Exercises Chapter 2

1. Let $P(A) = 0$ and show mathematically that in this case A is independent of every other event B.

2. Suppose we toss a fair coin until we get exactly two Heads (H).
   Examples: $\omega = TTTTTTHH$, $\omega = HTTH$, $\omega = HH$, $\omega = THTTTTH$, etc.

   (a) Describe formally the corresponding sample space Ω.

   (b) Let $X(\omega) = m$ denote the random variable that gives the number of coin tosses m for each $\omega \in \Omega$. What is the probability mass function $P(X = m)$ for $m = 0, 1, 2, \ldots$?

3. Consider a univariate, continuous random variable $X \in \mathbb{R}$ with density function f_X, where

   - f_X has a compact support $[0, 1] \subset \mathbb{R}$, i.e.,
     $f_X(x) > 0$ for all $x \in [0, 1]$ and $f_X(x) = 0$ for all $x \notin [0, 1]$, and
   - f_X is bounded, i.e., $\max_{x \in [0,1]} f_X(x) \leq c$, for some constant $0 < c < \infty$.

   (a) Show that the kth moment $E(X^k)$ is finite for each $k = 1, 2, \ldots$.

   (b) What is the value of $\lim_{k \to \infty} E(X^k)$?