Chapter 1
Introduction

There are certain activities, which we require in our daily lives. For example, we have to heat the water to take bath. Before the advent of control systems, we did it manually by using firewood. That requires time and effort, whereas, nowadays, we set the desired temperature in an electric water heater and turn it on. The task is done automatically without further human effort. The use of control systems is widespread in our homes as well as in industries. Control systems carry out the task automatically in the most efficient manner. In this chapter, we just introduce control systems first. One of the important tasks in control system design is to test that its performance is as required. For that purpose, some standard signals are used, while the actual signals in control systems have arbitrary amplitude profile.

1.1 Basics of Control Systems

A system carries out some task in response to an input signal. Control system is an interconnection of components, such as the controller, actuator, and plant, to produce a desired response. For example, an electric motor delivers mechanical rotational power when we energize it with electrical power. Apart from large number of industrial applications, control systems are often used for our comfort in our homes, such as room temperature control, water heater control, and voltage stabilizers for voltage control. An open-loop control system is a system with a controller and actuator to provide a desired response without any feedback, as shown in Fig. 1.1. Open-loop control system is like driving a car in a zigzag road with our eyes closed (without feedback from our eyes). A closed-loop control system is a system with a controller and actuator to provide a desired response with some feedback, as shown in Fig. 1.2. The controller produces the control signal. Its function is to hold the desired response at a desired value regardless of the changing environment around it. Closed-loop control system is like driving a car in a zigzag road with our eyes open (with feedback from our eyes).

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Fig. 1.1 Block diagram of an open-loop control system

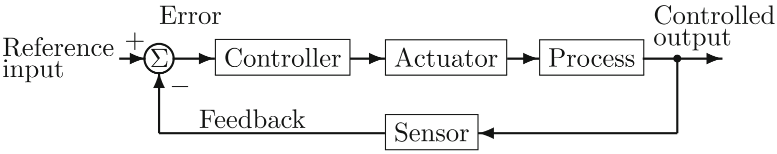

Fig. 1.2 Block diagram of a closed-loop control system

Room Temperature Control

A control system is used to maintain a desired temperature in the room, regardless of the changing temperature environment around it. In the open-loop control system, we set a desired temperature in the controller. It applies continuous heating with no regard to the actual output temperature. This type of system may not be satisfactory, at both day and night time, as the outside temperature varies considerably. In the closed-loop control system, the controller monitors the temperature in the room and if it goes high or low from the desired value, it sends a signal to activate the heater in such a way to reduce the error signal. The feedback input value to a temperature controller is an electric signal proportional to the temperature.

As control systems are used in several different applications, a transducer that transfers signals generated in one domain to another domain is often used. Transducers are used to sense signals such as flow, level, pressure, temperature, velocity, acceleration, etc. In particular, transducers those produce electrical output are most often used as electrical signals, which are very convenient for analysis, design, and simulation of systems. A tachometer is a transducer that provides a proportional voltage to the magnitude of the angular velocity of a shaft. Another commonly used transducer is the potentiometer. It is a resistor with three terminals, the third being an adjustable one. It is an electromechanical transducer that senses a mechanical displacement and provides a corresponding electrical signal. A thermistor is a transducer that is an electrical resistor, the resistance of which varies rapidly in a known manner with temperature. A thermocouple is also a temperature transducer that produces an electric current that is proportional to the temperature.

Tank Liquid (for Example, Water) Level Control

A common application of the control system is water tank level control. The control system prevents overflow of water, when the tank is full. When the water level goes down, the float also goes down and the attached valve opens the water inlet to the tank. When the water level reaches the set value, the float goes up and the attached valve closes the water inlet to the tank.

