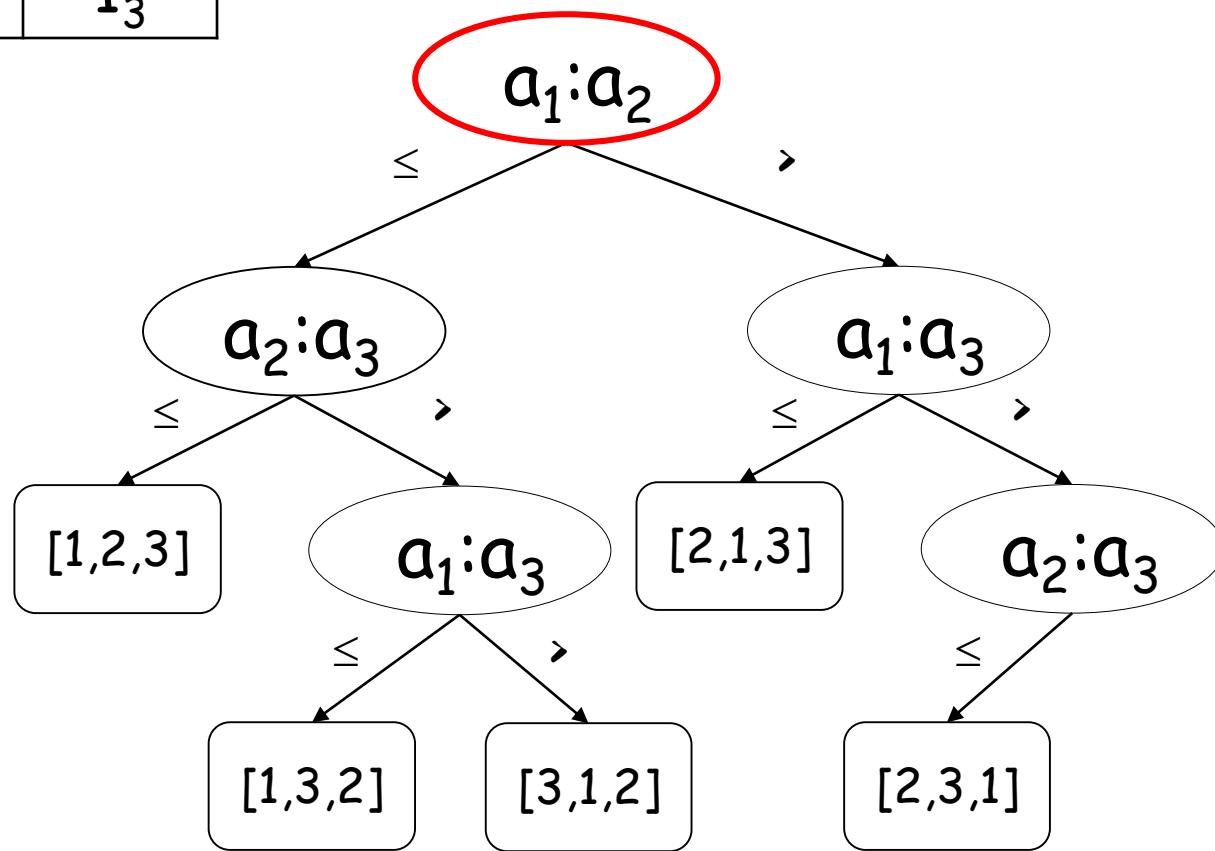
# עץ החלטה





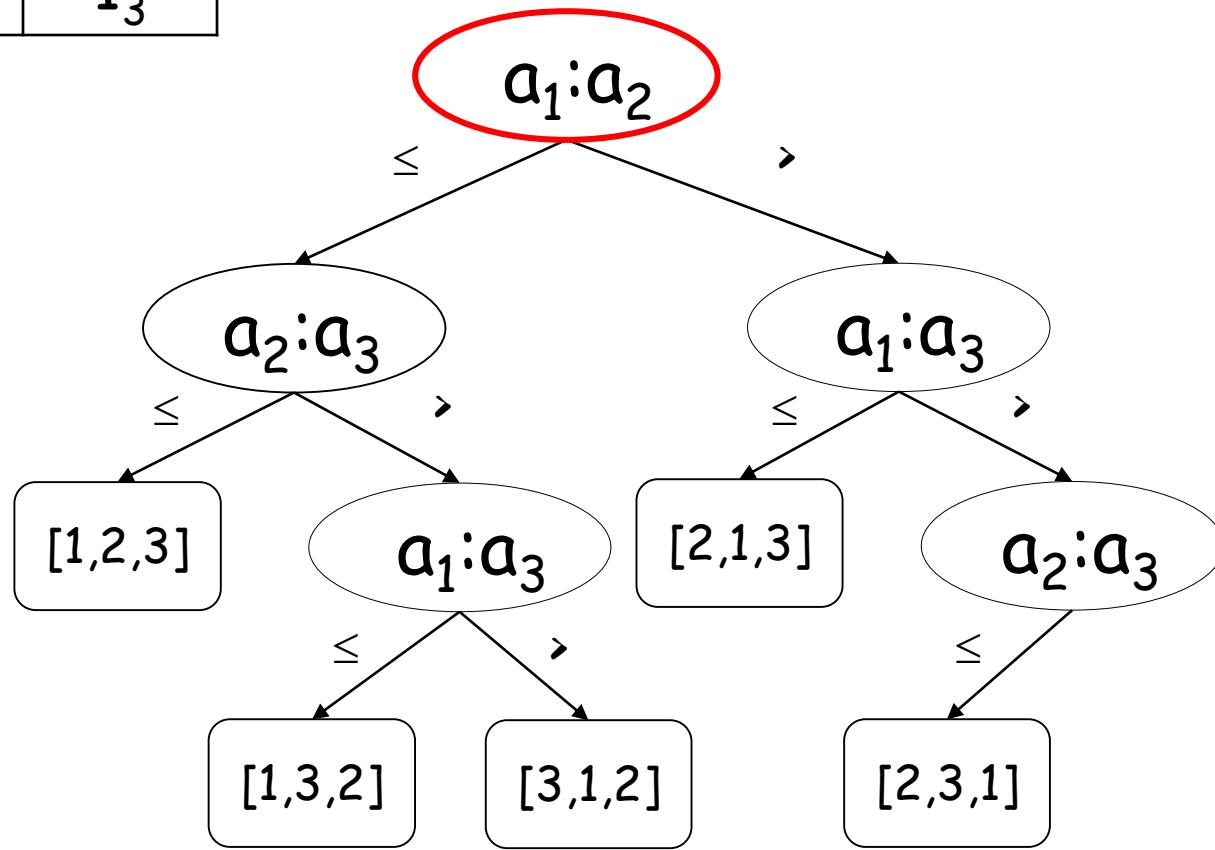
a[1]	a[2]	a[3]
3 <sub>1</sub>	2 <sub>2</sub>	1 <sub>3</sub>

