Squark Production in the R-Symmetric Supersymmetric Standardmodel

Master-Arbeit
zur Erlangung des Hochschulgrades
Master of Science
im Master-Studiengang Physik

vorgelegt von

Sebastian Liebschner
geboren am 08.05.1990 in Meerane

Institut für Kern- und Teilchenphysik Physik
Fachrichtung Physik
Fakultät Mathematik und Naturwissenschaften
Technische Universität Dresden
2016

Eingereicht am xx. Monat 20xx

1. Gutachter: Prof. Dr. XX
2. Gutachter: Prof. Dr. YY

Summary

Abstract

English: motivation: aesthetic: Coleman-Mandula –> Haag-Lopuszanski-Sohnius-Theorem

Abstract

Deutsch

Contents

1 Introduction

motivation: aesthetic: Coleman-Mandula –> Haag-Lopuszanski-Sohnius-Theorem
plots for exclusion of squarks in specific SUSY scenarios (from Michael) –> R-Symmetry could be possible explanation for that because:
MSSM-Lagrangian –> trafo rules for superfields under R-symm –> forbidden terms in MRSSM (write down Lagrangian for R-symmetric SUSYQCD)
suppression of squark production in MRSSM by less diagrams (m_{gluino}^{-4} suppression at low energies in MRSSM and only m_{gluino}^{-2} suppression in MSSM)
R-charges of all fields (show in diagram!) –> only if R-charges of final / initial particles are zero, a diagram is allowed in R-symm. model
references to build in

- "Matching Squark Pair Production at NLO with Parton Showers" from Gavin, Hangst, Krämer, Mühlleitner,.. for complete treatment of NLO calculation

- "dIRAC gAUGINOS IN susy - sUPPRESSED jETS + <met sIGNALS: a sNOWMASS wHITEPAPER" FROM kRIBS, mARTIN for Squark production at LO, allude to same result

- "Dirac Gaugino Masses and supersoft SUSY breaking" from Fox, Weiner, Nelson for the introduction of the MRSSM

2 The Standard Model

The Standard Model of particle physics is the commonly accepted theory describing the world's fundamental particles and their interactions. It is a gauge quantum field theory which is characterized by its invariance under different symmetry groups. The Standard Model contains different fields, whose quantized excitations are interpreted as particles.

2.1 Symmetries and Transformations

Spacetime Symmetries

The Standard Model is defined on Minkowski space, whose coordinates are label with x^μ $\mu = \{0, 1, 2, 4\}$. As a relativistic theory it is invariant under Poincaré transformations, i.e. it is invariant under Lorentz-transformations (with generators $J^{\mu\nu}$) and translations (with generators P^μ) in spacetime. The set of all Poincaré transformations form the Poincaré group, which is a Lie group. Its generators obey the Poincaré-algebra

$$\begin{aligned} [P^\mu, P^\nu] &= 0 \\ [P^\mu, J^{\nu\rho}] &= i(g^{\mu\nu}P^\rho - g^{\mu\rho}P^\nu) \\ [J^{\mu\nu}, J^{\rho\sigma}] &= i(g^{\nu\rho}J^{\mu\rho} + g^{\mu\sigma}J^{\nu\rho} - g^{\mu\rho}J^{\nu\sigma} - g^{\nu\sigma}J^{\mu\rho}). \end{aligned} \tag{2.1}$$

The fields of the Standard Model transform in different representations of the Poincaré-group [?].

Gauge Symmetries

In order to describe interactions of matter particles one uses gauge theories. In the Standard Model matter fields are described by Dirac spinors. The Lagrangian of a free Dirac field reads

$$\mathcal{L}_{Dirac} = \overline{\Psi}(i\partial\!\!\!/ - m)\Psi. \tag{2.2}$$

To include interactions one imposes a local group symmetry (gauge symmetry) upon this Lagrangian. A spinor transforms under a generic gauge transformation like

$$\Psi(x) \to U(x)\Psi(x), \tag{2.3}$$

where $U(x)$ is an element of the gauge group in question. Because the gauge group is a unitary matrix Lie group it can be written in the form $U(x) = \exp(-igT^a\theta^a(x))$. Here T^a are the self-adjoint generators of the associated Lie algebra which obey

$$[T^a, T^b] = if_{abc}T^c \tag{2.4}$$

where f_{abc} are the structure constants of a Lie algebra, g is the coupling constant of the gauge group and $\theta^a(x)$ are local parameters.

Because the parameters of the gauge group are local the derivative in 2.2 spoils the gauge invariance. In order to rectify gauge invariance of the Lagrangian one can introduce a further field for each index a of the generators - the gauge vector $G^{a\mu}$. Defining the transformation of the matrix valued gauge vector $G^\mu := G^{a\mu}T^a$ as

$$G^\mu(x) \to U(x)\left(G^\mu(x) + \frac{i}{g}\partial^\mu\right)U^{-1}(x) \tag{2.5}$$

and introducing the gauge covariant derivative

$$D^\mu = \partial^\mu + igT^aG^{a\mu} \tag{2.6}$$

one finds that the expression $D_\mu\Psi(x)$ transforms as

$$D_\mu\Psi(x) \to U(x)D_\mu\Psi(x) \tag{2.7}$$

Therefore gauge invariance in 2.2 is restored by replacing ∂_μ with D_μ. But if the gauge vector is interpreted as a physical field there must apart from the so far introduced interaction term also be a kinetic term associated with it. Using 2.7 one defines the field strength tensor[1] $F^{a\mu\nu}$ whose matrix valued form

$$F^{\mu\nu} = F^{a\mu\nu}T^a = \frac{1}{ig}[D^\mu, D^\nu] = \partial^\mu G^\nu - \partial^\nu G^\mu - if_{abc}.... \tag{2.8}$$

transforms as $F^{\mu\nu} \to U(x)F^{\mu\nu}U^{-1}(x)$. Using the cyclic property of the trace and the Dynkin index $T(F)$ defined in the appendix 11.6 one can write down a gauge invariant kinetic term for the gauge vector:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\mathrm{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right) = -\frac{T(F)}{2}F^{a\mu\nu}F^a_{\mu\nu} \tag{2.9}$$

This completes the construction of a Lagrangian which is invariant under non-abelian gauge group transformations. The result is the famous Yang and Mills Lagrangian[cite original paper]

$$\mathcal{L}_{Yang-Mills} = \overline{\Psi}i\not{D}\Psi - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} \tag{2.10}$$

[1] An alternative construction makes use of the gauge invariant Wilson loop. This gives some insights into the geometry of gauge transformations [?].

This Lagragian gives rise to spin-$\frac{1}{2}$ (matter) particles which interact with spin-1 (force mediator) particles with each other. Furthermore if the gauge group is non-abelian, i.e. $f_{abc} \neq 0$, there are self interactions among the spin-1 particles.
The gauge group of the SM is a direct product ??? of the three gauge groups: $U_Y(1)$, $SU_L(2)$ and $SU_C(3)$[2]. The elements $U(x)$ of those and the infinitesimal transformations of the spinors and gauge vectors are given in table 2.1.
These gauge groups give rise to 3 forces: the strong force, the weak force and the electromagnetic force.

$U_Y(1)$	$U(x) = \exp\left(-ig_Y\frac{\hat{Y}}{2}\theta_Y(x)\right)$ $\psi(x) \to \left(1 - ig_Y\frac{\hat{Y}}{2}\theta_Y(x)\right)\psi(x)$ $B^\mu(x) \to B^\mu(x) + \partial^\mu\theta_Y(x)$
$SU_L(2)$	$U(x) = \exp\left(-ig_w\vec{\tau}\cdot\vec{\theta}_w(x)\right)$ $\psi(x) \to \left(\mathbb{1} - ig_w\vec{\tau}\cdot\vec{\theta}_w(x)\right)\psi(x)$ $W^{a\mu}(x) \to W^{a\mu}(x) + \partial^\mu\theta^a_w(x) + g_w\varepsilon^{abc}\theta^b_w(x)W^{c\mu}(x)$
$SU_C(3)$	$U(x) = \exp\left(-ig_sT^a\cdot\theta^a_s(x)\right)$ $\psi(x) \to \left(\mathbb{1} - ig_sT^a\cdot\theta^a_s(x)\right)$ $G^{a\mu}(x) \to G^{a\mu}(x) + \partial^\mu\theta^a_s(x) + g_sf_{abc}\theta^b_s(x)G^{c\mu}$

Table 2.1: The table lists the explicit element $U(x)$ of the gauge groups $U_Y(1)$, $SU_L(2)$ and $SU_C(3)$ and the infinitesimal transformations of spinor and vector fields.
The hypercharge operator $\hat{Y}$ gives the eigenvalue of the hypercharge of the field it is applied to. $\vec{\tau}$ and T^a are the geneators of $SU_L(2)$ and $SU_C(3)$ respectively. In the fundamental representation one has $\vec{\tau} = \frac{\vec{\sigma}}{2}$ where $\vec{\sigma}$ has the 3 Pauli matrices as components and $T^a = \frac{\lambda^a}{2}$ where λ^a are the 8 Gell-Mann matrices. ε_{abc} and f_{abc} are the structure constants of $SU_L(2)$ and $SU_C(3)$ respectively.

[2]The subscript stands for the associated charge of the groups respectively: Y for hypercharge, L for left handedness (weak Isospin I_3) and C for color

2.2 The Particles of the Standard Model

In the Standard Model different matter particles take part in different interactions, i.e. their corresponding spinor couples to different gauge vectors.
If a spinor couples to a certain gauge vector it transforms non trivially (like indicated in table 2.1) under the gauge group which is associated with this gauge vector[3]. This means if a particle couples to a certain force its charge which is associated with this force is nonzero.
The charges of particles for a force are defined as the eigenvalues of the generators which correspond to the force.

The Quarks:

Quarks are strongly interacting fermions, which means their spinors transform non trivially under $SU_C(3)$. Because they transform in the fundamental representation of $SU_C(3)$ this means a quark spinor is built up by 3 spinors each carrying another color. This splitting of the quark spinor in colors is often suppressed for the sake of simplicity. This convention is adopted throughout this thesis.
Furthermore the left handed component of quarks interact weakly, which means that their spinors transform (in the fundamental representation) under $SU_L(2)$ meaning that 2 left handed quark spinors are assembled in a doublet.
Finally all quarks carry a hypercharge. In section ??? the mechanism of electroweak symmetry breaking is described. This mechanism explains how electromagnetism arises from the groups $SU_L(2)$ and $U_Y(1)$. All quarks interact electromagnetically.
After all there are 6 quarks which are listed in table2.2. They are categorized in 3 generations because their quantum numbers except (for their mass) is the same in each generation. The two types of quarks which have distinct quantum numbers are the up-type-quarks and the down-type quarks. A up-type-quark and the down-type quark of the same generation built up a doublet.

The Leptons:

Leptons do not interact strongly. They take part in the weak and the electromagnetic interaction, i.e. their spinors transform under the fundamental representation of $SU_L(2)$ and $U_Y(1)$. As for the quarks only the left handed components interact weakly.
As for the quarks there are 6 leptons which are classified into 3 generations (see table 2.2). In each generation is a lepton with a negative electrical charged and an electrically neutral lepton. The latter ones are referred to as neutrinos. Right handed neutrinos have not been observed (yet) and are therefore absent in the SM. The former are called electron, muon and tau. Each left handed leptons with an electric charge is assembled with its neutrino in a doublet.

[3]In the Standard Model all matter particles transform in the fundamental (or trivial) representation of gauge groups.

Quarks and Leptons are the matter particles of the SM. They are listed together with their charges for the different forces in table 2.3. One has the color for strong interactions, the third component of the weak isospin I_3 for weak interactions(the eigenvalue of the third generator of the $SU_L(2)$) and the half of the hypercharge $\frac{Y}{2}$ to obtain the electric charge Q via the Gell-Mann–Nishijima formula: $Q = I_3 + \frac{Y}{2}$.
Because the left and right-handed parts of spinors transform differently under the $SU_L(2)$ they are listed separately.
All quarks occur with three different colors.
In the last row the Higgs-boson is listed. Its associated field is responsible for the mass of elementary particles. That is explained in section 2.3.

Particle		1^{st} generation		2^{nd} generation		3^{rd} generation	
u_i	up-type-Quark	u	up-Quark	c	charm-Quark	t	top-Quark
d_i	down-type-Quark	d	down-Quark	s	strange-Quark	b	bottom-Quark
e_i	Charged Lepton	e	Electron	μ	Muon	τ	Tau
ν_i	Neutrino	ν_e	Electron Neutrino	ν_μ	Muon Neutrino	ν_τ	Tau Neutrino

Table 2.2: The matter particles of the SM. Listed are the symbol and the name of the particles.

Particle	Symbol	color	I_3	$\frac{Y}{2}$	Q
Left handed Quarks	$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$	red, green, blue	$\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$+\frac{1}{6}$	$\begin{pmatrix} +\frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
Right-handed Quarks	u_{iR}	red, green, blue	0	$+\frac{2}{3}$	$+\frac{2}{3}$
	d_{iR}	red, green, blue	0	$-\frac{1}{3}$	$-\frac{1}{3}$
Left-handed Leptons	$\ell_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$	-	$\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Right-handed Leptons	e_{iR}	-	0	+1	+1
Higgs	H	-	$-\frac{1}{2}$	$+\frac{1}{2}$	0

Table 2.3: This table lists all matter particles in the SM and the Higgs particle with their charges for all forces. This is the color, the weak isospin I_3, the half of their hypercharge and their electrical charge. The index $i = 1, 2, 3$ labels the generation of the matter particles and is written out in table 2.2

The Force Particles

The force particles are described by gauge fields. The gluon field is the gauge field of $SU_C(3)$. Because the $SU_C(3)$ has 8 generators there are 8 gluons. Their coupling constant is g_s.