Human Body Control

The human body is an ideal model of a control system, as well as in other applications of science and engineering. In practical applications, we try to emulate the way biological systems carry out various tasks. For example, our eyes adjust the gain to suit light levels and normalize responses to contrast. In image processing applications, we design algorithms to emulate visual systems. Some of the targets for control are:

- Maintenance of blood pressure and body temperature within the sustainable range
- Maintenance of water, salt, and electrolytes
- Discharge of waste materials from the body
- Closing of the eyelids, when an object approaches the cornea of the eye

Automatic Gain Control

Automatic gain control is a closed-loop control system in an amplifier to maintain a suitable signal amplitude at the output, despite wide variations in the signal amplitude at the input. A typical application is radio receivers. The gain control is necessary to adjust for the different amplitudes of the signals received from different radio stations. Further, gain control is required to receive the signal even from a single station due to fading because of atmospheric disturbances and other reasons. The gain control system reduces the volume for strong signals and raises it when the signal is weak.

Frequency Control in Electrical Power System

The frequency and voltage of the electrical power supply that is distributed to industries and homes have some specifications to comply with. That is, the frequency and voltage variations must be within some prescribed limits. A drop in speed of the generator due to increased load causes the control system to increase the input (admit more steam into the turbine in the case of steam power plants) resulting in an increase in the speed of the generator. This frequency control brings out the basics of closed-loop control systems.

1. Sensing of the actual response.
2. The sensor output is interpreted in terms of a deviation from a control point.
3. Necessary corrections are applied to restore the response to its desired form.

Basic Terminology Used in Control Systems

Actuator Device that provides motive power to the process
Comparator Computes the difference between the desired and actual output
Controller Device that computes the control signal. Composed of comparator and compensator
Output The output of the system to be controlled
Plant Combination of the actuator and the system under control
Process System or device, whose output is to be controlled
Reference input The desired output of the system. Also called set point or input

Sensor Device that detects and measures some physical effect and generates a signal proportional to the effect

Transfer function Ratio of the transforms of system output and input. That is, the transform of the input multiplied by the transfer function is the output in the transform domain

1.2 Basic Signals

Signals are a source of information, such as an audio or a temperature signal. As signals vary with time or some other independent variable, they are also referred as functions. For example, $x(t) = \sin(t)$, where $x(t)$ is its amplitude and t is the independent variable. The amplitude profiles of practical signals and those of the responses of practical systems are arbitrary. Therefore, signals have to be decomposed in terms of some well-defined basic signals, such as the impulse and sinusoid, for compact representation and easier processing. Systems can be characterized by their responses to the basic signals, impulse, unit-step, ramp, parabola type, and sinusoids. Then, using these responses, the system response for arbitrary signals can be easily determined by decomposing the input signal in terms of basic signals and summing the responses to all the components, assuming linearity property of linear systems. The basic signals are used as intermediaries in the analysis of signals and systems. They are not practical signals. For example, the duration of the sinusoid is infinite. The bandwidth of impulse is infinite. However, they can be approximated, in practice, to a required accuracy. The response of systems to them can be measured, in practice, with acceptable tolerance. While the responses of test signals of a system are related, the unit-step signal is the easiest to generate and is usually used for performance tests.

1.2.1 The Unit-Step Signal

On the interval it is defined, a function is continuous if a pen can trace its graph completely without being lifted from the page. While a sinusoid is a continuous function, the unit-step signal is discontinuous. The step signal is indispensable in the analysis of systems, for example, in modeling a switch. For this purpose, a generalized function, called the impulse, is defined to make the unit-step signal its integral. In control systems, the specifications of a system are usually given in terms of its unit-step response. The unit-step function $u(t)$, shown in Fig. 1.3a, is defined as

$$u(t) = \begin{cases} 1 \text{ for } t \geq 0 \\ 0 \text{ for } t < 0 \end{cases}$$