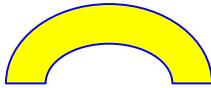
# עץ החלטה



$a[1]$	$a[2]$	$a[3]$
$2_2$	$3_1$	$1_3$

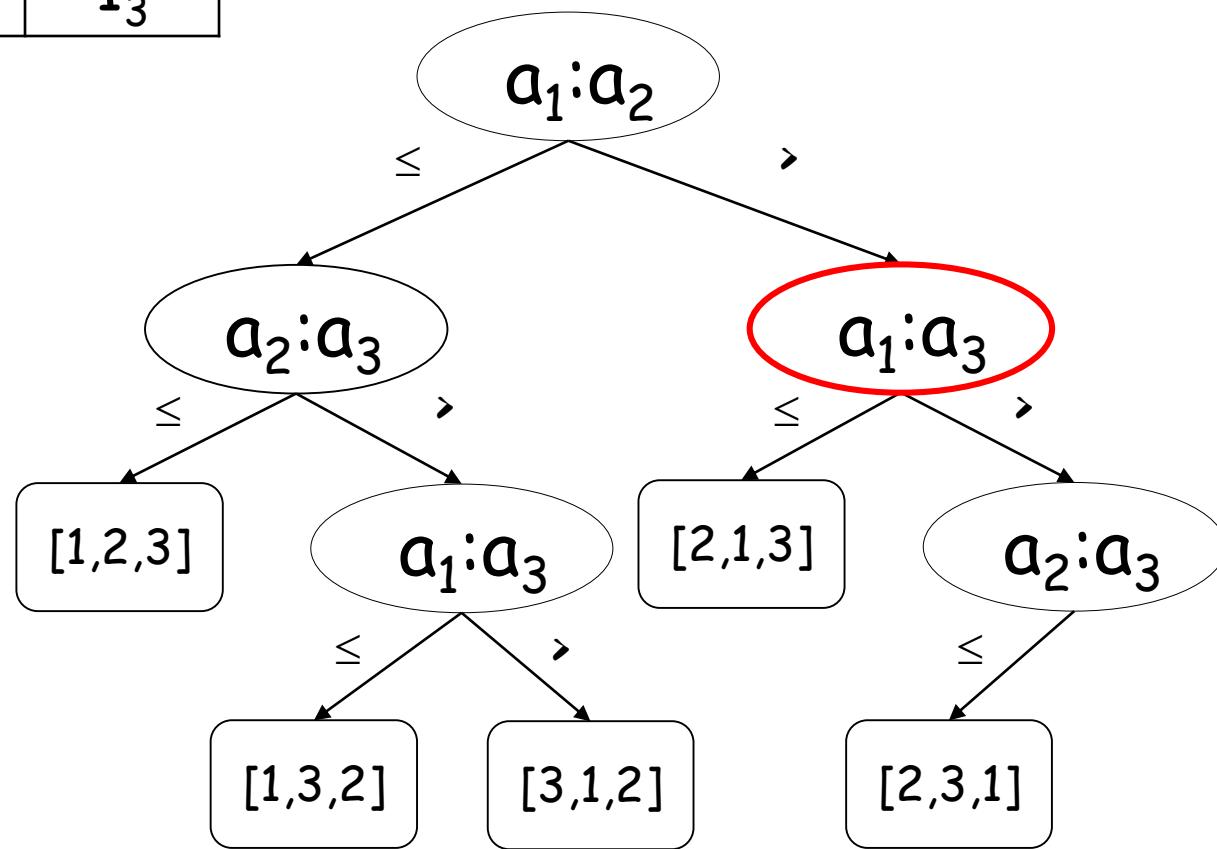
# עץ החלטה





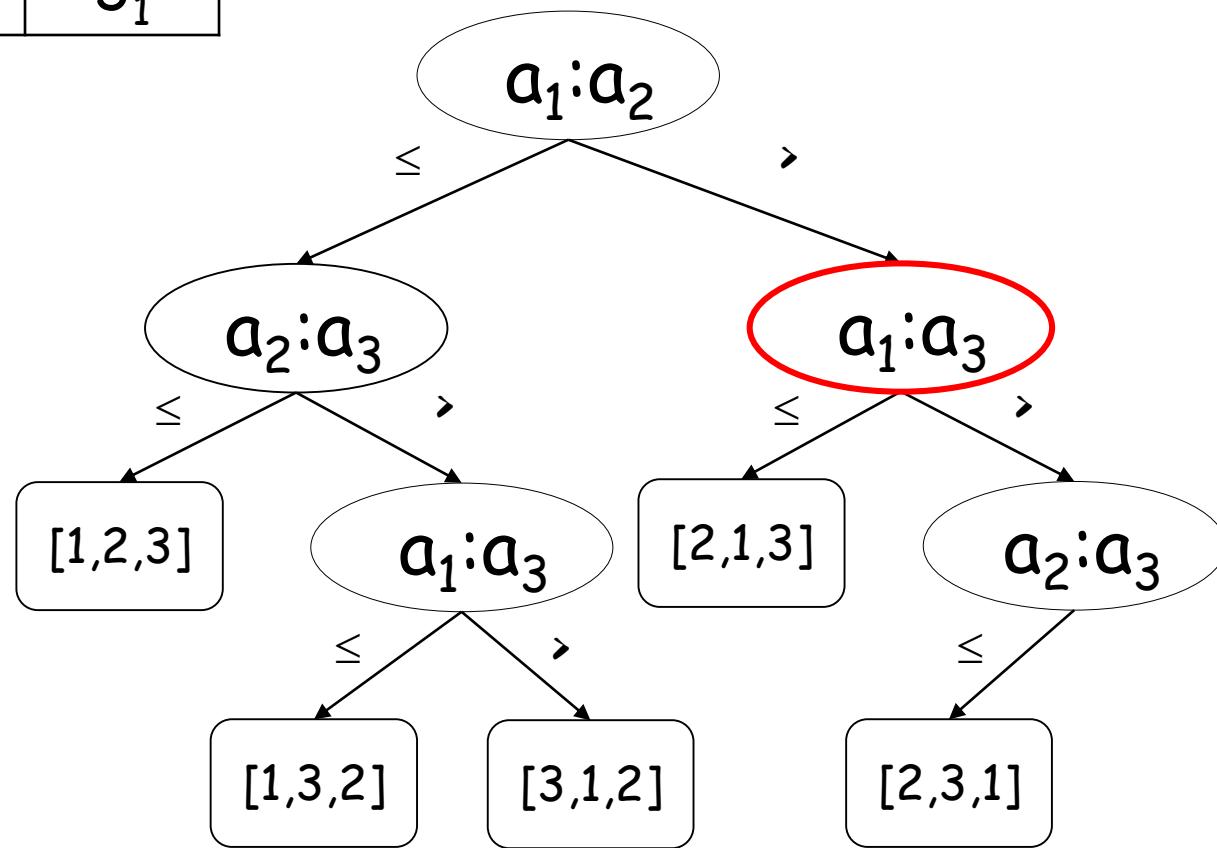
$a[1]$	$a[2]$	$a[3]$
$2_2$	$3_1$	$1_3$

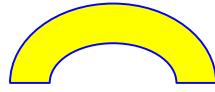
# עץ החלטה



$a[1]$	$a[2]$	$a[3]$
$2_2$	$1_3$	$3_1$

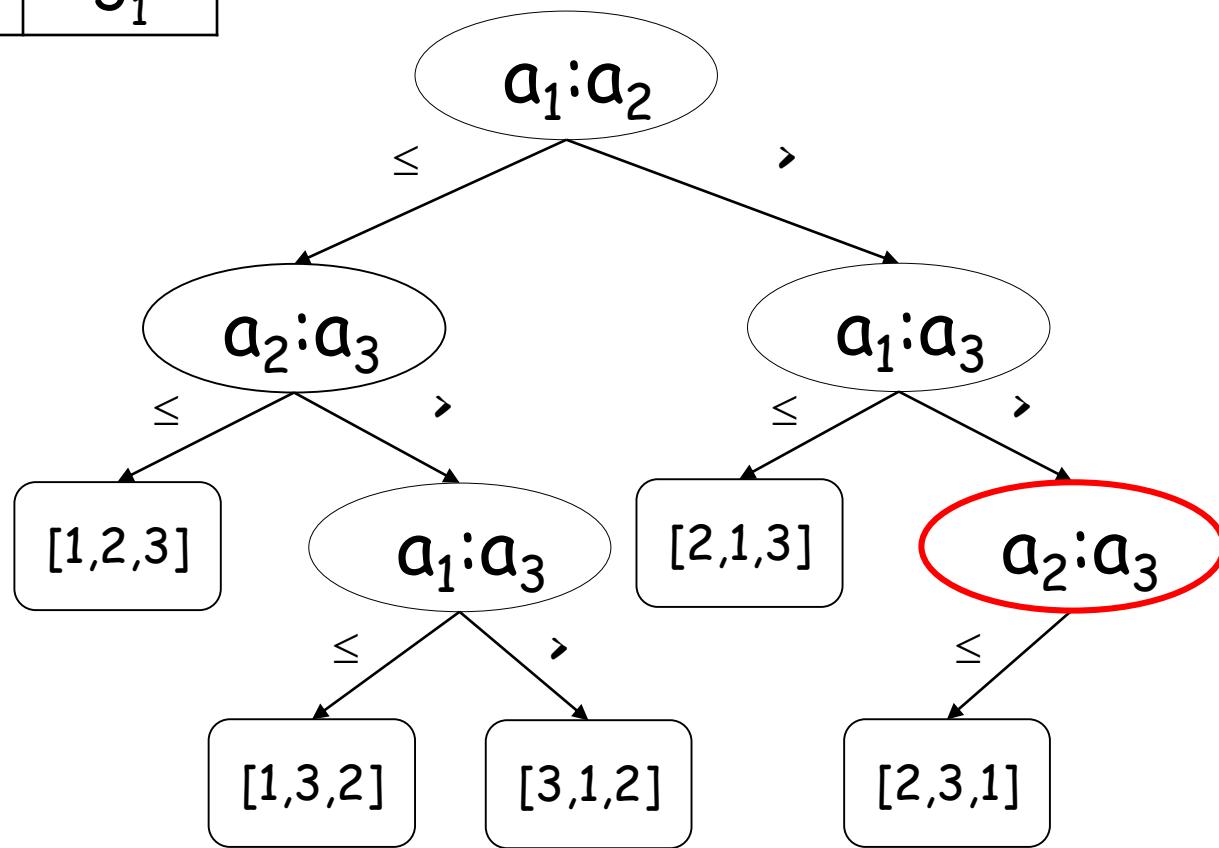
# עץ החלטה

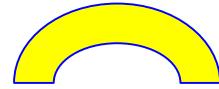




a[1]	a[2]	a[3]
2 <sub>2</sub>	1 <sub>3</sub>	3 <sub>1</sub>

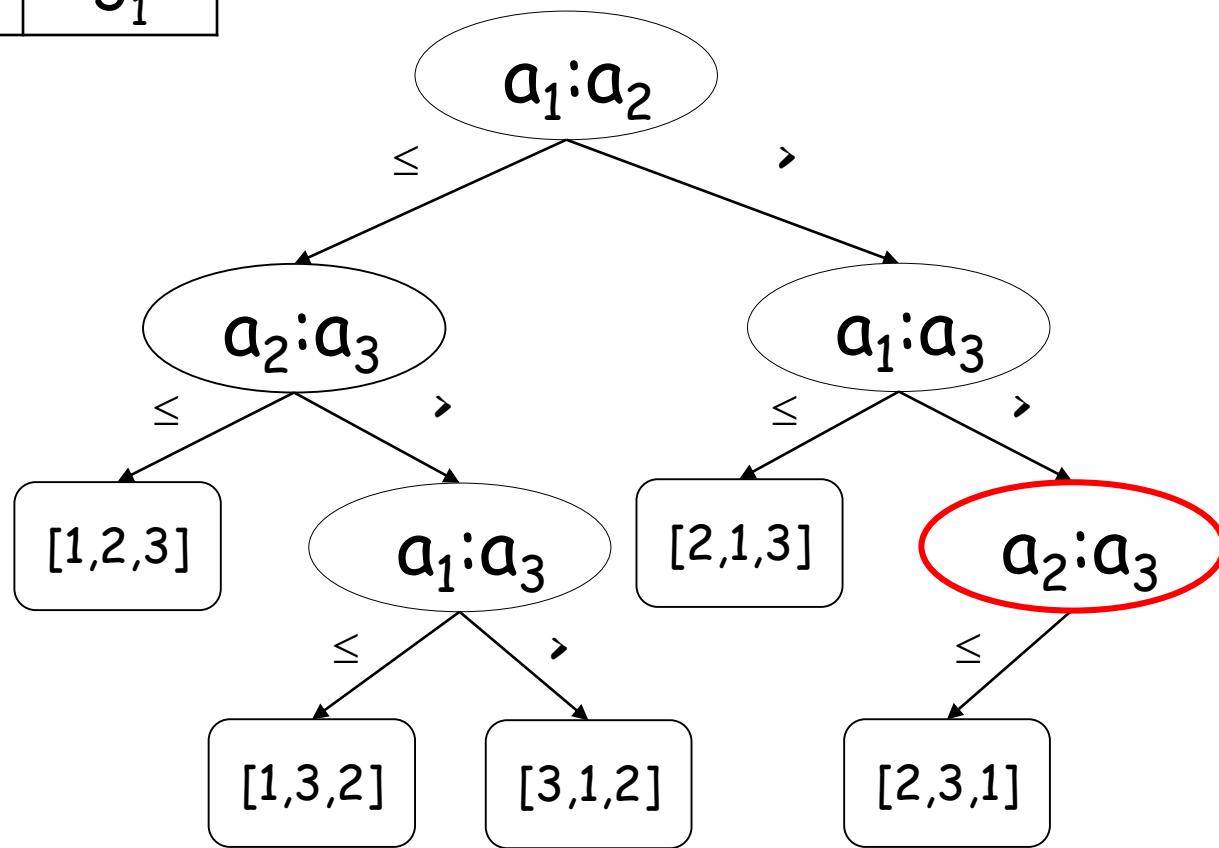
# עץ החלטה





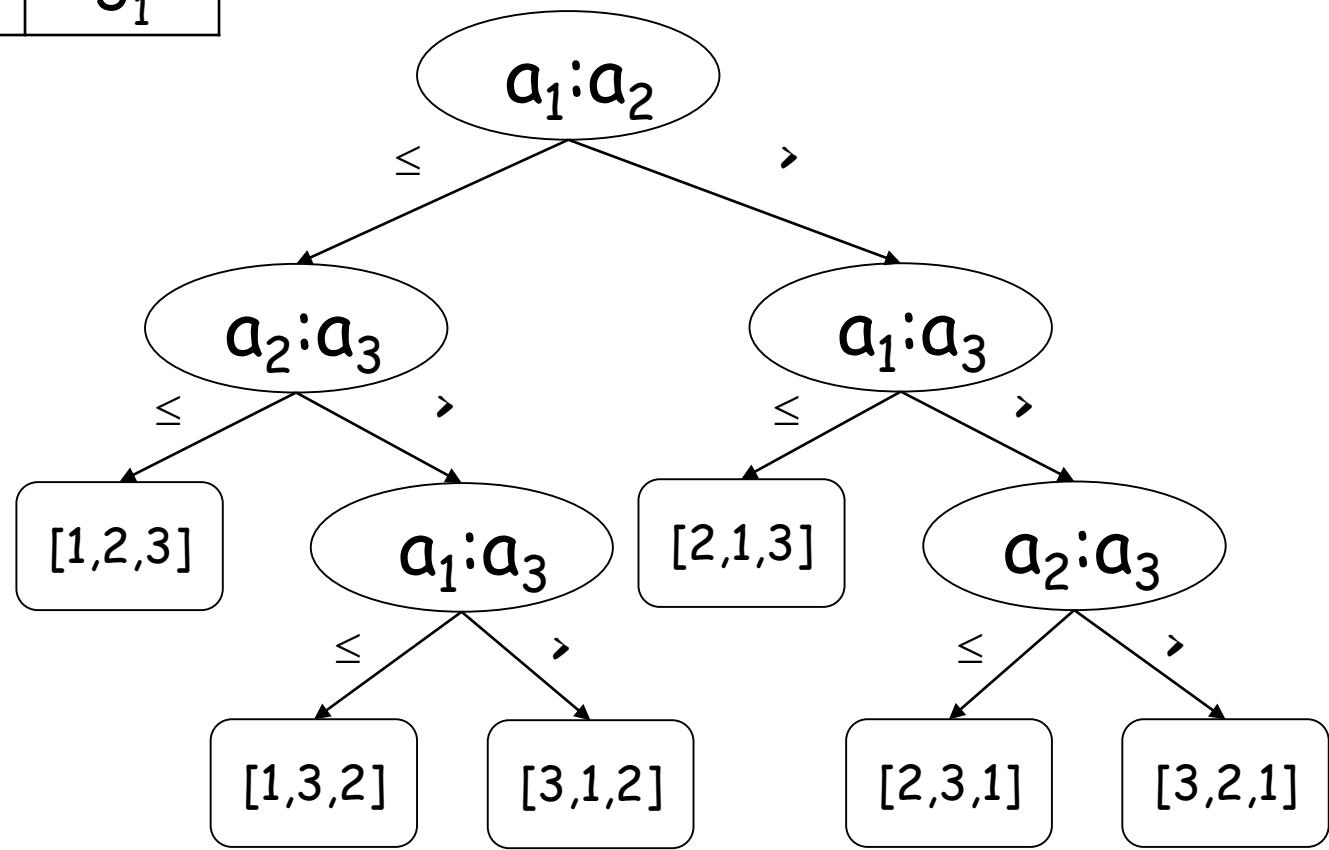
a[1]	a[2]	a[3]
1 <sub>3</sub>	2 <sub>2</sub>	3 <sub>1</sub>