For the other force particles in the SM - the $W^{\pm}$ bosons, the Z_0 boson and the photon the is situation is slightly more involved. They are obtained as a mixture of the W^b_μ ($b = 1, 2, 3$)and the B_μ field. This mixing procedure is referred to as electroweak symmetry breaking and is explained in section???.
For the moment being the coupling constants and gauge fields before and after EWSB are quoted in table 2.4

before EWSB			after EWSB		
group	coupling constant	gauge field	coupling constant	gauge field	Particle
$SU_C(3)$	g_s	G^a_μ	g_s	G^a_μ	Gluon
$SU_L(2)$	g_w	W^b_μ	$g_W = \sqrt{2} g_w,$	$W^{\pm}_\mu,$	$W^{\pm},$
			$g_Z = \sqrt{g_w^2 + g_Y^2}$	Z^0_μ	Z^0 Boson
$U_Y(1)$	g_Y	B_μ	$e = g_Y \cdot c_w$	A_μ	Photon

Table 2.4: The gauge fields and their coupling constants before and after EWSB. The Gluon field is not affected by EWSB. $a = 1, \ldots 8$ and $b = 1, 2, 3$ label the number of gauge fields. c_w is the cosine of the electroweak mixing angle defined in 2.3

The Lagrangian of the SM is built up by qualitatively different terms. Firstly there are the kinetic and minimal coupling terms of the matter fields

$$\mathcal{L}_{matter} = \sum_{i=1}^{3} \left(\bar{\ell}_{iL} i \not{D} l_{iL} + \bar{e}_{iR} i \not{D} e_{iR} + \bar{q}_{iL} i \not{D} q_{iL} + \bar{u}_{iR} i \not{D} u_{iR} \right) \tag{2.11}$$

where $i = 1, 2, 3$ labels the generations of matter. The gauge covariant derivative is given by

$$D_\mu = \partial_\mu + i g_Y \frac{\hat{Y}}{2} + i g_w \vec{\tau} \cdot \vec{W}^\mu + i g_s T^a G^\mu_a \tag{2.12}$$

where for each field the corresponding representation (fundamental or trivial) of the gauge group is to be inserted (see table 2.3). The hyper charge operator $\hat{Y}$ gives the eigenvalue of the hypercharge of the field it is applied to. These can be found in table 2.3. The kinetic terms of the gauge fields are given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} W^{a\mu\nu} W^a_{\mu\nu} - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} \tag{2.13}$$

2.3 Electroweak Symmetry Breaking

So far no mass terms like in the Dirac Lagrangian 2.2 have been introduced. The reason for that is that they are not gauge invariant. The same argument forbids terms like $-\frac{m^2}{2} A^\mu A_\mu$ for a generic gauge boson. EWSB gives masses to these particles while maintaining gauge invariance. To this end one considers a complex scalar doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.14}$$

which acquires a vacuum expectation value (VEV) $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ by the Higgs potential

$$V(\Phi^\dagger\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \tag{2.15}$$

where $\mu^2, \lambda > 0$. The Higgs sector of the SM reads

$$\mathcal{L}_{Higgs} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi^\dagger\Phi). \tag{2.16}$$

The Higgs doublet couples to the gauge fields of $SU_L(2)$ and $U_Y(1)$ in the fundamental representation. Inserting an expansion[4] around the VEV $\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + i\sigma(x)) \end{pmatrix}$ one obtains quadratic terms, i.e. mass terms, in the gauge fields in question. In order to get mass eigenstates for the gauge bosons one performs the transformation

$$\begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \qquad W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2), \tag{2.17}$$

where the electroweak mixing angle is given by $\cos\theta_w = \frac{g_w}{\sqrt{g_w^2+g_Y^2}}$ These gauge fields acquire masses:

$$m_W = \frac{g_w}{2}v \qquad m_Z = \frac{\sqrt{g_w^2+g_Y^2}}{2}v \qquad m_A = 0. \tag{2.18}$$

Apart from the massive $W_\mu^\pm$ and $Z_\mu^0(x)$ bosons one obtains the massless photon A_μ. As the photon is massless it is still associated with a gauge symmetry called $U_{em}(1)$. One therefore often writes EWSB as the braeking of the gauge group $SU_L(2) \times U_Y(1)$ to $U_Y(1)$.
Matter particles acquire mass via Yukawa coulings to the Higgs doublet. For up-type-quarks one uses that the charge conjugate of Φ: $\Phi^C = i\sigma^2\Phi^*$ transform as Φ.

$$\mathcal{L}_{Yukawa} = \sum_{i,j=1}^{3} \left(y_{ij}^e \bar{\ell}_L \Phi e_R + y_{ij}^d \bar{q}_L \Phi d_R + y_{ij}^u \bar{q}_L \Phi^C u_R \right) + h.c. \tag{2.19}$$

[4] The complex $\phi^+(x)$ and the real $\sigma(x)$ are so called massless Goldstone bosons. These degrees of freedom can be absorbed in the longitudinal polarized degrees of freedom of the arising gauge bosons $W^\pm$ and Z^0. The real $H(x)$ is the Higgs field, whose excitation is the Higgs boson.

where y^e, y^d, y^u are 3×3 matrices in generation space. The fermion mass matrices are therefore:

$$m_{ij}^e = \frac{y_{ij}^e}{\sqrt{2}}v \qquad m_{ij}^d = \frac{y_{ij}^d}{\sqrt{2}}v \qquad m_{ij}^u = \frac{y_{ij}^u}{\sqrt{2}}v. \tag{2.20}$$

The quark mass matrices are not diagonal (origin of CP - invariance). One therefore has to distinguish between interaction and mass eigenstates of the quark. The corresponding transformation matrix is the well known CKM-matrix [quote (original paper)].
The upshot of EWSB are masses for all matter particles except for the neutrinos and masses for the gauge bosons $W^\pm$ and Z^0.

2.4 Quantization

The Quantization of Spin 0 and Spin $\frac{1}{2}$ fields yield no complication in the Lagrangian formalism. To quantize Spin 1 fields it turns out that the usual gauge invariance needs to be replaced by the so called BRST invariance. This results in 2 extra terms in the Lagrangian. Firstly there is the gauge fixing term

$$\mathcal{L}_{R_\xi} = -\frac{1}{2\xi_A}(\partial^\mu A_\mu)^2 - \frac{1}{2\xi_W}|\partial^\mu W_\mu^+ - m_W\xi_W\phi^+|^2 - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu - m_Z\xi_Z\sigma)^2 \tag{2.21}$$

Here R_ξ is chosen, where the parameters ξ_i specify the gauge. The last 2 terms are modified with the Goldstone bosons from section 2.3 because of the massiveness of the $W^\pm$ an Z^0 bosons. [?]
Secondly there is a ghost Lagrangian

$$\mathcal{L}_{ghost} = \tag{2.22}$$

see [Denner Gauge Theories S622]

2.5 Lagrangian of the SM

The complete Lagrangian of the SM reads

$$\mathcal{L}_{SM} = \mathcal{L}_{matter} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{R_\xi} + \mathcal{L}_{ghost} \tag{2.23}$$

with the corresponding parts of the previous chapters

3 The Minimal Supersymmetric Standard Model

3.1 Supersymmetry as Extention of Poincaré Symmetry

The superalgebra is defined by

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} &= 0,\\
\{Q_\alpha, \overline{Q}_{\dot\alpha}\} &= 2\sigma^\mu_{\alpha\dot\alpha},\\
[P^\mu, Q_\alpha] = [P^\mu, \overline{Q}_{\dot\alpha}] &= 0,\\
[Q_\alpha, J^{\mu\nu}] &= \frac{1}{2}(\sigma^{\mu\nu})^\beta_\alpha Q_\beta
\end{aligned} \tag{3.1}$$

A representation in the form of differential operators is given by

$$\begin{aligned}
P^\mu &= i\partial^\mu\\
J^{\mu\nu} &= i(x^\mu\partial^\nu - x^\nu\partial^\mu)\\
Q_\alpha &= i(\partial_\alpha + i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_\mu\\
\overline{Q}_{\dot\alpha} &= i(-\overline{\partial}_{\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu).
\end{aligned} \tag{3.2}$$

INTRODUCE AND MOTIVATE SUSY COVARIANT DERIVATIVES $\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\overline{\theta})_\alpha\partial_\mu$ is the chiral covariant derivative and $\overline{\mathcal{D}\mathcal{D}} = \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}^{\dot\alpha}$

3.2 A Generic Supersymmetric Model in Superspace Formulation

This chapter outlines the generic ingredients and terms of a supersymmetric model. To this end it is practical to work in the language of superspace and superfields.
Superspace is a manifold obtained by enlarging Minkowski space, whose coordinates are label with x^μ, with four anticommuting numbers: θ^α and $\overline{\theta}^{\dot\alpha}$, where $\alpha,\ \dot\alpha = 1, 2$. Superfields are functions on superspace.
The for the MSSM relevant superfields[5] are the chiral superfield $\hat\Phi$, the antichiral superfield $\hat{\overline{\Phi}}$ and the vector superfield V. Chiral superfields are defined by the restriction $\overline{\mathcal{D}}_{\dot\alpha}\hat\Phi = 0$, antichiral superfields by $\mathcal{D}_\alpha\hat{\overline{\Phi}} = 0$ and vector superfields by the condition of being real $V^\dagger = V$. Their component decomposition reads[6]

$$\begin{aligned}
\hat\Phi(x,\theta,\overline{\theta}) &= A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) - i\theta\sigma^\mu\overline{\theta}\partial_\mu A(x) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial_\mu\partial^\mu A(x) - \frac{i}{\sqrt{2}}\theta\theta\overline{\theta}\overline{\sigma}^\mu\partial_\mu\psi(x)\\
\hat{\overline{\Phi}}(x,\theta,\overline{\theta}) &= A^\dagger(x) + \sqrt{2}\overline{\theta\psi}(x) + \overline{\theta\theta}F^\dagger(x) + i\theta\sigma^\mu\overline{\theta}\partial_\mu A^\dagger(x) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial_\mu\partial^\mu A^\dagger(x) - \frac{i}{\sqrt{2}}\overline{\theta\theta}\theta\sigma^\mu\partial_\mu\overline{\psi}(x)\\
\hat V(x,\theta,\overline{\theta}) &= \theta\sigma^\mu\overline{\theta}v_\mu + i\theta\theta\overline{\theta\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x),
\end{aligned} \tag{3.3}$$

[5]Superfield are throughout this thesis labeled with a hat.
[6]For tho vector superfield Wess-Zumino-gauge is applied.

where $A(x)$ and $F(x)$ are complex scalar fields, $\psi(x)$ and $\lambda(x)$ are left handed Weyl spinors and $D(x)$ being a real scalar field.
The superfields transform under a generic gauge transformation as

$$\begin{aligned} \hat{\Phi} &\to \mathrm{e}^{-2ig\Lambda}\Phi \\ \hat{\overline{\Phi}} &\to \hat{\overline{\Phi}}\mathrm{e}^{2ig\overline{\hat{\Lambda}}} \\ \mathrm{e}^{2g\hat{V}} &\to \mathrm{e}^{-2ig\hat{\overline{\Lambda}}}\mathrm{e}^{2g\hat{V}}\mathrm{e}^{2ig\hat{\Lambda}}, \end{aligned} \tag{3.4}$$

where $\hat{\Lambda} = \hat{\Lambda}^a T^a$ and $\hat{V} = \hat{V}^a T^a$. $\hat{\Lambda}^a$ is an arbitrary chiral superfield and the T^a are the generetors of the Lie algebra, which is associated to the gauge group in question. g is the gauge coupling constant of the gauge group.
One can therefore construct the important gauge invariant term $\int \mathrm{d}^4\theta\ \hat{\overline{\Phi}}\mathrm{e}^{2g\hat{V}}\hat{\Phi}$. If one introduces the gauge covariant derivative $D_\mu = \partial + igT^a v^a_\mu$ the component decomposition reads

$$\begin{aligned} \int \mathrm{d}^4\theta\hat{\overline{\Phi}}\mathrm{e}^{2g\hat{V}}\hat{\Phi} = {} & F^\dagger F + (D_\mu A)^\dagger (D^\mu A) + \overline{\psi}\overline{\sigma}^\mu i D_\mu \psi \\ & - \sqrt{2}g\left(-i(A^\dagger T^a A)\lambda^a + i\overline{\lambda}^a(AT^aA^\dagger)\right) + g(A^\dagger T^a A)D^a. \end{aligned} \tag{3.5}$$

Therefore this term gives rise to the kinetic terms of the components of the chiral and antichiral superfields A, $A^\dagger$, ψ and $\overline{\psi}$, their minimal coupling to the gauge fields v^a_μ and their superpartners λ^a and $\overline{\lambda}^a$ and terms involving the auxiliary fields F, $F^\dagger$ and D.
With the field-strength chiral superfields $\hat{W}_\alpha = -\frac{1}{4}\overline{\mathcal{D}\mathcal{D}}(\mathrm{e}^{-2gV}\mathcal{D}_\alpha \mathrm{e}^{2gV})$ one can write down a gauge invariant term yielding the kinetic terms of the gauge fields and their superpartners:

$$\int \mathrm{d}^2\theta \frac{1}{16g^2}\hat{W}^{\alpha a}\hat{W}^a_\alpha + h.c. = \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{i}{2}\overline{\lambda}^a\overline{\sigma}^\mu(D_\mu\lambda^a) + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\overline{\lambda}^a). \tag{3.6}$$

A third generic term in a supersymmetric theory arises from the superpotential $W(\hat{\Phi})$ which is a holomorphic function in the chiral superfields:

$$\int \mathrm{d}^2\theta\ W(\hat{\Phi}). \tag{3.7}$$

A renormalizable superpotential is given by $W(\hat{\Phi}) = c_i\hat{\Phi} + \frac{m_{ij}}{2}\hat{\Phi}_i\hat{\Phi}_j + \frac{g_{ijk}}{3!}\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k$. The component decomposition of the corresponding terms is

$$\begin{aligned} \int \mathrm{d}^2\theta\ \hat{\Phi}_1 &= F_1 \\ \int \mathrm{d}^2\theta\ \hat{\Phi}_1\hat{\Phi}_2 &= A_1F_2 + F_1A_2 - \psi_1\psi_2 \\ \int \mathrm{d}^2\theta\ \hat{\Phi}_1\hat{\Phi}_2\hat{\Phi}_3 &= F_1A_2A_3 + A_1F_2A_3 + A_1A_2F_3 - A_1\psi_2\psi_3 - \psi_1A_2\psi_3 - \psi_1\psi_2A_3. \end{aligned} \tag{3.8}$$

The Lagrangian for a supersymmetric theory is therefore given by

$$\begin{aligned}\mathcal{L}_{SUSY} &= \mathcal{L}_{matter} + \mathcal{L}_{gauge} + \mathcal{L}_{superpot} \\ &= \int \mathrm{d}^4\theta \hat{\bar{\Phi}} \mathrm{e}^{2g\hat{V}} \hat{\Phi} + \left(\int \mathrm{d}^2\theta \frac{1}{16g^2} \hat{W}^{\alpha a} \hat{W}^a_\alpha + h.c. \right) + \int \mathrm{d}^2\theta \; W(\hat{\Phi}) \end{aligned} \tag{3.9}$$

Observing the component decomposition 3.5,3.6, 3.8 of the 3 parts of this Lagrangian, one observes that the F and D fields have no kinetic term and are therefore auxiliary fields which can be eliminated by their equation of motion $\frac{\partial \mathcal{L}}{\partial \phi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}$ with $\phi = F, D$. Doing this one obtains

$$\begin{aligned}\mathcal{L}_D &= \frac{1}{2} D^a D^a + g A^\dagger T^a D^a A \qquad \Rightarrow \qquad D^a = -A^\dagger T^a A \\ \mathcal{L}_D &= -\frac{1}{2} \left(A^\dagger T^a A \right)^2 \end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\mathcal{L}_F &= F_i^\dagger F_i + \left(c_i F_i + m_{ij} F_i A_j + \frac{g_{ijk}}{2} F_i A_j A_k + h.c. \right) \qquad \Rightarrow \qquad F_i^\dagger = -\frac{\partial W(A)}{\partial A_i} \\ \mathcal{L}_F &= -\left| \frac{\partial W(A)}{\partial A_i} \right|^2 \end{aligned} \tag{3.11}$$

3.3 The Minimal Supersymmetric Standard Model in superspace formulation

The Lagrangian for the MSSM[7] reads

$$\begin{aligned}\mathcal{L}_{MSSM} = &\int \mathrm{d}^4\theta \left[\hat{\bar{Q}} \mathrm{e}^{2g'\hat{V}'+2g\hat{V}+2g_s\hat{V}_s} \hat{Q} + \hat{\bar{U}} \mathrm{e}^{2g'\hat{V}'+2g\hat{V}-2g_s\hat{V}_s^T} \hat{U} + \hat{\bar{D}} \mathrm{e}^{2g'\hat{V}'+2g\hat{V}-2g_s\hat{V}_s^T} \hat{D} \right. \\ &\qquad + \hat{\bar{L}} \mathrm{e}^{2g'\hat{V}'+2g\hat{V}} \hat{L} + \hat{\bar{E}} \mathrm{e}^{2g'\hat{V}'+2g\hat{V}} \hat{E} \\ &\qquad \left. + \hat{\bar{H}}_d \mathrm{e}^{2g'\hat{V}'+2g\hat{V}} \hat{H}_d + \hat{\bar{H}}_u \mathrm{e}^{2g'\hat{V}'+2g\hat{V}} \hat{H}_u \right] \\ &+ \int \mathrm{d}^2\theta \left[\frac{1}{16g'^2} \hat{W}'^\alpha \hat{W}'_\alpha + \frac{1}{16g^2} \hat{W}^{a\alpha} \hat{W}^a_\alpha + \frac{1}{16g_s^2} \hat{W}_s^{a\alpha} \hat{W}^a_{s\alpha} \right] + h.c. \\ &+ \int \mathrm{d}^2\theta \; W_{MSSM} + h.c. \\ &+ \mathcal{L}_{soft}. \end{aligned} \tag{3.12}$$