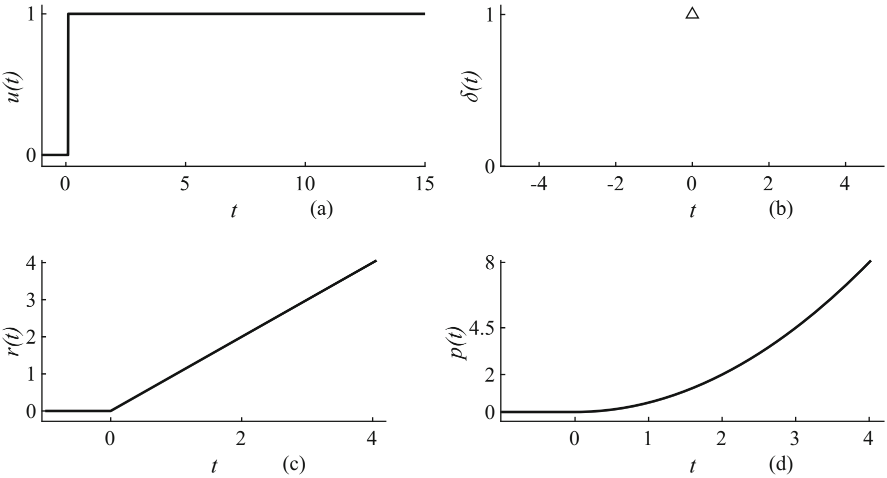

Fig. 1.3 (**a**) The unit-step signal, $u(t)$; (**b**) the unit-impulse signal, $\delta(t)$; (**c**) the unit-ramp signal, $r(t)$; (**d**) the unit-parabolic signal, $p(t)$

The unit-step signal has a value of one for positive values of its argument t and its value is zero otherwise. The delayed version of the unit-step signal is defined as

$$u(t-a) = \begin{cases} 1 \text{ for } t \geq a \\ 0 \text{ for } t < a \end{cases}$$

1.2.2 The Unit-Impulse Signal

Although step and impulse functions are of fundamental importance in signal and system analysis, there is no analytic way to define these functions, as they are not analytic. A simple, rigorous, and usable interpretation for the derivative of a function at a point of discontinuity is given by defining a generalized function—the impulse. A regular function is usually defined by its value. The impulse function is called a generalized function, since it is defined by the result of its operation (integration) on an ordinary function. The continuous unit-impulse signal $\delta(t)$, located at $t = 0$, is defined, in terms of an integral, as

$$\int_{-\infty}^{\infty} x(t)\delta(t)\, dt = x(0)$$

assuming that $x(t)$ is continuous at $t = 0$ (so that the value $x(0)$ is unique). The value of the function $x(t)$ at $t = 0$ has been sifted out or sampled by the defining operation. The signal is shown in Fig. 1.3b.

1.2.3 The Unit-Ramp Signal

The unit-ramp signal, shown in Fig. 1.3c, is defined as

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

The unit-ramp signal linearly increases, with unit slope, for positive values of its argument and its value is zero for negative values of its argument. The unit-ramp signal is the integral of the unit-step signal.

1.2.4 The Unit-Parabolic Signal

The graph of any quadratic function is called a parabola. The unit-parabolic signal, shown in Fig. 1.3d, is defined as

$$p(t) = \begin{cases} \frac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

The denominator 2 makes its Laplace transform $1/s^3$. The unit-parabolic signal quadratically increases for positive values of its argument and its value is zero for negative values of its argument. The unit-parabolic signal is the integral of the unit-ramp signal.

1.3 Sinusoids

A general sinusoidal waveform is a linear combination of trigonometric sine and cosine waveforms or shifted sine and cosine functions. Sinusoidal representation of signals is indispensable in the analysis of signals and systems for the following most cogent reasons. The steady-state waveform, due to an input sinusoid, in any part of a linear system, however, complex it may be, is also a sinusoid of the same frequency as that of the input differing only in its amplitude and phase. No other periodic waveform has this distinction. Linear systems can be modeled by a linear differential equation with constant coefficients and the particular integral of the equation is a sinusoid when the input is sinusoidal. The sum of any number of sinusoids having the same frequency but arbitrary amplitudes and phases is also a sinusoid of the same frequency. No other periodic function can lay claim to this property either. The integral and derivative of a sinusoid is again a sinusoid of the same frequency. Due to these properties, system models, such as differential equation and convolution, reduce to algebraic equations for a sinusoidal input for linear systems. Further, due to the orthogonal property, any practical signal, with arbitrary amplitude profile, can be decomposed into a set of sinusoids using fast algorithms. Physical systems also, such as a combination of an inductor and a capacitor, produce an output of sinusoidal nature. The motion of a simple pendulum is approximately sinusoidal.