# עץ החלטה

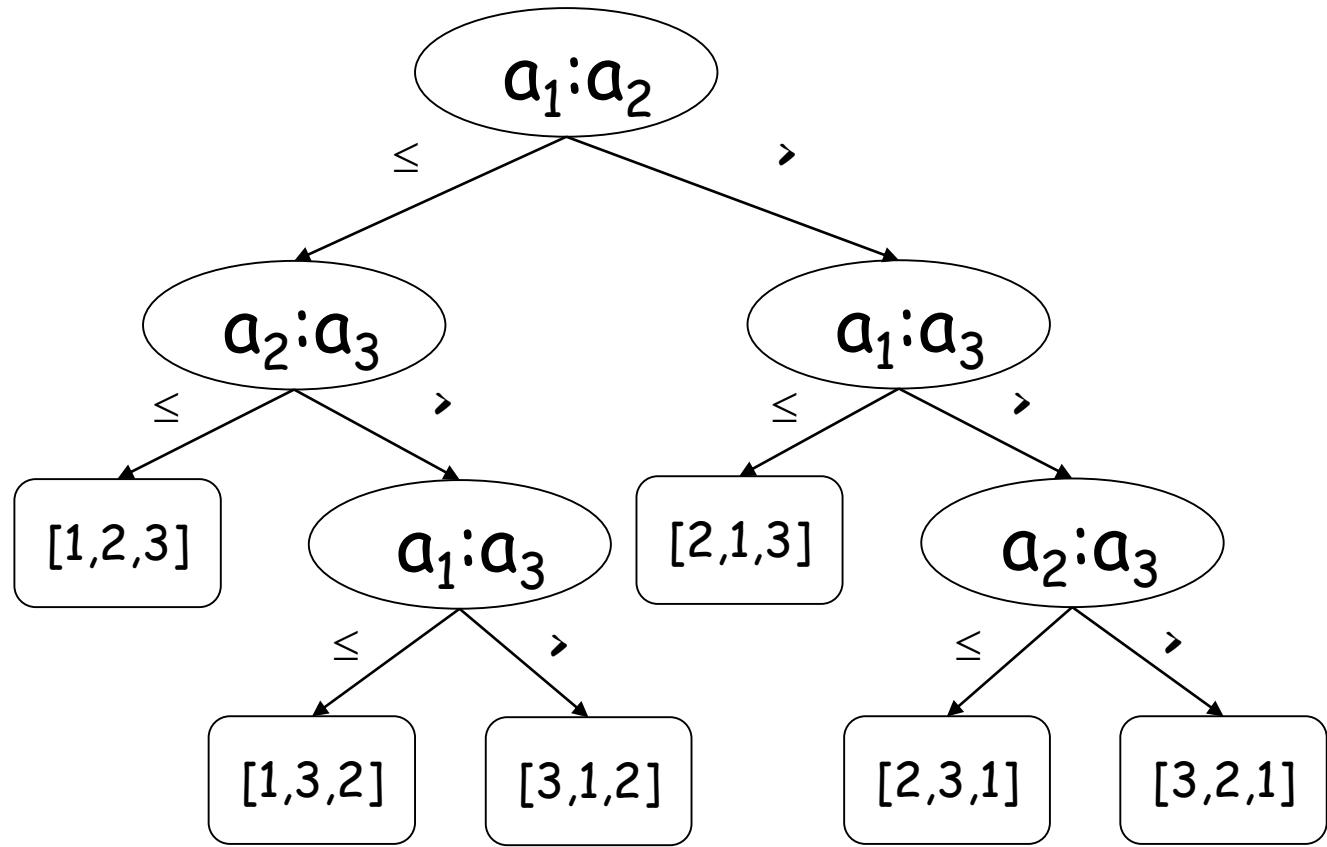


$a[1]$	$a[2]$	$a[3]$
$1_3$	$2_2$	$3_1$

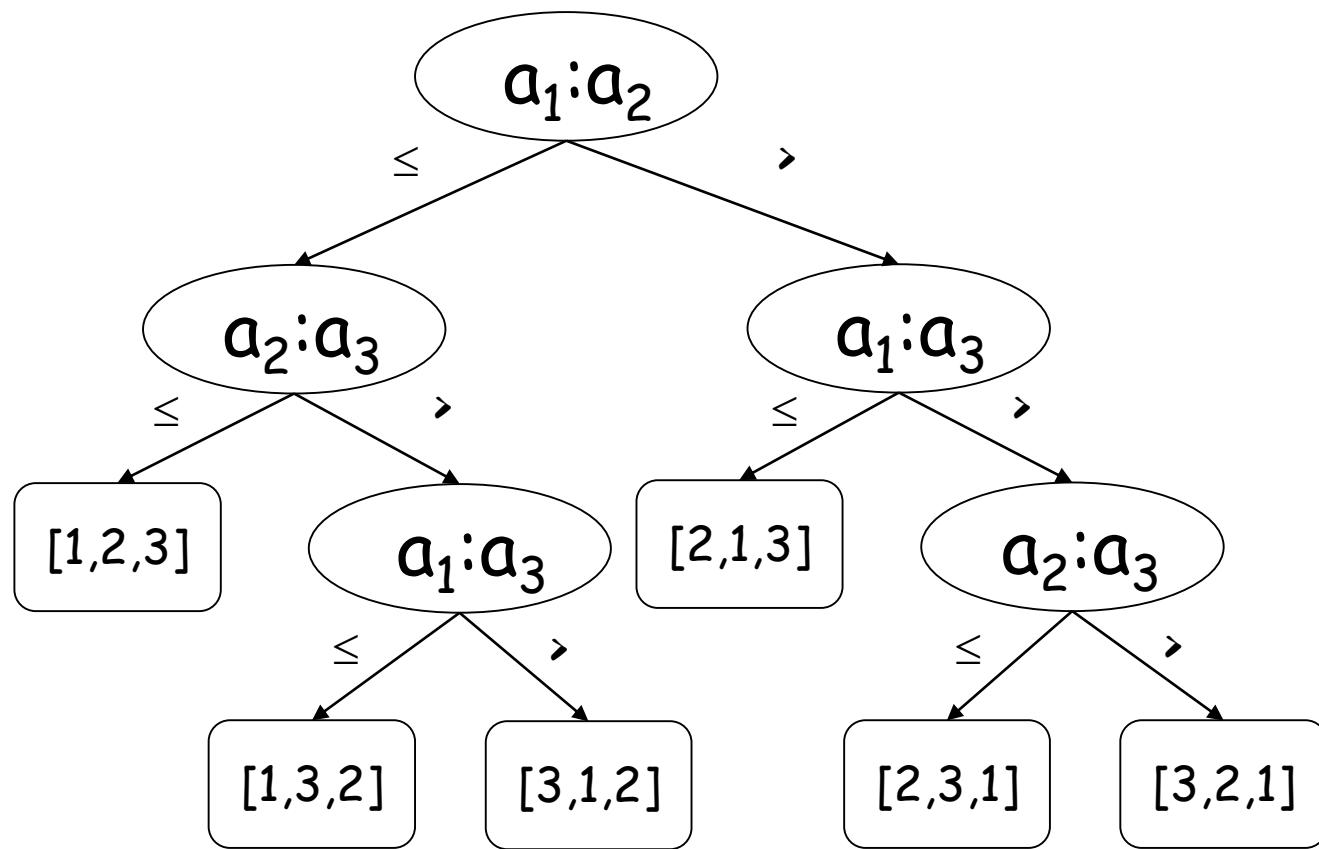
# עץ החלטה



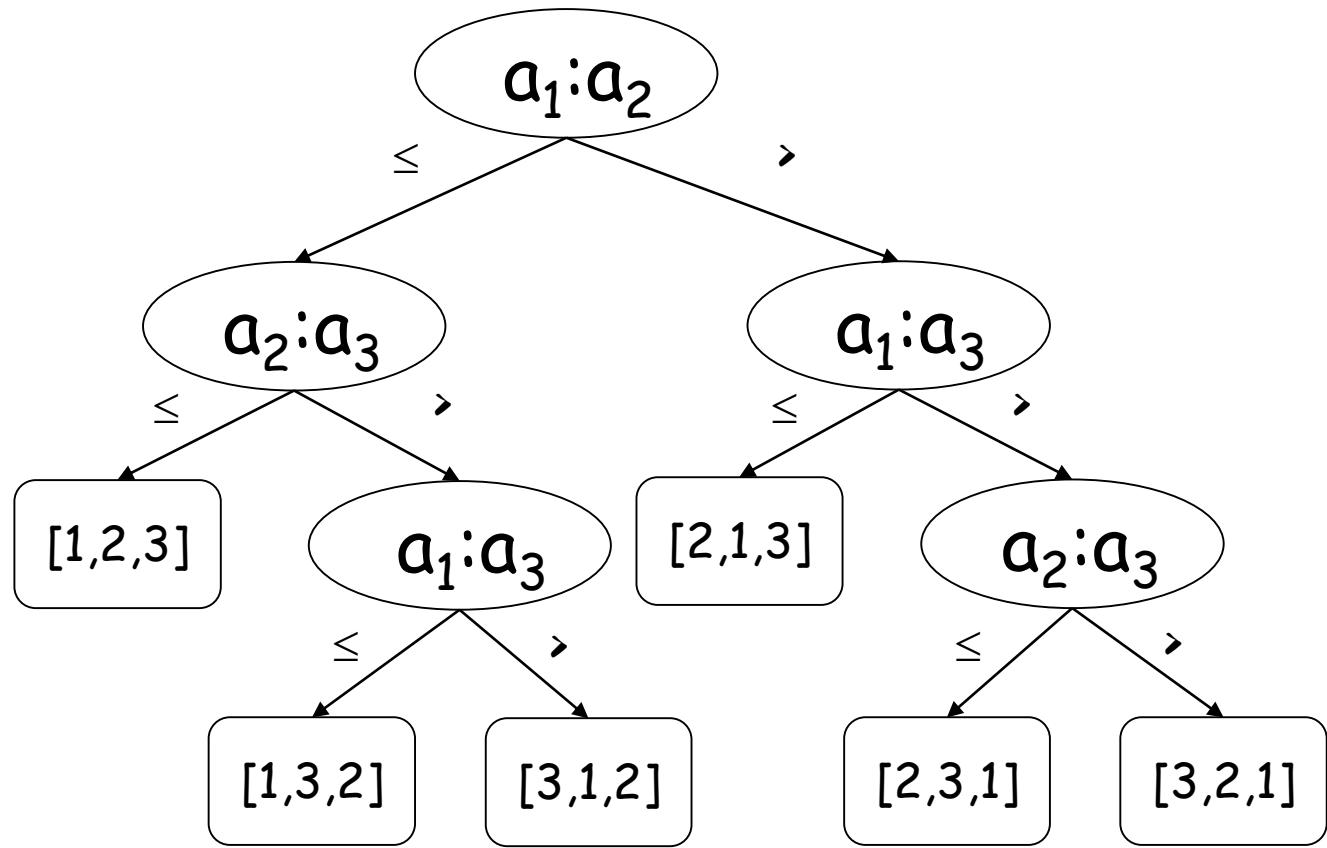
# איזה סוג עץ החלטה ?



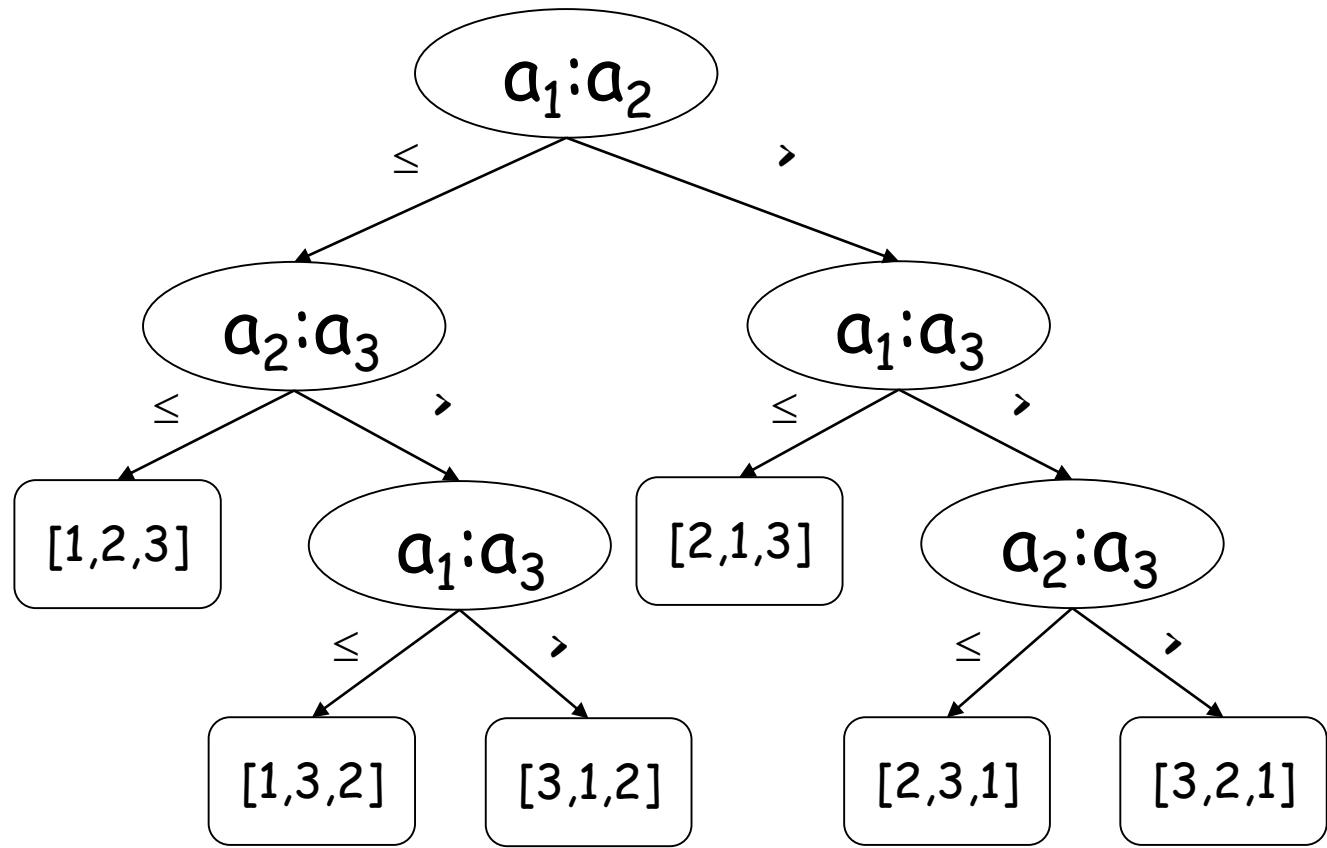
# איזה סוג עץ הוא עץ החלטה ? עץ בינרי מלא



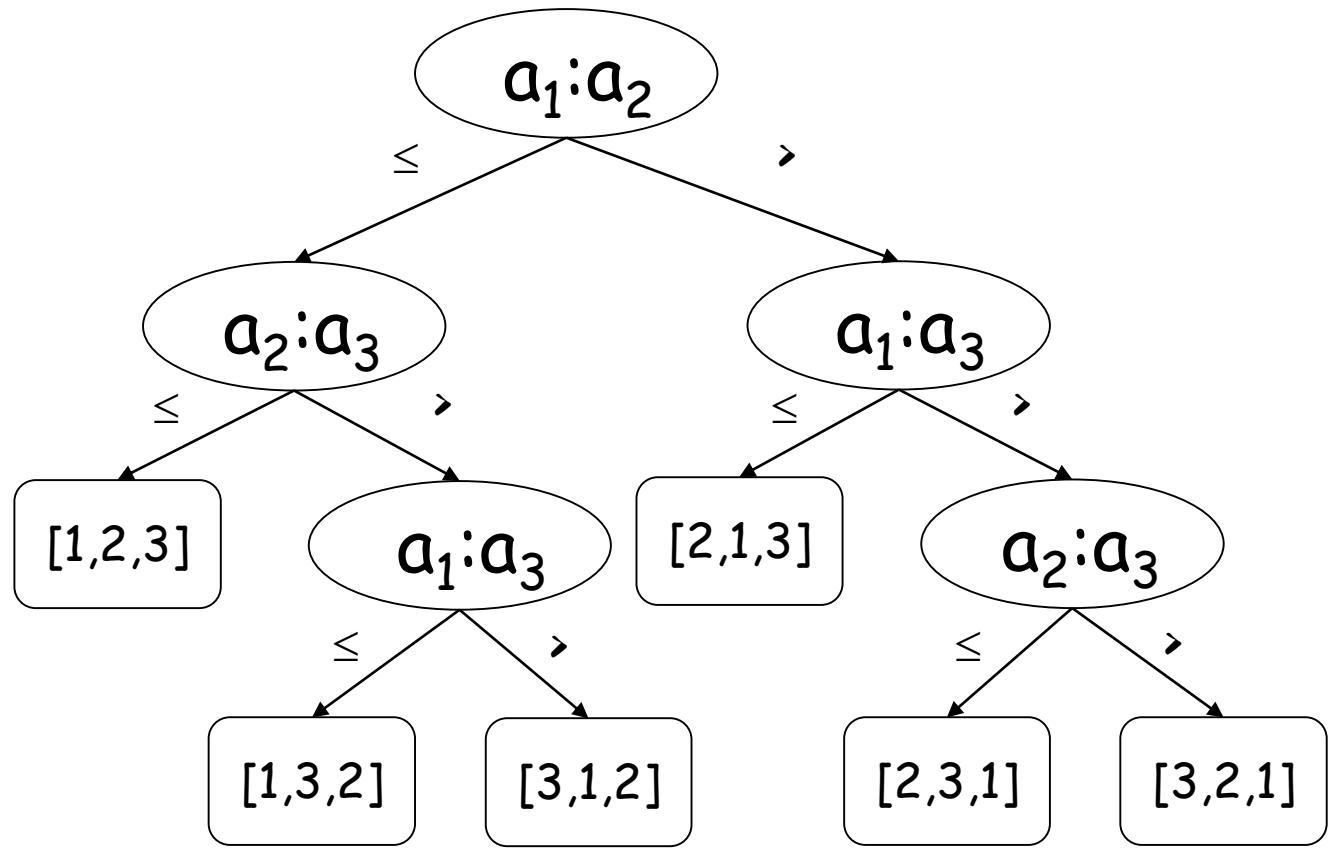
# מהו מסלול בעץ?