Apart from the already discussed terms in the first 4 lines of 3.12 there is a superpotential W_{MSSM}:

$$W_{MSSM} = y_d \hat{H}_d \hat{Q} \hat{D} + y_u \hat{H}_u \hat{Q} \hat{U} + y_e \hat{H}_d \hat{L} \hat{E} - \mu \hat{H}_d \hat{H}_u \tag{3.13}$$

[7] This is the Lagrangian on the classical level, i.e. there are neither gauge fixing nor ghost terms.

and terms which break supersymmetry softly, i.e. terms with coupling constants with positive mass dimension.

$$\begin{aligned}\mathcal{L}_{soft} = & -M_{\tilde{Q}}^2|\tilde{q}_L|^2 - M_{\tilde{U}}^2|\tilde{u}_R|^2 - M_{\tilde{D}}^2|\tilde{d}_R|^2 \\ & - M_{\tilde{L}}^2|\tilde{l}_L|^2 - M_{\tilde{E}}^2|\tilde{e}_R|^2 - M_{H_d}^2|H_d|^2 - M_{H_u}^2|H_u|^2 \\ & + \frac{1}{2}\left(M_1\lambda\lambda + M_2\lambda^a\lambda^a + M_3\lambda_s^a\lambda_s^a\right) + h.c. \\ & - \Big(A_d y_d H_d \tilde{q}_L \tilde{d}_R^\dagger + A_u y_u H_u \tilde{q}_L \tilde{u}_R^\dagger + A_e y_e H_d \tilde{l}_L \tilde{e}_R^\dagger - B\mu H_d H_u\Big) + h.c.\end{aligned} \tag{3.14}$$

The field content of the MSSM is summarized in 3.1

Superfield	Components	$SU_{\mathrm{C}}(3) \times SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$
$\hat{\Phi}$	$A,\ \psi$	
$\hat{V}$	$\lambda,\ v_\mu$	
$\hat{Q}$	$\tilde{q}_L = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix},\ q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(3,\ 2,\ \frac{1}{6})$
$\hat{U}$	$\tilde{u}_R^\dagger,\ u_R$	$(3^*,\ 1,\ -\frac{2}{3})$
$\hat{D}$	$\tilde{d}_R^\dagger,\ d_R$	$(3^*,\ 1,\ +\frac{1}{3})$
$\hat{L}$	$\tilde{l}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix},\ l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(1,\ 2,\ -\frac{1}{2})$
$\hat{E}$	$\tilde{e}_R^\dagger,\ e_R$	$(1,\ 1,\ 1)$
$\hat{H}_d$	$H_d,\ \tilde{H}_d$	$(1,\ 2,\ -\frac{1}{2})$
$\hat{H}_u$	$H_u,\ \tilde{H}_u$	$(1,\ 2,\ +\frac{1}{2})$
$\hat{V}'$	$\lambda',\ B_\mu$	$(1,\ 1,\ 0)$
$\hat{V}^a$	$\lambda^a,\ W_\mu^a$	$(1,\ 3,\ 0)$
$\hat{V}_s^a$	$\lambda_s^a,\ G_\mu^a$	$(8,\ 1,\ 0)$

Table 3.1: The table shows the field content of the MSSM in terms of the superfields and their component decomposition. The first two lines show the decomposition of the generic superfields (cf. 3.3).
The third column shows the representation (for $SU_{\mathrm{C}}(3)$ and $SU_{\mathrm{L}}(2)$) in which the fields transform and the charges of the fields for $U_{\mathrm{Y}}(1)$.

4 R-Symmetry

4.1 R-Symmetry Transformation

R-symmetry is a global $U(1)$ symmetry. R-symmetry should not be confused with R-parity which is a discrete Z_2 symmetry. A continuous global symmetry implies according to Noether's theorem a conserved charge. In the case of R-symmetry this charge is called R-charge and one therefore refers to R-symmetry as $U_R(1)$.
The defining property of $U_R(1)$ is that the anticommuting coordinates θ^α and $\overline{\theta}^{\dot{\alpha}}$ transform like

$$\theta \to \mathrm{e}^{i\alpha}\theta \qquad\qquad \overline{\theta} \to \mathrm{e}^{-i\alpha}\overline{\theta}, \tag{4.1}$$

where α parametrizes the transformation. This in turn implies that R-symmetry does not commute with supersymmetry, meaning that superpartners do not have the same R-charge.
The transformation of chiral and vector superfields reads

$$\begin{aligned} \hat{\Phi}(x,\theta,\overline{\theta}) &\to \mathrm{e}^{ir_{\hat{\Phi}}\alpha}\ \hat{\Phi}(x,\mathrm{e}^{i\alpha}\theta,\mathrm{e}^{-i\alpha}\overline{\theta}) \\ \hat{V}(x,\theta,\overline{\theta}) &\to \hat{V}(x,\mathrm{e}^{i\alpha}\theta,\mathrm{e}^{-i\alpha}\overline{\theta}). \end{aligned} \tag{4.2}$$

If one inserts the component decomposition 3.3 of the superfields one can read off the R-charges of the component fields.

4.2 The Minimal R-symmetric Supersymmetric Standard Model

The MSSM with additional R-symmetry is called minimal R-symmetric supersymmetric standard model (MRSSM). If one imposes R-symmetry upon the MSSM one is faced with a certain arbitrariness, i.e. the choice of the R-charges of the chiral superfields. In this thesis the R-charges are chosen in that way, that every Standard model particle has R-charge zero. Following this one obtains the R-charges of all particles which are summed up in table 4.2. The gauge, matter and H-Higgs fields are the fields of the MSSM. Below the horizontal line one finds the fields which are not present in the MSSM, i.e. the R-Higgs and adjoint chiral fields. These occur for the following reason.
Since in the MSSM the gauginos are Majorana particles their mass terms reads

$$\mathcal{L}_{Majorana\ mass} = -m\lambda\lambda + h.c. \tag{4.3}$$

which is not R-invariant because the Weyl fermion λ has R-charge $+1$. The only other way to account for a fermion mass is to write down a Dirac mass term.

$$L_{Dirac\ mass} = -m\chi\lambda + h.c. \tag{4.4}$$

In order to get a R-symmetric mass term on has to choose the R-charge of the new Weyl-spinor χ to be the opposite of λ.
This explains the necessity of enlarging the field content if one imposes R-symmetry.
Of course the new Weyl-spinor χ must have also a superpartner. One chooses this superpartner to be a scalar, i.e. the additional Weyl fermion comes from a chiral superfield. In order to maintain gauge invariance this chiral superfield has to transform in the adjoint representation, hence the name adjoint chiral in table 4.2. To fix notation the component decomposition of the 8 chiral supermultipletts assosiated to the gluons is given by

$$\hat{O}^a(x,\theta,\overline{\theta}) = \sigma^a + \sqrt{2}\theta i\chi^a + \dots \qquad\qquad a = 1,\dots,8. \tag{4.5}$$

superfield		boson		fermion	
$\hat{\Phi}$	$r_{\hat{\Phi}}$	A	$r_{\hat{\Phi}}$	ψ	$r_{\hat{\Phi}} - 1$
$\hat{V}$	0	v^μ	0	λ	$+1$

Table 4.1: This table shows the R-charges of a generic chiral and vector superfield.

Field	Superfield		Boson		Fermion	
Gauge Vector	$\hat{g}$, $\hat{W}$, $\hat{B}$	0	g, W, B	0	$\tilde{g}$, $\tilde{W}$, $\tilde{B}$	+1
Matter	$\hat{L}$, $\hat{E}$	0	$\tilde{l}$, $\tilde{e}_R$	+1	l, e_R	0
	$\hat{Q}$, $\hat{D}$, $\hat{U}$	+1	$\tilde{q}$, $\tilde{d}_R^\dagger$, $\tilde{u}_R^\dagger$	+1	q, d_R, u_R	0
H-Higgs	$\hat{H}_{d,u}$	0	$H_{d,u}$	0	$\tilde{H}_{d,u}$	-1
R-Higgs	$\hat{R}_{d,u}$	+2	$R_{d,u}$	+2	$\tilde{R}_{d,u}$	+1
Adjoint Chiral	$\hat{O}$, $\hat{T}$, $\hat{S}$	0	O, T, S	0	$\tilde{O}$, $\tilde{T}$, $\tilde{S}$	-1

Table 4.2: This table shows the R-charges of all particles in the MRSSM.

The scalar components σ^a are called scalar gluons and the Weyl spinors χ^a are called octinos. The same argument as for the adjoint chiral explains the existence of additional Higgs-superfields which are referred to as R-Higgs fields.
But instead of including more fields in the model R-symmetry also forbids terms which are allowed by supersymmetry. For the above choice of R-charges the μ-term in 3.13 and the A-terms in the last line of 3.14 are excluded. As a consequence terms which allow flavor violating processes like $\mu \to e\gamma$ are allowed in the MSSM but forbidden in the MRSSM [Kribs, Popitz, Weiner].

4.3 The R-Symmetric Supersymmetric Quantum Chromodynacis

The subject of this thesis is the phenomenology of the strongly coupling sector of the MRSSM. The R-symmetric supersymmetric quantumchromodynamics (RSQCD) is therefore considered closer. Its Lagrangian reads

$$\begin{aligned}\mathcal{L}_{RSQCD} = &\int \mathrm{d}^4\theta \, \left(\bar{\hat{Q}}_L \mathrm{e}^{2g_s\hat{V}_s}\hat{Q}_L + \bar{\hat{Q}}_R \mathrm{e}^{-2g_s\hat{V}_s^T}\hat{Q}_R + \bar{\hat{O}}\mathrm{e}^{2g_s\hat{V}_s^{fund}}\hat{O}\right) \\ &+ \left(\int \mathrm{d}^2\theta \frac{1}{16g_s^2}\hat{W}_s^{a\alpha}\hat{W}_{s\alpha}^a + h.c.\right) + \mathcal{L}_{soft} \qquad (4.6)\end{aligned}$$

where in terms of component fields the terms are given by

$$\int \mathrm{d}^4\theta\, \hat{\overline{Q}}_L \mathrm{e}^{2g_s\hat{V}_s}\hat{Q}_L = F_L^\dagger F_L + (D_\mu\tilde{q}_L)^\dagger(D^\mu\tilde{q}_L) + \overline{q}_L\overline{\sigma}^\mu iD_\mu q_L$$
$$- \sqrt{2}g_s\left(-i(\tilde{q}_L^\dagger T^a q_L)\lambda^a + i\overline{\lambda}^a(\overline{q}_L T^a\tilde{q}_L)\right) + g_s\tilde{q}_L^\dagger T^a D^a\tilde{q}_L \tag{4.7}$$

$$\int \mathrm{d}^4\theta\, \hat{\overline{Q}}_R \mathrm{e}^{-2g_s\hat{V}_s^T}\hat{Q}_R = F_R^\dagger F_R + (D_\mu\tilde{q}_R)^\dagger(D^\mu\tilde{q}_R) + \overline{q}_R\overline{\sigma}^\mu iD_\mu q_R$$
$$+ \sqrt{2}g_s\left(-i(\tilde{q}_R T^{*a} q_R)\lambda^a + i\overline{\lambda}^a(\overline{q}_R T^{*a}\tilde{q}_R^\dagger)\right) - g_s\tilde{q}_R T^{*a} D^a\tilde{q}_R^\dagger \tag{4.8}$$

$$\int \mathrm{d}^4\theta\, \hat{\overline{O}} \mathrm{e}^{2g_s\hat{V}_s^{fund}}\hat{O} = F_O^\dagger F_O + (D_\mu\sigma^a)^\dagger(D^\mu\sigma^a) + \overline{\chi}\overline{\sigma}^\mu iD_\mu\chi$$
$$- \sqrt{2}g_s\left(-i(\sigma_{b\dagger}(-if_{abc})(-i\chi^c))\lambda^a + i\overline{\lambda}^a(i\overline{\chi}_b(+if_{abc})\sigma^{c\dagger})\right)$$
$$- ig_s\sigma^{b^\dagger} f^{abc} D^a\sigma^c \tag{4.9}$$

where in the gauge covariant derivative $D_\mu = \partial_\mu + ig_s T^a G^a_\mu$ the generator T^a needs to be replaced by $-T^{*a}$ or $-if^{abc}$ if applied to a field transforming in the antifundamental or adjoint representation respectively.

The soft breaking Lagrangian accounts for the squark, gaugino and scalar gluon masses. These mass terms arise from a hidden sector spurion. For the gauginos the D-type spurion is given by $\hat{W}'_\alpha = \theta_\alpha D$ and mediates super symmetry breaking at the mediation scale M: $\int \mathrm{d}\theta^2 \frac{\hat{W}'_\alpha}{M} W_s^\alpha \hat{O}$. After integrating out the spurion one obtains

$$\mathcal{L}_{soft} = -\frac{m_{\tilde{q}}^2}{2}(|\tilde{q}_L|^2 + |\tilde{q}_R|^2)$$
$$-\frac{m_{\sigma_1}^2}{2}\sigma_1^2 - \frac{m_{\sigma_2}^2}{2}\sigma_2^2 - m_g(\lambda\chi - \sqrt{2}D^a\sigma^a + h.c.) \tag{4.10}$$

where the complex scalar gluons $\sigma = \frac{\sigma_1 + i\sigma_2}{\sqrt{2}}$ constitutes of two real scalar gluons with different masses. The equations of motion for the auxiliary fields are

$$D^a = -g_s\tilde{q}_L^\dagger T^a\tilde{q}_L + g_s\tilde{q}_R T^a\tilde{q}_R^\dagger + ig_s\sigma^{\dagger b} f^{abc}\sigma^c - \sqrt{2}m_g(\sigma^a + \sigma^{\dagger a}) \tag{4.11}$$

$$F_i = 0 \qquad \text{for} \qquad i = L, R, O \tag{4.12}$$

where D^a is still real as the purely imaginary parts to not contribute by virtue of the antisymmetry of the structure constants. After eliminating the auxiliary fields the complete

Lagrangian in 4 spinor notation[8] reads

$$
\begin{aligned}
\mathcal{L}_{RSQCD} =& |D_\mu\sigma|^2 + |D_\mu\tilde{q}_R|^2 + |D_\mu\tilde{q}_L|^2 + \overline{q} i \not{D} q + \overline{\tilde{g}}^a i\not{D} P_L \tilde{g}^a + \overline{\tilde{g}}^a i\not{D} P_R \tilde{g}^a - \frac{1}{4}(F^{\mu\nu}_a)^2 \\
& - \sqrt{2} g_s \left(\overline{\tilde{g}}^a P_R (q^C T^a \tilde{q}_L) + (\tilde{q}_L^\dagger T^a \overline{q}^C) P_L \tilde{g}^a \right) \\
& + \sqrt{2} g_s \left(\overline{\tilde{g}}^a P_R (q T^{*a} \tilde{q}_R^\dagger) + (\tilde{q}_R T^{*a} \overline{q}) P_L \tilde{g}^a \right) \\
& - \sqrt{2} g_s \left(\overline{\tilde{g}}^a P_R (\tilde{g}^b (i f^{abc}) \sigma^c) + (\sigma^{\dagger b}(-i f^{abc}) \overline{\tilde{g}}^c) P_L \tilde{g}^a \right) \\
& - \frac{m^2_{\tilde{q}}}{2} (|\tilde{q}_L|^2 + |\tilde{q}_R|^2) - \frac{m^2_{\sigma_1}}{2} \sigma_1^2 - \frac{m^2_{\sigma_2}}{2}\sigma_2^2 - m_g \overline{\tilde{g}}^a \tilde{g}^a \\
& - \frac{1}{2} \left(g_s \tilde{q}_L^\dagger T^a \tilde{q}_L - g_s \tilde{q}_R T^{*a} \tilde{q}_R^\dagger - i g_s \sigma^{\dagger b} f^{abc} \sigma^c + \sqrt{2} m_g (\sigma^a + \sigma^{\dagger a}) \right)^2
\end{aligned}
\tag{4.13}
$$

Observe that there is no 3 sgluon vertex, because of the antisymmetry of the structure constants f^{abc}.