1.3.1 The Polar Form of Sinusoids

There are two forms of representation of real sinusoids. At a given angular frequency ω, a sinusoid is characterized by its amplitude A and its phase θ (called the polar form) or by the amplitudes of its sine and cosine components (called the rectangular form). Signal amplitude can be either positive or negative.

The polar form of a sinusoid is

$$x(t) = A\cos(\omega t + \theta), \qquad -\infty < t < \infty$$

A sinusoidal waveform has a positive peak and a negative peak in each cycle. The distance of either peak of the waveform from the horizontal axis is its amplitude A. The cosine function is periodic,

$$\cos(\omega t) = \cos(\omega t + 2\pi)$$

Any function defined on a circle will be a periodic function of an angular variable ω. It repeats its values for $t = t + T = t + 2\pi/\omega$, where T is its period in seconds. Then, the cyclic frequency of the sinusoid is $f = 1/T$ Hz (cycles/second). The independent variable t, while time in most applications, can be anything else also, such as distance.

Sinusoids $x(t) = 2\cos(\frac{2\pi}{8}t)$ and $x(t) = \sin(\frac{2\pi}{8}t)$ are shown in Fig. 1.4a. Cosine and sine waveforms are special cases of a sinusoidal waveform. Cosine waveform has its peak value 2 at $t = 0$. Taking it as a reference, its phase is defined as

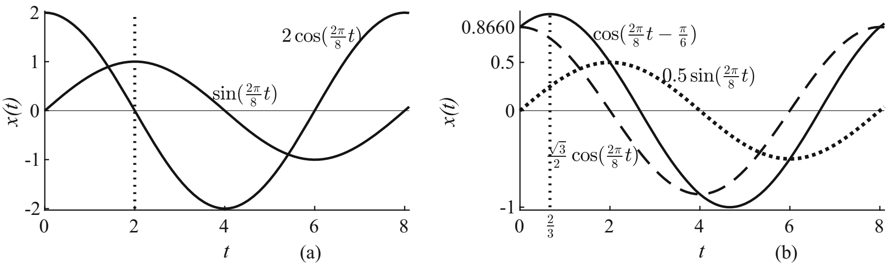

Fig. 1.4 (**a**) $x(t) = \sin(\frac{2\pi}{8}t)$ and $x(t) = 2\cos(\frac{2\pi}{8}t)$; (**b**) $x_o(t) = 0.5\sin(\frac{2\pi}{8}t)$, $x_e(t) = \frac{\sqrt{3}}{2}\cos(\frac{2\pi}{8}t)$ and $x(t) = \cos(\frac{2\pi}{8}t - \frac{\pi}{6})$

zero radians. The radian frequency is $\omega = 2\pi/8$ radians/second. Its period is $T = 2\pi/\omega = 8$ seconds. That is, it makes one complete cycle in 8 s, as shown in the figure, and repeats indefinitely for $-\infty < t < \infty$. Its cyclic frequency is $f = 1/8$ Hz. The sine waveform $x(t) = \sin(\frac{2\pi}{8}t)$ has its peak value 1 at $t = 2$. Taking the cosine waveform as the reference, its first peak occurs after a delay of 2 s, which is one-fourth of a cycle in the period 8. Since one complete cycle corresponds to 2π radians or 360°, its phase is defined as $-\pi/2$ radians or $-90°$. That is,

$$\sin\left(\frac{2\pi}{8}t\right) = \cos\left(\frac{2\pi}{8}t - \frac{\pi}{2}\right)$$

Therefore, given a sinusoidal waveform in terms of sine waveform, it can be expressed, in terms of cosine waveform as $A\sin(\omega t + \theta) = A\cos(\omega t + (\theta - \frac{\pi}{2}))$. Similarly, $A\cos(\omega t + \theta) = A\sin(\omega t + (\theta + \frac{\pi}{2}))$. Sinusoids remain the same by a shift of an integral number of their periods, as they are periodic.