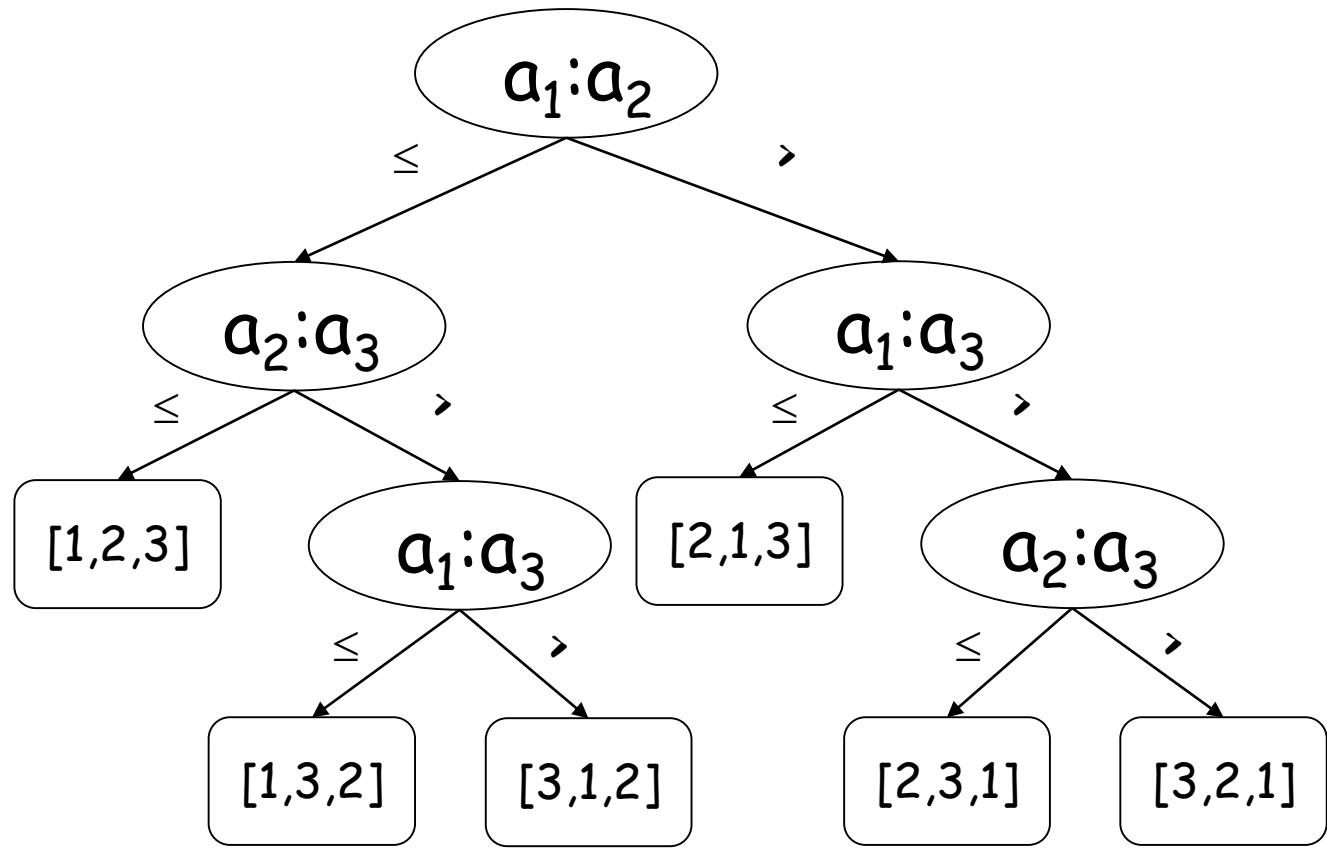
# מהו מסלול בעץ? ריצה מסוימת



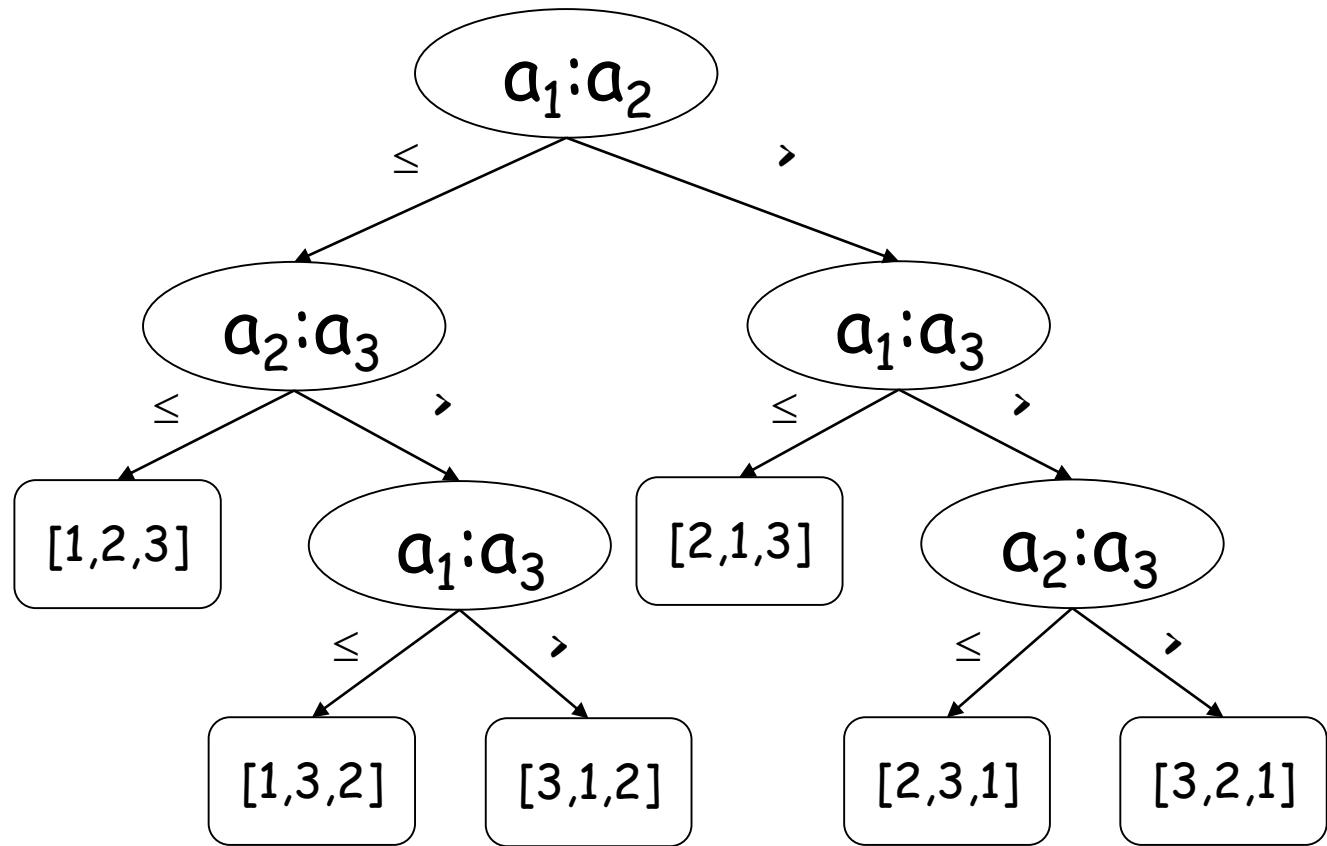
# מה מצין עליה?



# מה מצין עליה? פלט אפשרי של המילון

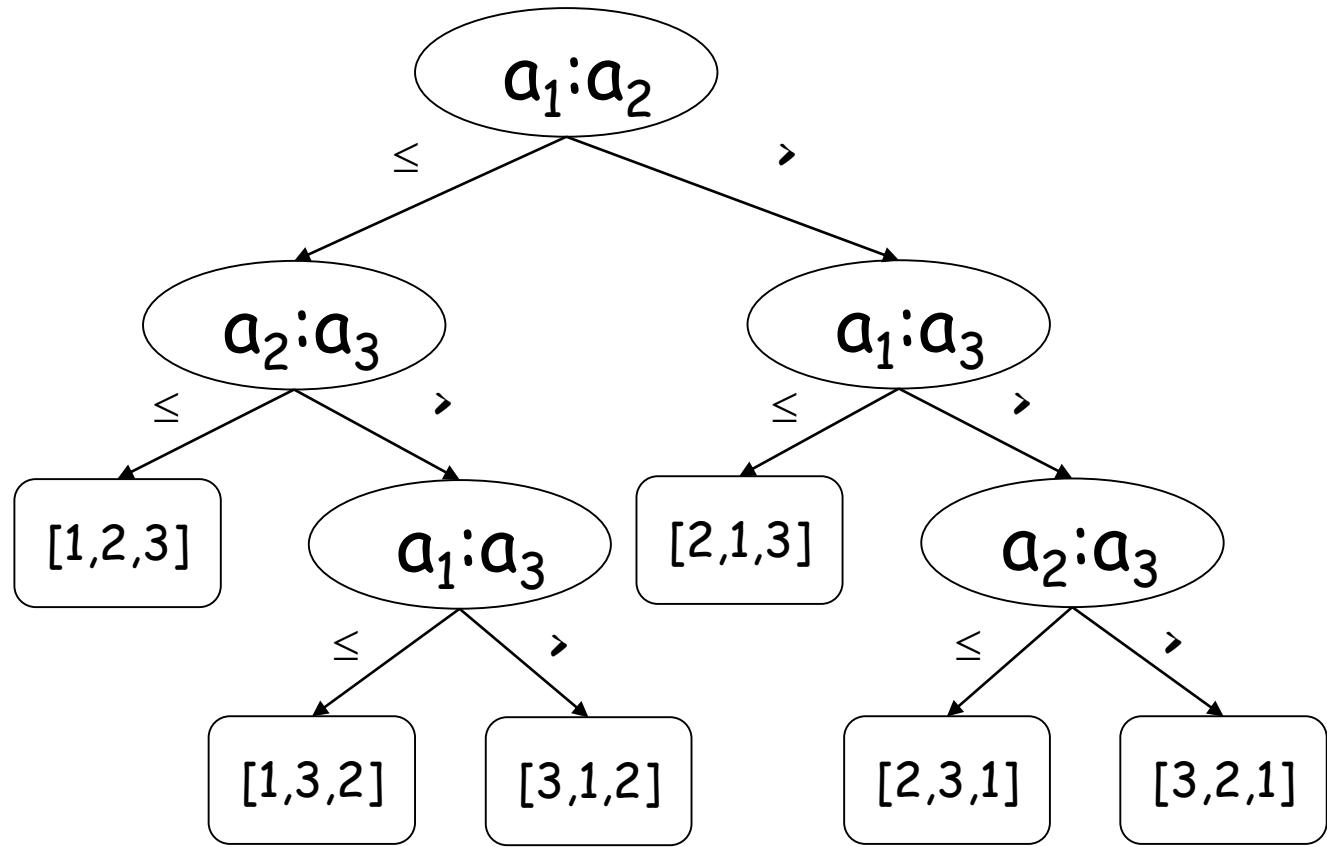


# מקנה: כמות העלים =

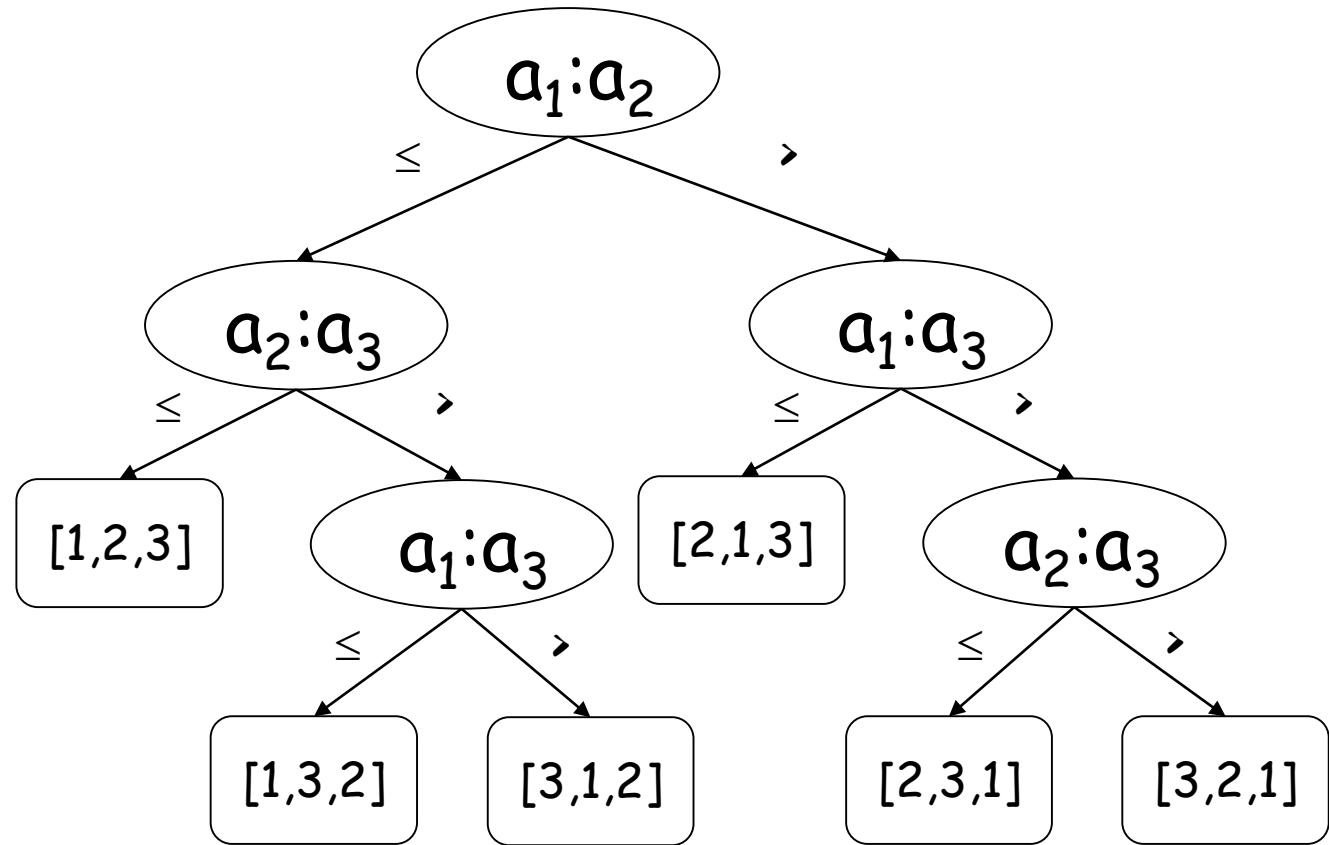


# מקנה:

כמות העלים =  $n!$

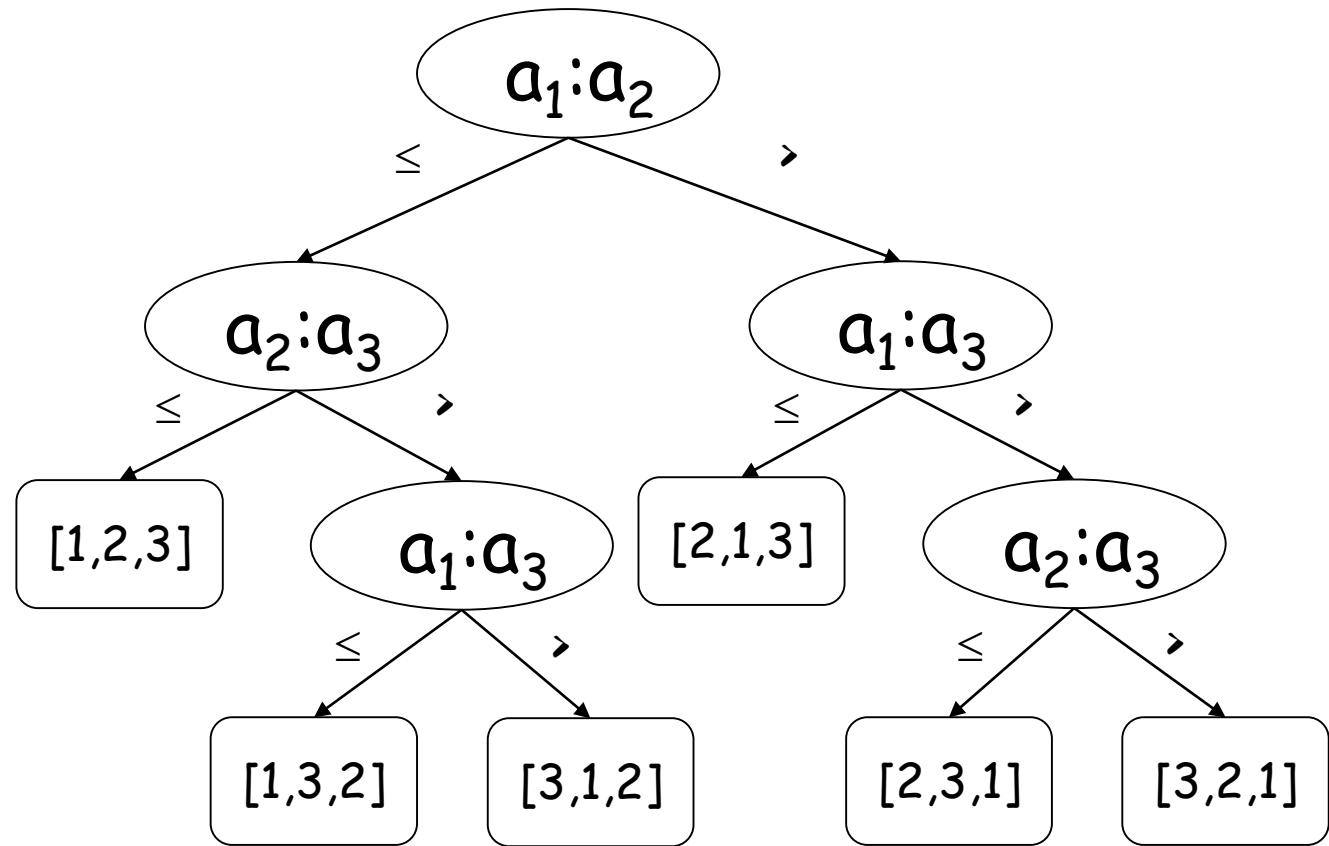


# מה מייצג המסלול הארוך ביותר?



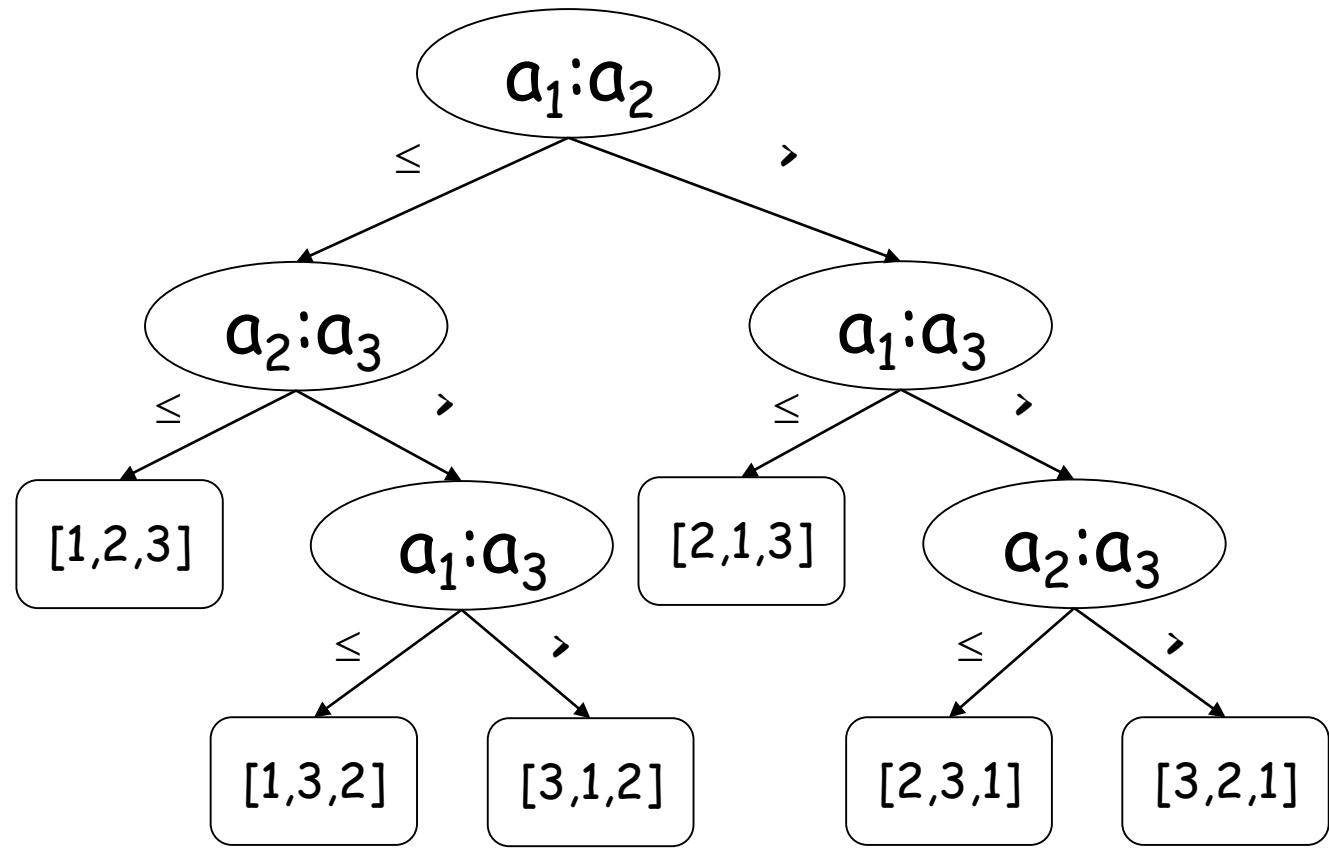
# מה מייצג המסלול הארוך ביותר?