[8] How a 4 spinor is composed of Weyl-spinors is given in the Appendix 11.3

5 Squark and Gluino Production at Tree Level

In this chapter the production of strongly interacting supersymmetric particles[9] in the MRSSM is considered. The various processes and their associated cross section is compared to their analogues in the MSSM.

5.1 Partonic Processes

For the calculation the top quark is excluded from the initial state as it is too heavy to be significantly present in hadrons. For consistency reasons also the stop is excluded from the final states. One therefore deals with $n_f - 1 = 5$ quark flavors. Using the Feynman rules in the Appendix one obtains the following sums over absolute squared Feynman amplitudes.

$$\begin{aligned}
\sum |\mathcal{M}^B|^2(q_i\bar{q}_j \to \tilde{q}\tilde{q}^\dagger) = {} & \delta_{ij}\left[8N_cC(F)g_s^4\frac{(n_f-1)}{s^2} + 4N_cC(F)\hat{g}_s^4\frac{1}{t_{\tilde{g}}^2} - 8C(F)g_s^2\hat{g}_s^2\frac{1}{t_{\tilde{g}}s}\right](tu - m_{\tilde{q}}^4) \\
& + (1-\delta_{ij})4N_cC(F)\hat{g}_s^4\frac{tu - m_{\tilde{q}}^4}{t_{\tilde{g}}^2}
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
\sum |\mathcal{M}^B|^2(GG \to \tilde{q}\tilde{q}^\dagger) = {} & 4(n_f-1)g_s^4\left[2N_c^2C(F)\left(1 - 2\frac{t_{\tilde{q}}u_{\tilde{q}}}{s^2}\right) - 2C(F)\right] \\
& \left[1 - \epsilon - 2\frac{sm_{\tilde{q}}^2}{t_{\tilde{q}}u_{\tilde{q}}}\left(1 - \frac{sm_{\tilde{q}}^2}{t_{\tilde{q}}u_{\tilde{q}}}\right)\right]
\end{aligned} \tag{5.2}$$

$$\begin{aligned}
\sum |\mathcal{M}^B|^2(q_iq_j \to \tilde{q}\tilde{q}) = {} & \delta_{ij}2\hat{g}_s^4N_cC(F)\left[\frac{1}{t_{\tilde{g}}^2} + \frac{1}{u_{\tilde{g}}^2}\right](tu - m_{\tilde{q}}^4) \\
& + (1-\delta_{ij})4\hat{g}_s^4N_cC(F)\frac{tu - m_{\tilde{q}}^4}{t_{\tilde{g}}^2}
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
\sum |\mathcal{M}^B|^2(q\bar{q} \to \tilde{g}\bar{\tilde{g}}) = {} & 8N_c^2C(F)g_s^4\left[\frac{2m_{\tilde{g}}^2s + t_{\tilde{g}}^2 + u_{\tilde{g}}^2}{s^2} - \epsilon\right] \\
& + 4N_c^2C(F)g_s^2\hat{g}_s^2\left[\frac{m_{\tilde{g}}^2s + t_{\tilde{g}}^2}{st_{\tilde{q}}} + \frac{m_{\tilde{g}}^2s + u_{\tilde{g}}^2}{su_{\tilde{q}}} + \epsilon\left(\frac{t_{\tilde{g}}}{t_{\tilde{q}}} + \frac{u_{\tilde{g}}}{u_{\tilde{q}}}\right)\right] \\
& + 2C(F)(N_c^2-1)\hat{g}_s^4\left(\frac{t_{\tilde{g}}^2}{t_{\tilde{q}}^2} + \frac{u_{\tilde{g}}^2}{u_{\tilde{q}}^2}\right)
\end{aligned} \tag{5.4}$$

$$\begin{aligned}
\sum |\mathcal{M}^B|^2(GG \to \tilde{g}\bar{\tilde{g}}) = {} & 16N_c^3C(F)g_s^4\left(1 - \frac{t_{\tilde{g}}u_{\tilde{g}}}{s^2}\right) \\
& \left[\frac{s^2}{t_{\tilde{g}}u_{\tilde{g}}}(1-\epsilon)^2 - 2(1-\epsilon) + 4\frac{m_{\tilde{g}}^2s}{t_{\tilde{g}}u_{\tilde{g}}}\left(1 - \frac{m_{\tilde{g}}^2s}{t_{\tilde{g}}u_{\tilde{g}}}\right)\right]
\end{aligned} \tag{5.5}$$

$$\begin{aligned}
\sum |\mathcal{M}^B|^2(qg \to \tilde{q}\tilde{g}) = {} & 2g_s^2\hat{g}_s^2\left[2N_c^2C(F)\left(1 - 2\frac{su_{\tilde{q}}}{t_{\tilde{g}}^2}\right) - 2C(F)\right] \\
& \left[(-1+\epsilon)\frac{t_{\tilde{g}}}{s} + \frac{2(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)t_{\tilde{g}}}{su_{\tilde{q}}}\left(1 + \frac{m_{\tilde{q}}^2}{u_{\tilde{q}}} + \frac{m_{\tilde{g}}^2}{t_{\tilde{g}}}\right)\right]
\end{aligned} \tag{5.6}$$

[9] The production of sgluons is excluded from the analysis for their mass is chosen to be too large to significantly produce them.

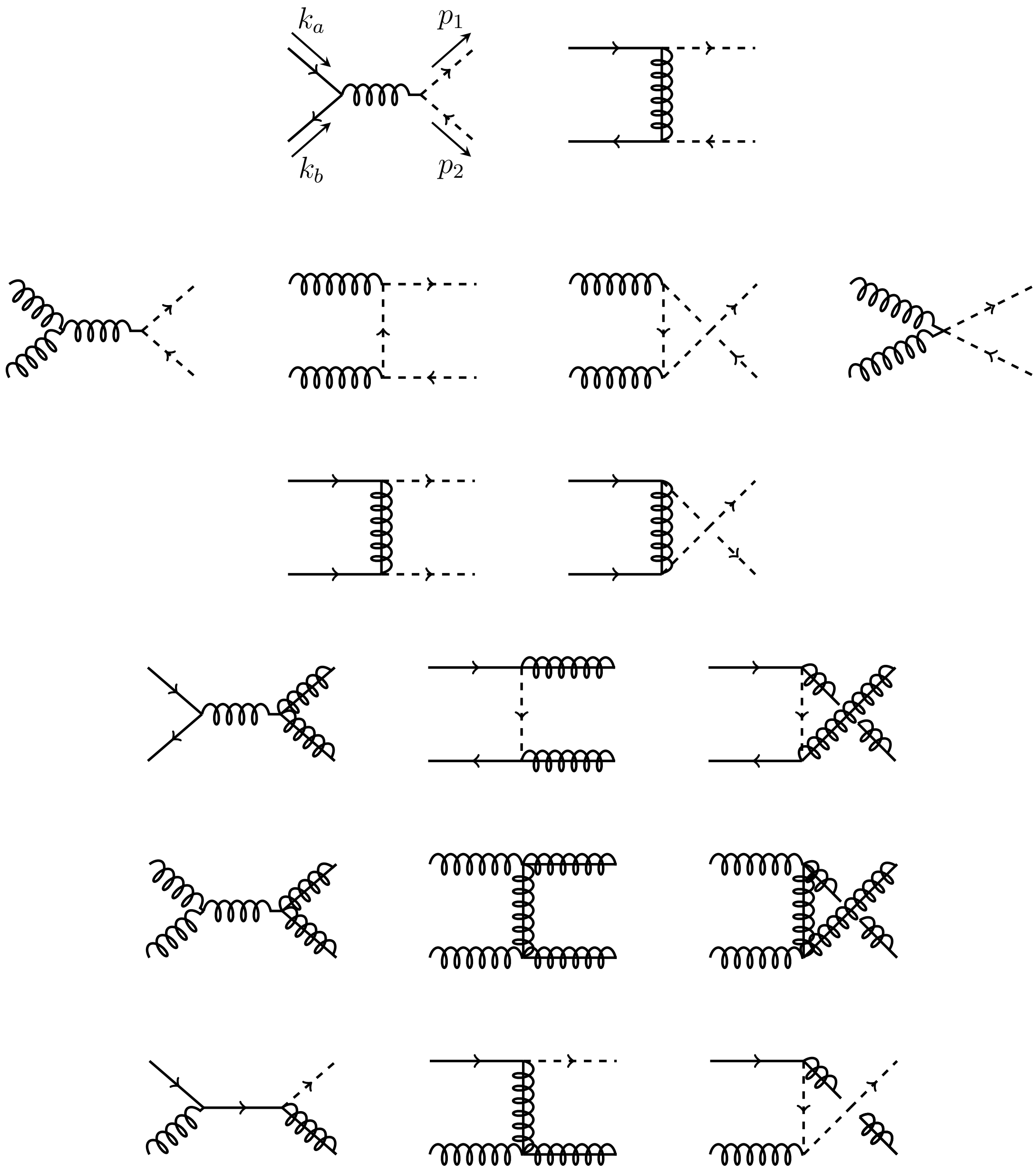


Figure 5.1: Tree level diagrams for squark and gluino production at tree level in the MRSSM. The processes $GG \to \tilde{q}\overline{\tilde{q}}$ and $qG \to \tilde{q}\tilde{g}$ are identical to those in the MSSM. The processes $q\overline{q} \to \tilde{q}\overline{\tilde{q}}$ and $qq \to \tilde{q}\tilde{q}$ involve the production of less chiralities than in the MSSM. Also in the $q\tilde{q} \to \tilde{g}\overline{\tilde{g}}$ channel only half of the t-channel squark chiralities occur in the MRSSM. (Anti-)Gluino production via initial gluons $GG \to \tilde{g}\overline{\tilde{g}}$ proceeds via the same diagrams like in the MSSM but its cross section is twice as much in the MRSSM for gluino and antigluino are distinguishable particles.

In view of the renormalization to be performed at the 1-loop level the results are given in $D = 4 - 2\epsilon$ dimensions and it has been distinguished between the gauge coupling g_s from the gluon-quark-quark vertex and its supersymmetric analogue $\hat{g}_s$ from the gluino-squark-quark vertex. The usual Mandelstam variables s, t, u and the following modifications of them are used[10]

$$
\begin{aligned}
&s = (k_a + k_b)^2 = (p_1 + p_2)^2 \\
&t = (k_a - p_1)^2 = (k_b - p_2)^2 \\
&u = (k_a - p_2)^2 = (k_a - p_2)^2 \\
&t_{\tilde{g}} = t - m_{\tilde{g}}^2 \qquad\qquad\qquad\qquad t_{\tilde{q}} = t - m_{\tilde{q}}^2 \\
&u_{\tilde{g}} = u - m_{\tilde{g}}^2 \qquad\qquad\qquad\qquad u_{\tilde{q}} = u - m_{\tilde{q}}^2
\end{aligned}
\tag{5.7}
$$

5.2 Partonic Cross Sections

Having calculated the absolute squared Feynman amplitudes one obtains the partonic cross sections (see Appendix 11.7) via

$$
\begin{aligned}
\frac{\mathrm{d}^2\sigma^B}{\mathrm{d}t\ \mathrm{d}u} =& \frac{K_{ab}}{s^2}\frac{\pi S_\epsilon}{\Gamma(1-\epsilon)}\left[\frac{tu - m_1^2 m_2^2}{\mu^2 s}\right]^{-\epsilon}\Theta(tu - m_1^2 m_2^2) \\
&\Theta(s - 4m^2)\delta(s + t + u - m_1^2 - m_2^2)\sum|\mathcal{M}^B|^2
\end{aligned}
\tag{5.8}
$$

[10]The kinematics of the process is like denoted in fig. 5.1.

The leading order cross sections are

$$\begin{aligned}
\sigma^B(q_i\overline{q}_j \to \tilde{q}\tilde{q}^\dagger) &= \delta_{ij}\frac{g_s^4}{16\pi s}(n_f-1)\left[\frac{4}{27}-\frac{16m_{\tilde{q}}^2}{27s}\right] \\
&+ \delta_{ij}\frac{g_s^2\hat{g}_s^2}{16\pi s}\left[\left(\frac{4}{27}+\frac{8m_-^2}{27s}\right)\beta_{\tilde{q}}+\left(\frac{8m_{\tilde{g}}^2}{27s}+\frac{8m_-^4}{27s^2}\right)L_1\right] \\
&+ \frac{\hat{g}_s^4}{16\pi s}\left[-\frac{8}{9}\beta_{\tilde{q}}+\left(-\frac{4}{9}-\frac{8m_-^2}{9s}\right)L_1\right] &\quad (5.9)\\
\sigma^B(GG \to \tilde{q}\tilde{q}^\dagger) &= \frac{(n_f-1)g_s^4}{16\pi s}\left[\left(\frac{5}{24}+\frac{31m_{\tilde{q}}^2}{12s}\right)\beta_{\tilde{q}}+\left(\frac{4m_{\tilde{q}}^2}{3s}+\frac{m_{\tilde{q}}^4}{3s^2}\right)\ln\frac{1-\beta_{\tilde{q}}}{1+\beta_{\tilde{q}}}\right] &\quad (5.10)\\
\sigma^B(q_iq_j \to \tilde{q}\tilde{q}) &= \frac{\hat{g}_s^4}{16\pi s}\left[-\frac{8}{9}\beta_{\tilde{q}}+\left(-\frac{4}{9}-\frac{8m_-^2}{9s}\right)L_1\right] &\quad (5.11)\\
\sigma^B(q\overline{q} \to \tilde{g}\overline{\tilde{g}}) &= \frac{g_s^4}{16\pi s}\left[\frac{16}{9}+\frac{32m_{\tilde{g}}^2}{9s}\right]\beta_{\tilde{g}} \\
&+ \frac{\hat{g}_s^2g_s^2}{16\pi s}\left[\left(-\frac{4}{3}-\frac{8m_-^2}{3s}\right)\beta_{\tilde{g}}+\left(\frac{8m_{\tilde{g}}^2}{3s}+\frac{8m_-^4}{3s^2}\right)L_2\right] \\
&+ \frac{\hat{g}_s^4}{16\pi s}\left[\left(\frac{32}{27}+\frac{32m_-^4}{27(m_-^4+m_{\tilde{q}}^2s)}\right)\beta_{\tilde{g}}-\frac{64m_-^2}{27s}L_2\right] &\quad (5.12)\\
\sigma^B(GG \to \tilde{g}\overline{\tilde{g}}) &= \frac{g_s^4}{16\pi s}\left[\left(-6-\frac{51m_{\tilde{g}}^2}{2s}\right)\beta_{\tilde{g}}+\left(-\frac{9}{2}-\frac{18m_{\tilde{g}}^2}{s}+\frac{18m_{\tilde{g}}^4}{s^2}\right)\ln\frac{1-\beta_{\tilde{g}}}{1+\beta_{\tilde{g}}}\right] &\quad (5.13)\\
\sigma^B(qG \to \tilde{q}\tilde{g}) &= \frac{g_s^2\hat{g}_s^2}{16\pi s}\left[\frac{\kappa}{s}\left(-\frac{7}{9}-\frac{32m_-^2}{9s}\right)+\left(-\frac{8m_-^2}{9s}+\frac{2m_{\tilde{q}}^2m_-^2}{s^2}+\frac{8m_-^4}{9s^2}\right)L_3\right. \\
&\left.+\left(-1-\frac{2m_-^2}{s}+\frac{2m_{\tilde{q}}m_-^2}{s^2}\right)L_4\right] &\quad (5.14)
\end{aligned}$$