1.3.2 The Rectangular Form of Sinusoids

Figure 1.4b shows the sinusoid

$$x(t) = \cos(\frac{2\pi}{8}t - \frac{\pi}{6})$$

in solid line. Its peak value of 1 occurs at $t = 2/3$ s. Therefore, its phase is $-((2/3)/8)2\pi = -\pi/6$ radians or $-30°$. Using the trigonometric subtraction formula, we get the rectangular form as

$$\begin{aligned}\cos\left(\frac{2\pi}{8}t - \frac{\pi}{6}\right) &= \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{2\pi}{8}t\right) + \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{2\pi}{8}t\right)\\ &= \frac{1}{2}\sin\left(\frac{2\pi}{8}t\right) + \frac{\sqrt{3}}{2}\cos\left(\frac{2\pi}{8}t\right)\end{aligned}$$

The rectangular form expresses a sinusoid as the sum of its sine and cosine components, which are also, respectively, its odd and even components. The sine and cosine components are shown, respectively, by dotted and dashed lines in the figure. In general, we get

$$A\cos(\omega t + \theta) = A\cos(\theta)\cos(\omega t) - A\sin(\theta)\sin(\omega t) = C\cos(\omega t) + D\sin(\omega t)$$

where

$$C = A\cos\theta \quad \text{and} \quad D = -A\sin\theta$$

The inverse relation is

$$A = \sqrt{C^2 + D^2} \quad \text{and} \quad \theta = \cos^{-1}\left(\frac{C}{A}\right) = \sin^{-1}\left(\frac{-D}{A}\right)$$

Sum of Sinusoids with the Same Frequency

An important property of the sinusoids is that the sum of sinusoids of the same frequency, but with arbitrary amplitudes and phases, is also a sinusoid of the same frequency. In order to find the sum, we have to express the sinusoids in their rectangular form and sum the respective amplitudes of the sine and cosine components. Consider the two sinusoids

$$x(t) = A\cos(\omega t + \theta) \qquad \text{and} \qquad y(t) = B\cos(\omega t + \phi)$$

Then,

$$\begin{aligned} z(t) = x(t) + y(t) = C\cos(\omega t + \psi) &= A\cos(\omega t + \theta) + B\cos(\omega t + \phi) \\ &= \cos(\omega t)(A\cos(\theta) + B\cos(\phi)) - \sin(\omega t)(A\sin(\theta) + B\sin(\phi)) \\ &= \cos(\omega t)(C\cos(\psi)) - \sin(\omega t)(C\sin(\psi)) \end{aligned}$$

Solving for C and ψ, we get

$$C = \sqrt{A^2 + B^2 + 2AB\cos(\theta - \phi)}$$

$$\psi = \tan^{-1}\frac{A\sin(\theta) + B\sin(\phi)}{A\cos(\theta) + B\cos(\phi)}$$

With $\theta = 0$ and $\phi = -\pi/2$ (one sinusoid being the cosine and the other being sine), the formula reduces to relation between the polar and the rectangular form of a sinusoid.

Example 1.1 Determine the sinusoid that is the sum of two sinusoids

$$x(t) = 3\cos\left(\omega t + \frac{\pi}{3}\right) \quad \text{and} \quad y(t) = 2\sin\left(\omega t - \frac{\pi}{6}\right)$$

Solution The second sinusoid can also be expressed as

$$y(t) = 2\cos\left(\omega t - \frac{\pi}{6} - \frac{\pi}{2}\right) = 2\cos\left(\omega t - \frac{2\pi}{3}\right)$$