מספר ההשוואות שאלגוריתם מינו **הכי טוב** צריך **במקרה הגרוע**

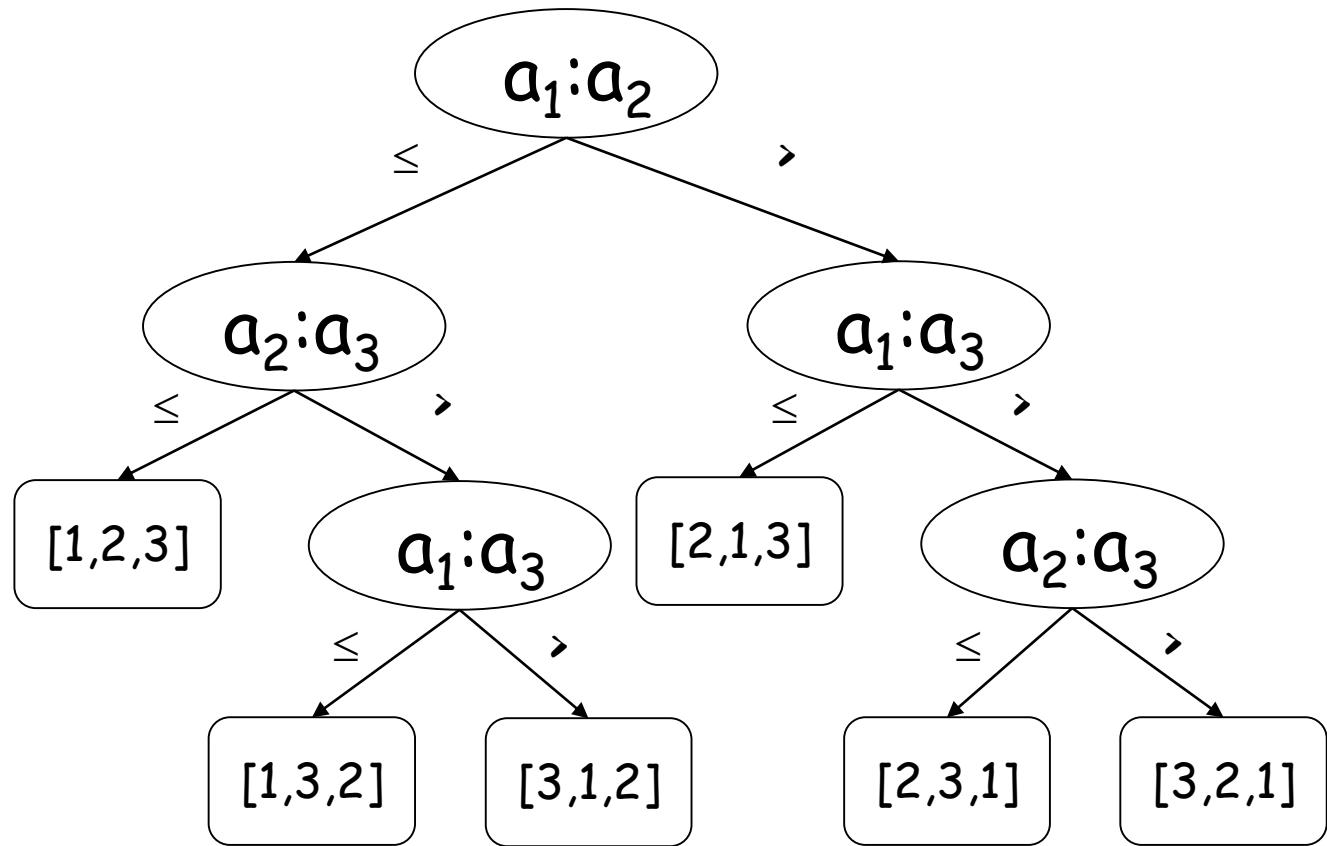


## מסקנה:

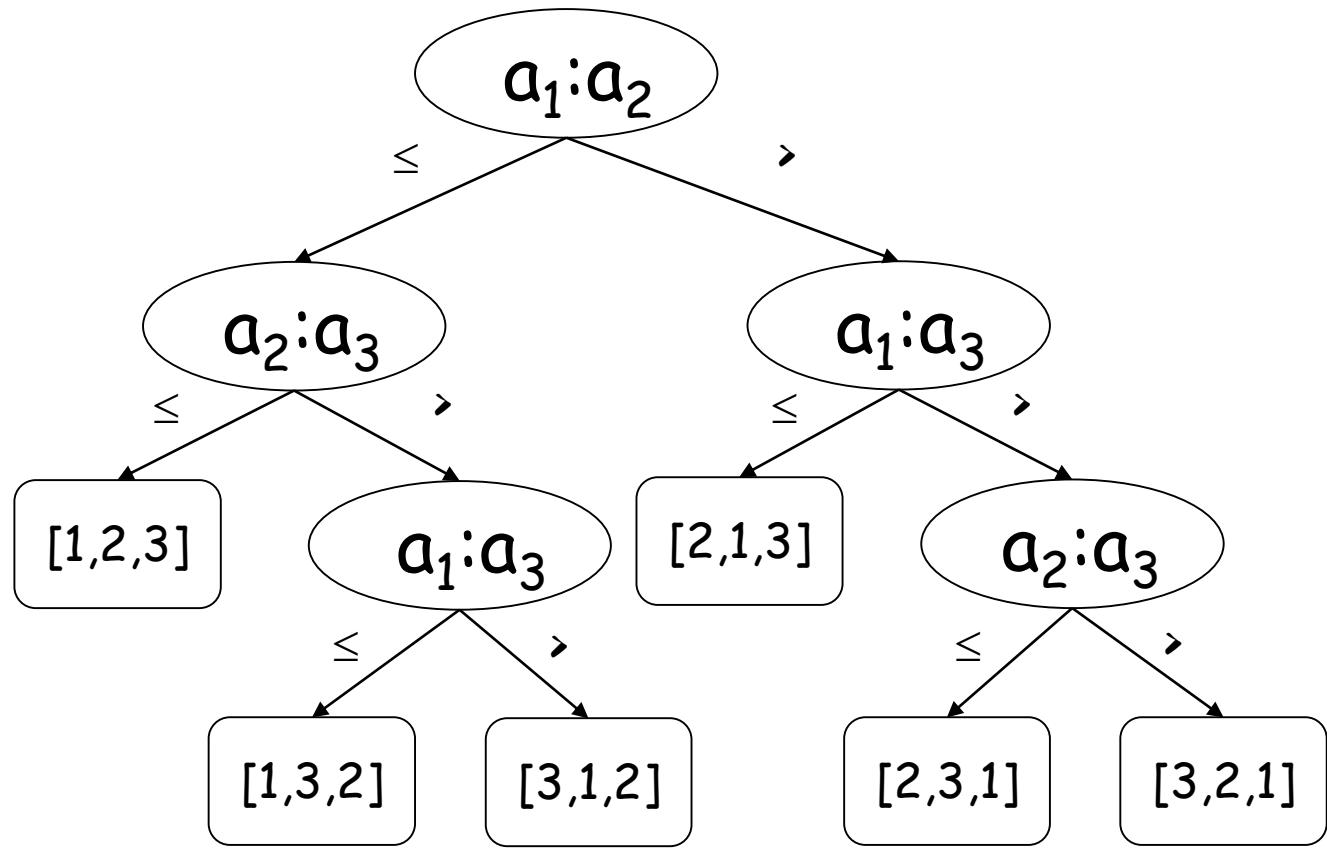
**גובה העץ** מייצג את מספר ההשוואות המבוצעות במקרה הגרוע  
באלגוריתם הći טוב



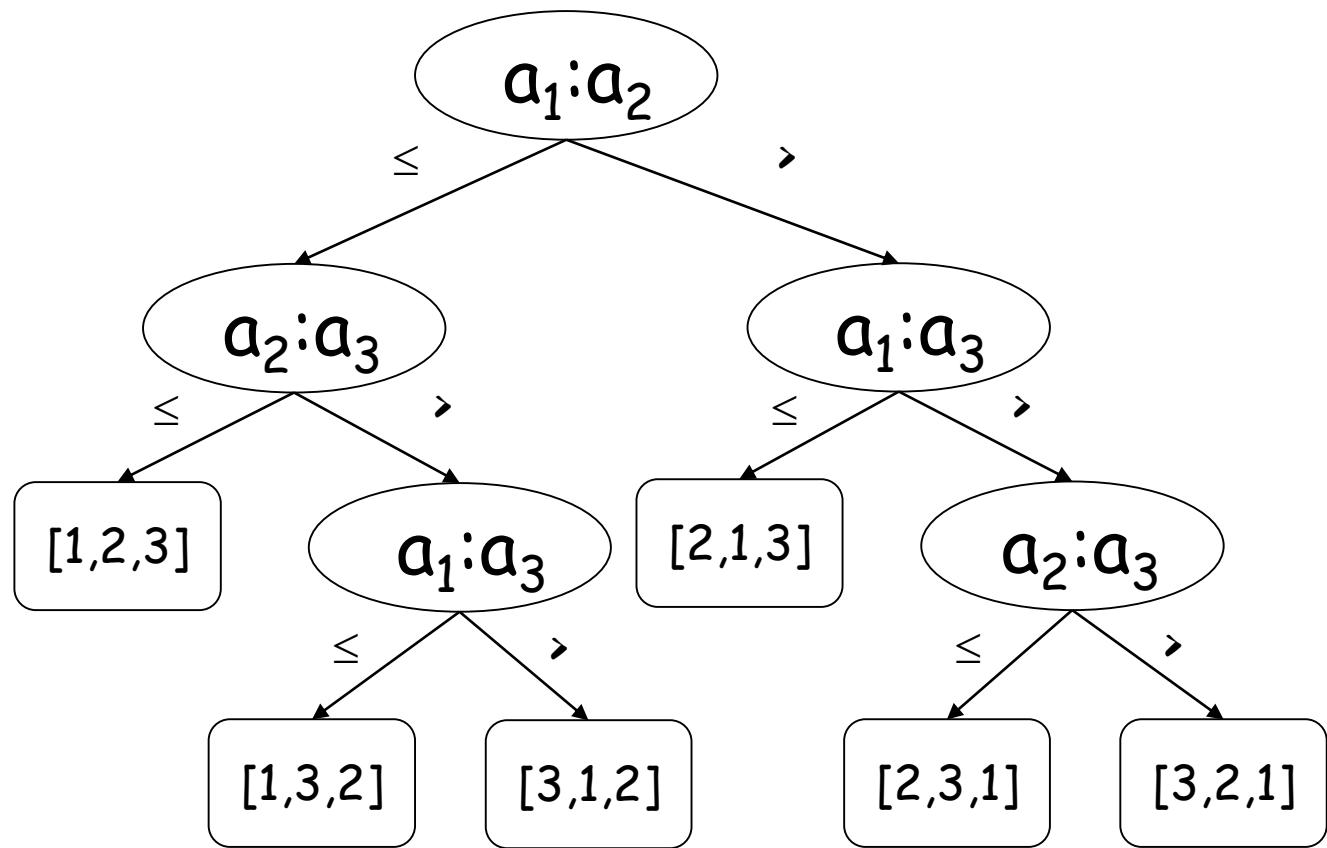
מה גובה העץ במקורה הטוב ביותר?  
ומה נוכל ללמידה מכך?