where the abbreviations

$$\begin{aligned}
\beta_{\tilde{q}} &= \sqrt{1-\frac{4m_{\tilde{q}}^2}{s}} & \beta_{\tilde{g}} &= \sqrt{1-\frac{4m_{\tilde{g}}^2}{s}} \\
m_-^2 &= m_{\tilde{g}}^2-m_{\tilde{q}}^2 & \kappa &= \sqrt{(s-m_{\tilde{g}}^2-m_{\tilde{q}}^2)^2-4m_{\tilde{g}}^2m_{\tilde{q}}^2} \\
L_1 &= \ln\frac{s+2m_-^2-s\beta_{\tilde{q}}}{s+2m_-^2+s\beta_{\tilde{q}}} & L_2 &= \ln\frac{s-2m_-^2-s\beta_{\tilde{g}}}{s-2m_-^2+s\beta_{\tilde{g}}} \\
L_3 &= \ln\frac{s-m_-^2-\kappa}{s-m_-^2+\kappa} & L_4 &= \ln\frac{s+m_-^2-\kappa}{s+m_-^2+\kappa} &\quad (5.15)
\end{aligned}$$

are used [?].

The process $q_i\overline{q}_j \to \tilde{q}\tilde{q}^\dagger$

The production of a squark and an antisquark through a quark and an antiquark in the initial state origins from two diagrams. The first one with an s-channel gluon is the same in MSSM and MRSSM whereas the second one exhibits a difference due to the gluino which is no Majorana particle in the MRSSM. As the consequence contributions from this diagram are suppressed in comparison to the MSSM. One can also think of the difference as a result of the conservation of R-charge. As left handed squarks have R-charge +1 they must be produced with a right handed squark which carries R-charge -1 in order to meet the total R-charge of 0 zero in the initial state.

The process $GG \to \tilde{q}\tilde{q}^\dagger$

This process has the same cross section in the MRSSM and in the MSSM for also in the MSSM only like chirality squark and antisquark $\tilde{q}_A\tilde{q}_A^\dagger$ with $A \in L, R$ can be produced

The process $q_i q_j \to \tilde{q}\tilde{q}$

In the MRSSM only the production of unlike chirality squarks $\tilde{q}_L + \tilde{q}_R$ is allowed whilst in the MSSM also like chirality squarks can be produced. This is again a consequence of the conservation of R-charge. The upshot of this is a suppression of squark production in the MRSSM in comparison to the MSSM. To be more explicit the suppression of squark production in the MRSSM grows with the gluino mass. This can be understood as follows: As in the MRSSM a left handed squark needs to be produced with a right handed squark one can read off from the Feynman rules given in Appendix ??? that the gluino propagator $i\frac{\not{p}+m_{\tilde{g}}}{p^2-m_{\tilde{g}}^2}$ is sandwiched between the projectors P_L and P_R which leads to the cancellation of the gluino mass in the numerator. Therefore for small momenta of the gluino one gets for $\mathcal{M}$ a $\sim \frac{1}{m_{\tilde{g}}^2}$ suppression in the MRSSM while in the MSSM one finds a $\frac{1}{m_{\tilde{g}}}$ suppression.
Furthermore because of the absence of chirality like squarks in the final state in the MRSSM the cross section of flavor like and unlike squarks is the same, i.e. on the partonic level $\sigma^B(\tilde{u}_L\tilde{u}_R) = \sigma^B(\tilde{u}_L\tilde{d}_R)$. That is however not true in the MSSM.

The process $q\overline{q} \to \tilde{g}\overline{\tilde{g}}$

In contrast to the MSSM no statistical factor of $\frac{1}{2}$ is taken into account when turning from $|\mathcal{M}|^2$ to σ. This is because gluino and antigluino are distinguishable particles. Still in comparison to the MSSM cross section only the first line in 5.12 is doubled up as the other two lines origin from an t or u channel squark which occurs in only one instead of two chiralities. Furthermore an interference term from the t or u channel diagram which occurs in the MSSM is absent in the MRSSM.

The process $GG \to \tilde{g}\overline{\tilde{g}}$

As in the previous process the MRSSM cross section for $GG \to \tilde{g}\overline{\tilde{g}}$ receives no statistical factor of $\frac{1}{2}$. As there are no further differences between MSSM and MRSSM in this channel, the MRSSM cross section is simply twice as large as in the MSSM.

The process $qG \to \tilde{q}\tilde{g}$

This process is exactly the same in the MSSM and MRSSM.

5.3 Hadronic Cross Section

factorization (pictorial explanation in factorization paper chapter 1.4 and Dissertori and Seymour, Marx page 24 below)
The hadronic cross section for the production of a final state X, e.g. $X = \tilde{q}\tilde{q}$, can be obtained by convolving the partonic cross section with the parton density function of the initial partons.

$$\sigma^B_{\mathrm{Had}}(ij \to X) = \int \mathrm{d}x_1 \mathrm{d}x_2 \; f_i(x_1) f_j(x_2) \; \sigma^B_{\mathrm{Part}}(ij \to X, s = x_1 x_2 S). \tag{5.16}$$

As the production of the final state X may proceed with various initial partons one has to sum over all possible possibilities arising from the initial hadrons H_1 and H_2:

$$\sigma^B_{\mathrm{Had}}(H_1 H_2 \to X) = \sum_{i,j} \sigma^B_{\mathrm{Had}}(i, j \to X). \tag{5.17}$$

To get an intuitive idea of the factorization consider the hadrons as extended objects consisting of partons[11] which are permanently interacting which each other. Now consider two colliding hadrons in their center-of-mass frame. Due to Lorentz contraction the hadrons appear as thin discs. Furthermore the parton's mutual interactions are time-delayed. This effectively means that a hadron at the time of collision is virtually frozen.
refer to paper in Peskin on how to get pdf's
refer to Kribs, Martin
discussion of channels, maybe copy from above
refer to Vernoica Sanz: How many SUpersymmetries?

[11] The proton is considered as composed of three valence quarks: Two up-quarks and one down-quark. In addition there are gluons and seaquarks, i.e. virtual quark-antiquark pairs.

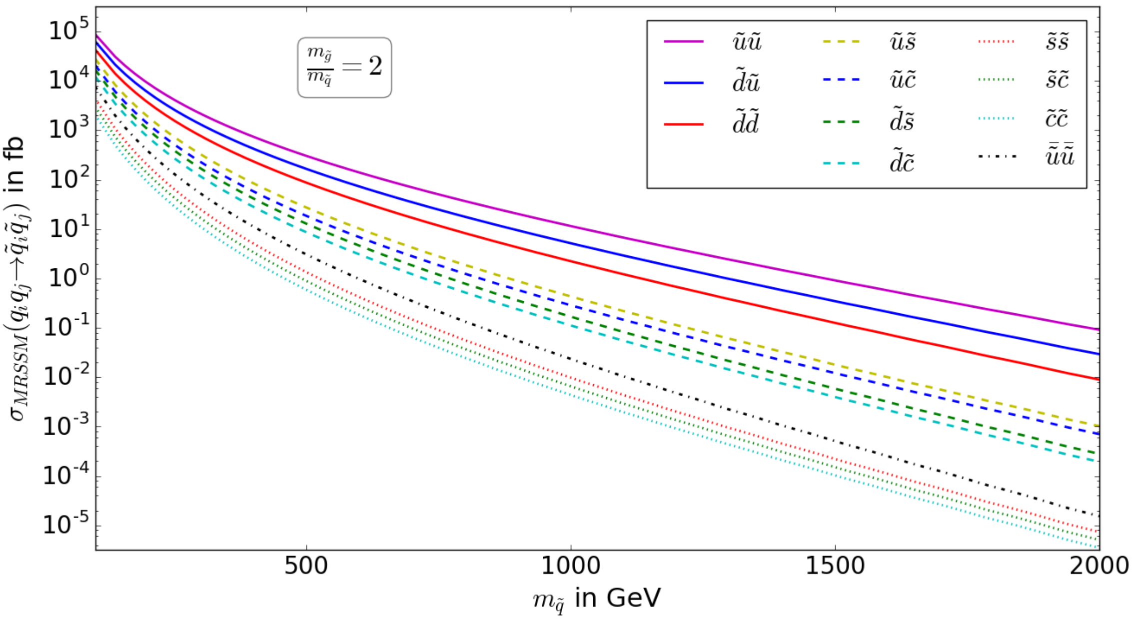


Figure 5.2: Hadronic cross section for squark production in the MRSSM at the LHC at $\sqrt{S} = 13\,\text{GeV}$. The ratio of gluino and squark mass is fixed to 1.5. The parton densities used are `MMHT2014` LO with $\alpha_s(M_Z) = 0.135$ in the 5-flavor scheme [**?**]. As renormalization and factorization scale $\mu_R = \mu_F = \frac{m_1+m_2}{2}$ has been chosen, where m_i are the final state particle masses.

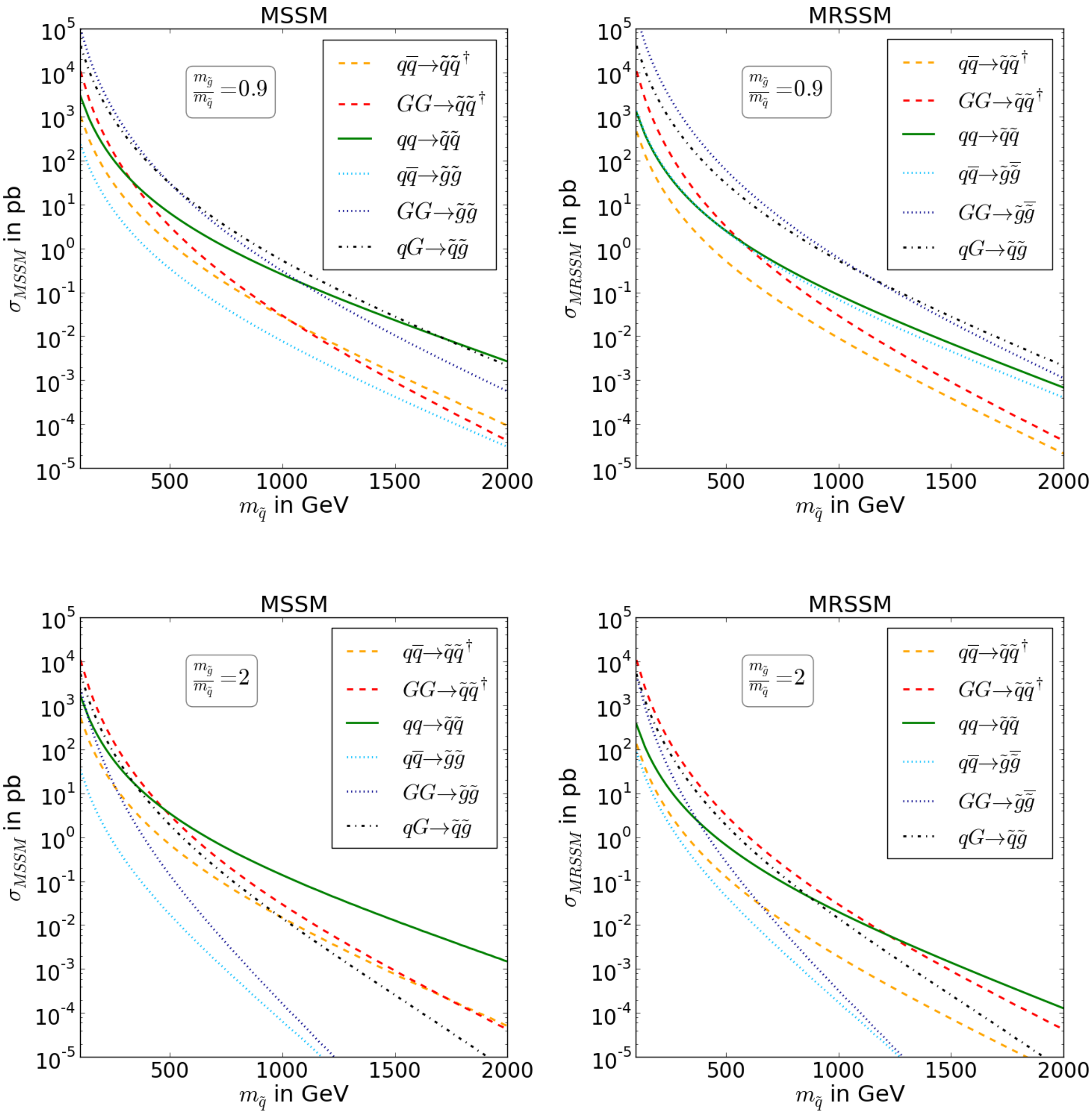


Figure 5.3: Hadronic cross section for squark and gluino production in the MSSM (left-hand side) and MRSSM (right-hand side) at the LHC at $\sqrt{S} = 13\,\text{GeV}$. The ratio of gluino and squark mass is fixed to 0.9 (first row) and 2 (second row). In the final state it has been summed over all squark flavors expect for staus. For the channels $qq \to \tilde{q}\tilde{q}$ and $qG \to \tilde{q}\tilde{G}$ also the charge conjugated process is included. The parton densities used are `MMHT2014` LO with $\alpha_s(M_Z) = 0.135$ in the 5-flavor scheme [?]. As renormalization and factorization scale $\mu_R = \mu_F = \frac{m_1+m_2}{2}$ has been chosen, where m_i are the final state particle masses.