Now,

$$A = 3,\ B = 2,\ \theta = \frac{\pi}{3},\ \phi = -\frac{2\pi}{3}$$

Substituting the numerical values in the equations, we get

$$C = \sqrt{3^2 + 2^2 + 2(3)(2)\cos\left(\frac{\pi}{3} + \frac{2\pi}{3}\right)} = 1$$

$$\psi* = \cos^{-1}\frac{3\cos\left(\frac{\pi}{3}\right) + 2\cos\left(-\frac{2\pi}{3}\right)}{1} = \sin^{-1}\frac{3\sin\left(\frac{\pi}{3}\right) + 2\sin\left(-\frac{2\pi}{3}\right)}{1}$$

$$= 1.0472 \ \text{ radians} = 60^\circ$$

$$z(t) = x(t) + y(t) = \cos(\omega t + 1.0472)$$

∎

1.3.3 The Complex Sinusoids

While the sinusoidal waveform is generated by practical systems, its mathematically equivalent form, called the complex sinusoid,

$$v(t) = Ve^{j(\omega t+\theta)} = Ve^{j\theta}e^{j\omega t}, \qquad -\infty < t < \infty$$

is found to be indispensable for analysis due to its compact form and ease of manipulation of the exponential function. $e^{j\omega t}$ is the complex sinusoid with unit magnitude and zero phase. The complex (amplitude) coefficient is $Ve^{j\theta}$. The amplitude and phase of the sinusoid is represented by the single complex number $Ve^{j\theta}$, in contrast to using two real values in the real sinusoid. Due to Euler's identity, we get

$$v(t) = \frac{V}{2}\left(e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right) = V\cos(\omega t + \theta)$$

The complex exponential functions separately have no physical significance. Their sum represents a physical variable, such as voltage. However, the response of a system to $Ve^{j\omega t}$ yields enough information with ease to deduce the response to real sinusoids.

Real Causal Exponential Signal

Another commonly encountered signal in signal and systems is the real causal exponential signal, for example, $e^{-2t}u(t)$, shown in Fig. 1.5a.

The **time constant**, which is the inverse of the coefficient associated with the independent variable t, is 1/2. The peak value is 1 at $t = 0$. At $t = 1/2$ (one

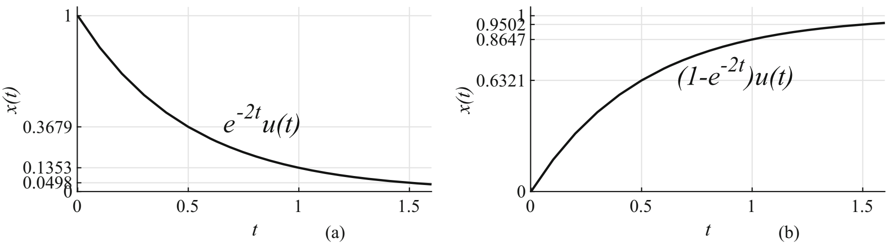

Fig. 1.5 Real causal exponential signals. (**a**) $e^{-2t}u(t)$; (**b**) $(1-e^{-2t})u(t)$

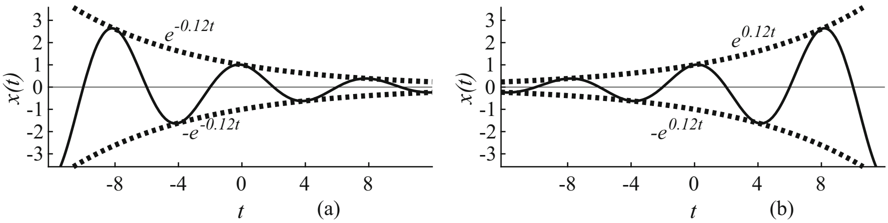

Fig. 1.6 (**a**) Exponentially decreasing amplitude cosine wave, $x(t) = e^{-0.12t}\cos(\frac{2\pi}{8}t)$; (**b**) exponentially increasing amplitude cosine wave, $x(t) = e^{0.12t}\cos(\frac{2\pi}{8}t)$

time constant), its value is $1/e \approx 0.37$. At $t = 1$ (two time constants), its value is $(1/e)^2 \approx 0.135$ and so on. Figure 1.5b shows the graph of $(1-e^{-2t})u(t)$.