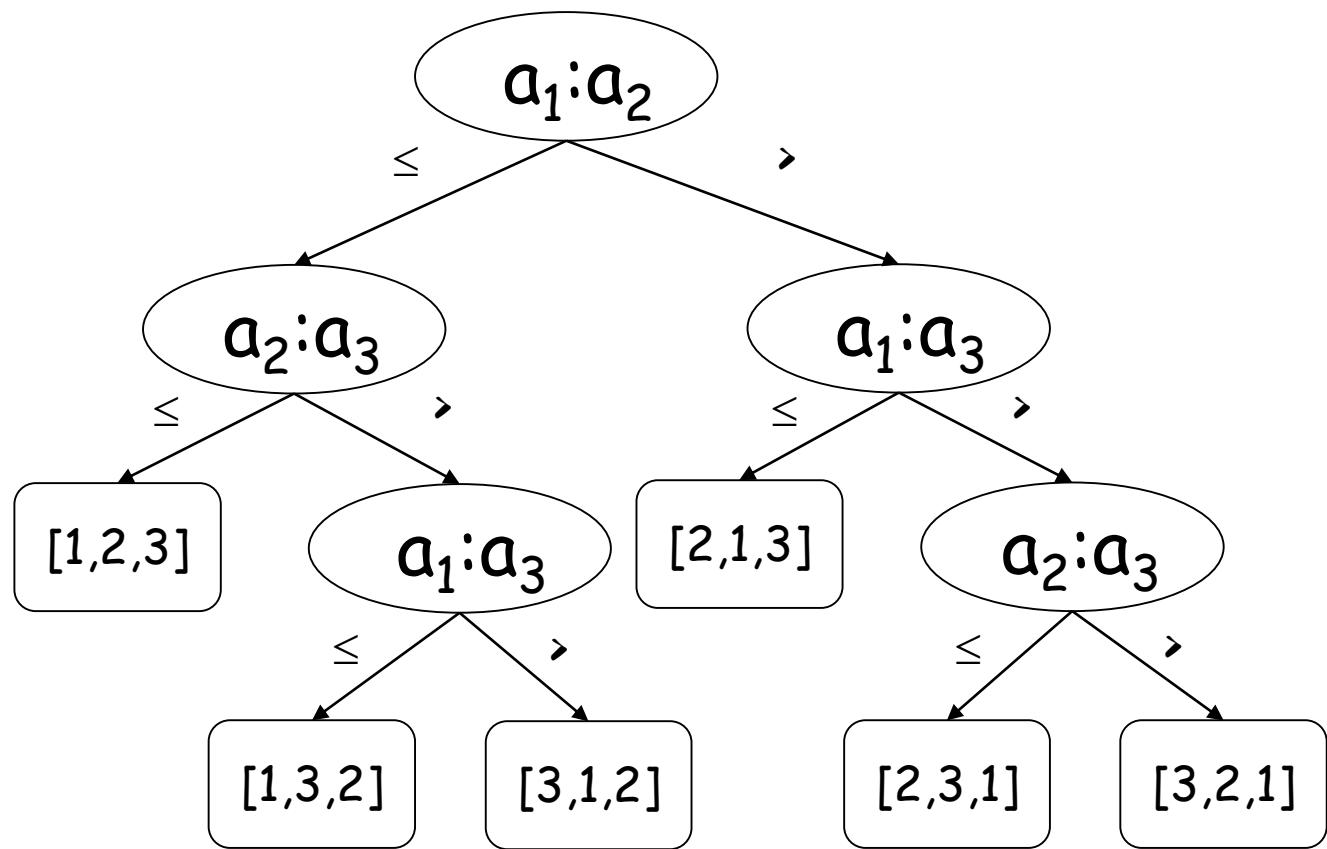
**בעז החלטה אַ עליים ולקן גובהו לפחות אַ  $\log$**



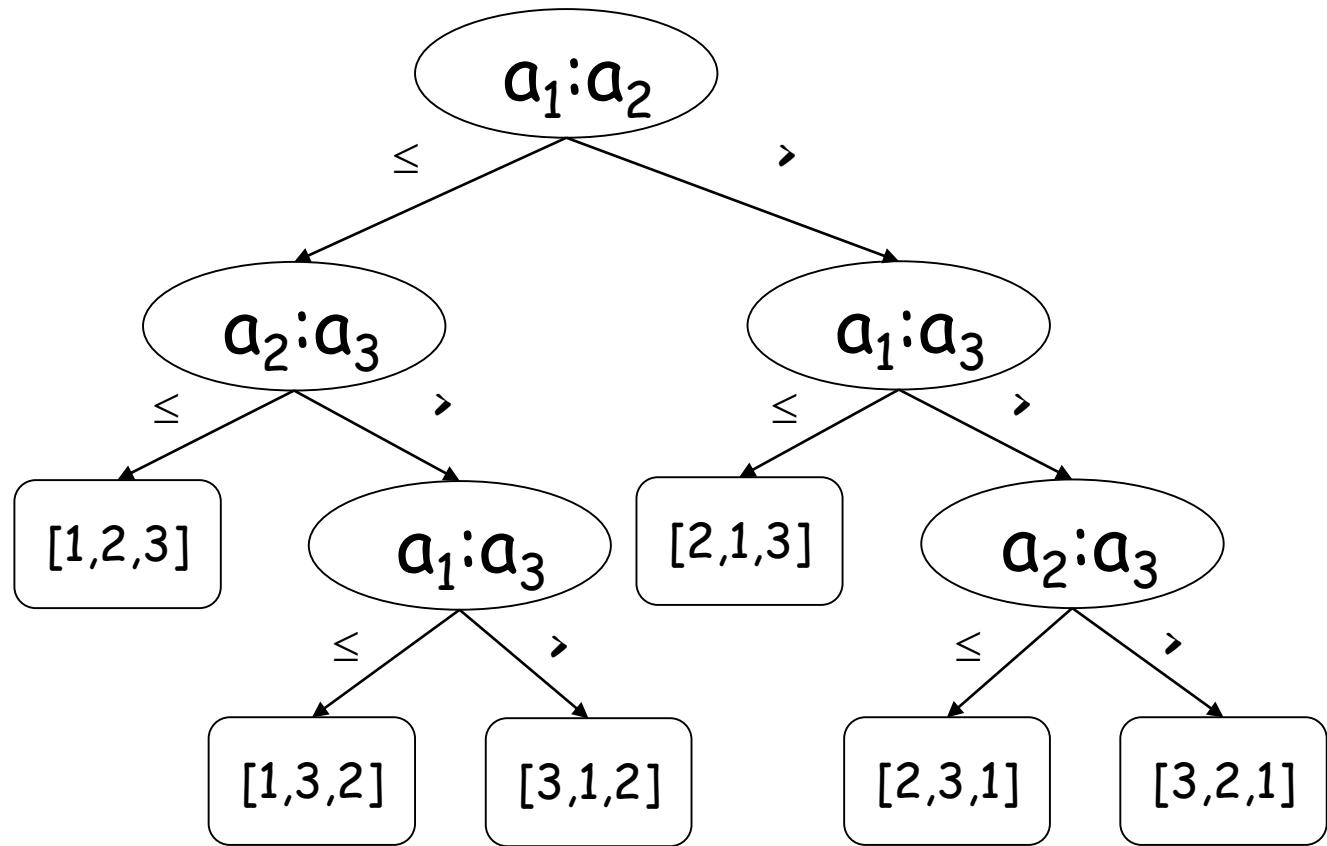
$n! =$



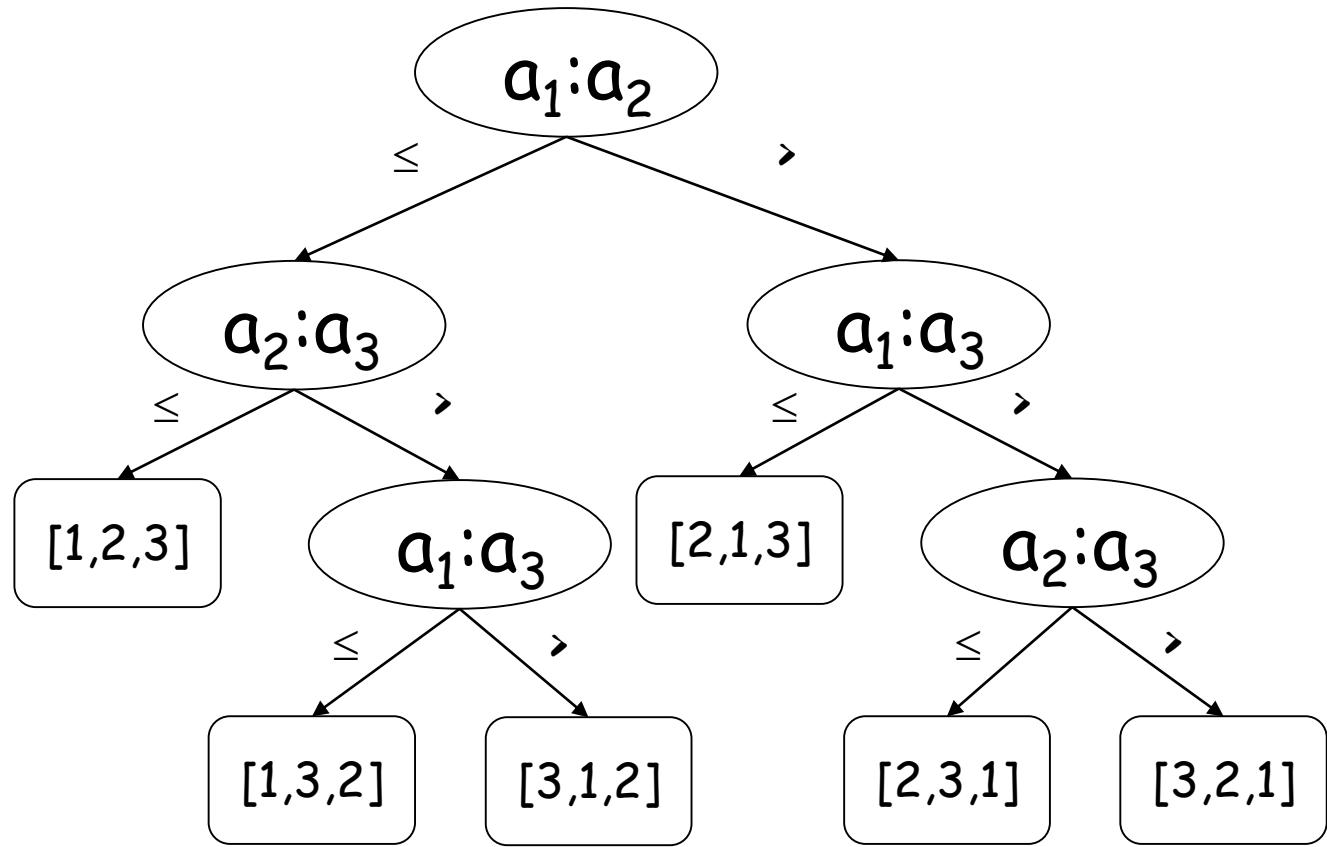
$$n! = 1 \cdot 2 \cdot 3 \dots n$$



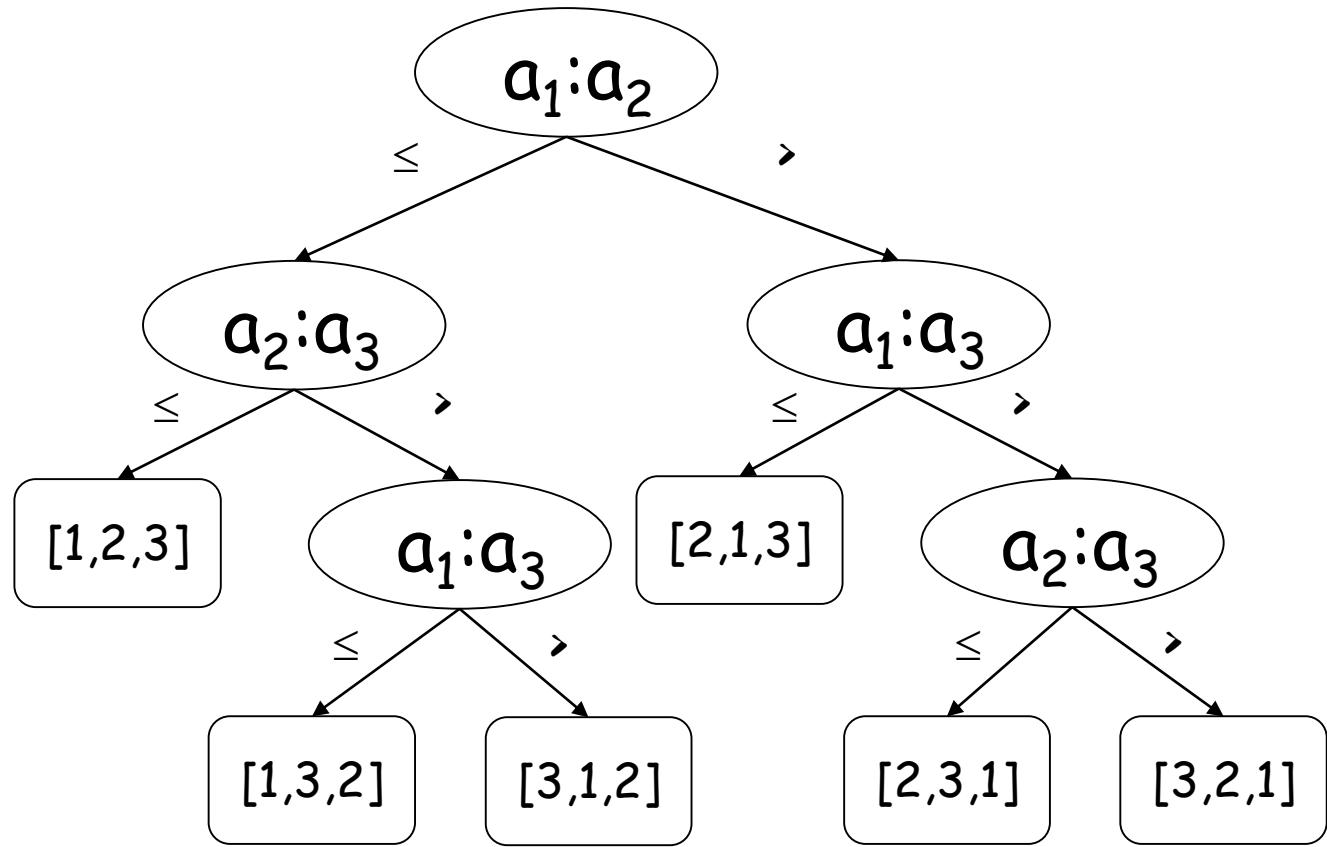
$$n! = 1 \cdot 2 \cdot 3 \dots n < n \cdot n \dots n$$



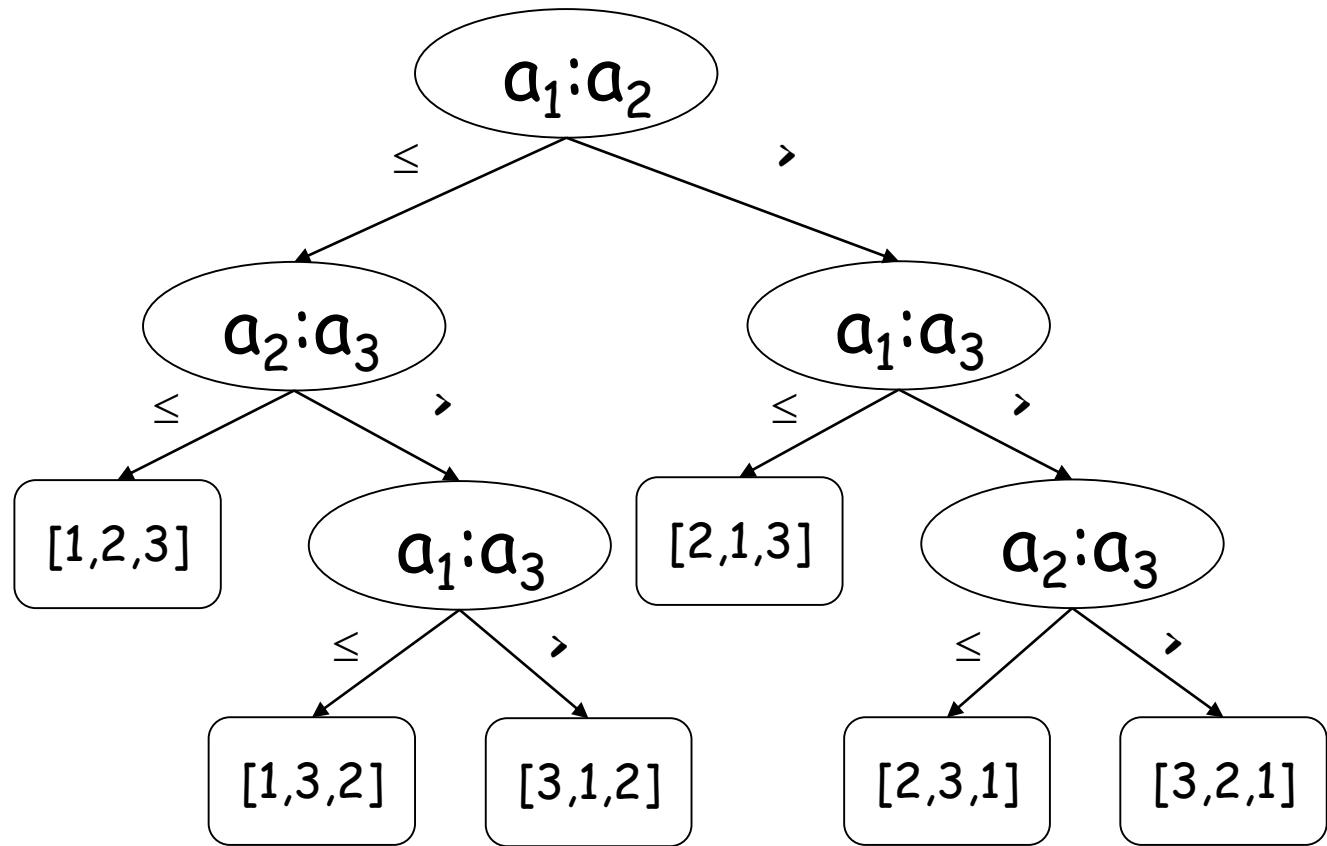
$$n! = 1 \cdot 2 \cdot 3 \dots n < n \cdot n \dots n = n^n$$



