Lecture Proofs

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Review of Probability and Statistics

Theorem: For any events A and B, we have that

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

PROOF. Write $A \cup B = (AB^c) \cup (AB) \cup (A^cB)$ and note that these events are disjoint. Hence, making repeated use of the fact that P is additive for disjoint events, we see that

$$\begin{aligned}
P(A \cup B) &= P\left((AB^r) \cup (AB) \cup (A^cB)\right) \\
&= P\left(AB^c\right) + P(AB) + P\left(A^cB\right) \\
&= P\left(AB^c\right) + P(AB) + P\left(A^cB\right) + \Big(P(AB) - P(AB)\Big) \\
&= P\Big(\left(AB^r\right) \cup (AB)\Big) + P\Big(\left(A^cB\right) \cup (AB)\Big) - P(AB) \\
&= P(A) + P(B) - P(AB)
\end{aligned}$$

qed