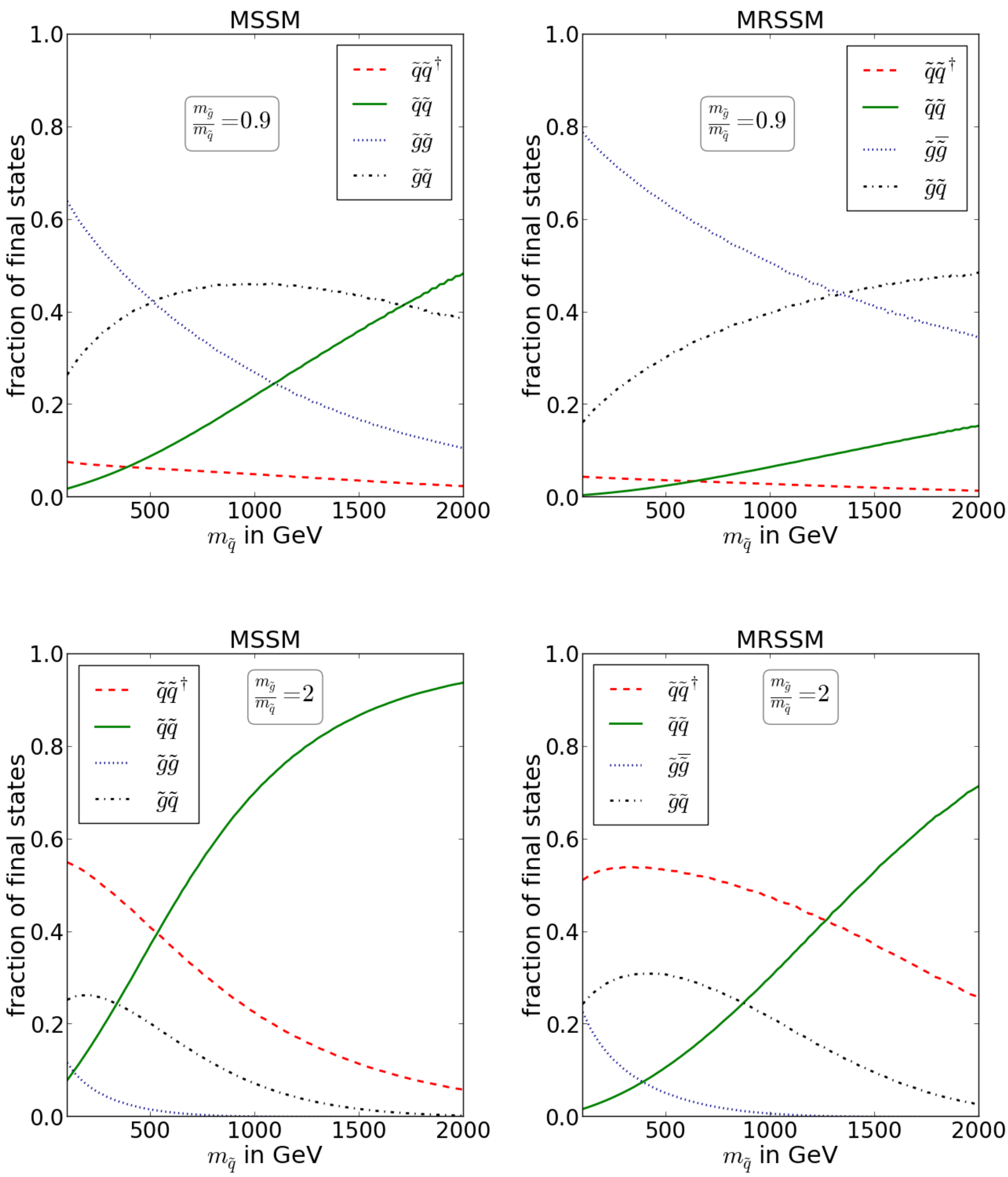


Figure 5.4: Relative contributions of the indicated final states on the total hadronic cross section in the MSSM (left-hand side) and MRSSM (right-hand side) at the LHC at $\sqrt{S} = 13\,\text{GeV}$. The ratio of gluino and squark mass is fixed to 0.9 (first row) and 2 (second row). In the final state it has been summed over all squark flavors expect for staus. For the channels $qq \to \tilde{q}\tilde{q}$ and $qG \to \tilde{q}\tilde{G}$ also the charge conjugated process is included. The parton densities and the renormalization and factorization scale are chosen as in fig. 5.3.

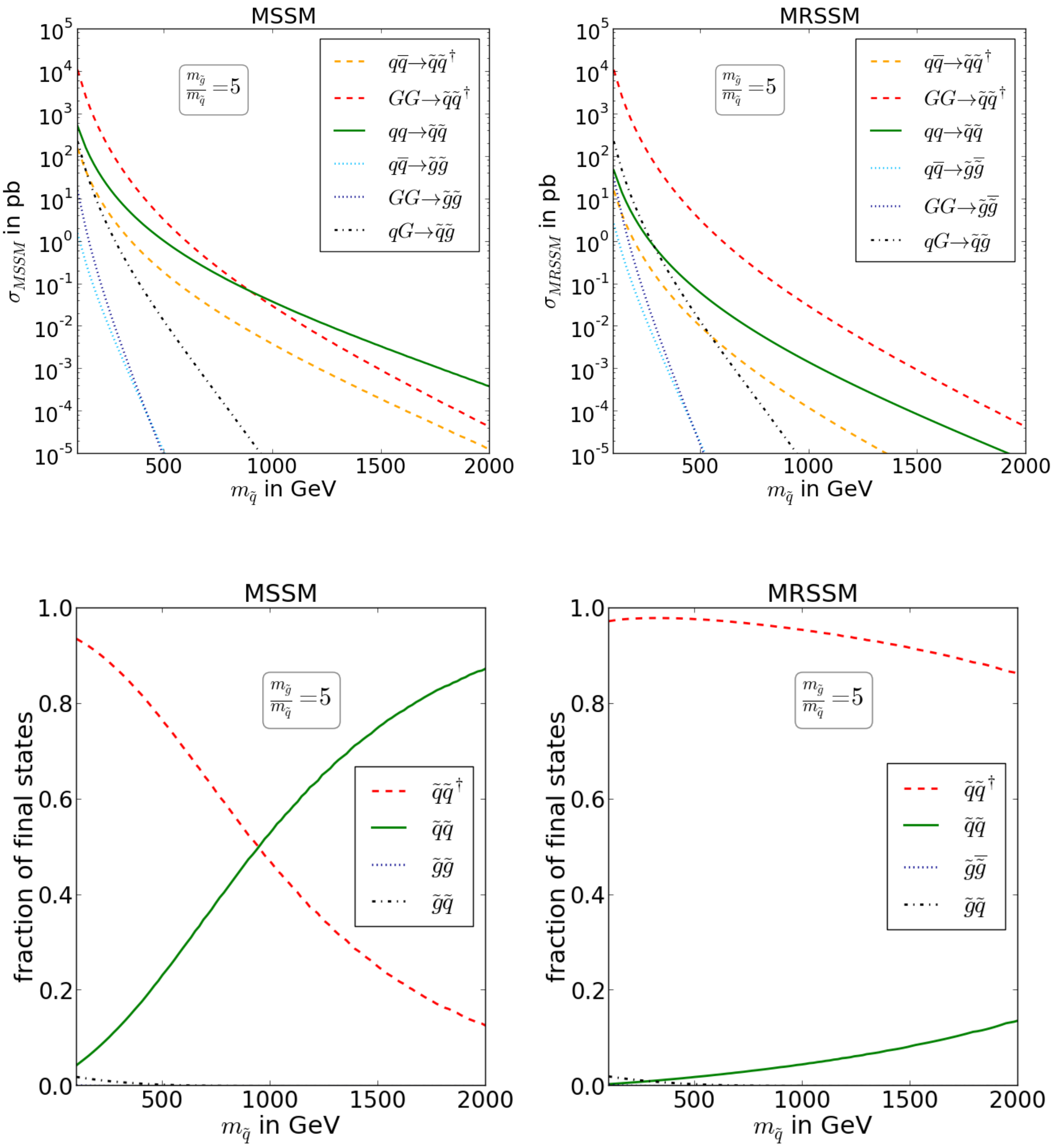
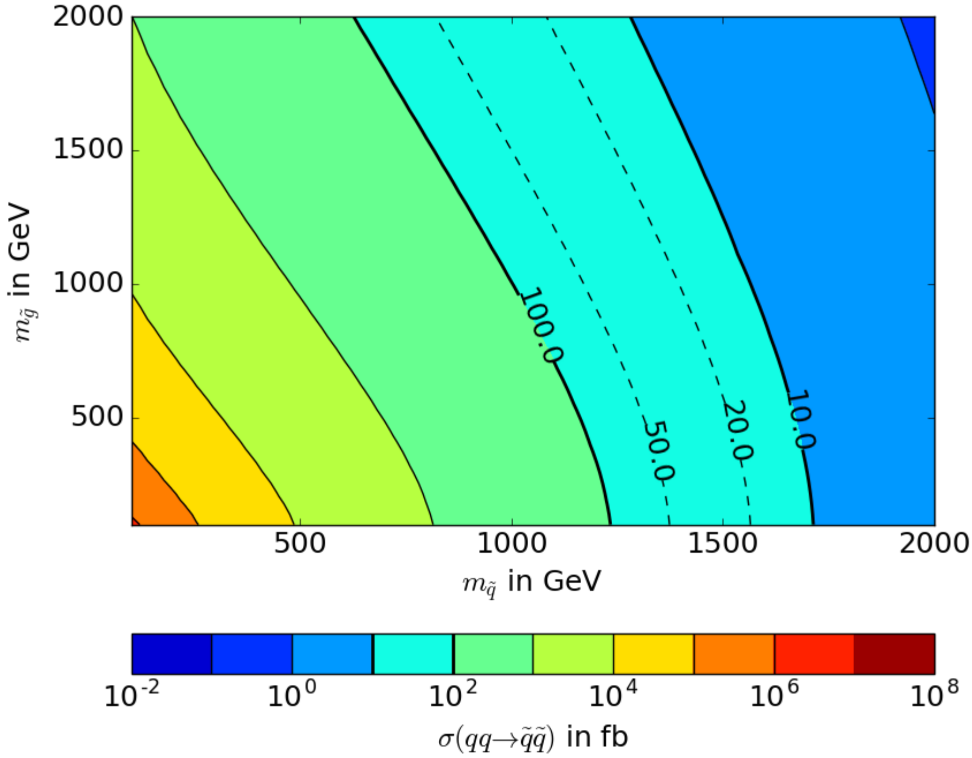


Figure 5.5: Hadronic cross section for squark and gluino production (in the first row) and relative contributions of the indicated final states on the total hadronic cross section in the MSSM (left-hand side) and MRSSM (right-hand side) at the LHC at $\sqrt{S} = 13\,\text{GeV}$. The ratio of gluino and squark mass is fixed to 5. In the final state it has been summed over all squark flavors expect for staus. For the channels $qq \to \tilde{q}\tilde{q}$ and $qG \to \tilde{q}\tilde{G}$ also the charge conjugated process is included. The parton densities and the renormalization and factorization scale are chosen as in fig. 5.3.



2000
1500
1000
500
$m_{\tilde{g}}$ in GeV
500
1000
1500
2000
$m_{\tilde{q}}$ in GeV
100.0
50.0
20.0
10.0
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
$\sigma(qq \rightarrow \tilde{q}\tilde{q})$ in fb

Figure 5.6

6 Virtual and Real Corrections

This section describes the necessary steps in the calculation of the squark production cross section at next-to-leading-order. The $\mathcal{O}(\alpha_s)$ correction to the tree level process includes the computation of one-loop diagrams such as shown in fig. ???. To yield $\mathcal{O}(\alpha_s)$ corrections in the cross section the interference term between the one-loop amplitude $\mathcal{M}^{1\mathrm{L}}$ and the Born amplitude $\mathcal{M}^{\mathrm{B}}$ needs to be considered. However the virtual amplitude

$$\mathcal{M}^{\mathrm{V}} = \mathcal{M}^{\mathrm{B}}\mathcal{M}^{1\mathrm{L}*} + \mathcal{M}^{1\mathrm{L}}\mathcal{M}^{\mathrm{B}*} = 2\Re\left(\mathcal{M}^{\mathrm{B}}\mathcal{M}^{1\mathrm{L}*}\right) \tag{6.1}$$

contains divergences. These come from integrals over undetermined momentum and energy of particles running in loops. Due to their origin these divergences are referred to as ultraviolet and infrared divergences. Ultraviolet divergences occur when the loop momentum tends to infinity which corresponds to arbitrary short distance interactions. However, these divergences are cured by first regularizing them and then introducing appropriate counterterms in the Lagrangian to cancel the extracted singularities. The second step is called renormalization and can be understood as a redefinition or rescaling of parameters and fields in the Lagrangian in the first place. This procedure will be discussed in detail in section .

Having removed the ultraviolet divergences one performs the 2-body phase space integration to arrive at the cross section. However, this is not finite as it comprises infrared divergences. These split into soft and collinear (or mass) singularities[12] which cannot be removed by means of renormalization.

Naunberg

6.1 Virtual Correction

[12] The names differ in the literature. Often infrared divergences are used as a synonym for soft divergences.

7 Renormalization of the MRSSM

In order to improve the prediction of the cross section of the previous chapter one has to take quantum corrections into account. These are associated with loops in the corresponding Feynman diagrams. Computing these loop diagrams one might encounter infinities which arise from certain momentum configurations of the unspecified loop momentum. These infinities can be classified due to their origin. Infinities which are associated with loop momenta which tend to infinity are referred to as ultraviolet(UV) divergences. Infinities arising from loop momenta approaching zero can occur in loops with massless particles and are called infrared(IR) singularities. Furthermore there are collinear singularities with occur when a massless particle splits into two massless collinear particles.
These infinities are not physical and must therefore be removed to get sensible predictions. To this end one regularizes them to extract them from the quantity in question. UV-divergences can be removed by means of renormalization, i.e. counterterms are inserted into the Lagrangian to cancel UV-divergences. Infrared and collinear divergenves are removed by adding up all possible contributions which give rise to the considered observable.

7.1 Regularization Schemes

Dimensional Regularization(DREG)

Dimensional regularization(DREG) is a very common procedure for regularizing infinities which was devised by t'Hooft and Veltman [?]. In this scheme loop momenta, gamma- and epsilon-tensors, phase space and fields are defined in D dimensions. As in every regularization scheme a parameter with mass dimension needs to be introduced. In DREG that is the μ parameter which ensures that the loop integrals still have mass dimension 4:

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \to \mu^{4-D} \int \frac{\mathrm{d}^D p}{(2\pi)^D}. \tag{7.1}$$

One often writes $D = 4 - 2\epsilon$. Then the divergences of the loop integral manifest in $\frac{1}{\epsilon}$ poles. However DREG suffers a flaw in supersymmetry. As the degrees of freedom for a massless gauge boson are $D-2$ but the degrees of freedom for its superpartner are 2 there is a mismatch if $D \neq 4$. As a consequence there are 2ϵ degrees of freedom associated with the gluon[13] which do not have a supersymmetric partner. Therefore DREG violates supersymmetry.