Exponentially Varying Amplitude Sinusoids

An exponentially varying amplitude sinusoid, $Ae^{at}\cos(\omega t + \theta)$, is obtained by multiplying a sinusoid, $A\cos(\omega t + \theta)$, by a real exponential, e^{at}. The more familiar constant amplitude sinusoid results when $a = 0$. If ω is equal to zero, then we get a real exponential. Sinusoids, $x(t) = e^{-0.12t}\cos(\frac{2\pi}{8}t)$ and $x(t) = e^{0.12t}\cos(\frac{2\pi}{8}t)$, with exponentially varying amplitudes are shown, respectively, in Fig. 1.6a and b.

The complex exponential representation of an exponentially varying amplitude sinusoid is given as

$$x(t) = \frac{A}{2}e^{at}\left(e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right) = Ae^{at}\cos(\omega t + \theta)$$

1.4 System Modeling

Practical systems have to be modeled for their analysis and design. Further, these models have to be simulated for verification of the response. Various physical systems, such as mechanical, electrical, and hydraulic, are composed of different components. A system is called dynamic, if its present output depends on past inputs. An example is the input–output relationship of an inductor or a spring. The output varies until equilibrium condition is reached. The output of a static system depends only on current input. An example is the input–output relationship of a resistor or a dashpot.

A mathematical model of a system, that is sufficiently accurate and of lowest possible complexity, has to be first developed. Neither it is necessary nor it is possible to develop a precise model of a practical system as the required performance of engineering systems is always specified with some tolerance levels (not exact).

For many practical systems, a constant-coefficient linear differential equation model can be obtained that represents the system with acceptable tolerances. While resistors and dashpots are characterized by a linear equation, springs, masses, inductors, and capacitors need derivative terms to characterize them. Once the mathematical model of components of a system is known, depending on the configuration, a model can be developed. The procedure for obtaining an adequate model is to start with a simple model and keep on refining it until we get an acceptable one. The response of the mathematical model must be compared with simulation results for the validation of the model.

A linear time-invariant differential equation, with which we are mostly involved in this book, has an input, an output, and their derivatives in a linear combination. For example,

$$\frac{dy}{dt} + 4y = \cos(t)$$

is a first-order (contains only the first derivative of y) constant-coefficient differential equation. While we can use the differential equation model for complete analysis and design of systems, as it is difficult for practical systems, it is not used in this form. The total response of a linear system consists of a component called, zero-input response, that is due to the initial conditions alone. Another component, called zero-state response, is due to the input alone. From these responses, we can derive the transient and steady-state components. Both components are important in the analysis and design of control systems. For stable systems, the transient response decays to negligible levels in a short time and, after that, what is remaining is essentially steady-state response.

In general, the highest derivative of y present determines the order of the equation. While the first model we derive is a differential equation, it is the most difficult model to solve. Therefore, out of necessity, other models have been developed. Out of these models, we use transform and state-space models extensively in the analysis and design of control systems. The transform model uses the complex exponentials to represent arbitrary signals and system responses. The input and output of systems are related by the transfer function. While the transfer function model can be used for somewhat higher-order models and excellent for understanding the theory of linear systems, it cannot be used for practical systems, since the order involved is very high. Further, the model based on transform is applicable only for linear systems. The ultimate solution for practical use is the state-space model. This model is more general than other models with several advantages. In this model, a N-th order differential equation is decomposed into a set of N simultaneous first-order differential equations. Control system analysis and design is highly mathematical. But, one can become sufficiently proficient: (i) through practicing with paper-and-pencil method for lower-order systems and (ii) using widely prevalent software packages, such as MATLAB®, for computation and simulation of higher-order systems. In the rest of the book, we have provided both the analytical and simulation methods with numerical examples. While the state-space model will be presented in later chapters, the Laplace transform is introduced in the next chapter and, subsequently, used for system analysis and design.