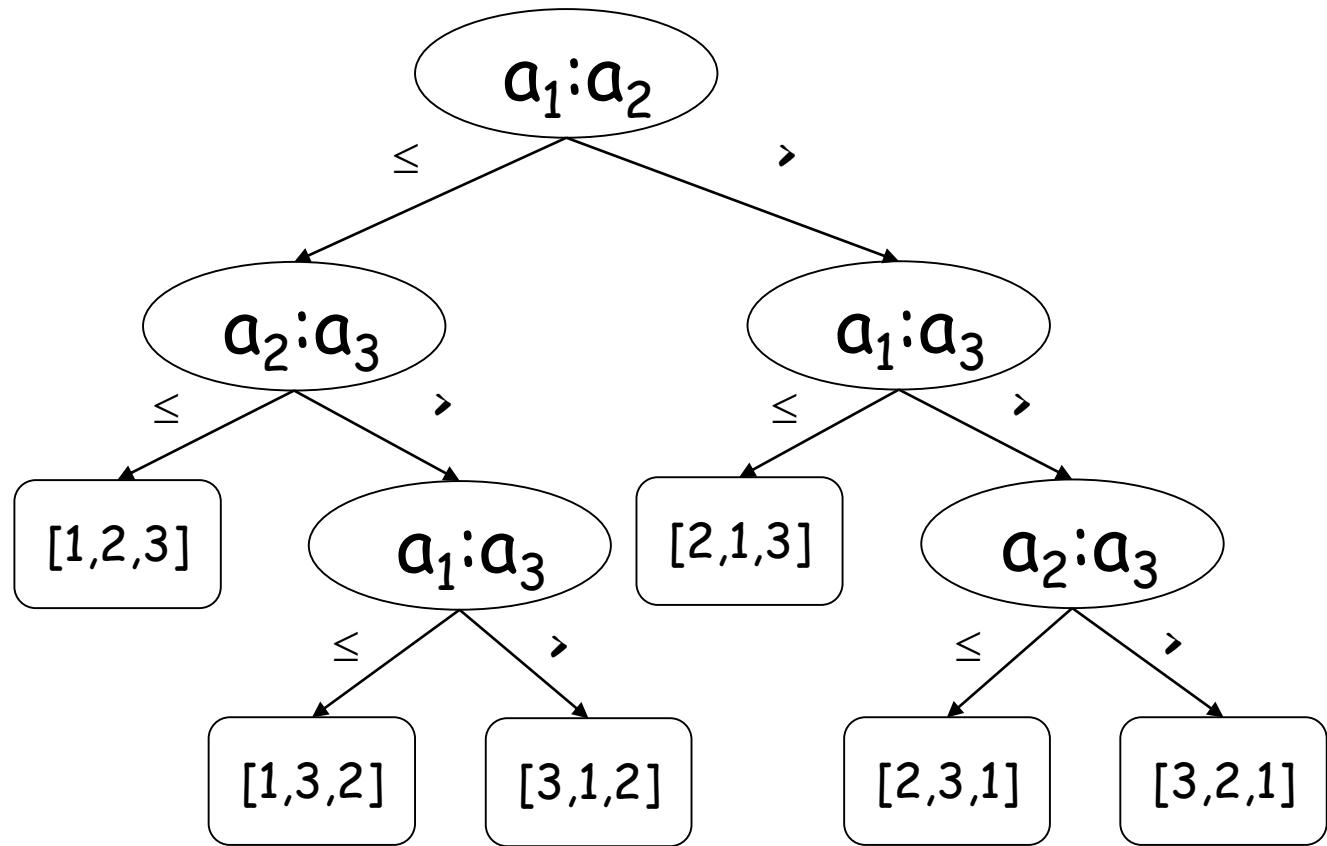
$$n! < n^n$$



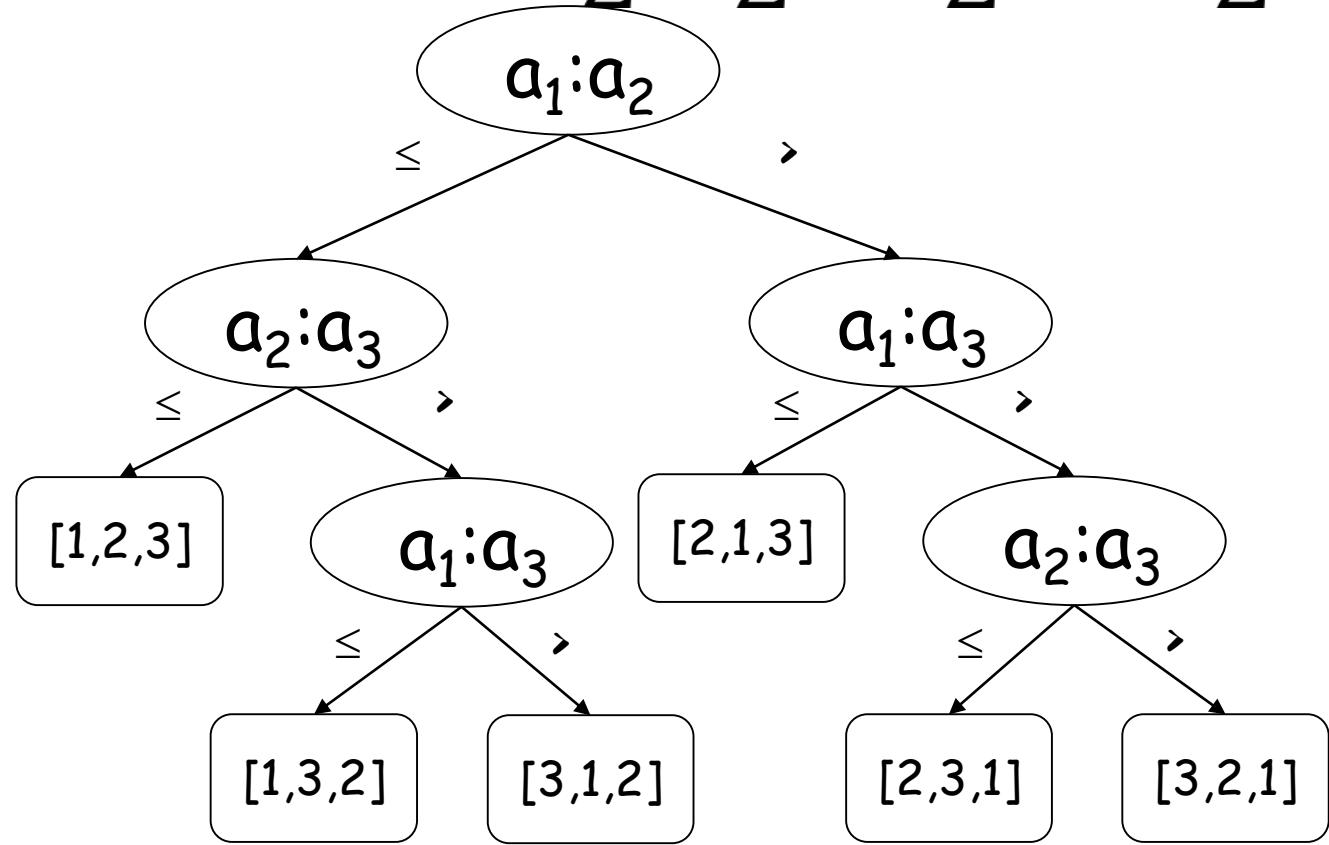
$$\log n! < \log n^n$$



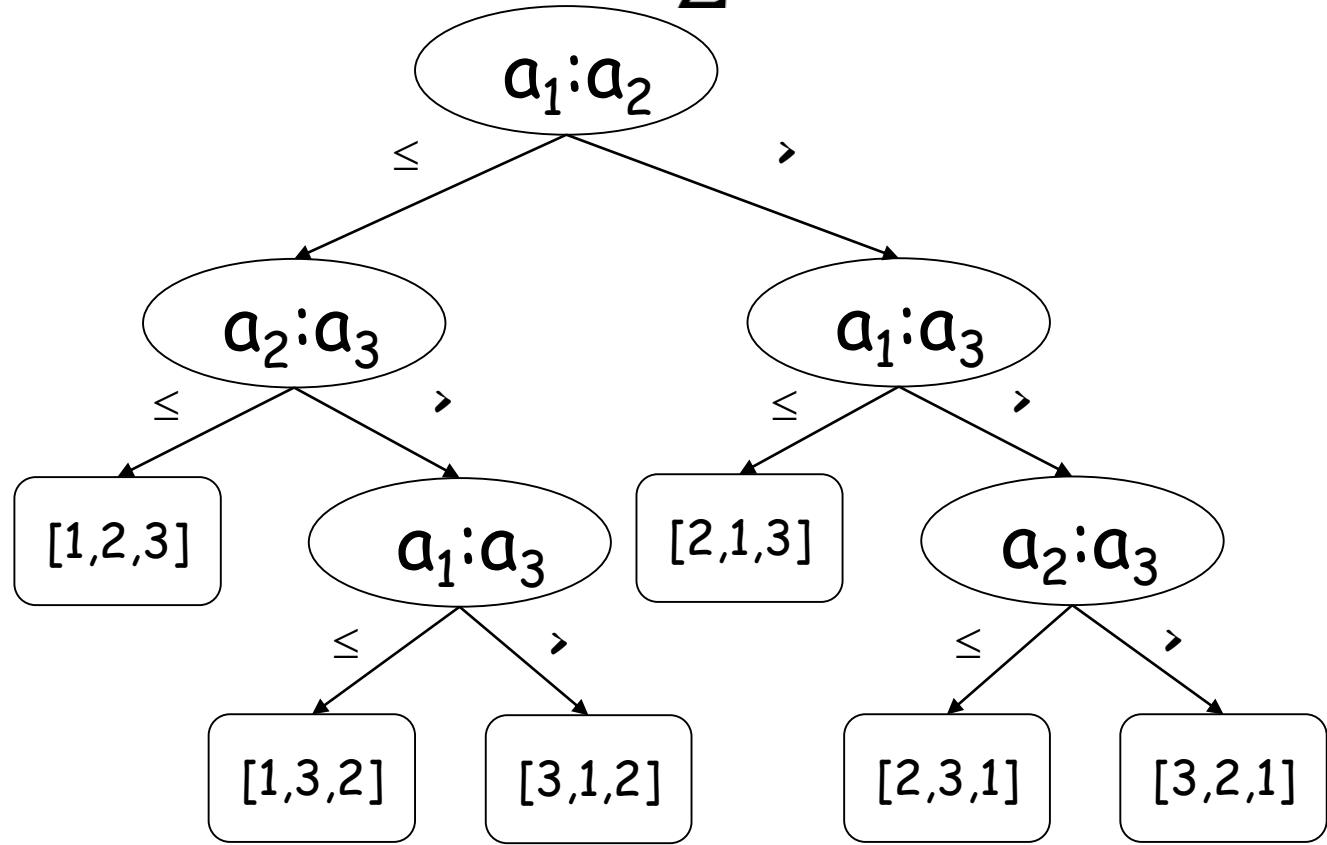
$$\log n! < n \cdot \log n$$



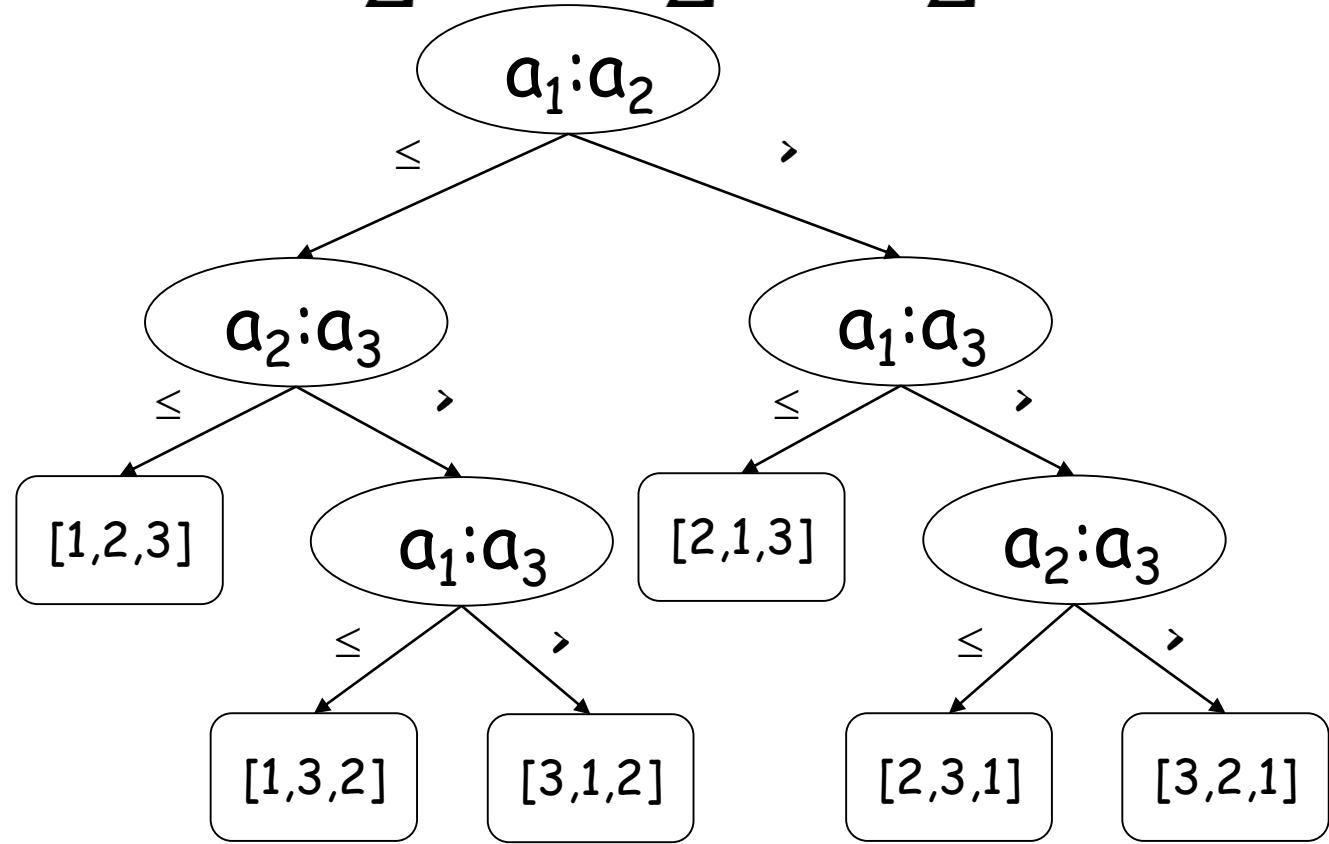
$$n! = 1 \cdot 2 \cdots n > \frac{n}{2} \cdot \frac{n}{2} \cdots \frac{n}{2} = \left(\frac{n}{2}\right)^{\frac{n}{2}}$$