[13]These degrees of freedom are identified with scalars and are therefore referred to as ϵ scalars

Dimensional Reduction (DRED)

Dimensional reduction (DRED) was introduced to rectify the imperfections of DREG, i.e. it preserves supersymmetry[14]. DRED promotes only loop momenta to D dimensions. All other quantities which are D dimensional in DREG stay in 4 dimensions.
maybe refer to Collins: Renormalization

7.2 Regularization Scheme Dependences

It is useful to introduce the effective action Γ to discuss the subject of this and ensuing subchapters. A formal introduction of Γ can be found in [?]. In short Γ can be viewed as a modification of the classical action $\Gamma_{cl} = \int \mathcal{L}_{cl}$ by quantum effects:

$$\Gamma = \Gamma_{cl} + \mathcal{O}(\hbar) \tag{7.2}$$

This means that in addition to the vertices in the classical Lagrangian new vertices arise due to loop effects. As already suggested loop corrections might a priori not be finite and then need to be made finite by the addition of counterterms. For $\mathcal{O}(\hbar)$ corrections one writes

$$\Gamma^{(\leq 1)} \to \Gamma^{(\leq 1)} + \Gamma^{(1),ct} \tag{7.3}$$

These counterterms depend on the regularization (and renormalization) scheme. If one chooses to work with DREG supersymmetry will not be preserved at 1-loop order, i.e. $\Gamma^{(\leq 1)}_{DREG}$ is not supersymmetric. To maintain supersymmetry invariance of the renormalized effective action the counterterms will not only consist of supersymmetric counterterms $\Gamma^{(1),ct,sym}_{DREG}$ but also of counterterms restoring supersymmetry $\Gamma^{(1),ct,restore}_{DREG}$.

$$\Gamma^{(1),ct}_{DREG} = \Gamma^{(1),ct,sym}_{DREG} + \Gamma^{(1),ct,restore}_{DREG} \tag{7.4}$$

Fortunately a supersymmetry conserving regularization scheme (at 1-loop level) is given by DRED [?]. One way to acquire supersymmetry restoring counterterms is therefore given by

$$\Gamma^{(\leq 1)}_{DRED} + \Gamma^{(1),ct}_{DRED} \stackrel{!}{=} \Gamma^{(\leq 1)}_{DREG} + \Gamma^{(1),ct}_{DREG}. \tag{7.5}$$

Setting also the finite terms in $\Gamma^{(1),ct,sym}$ equal in DRED and DREG the choice of the supersymmetry restoring counterterms is fixed by:

$$\Gamma^{(1),\mathrm{ct},\mathrm{restore}}_{\mathrm{DREG}} = \Gamma^{(\leq 1)}_{\mathrm{DREG}} - \Gamma^{(\leq 1)}_{\mathrm{DREG}}. \tag{7.6}$$

[14] It is not clear if DRED preserves supersymmetry at all orders in perturbation theory but it does preserve supersymmetry at the 1-loop level.

This way supersymmetry is preserved by the renormalization constants.
In the case of the MRSSM it will turn out that the only supersymmetry violation comes from correction associated with the gluon as already alluded to in 7.1. However supersymmetry restoring is always already included in δZ^{DREG}. This is $\delta Z^{\text{DREG}} = \delta Z^{\text{DREG,sym}} + \delta Z^{\text{trans}}$ where $\delta Z^{\text{trans}} = \delta Z^{\text{DREG}} - \delta Z^{\text{DRED}}$ is the supersymmetry restoring renormalization constant. The only point where particularly care is required is the coupling: The gauge coupling g_s and the Yukawa coupling $\hat{g}_s$ receive different supersymmetry restoring counterterms. Therefore one has to make a difference between these couplings at 1-loop level. In order to match g_s to the experimentally measured coupling it is renormalized in $\overline{\text{MS}}$. The Yukawa coupling $\hat{g}_s$ therefore needs to be added with the difference of the supersymmetry restoring counterterms at 1-loop in order to be renormalized the same way.

7.3 On-Shell Renormalization

A part of the computation of NLO processes is the calculation of renormalization constants. The field and mass renormalization constants have been calculated in DREG in the on-shell scheme. This has the advantaged that when turning to the cross section no manipulation of the Green function to the S-matrix element has to be done.

7.3.1 The Quark Self-Energy

The quark self-energy splits into contribution from the SM as well as a supersymmetric analogue which is already present in the MSSM.

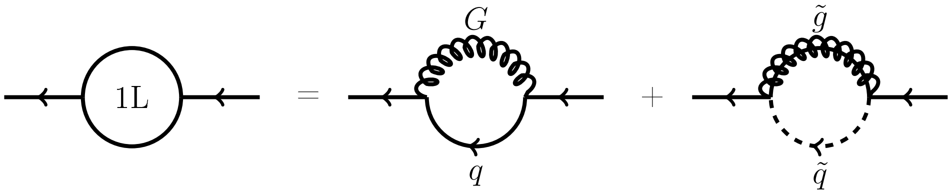


Figure 7.1: diagrammatic contributions to the self-energy of the quark at 1-loop level

The 1-PI diagrams evaluate to

$$i\Gamma^{1L}_{q_i\overline{q}_j} = i\frac{g_s^2}{16\pi^2}\delta_{ij}C(F)\left[2\left(B_0(p^2,0,0)+B_1(p^2,0,0)-\frac{1}{2}\right)\not p - 2B_1(p^2,m_{\tilde{g}}^2,m_{\tilde{q}}^2)\not p\right]. \tag{7.7}$$

With the counterterm Feynman rule

$$i\Gamma^{1L,ct}_{q_i\overline{q}_j} \mathrel{\hat{=}} i \longleftarrow\!\!\!\times\!\!\!\longleftarrow j \mathrel{\hat{=}} i\delta_{ij}\delta Z_q \not p$$

and the on-shell renormalization condition

$$\frac{\partial}{\partial \not{p}}\left[\Re(\Gamma^{1\mathrm{L}}_{q_i\overline{q}_j})+\Gamma^{1\mathrm{L,ct}}_{q_i\overline{q}_j}\right]_{p^2=0}=0 \tag{7.8}$$

where $\Re(\ldots)$ denotes the real part of ... one finds

$$\delta Z_q = 2C(F)\frac{g_s^2}{16\pi^2}\Re\left[B_1(0,m_{\tilde{g}}^2,m_{\tilde{q}}^2)+\frac{1}{2}???\right]. \tag{7.9}$$

Doing the same calculation in DRED one finds that the second term in the squared brackets is absent. Therefore the transition counterterm between DREG and DRED is given by

$$\delta Z_q^{\mathrm{trans}} = \delta Z_q^{\mathrm{DREG}} - \delta Z_q^{\mathrm{DRED}} = C(F)\frac{g_s^2}{16\pi^2}. \tag{7.10}$$

7.3.2 The Squark Self-Energy

The contributions to the self-energy of the left- and right-handed squark are the same. Therefore to avoid unnecessary labeling $\Gamma_{\tilde{q}\tilde{q}^\dagger}$ stands in the following for $\Gamma_{\tilde{q}_L\tilde{q}_L^\dagger}=\Gamma_{\tilde{q}_R\tilde{q}_R^\dagger}$.

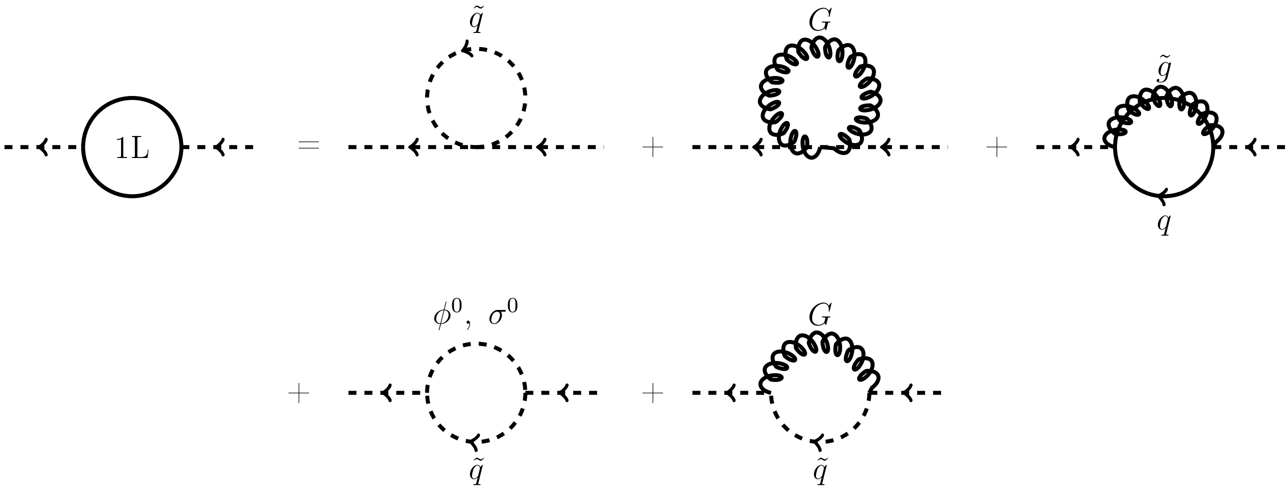
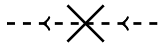


Figure 7.2: diagrammatic contributions to the self-energy of the squark at 1-loop level

$$\begin{aligned} i\Gamma^{1L}_{\tilde{q}_i\tilde{q}_j^\dagger} = i\frac{g_s^2}{16\pi^2}\delta_{ij}C(F)\,\big[&A_0(m_{\tilde{q}}^2)+0-\big(4A_0(m_{\tilde{g}}^2)+4B_1(p^2,0,m_{\tilde{g}}^2)p^2\big)\\ &+4m_{\tilde{g}}^2B_0(p^2,m_{\phi^0}^2,m_{\tilde{q}}^2)-\big(2B_1(p^2,0,m_{\tilde{q}}^2)p^2+B_0(p^2,0,m_{\tilde{q}}^2)(m_{\tilde{q}}^2+3p^2)\big)\big]. \end{aligned} \tag{7.11}$$

Suppressing δ_{AB} with $A,B=L,R$ which is present in the tree level propagator the counterterm Feynman rule is given by

$i\Gamma^{1L,ct}_{q_i\bar{q}_j} \hat{=}\ i$  $j \hat{=}\ i\delta_{ij}\left[\delta Z_{\tilde{q}}(p^2 - m^2_{\tilde{q}}) - \delta m^2_{\tilde{q}}\right].$

The on-shell renormalization conditions read

$$\frac{\partial}{\partial p^2}\left[\Re(\Gamma^{1\mathrm{L}}_{\tilde{q}_i\tilde{q}^\dagger_j}) + \Gamma^{1\mathrm{L,ct}}_{\tilde{q}_i\tilde{q}^\dagger_j}\right]_{p^2=m^2_{\tilde{q}}} = 0 \qquad \left[\Re(\Gamma^{1\mathrm{L}}_{\tilde{q}_i\tilde{q}^\dagger_j}) + \Gamma^{1\mathrm{L,ct}}_{\tilde{q}_i\tilde{q}^\dagger_j}\right]_{p^2=m^2_{\tilde{q}}} = 0. \tag{7.12}$$

This results in the following renormalization constants

$$\begin{aligned}\delta Z_{\tilde{q}} = \frac{g_s^2}{16\pi^2}C(F)\Re\Big[&4B_1(p^2,0,m^2_{\tilde{g}}) + 2B_1(B_1(p^2,0,m^2_{\tilde{q}})) + 3B_0(p^2,0,m^2_{\tilde{q}}) \\ &+ 4m^2_{\tilde{q}}\frac{\partial}{\partial p^2}B_1(p^2,0,m^2_{\tilde{g}}) - 4m^2_{\tilde{g}}\frac{\partial}{\partial p^2}B_0(p^2,m^2_{\phi^0},m^2_{\tilde{q}}) + 2m^2_{\tilde{q}}\frac{\partial}{\partial p^2}B_1(p^2,0,m^2_{\tilde{q}}) \\ &+4m^2_{\tilde{q}}\frac{\partial}{\partial p^2}B_0(p^2,0,m^2_{\tilde{q}})\Big]_{p^2=m^2_{\tilde{q}}}\end{aligned} \tag{7.13}$$

$$\begin{aligned}\delta m^2_{\tilde{q}} = \frac{g_s^2}{16\pi^2}C(F)\Big[&A_0(m^2_{\tilde{q}}) - (4A_0(m^2_{\tilde{g}}) + 4B_1(m^2_{\tilde{q}},0,m^2_{\tilde{g}})m^2_{\tilde{q}}) + 4m^2_{\tilde{g}}B_0(m^2_{\tilde{q}},m^2_{\phi^0},m^2_{\tilde{q}}) \\ &-(2B_1(m^2_{\tilde{q}},0,m^2_{\tilde{q}})m^2_{\tilde{q}} + 4B_0(m^2_{\tilde{q}},0,m^2_{\tilde{q}})m^2_{\tilde{q}})\Big]\end{aligned} \tag{7.14}$$

The squark self-energy exhibits no regularization dependence. The transition counterterms are therefore

$$\delta Z^{\mathrm{trans}}_{\tilde{q}} = \delta m^{2\ \mathrm{trans}}_{\tilde{q}} = 0. \tag{7.15}$$

7.3.3 The Glunio Self-Energy

The 4-spinor of the gluino comprises two Weyl spinors which describe very different particles.

$$\tilde{g}^a = \begin{pmatrix}\lambda^a \\ \overline{\chi}^a\end{pmatrix} \tag{7.16}$$

The left-handed part λ^a is associated with the superpartner of the gluon and therefore the "actual" gluino whereas the right-handed part $\overline{\chi}^a$ was introduced to assign a Dirac-mass to the gluino and is referred to as the octino.

From the Lagrangian 4.13 one can see that the couplings of the two particles are quite distinct. This is reflected by different field renormalization constants of the left- and right-handed part of the gluino. As for the quarks the fermion (and momentum) flow is from the right to the

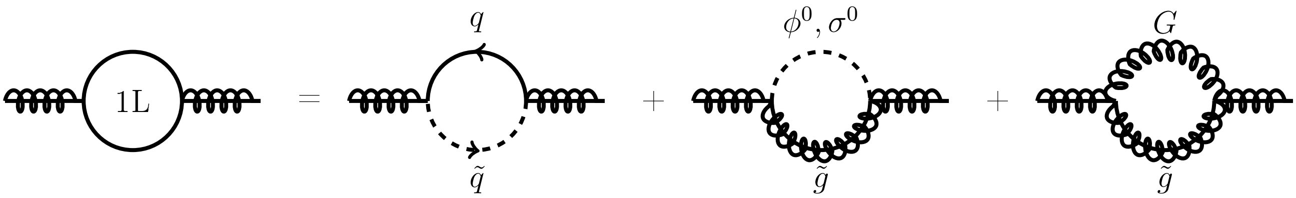


Figure 7.3: diagrammatic contributions to the self-energy of the squark at 1-loop level

left.

$$
\begin{aligned}
i\Gamma^{1L}_{\tilde{g}^a\bar{\tilde{g}}^b} = i\frac{g_s^2}{16\pi^2}\delta_{ab}\big[&-4T(F)\left((n_f-1)B_1(p^2,0,m_{\tilde{q}}^2)+B_1(p^2,m_t^2,m_{\tilde{q}}^2)\right)P_L\not{p} \\
&+C(A)\left((B_0(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)-B_0(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2))m_{\tilde{g}}-(B_1(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)+B_1(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2))\not{p}\right) \\
&+C(A)\left((2-4B_0(p^2,0,m_{\tilde{g}}^2))m_{\tilde{g}}-(1-2(B_0(p^2,0,m_{\tilde{g}}^2)+B_1(p^2,0,m_{\tilde{g}}^2)))\not{p}\right)\big]
\end{aligned}
\tag{7.17}
$$

Where $n_f = 6$ is the number of quark flavors.
The counterterm Feynman rule reads
$i\Gamma^{1L,ct}_{\tilde{g}^a\bar{\tilde{g}}^b} \mathrel{\hat{=}} a \quad b \mathrel{\hat{=}} i\delta_{ab}\left[(\delta Z^L_{\tilde{g}}P_L+\delta Z^R_{\tilde{g}}P_R)\not{p}-\left(\frac{\delta Z^L_{\tilde{g}}+\delta Z^R_{\tilde{g}}}{2}m_{\tilde{g}}+\delta m_{\tilde{g}}\right)\right].$