1.5 Summary

- The use of control systems is widespread in our homes as well as in industries. Control systems carry out the task automatically in the most efficient manner.
- A system carries out some task in response to an input signal. Control system is an interconnection of components, such as the controller, actuator, and plant, to produce a desired response.
- An open-loop control system is a system with a controller and actuator to provide a desired response without any feedback.
- A closed-loop control system is a system with a controller and actuator to provide a desired response with some feedback.
- Transducers are used to sense signals such as flow, level, pressure, temperature, velocity, acceleration, etc.
- Signals have to be decomposed in terms of some well-defined basic signals, such as the impulse and sinusoid, for compact representation and easier processing. Systems can be characterized by their responses to the basic signals, impulse, unit-step, ramp, parabola type, and sinusoids.
- In control systems, the specifications of a system are usually given in terms of its unit-step response. The unit-step signal has a value of one for positive values of its argument t and its value is zero otherwise.
- The impulse function is called a generalized function, since it is defined by the result of its operation (integration) on an ordinary function.
- The unit-ramp signal linearly increases, with unit slope, for positive values of its argument and its value is zero for negative values of its argument.
- The unit-parabolic signal quadratically increases for positive values of its argument and its value is zero for negative values of its argument. The unit-parabolic signal is the integral of the unit-ramp signal.
- A general sinusoidal waveform is a linear combination of trigonometric sine and cosine waveforms or shifted sine and cosine functions.

- While the sinusoidal waveform is generated by practical systems, its mathematically equivalent form, called the complex sinusoid,

$$v(t) = Ve^{j(\omega t+\theta)} = Ve^{j\theta}e^{j\omega t}, \qquad -\infty < t < \infty$$

  is found to be indispensable for analysis due to its compact form and ease of manipulation of the exponential function.
- Another commonly encountered signal in signal and systems is the real causal exponential signal.
- Practical systems have to be modeled for their analysis and design. Further, these models have to be simulated for verification of the response.
- The procedure for obtaining an adequate model is to start with a simple model and keep on refining it until we get an acceptable one.

Exercises

1.1 Draw the graphs of the signals $2u(t-1)$ and $3(\frac{(t-1)^2}{2})u(t-1)$.

1.2 Express the sinusoid in rectangular form. Get back the polar form from the rectangular form and verify that the given sinusoid is obtained. Find the sample values of the sinusoid for $t = 0$, $t = 2$, $t = 4$, and $t = 6$. Find the value of t nearest to $t = 0$, where the first positive peak of the sinusoid occurs?

***1.2.1** $x(t) = 2\sin(\frac{2\pi}{8}t - \frac{\pi}{6})$

1.2.2 $x(t) = 3\cos(\frac{2\pi}{8}t + \frac{\pi}{3})$

1.2.3 $x(t) = -3\cos(\frac{2\pi}{8}t - \frac{\pi}{2})$

1.3 Determine the sinusoid $c(t)$ that is the sum of the pair of given sinusoids, $a(t)$ and $b(t)$, $c(t) = a(t) + b(t)$. Find the sample values of the sinusoid $c(t)$ over one cycle starting from $t = 0$ at intervals of 1 s and verify that the sample values are the same as the sum of the sample values of $a(t)$ and $b(t)$, $a(t) + b(t)$.

1.3.1 $a(t) = \cos(\frac{\pi}{4}t + \frac{\pi}{3}),\ b(t) = -\sin(\frac{\pi}{4}t + \frac{\pi}{3})$

1.3.2 $a(t) = 2\cos(\frac{\pi}{4}t),\ b(t) = 2\sin(\frac{\pi}{4}t)$

***1.3.3** $a(t) = 3\cos(\frac{\pi}{4}t + \frac{\pi}{3}),\ b(t) = \cos(\frac{\pi}{4}t - \frac{\pi}{4})$

1.4 Express the signal in terms of complex exponentials. Find the sample values of the two forms over one cycle starting from $t = 0$ at intervals of 1 s and verify that they are the same.

1.4.1 $x(t) = \sin(\frac{2\pi}{8}t + \frac{\pi}{6})$

***1.4.2** $x(t) = 2\cos(\frac{2\pi}{8}t - \frac{\pi}{3})$

1.4.3 $x(t) = 3\cos(\frac{2\pi}{8}t + \frac{\pi}{3})$