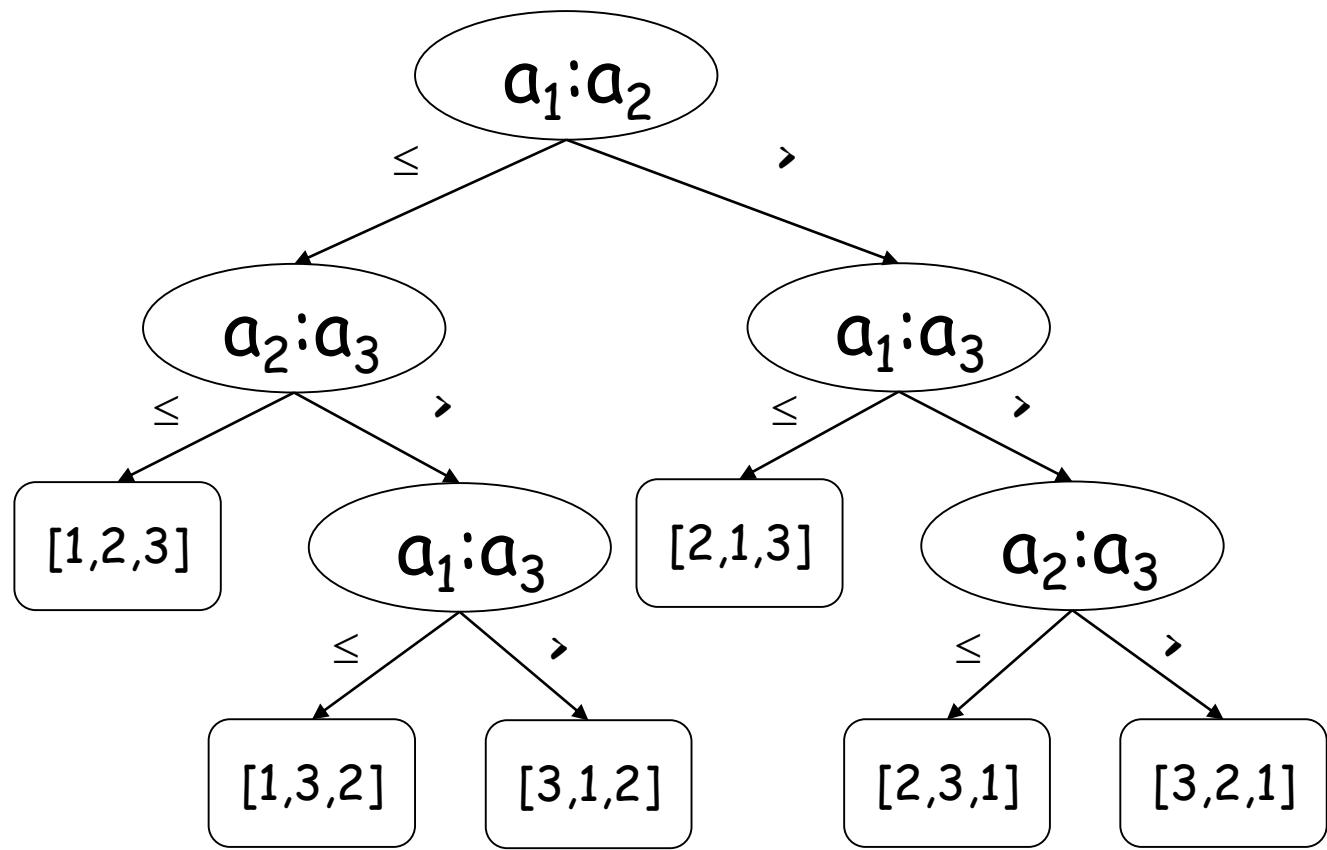
$$n! > \left(\frac{n}{2}\right)^{\frac{n}{2}}$$



$$\log n! > \log\left(\frac{n}{2}\right)^{\frac{n}{2}} > \frac{n}{2} \log \frac{n}{2}$$

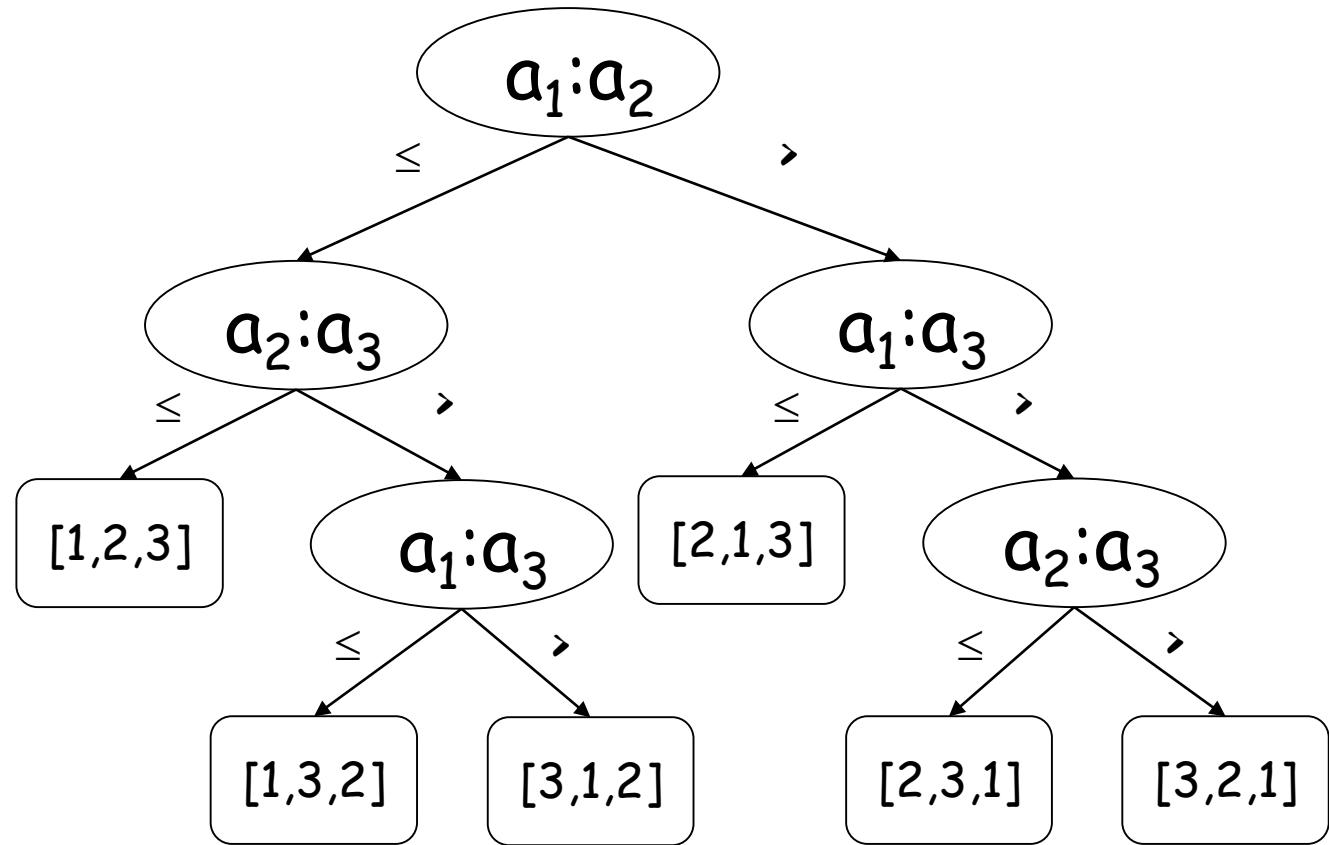


$$\log n! = \theta(n \log n)$$



**משפט:**

כל אלגוריתם למיון מבוסס השוואות צריך  $\Omega(n \log n)$  השוואות במקרה הגרוע



# מיון מנייה

הנחות:

- איברי הקלט מספרים שלמים בתחום  $0 \leq k$ .
- הקלט הוא מערך  $A[1..n]$ .
- הפלט: מערך  $B[1..n]$  איברי המערך  $A$  ממוחנים.
- מערך עזר:  $C[0..k]$ .

# Counting sort( $A, B, k$ )

for  $i \leftarrow 0$  to  $k$  do  $C[i] \leftarrow 0$

for  $j \leftarrow 1$  to  $n$  do

$C[A[j]] \leftarrow C[A[j]] + 1 ; \quad C[i] = |\{key = i\}|$

for  $i \leftarrow 1$  to  $k$  do

$C[i] \leftarrow C[i] + C[i-1] ; \quad C[i] = |\{key \leq i\}|$

for  $j \leftarrow n$  down to 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

# Counting sort( $A, B, k$ )

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$						

**for**  $i \leftarrow 0$  **to**  $k$  **do**  $C[i] \leftarrow 0$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	0	0	0	0	0

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	0	1	0	0	0

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	0	1	0	0	1

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	0	1	1	0	1

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	1	0	1	1	0	1

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	1	0	2	1	0	1

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	1	0	2	2	0	1

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	0	2	2	0	1

for  $j \leftarrow 1$  to  $n$  do  $C[A[j]] \leftarrow C[A[j]] + 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	0	2	3	0	1

for  $i \leftarrow 1$  to  $k$  do  $C[i] \leftarrow C[i] + C[i-1]$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	0	2	3	0	1

for  $i \leftarrow 1$  to  $k$  do  $C[i] \leftarrow C[i] + C[i-1]$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	2	3	0	1

for  $i \leftarrow 1$  to  $k$  do  $C[i] \leftarrow C[i] + C[i-1]$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	4	3	0	1

for  $i \leftarrow 1$  to  $k$  do  $C[i] \leftarrow C[i] + C[i-1]$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	4	7	0	1

for  $i \leftarrow 1$  to  $k$  do  $C[i] \leftarrow C[i] + C[i-1]$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	4	7	7	1

for  $i \leftarrow 1$  to  $k$  do  $C[i] \leftarrow C[i] + C[i-1]$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$								

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	4	7	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$							3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	4	7	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
A	2	5	3	0	2	3	0	3

	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]
B							3	

	C[0]	C[1]	C[2]	C[3]	C[4]	C[5]
C	2	2	4	6	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$		0					3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	2	2	4	6	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
A	2	5	3	0	2	3	0	3

	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]
B		0					3	

	C[0]	C[1]	C[2]	C[3]	C[4]	C[5]
C	1	2	4	6	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3



	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$		0				3	3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	1	2	4	6	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
A	2	5	3	0	2	3	0	3



	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]
B		0				3	3	

	C[0]	C[1]	C[2]	C[3]	C[4]	C[5]
C	1	2	4	5	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3



	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$		0		2		3	3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	1	2	4	5	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
A	2	5	3	0	2	3	0	3

	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]
B		0		2		3	3	

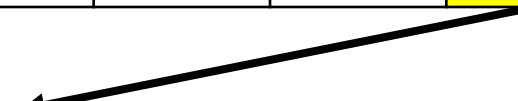
	C[0]	C[1]	C[2]	C[3]	C[4]	C[5]
C	1	2	3	5	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3



	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0		2		3	3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	1	2	3	5	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0		2		3	3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	2	3	5	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0		2	3	3	3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	2	3	5	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0		2	3	3	3	

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	2	3	4	7	8

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3



	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0		2	3	3	3	5

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	2	3	4	7	8

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3



	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0		2	3	3	3	5

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	2	3	4	7	7

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3



	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0	2	2	3	3	3	5

	$C[0]$	$C[1]$	$C[2]$	$C[3]$	$C[4]$	$C[5]$
$C$	0	2	3	4	7	7

```

for j←n downto 1 do
    B[C[A[j]]] ← A[j]
    C[A[j]] ← C[A[j]] -1

```

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
A	2	5	3	0	2	3	0	3



	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	B[7]	B[8]
B	0	0	2	2	3	3	3	5

	C[0]	C[1]	C[2]	C[3]	C[4]	C[5]
C	0	2	2	4	7	7

# Counting sort( $A, B, k$ )

```
Θ(k) for i←0 to k do  $C[i] \leftarrow 0$ 
      for j←1 to n do
           $C[A[j]] \leftarrow C[A[j]] + 1$ ;  $C[i] = |\{key = i\}|$ 
      for i←1 to k do
           $C[i] \leftarrow C[i] + C[i-1]$ ;  $C[i] = |\{key \leq i\}|$ 
      for j←n downto 1 do
           $B[C[A[j]]] \leftarrow A[j]$ 
           $C[A[j]] \leftarrow C[A[j]] - 1$ 
```

# Counting sort( $A, B, k$ )

$\Theta(k)$  for  $i \leftarrow 0$  to  $k$  do  $C[i] \leftarrow 0$

$\Theta(n)$  for  $j \leftarrow 1$  to  $n$  do

$C[A[j]] \leftarrow C[A[j]] + 1 ; C[i] = |\{key = i\}|$

for  $i \leftarrow 1$  to  $k$  do

$C[i] \leftarrow C[i] + C[i-1] ; C[i] = |\{key \leq i\}|$

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

# Counting sort( $A, B, k$ )

$\Theta(k)$  for  $i \leftarrow 0$  to  $k$  do  $C[i] \leftarrow 0$

$\Theta(n)$  for  $j \leftarrow 1$  to  $n$  do

$C[A[j]] \leftarrow C[A[j]] + 1$  ;  $C[i] = |\{key = i\}|$

$\Theta(k)$  for  $i \leftarrow 1$  to  $k$  do

$C[i] \leftarrow C[i] + C[i-1]$  ;  $C[i] = |\{key \leq i\}|$

for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

# Counting sort( $A, B, k$ )

$\Theta(k)$  for  $i \leftarrow 0$  to  $k$  do  $C[i] \leftarrow 0$

$\Theta(n)$  for  $j \leftarrow 1$  to  $n$  do

$C[A[j]] \leftarrow C[A[j]] + 1 ; C[i] = |\{key = i\}|$

$\Theta(k)$  for  $i \leftarrow 1$  to  $k$  do

$C[i] \leftarrow C[i] + C[i-1] ; C[i] = |\{key \leq i\}|$

**$\Theta(n)$**  for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

# Counting sort( $A, B, k$ )

$\Theta(k)$  for  $i \leftarrow 0$  to  $k$  do  $C[i] \leftarrow 0$

$\Theta(n)$  for  $j \leftarrow 1$  to  $n$  do

$C[A[j]] \leftarrow C[A[j]] + 1 ; C[i] = |\{key = i\}|$

$\Theta(k)$  for  $i \leftarrow 1$  to  $k$  do

$C[i] \leftarrow C[i] + C[i-1] ; C[i] = |\{key \leq i\}|$

$\Theta(n)$  for  $j \leftarrow n$  downto 1 do

$B[C[A[j]]] \leftarrow A[j]$

$C[A[j]] \leftarrow C[A[j]] - 1$

$\Theta(n+k)$

# כיצד יתכן שקיבלנו מילון לינארי?



# **כִּי הַמֵּין אֵינוֹ מְבוּסׁוּשׁ שׂוֹאָות!**

**מיון מניה הוא רק דוגמא אחת אפשרית למינן  
שאינו מבויסו שׂוֹאָות**



**אבל נשים לב שהוא מבויסו  
הנחה על הקלט!**

# מיון יציב

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3
	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0	2	2	3	3	3	5

# מיון יציב

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0	2	2	3	3	3	5

# מיון יציב

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3
	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0	2	2	3	3	3	5

# מיון יציב

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3
	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0	2	2	3	3	3	5

The diagram illustrates a stable sorting process. Array A contains elements [2, 5, 3, 0, 2, 3, 0, 3]. Array B contains elements [0, 0, 2, 2, 3, 3, 3, 5]. Three green arrows map elements A[1], A[5], and A[6] to B[3], indicating that these elements are mapped to the same position in array B.

# מיון יציב

	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$	$A[7]$	$A[8]$
$A$	2	5	3	0	2	3	0	3

	$B[1]$	$B[2]$	$B[3]$	$B[4]$	$B[5]$	$B[6]$	$B[7]$	$B[8]$
$B$	0	0	2	2	3	3	3	5