The on-shell renormalization conditions for the fields are

$$
\frac{\partial}{\partial(P_L\not{p})}\left[\Re(\Gamma^{\mathrm{1L}}_{\tilde{g}^a\bar{\tilde{g}}^b})+\Gamma^{\mathrm{1L,ct}}_{\tilde{g}^a\bar{\tilde{g}}^b}\right]_{\not{p}=m_{\tilde{g}}}=0 \qquad \frac{\partial}{\partial(P_R\not{p})}\left[\Re(\Gamma^{\mathrm{1L}}_{\tilde{g}^a\bar{\tilde{g}}^b})+\Gamma^{\mathrm{1L,ct}}_{\tilde{g}^a\bar{\tilde{g}}^b}\right]_{\not{p}=m_{\tilde{g}}}=0
\tag{7.18}
$$

where the derivative of $\Sigma = \Sigma^{VL}P_L\not{p}+\Sigma^{VR}P_R\not{p}+\Sigma^{SL}P_L+\Sigma^{SR}P_R$ with respect to $P_A\not{p}$ $(A = L, R)$ is defined by

$$
\frac{\partial}{\partial(P_A\not{p})}\Sigma\bigg|_{\not{p}=m} = \Sigma^{VA}+\frac{\partial}{\partial p^2}\left(m^2\Sigma^{VL}+m^2\Sigma^{VR}+m\Sigma^{SL}+m\Sigma^{SR}\right).
\tag{7.19}
$$

This leads to the following renormalization constants

$$
\begin{aligned}
\delta Z^L_{\tilde{g}} = \frac{g_s^2}{16\pi^2}\Re\big[&4T(F)\left((n_f-1)B_1(m_{\tilde{g}}^2,0,m_{\tilde{q}}^2)+B_1(m_{\tilde{g}}^2,m_t^2,m_{\tilde{q}}^2)\right) \\
&+C(A)(B_1(m_{\tilde{g}}^2,m_{\tilde{g}}^2,m_{\phi^0}^2)+B_1(m_{\tilde{g}}^2,m_{\tilde{g}}^2,m_{\sigma^0}^2)) \\
&+C(A)(1-2(B_0(m_{\tilde{g}}^2,0,m_{\tilde{g}}^2)+B_1(m_{\tilde{g}}^2,0,m_{\tilde{g}}^2))) \\
&+4T(F)m_{\tilde{g}}^2\frac{\partial}{\partial p^2}\left(((n_f-1))B_1(p^2,0,m_{\tilde{q}}^2)+B_1(p^2,m_t^2,m_{\tilde{q}}^2)\right) \\
&-2C(A)m_{\tilde{g}}\frac{\partial}{\partial p^2}\left(B_0(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)-B_0(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2)-B_1(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)-B_1(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2)\right) \\
&-4C(A)m_{\tilde{g}}^2\frac{\partial}{\partial p^2}\left(-B_0(p^2,0,m_{\tilde{g}}^2)+B_0(p^2,0,m_{\tilde{g}}^2)\right)\big]_{p^2=m_{\tilde{g}}^2}
\end{aligned}
\tag{7.20}
$$

and

$$\begin{aligned}
\delta Z^R_{\tilde g} = &\frac{g_s^2}{16\pi^2}\Re\big[C(A)(B_1(m^2_{\tilde g}, m^2_{\tilde g}, m^2_{\phi^0}) + B_1(m^2_{\tilde g}, m^2_{\tilde g}, m^2_{\sigma^0}))\\
&+ C(A)(1 - 2(B_0(m^2_{\tilde g}, 0, m^2_{\tilde g}) + B_1(m^2_{\tilde g}, 0, m^2_{\tilde g})))\\
&+ 4T(F)m^2_{\tilde g}\frac{\partial}{\partial p^2}\big(((n_f - 1))B_1(p^2, 0, m^2_{\tilde q}) + B_1(p^2, m^2_t, m^2_{\tilde q})\big)\\
&- 2C(A)m_{\tilde g}\frac{\partial}{\partial p^2}\big(B_0(p^2, m^2_{\tilde g}, m^2_{\phi^0}) - B_0(p^2, m^2_{\tilde g}, m^2_{\sigma^0}) - B_1(p^2, m^2_{\tilde g}, m^2_{\phi^0}) - B_1(p^2, m^2_{\tilde g}, m^2_{\sigma^0})\big)\\
&- 4C(A)m^2_{\tilde g}\frac{\partial}{\partial p^2}\big(-B_0(p^2, 0, m^2_{\tilde g}) + B_0(p^2, 0, m^2_{\tilde g})\big)\big]_{p^2=m^2_{\tilde g}}\,. \qquad (7.21)
\end{aligned}$$

As for the quark there are constant terms amid the Passarino-Veltman integrals. These arise only in DREG and not in DRED. The transition counterterms are

$$\delta Z^{A\ \mathrm{trans}}_{\tilde g} = \delta Z^{A\ \mathrm{DREG}}_{\tilde g} - \delta Z^{A\ \mathrm{DRED}}_{\tilde g} = C(A)\frac{g_s^2}{16\pi^2} \qquad (7.22)$$

for $A \in \{L, R\}$. The gluino mass counterterm is ascertained by the condition

$$\Big[\Re(\Gamma^{1\mathrm{L}}_{\tilde g^a\bar{\tilde g}^b}) + \Gamma^{1\mathrm{L,ct}}_{\tilde g^a\bar{\tilde g}^b}\Big]_{\not p=m_{\tilde g}} = 0 \qquad (7.23)$$

which is equivalent to

$$\delta m_{\tilde g} = \Re\left(m_{\tilde g}\frac{\Sigma^{VL} + \Sigma^{VR}}{2} + \frac{\Sigma^{SL} + \Sigma^{SR}}{2}\right) \qquad (7.24)$$

and yields

$$\begin{aligned}
\delta m_{\tilde g} = &\frac{g_s^2}{16\pi^2}m^2_{\tilde g}\ \Re\big[-2T(F)\big((n_f - 1)B_1(m^2_{\tilde g}, 0, m^2_{\tilde q}) + B_1(m^2_{\tilde g}, m^2_t, m^2_{\tilde q})\big)\\
&+ C(A)\ \big(B_0(m^2_{\tilde g}, m^2_{\tilde g}, m^2_{\phi^0}) - B_0(m^2_{\tilde g}, m^2_{\tilde g}, m^2_{\sigma^0}) - B_1(m^2_{\tilde g}, m^2_{\tilde g}, m^2_{\phi^0}) - B_1(m^2_{\tilde g}, m^2_{\tilde g}, m^2_{\sigma^0})\big)\\
&+ C(A)\ \big(1 - 2B_0(m^2_{\tilde g}, 0, m^2_{\tilde g}) + 2B_1(m^2_{\tilde g}, 0, m^2_{\tilde g})\big)\big]\,. \qquad (7.25)
\end{aligned}$$

Again there is a transition counterterm

$$\delta m^{\mathrm{trans}}_{\tilde g} = \delta m^{\mathrm{DREG}}_{\tilde g} - \delta m^{\mathrm{DRED}}_{\tilde g} = C(A)\frac{g_s^2}{16\pi^2}m_{\tilde g}. \qquad (7.26)$$

7.4 Renormalization of the Gauge Coupling

The gauge coupling g_s is renormalized in the $\overline{\mathrm{MS}}$-scheme with the modification that additional logarithms are subtracted, i.e. light particles are treated in the $\overline{\mathrm{MS}}$-scheme and heavy particles in the zero-momentum subtraction scheme. This is to decouple heavy particles from the running of $\alpha_s = \frac{g_s^2}{4\pi}$. This renormalization procedure allows to adopt the experimental values

of α_s from the PDF's. The running due to effects of heavy particles is then encoded in the logarithms of δg_s.
Extracting δg_s from the quark-quark-gluon vertex requires not only the computation of $i\Gamma^{1L}_{q_i\bar{q}_j G^\mu_a}$ but also the (re)evaluation of auxiliary field renormalization constants δZ_q^{aux} and δZ_G^{aux} in the above mentioned scheme. These will not be the same as the on-shell scheme.

7.4.1 The Quark Self-Energy Revisited

The quark self-energy has two contribution which are shown in figure 7.1. The first one corresponds to light particles the second one to heavy particles: $i\Gamma^{1L}_{q_i\bar{q}_j} = i\Gamma^{1L,\text{light}}_{q_i\bar{q}_j} + i\Gamma^{1L,\text{heavy}}_{q_i\bar{q}_j}$. For light particles only the UV-divergent[15] part is kept. The self energy corresponding to the heavy particles is taken at zero momentum $p^2 = 0$.

$$i\Gamma^{1L,\text{light}}_{q_i\bar{q}_j}\Big|_{\text{UV-div}} = iC(F)\frac{g_s^2}{16\pi^2}\Delta_\epsilon \not{p}\delta_{ij} \tag{7.27}$$

$$i\Gamma^{1L,\text{heavy}}_{q_i\bar{q}_j}(p^2=0) = -iC(F)\frac{g_s^2}{16\pi^2}2B_1(0,m^2_{\tilde{g}},m^2_{\tilde{q}})\not{p}\delta_{ij} \tag{7.28}$$

The renormalization constant for the evaluation of δg_s is determined by the condition

$$\frac{\partial}{\partial\not{p}}\left[\Gamma^{1L,\text{light}}_{q_i\bar{q}_j}\Big|_{\text{UV-div}} + \Gamma^{1L,\text{heavy}}_{q_i\bar{q}_j}(p^2=0) + \Gamma^{1L,\text{ct}}_{q_i\bar{q}_j}\right] = 0 \tag{7.29}$$

and computes to

$$\delta Z_q^{\text{aux}} = -\frac{g_s^2}{16\pi^2}C(F)\left[\Delta_\epsilon + B_1(0,m^2_{\tilde{g}},m^2_{\tilde{q}})\right]. \tag{7.30}$$

7.4.2 The Gluon Self-Energy

As for the quark self-energy there are again contributions to the self-energy originating from light and heavy particles. Again these are differently dealt with.

$$\begin{aligned}
i\Gamma^{1L,\text{light}}_{G^a_\mu G^b_\nu}\Big|_{\text{UV-div}} &= i\frac{g_s^2}{16\pi^2}\Delta_\epsilon\left[0 - \frac{4(n_f-1)}{3}T(F)(p^2g^{\mu\nu}-p^\mu p^\nu) + \frac{C(A)}{12}(p^2g^{\mu\nu}+2p^\mu p^\nu)\right.\\
&\quad\left. + \frac{C(A)}{12}(19p^2g^{\mu\nu}-22p^\mu p^\nu)\right]\delta_{ab}\\
&= i\frac{g_s^2}{16\pi^2}\Delta_\epsilon\left[-\frac{4(n_f-1)}{3}T(F)+\frac{5}{3}C(A)\right](p^2g^{\mu\nu}-p^\mu p^\nu)\delta_{ab}
\end{aligned} \tag{7.31}$$

Both contributions the one from the gluon loop and the one from the ghost loop are not proportional to $(p^2g^{\mu\nu}-p^\mu p^\nu)$ but their sum is. The heavy particle contributions are calculated

[15] In contrast to the MS-scheme in the $\overline{\text{MS}}$-scheme not only the pure ultraviolet divergence but also two additional transcendent numbers are subtracted. It is therefore common to define $\Delta_\epsilon = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$.

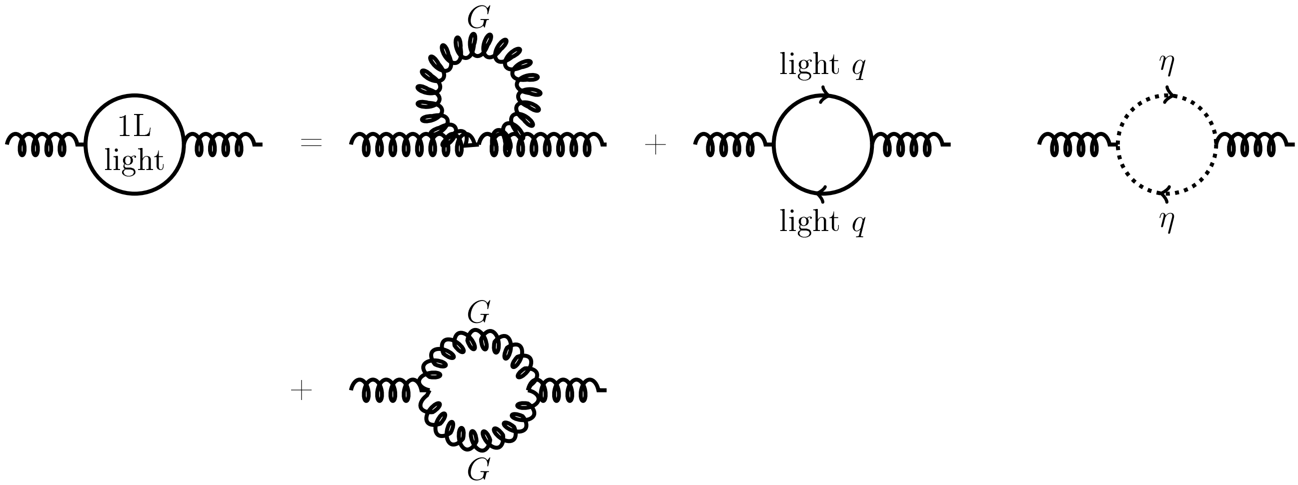


Figure 7.4: Contribution to the self-energy of the gluon originating from light particles

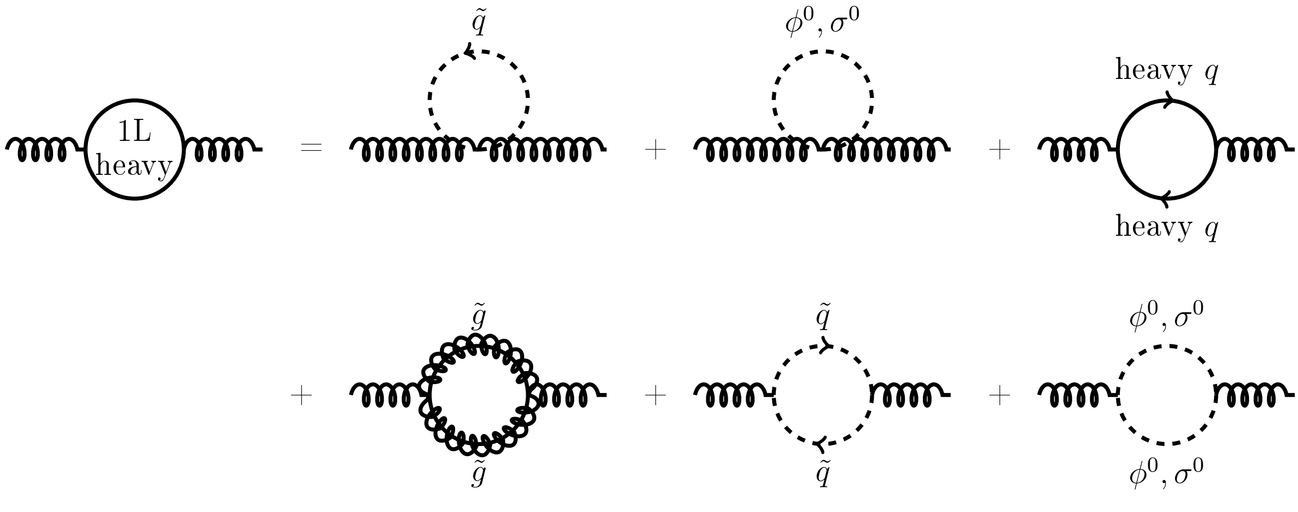


Figure 7.5: contribution to the self-energy of the gluon originating from heavy particles, in the last diagram either ϕ^0 or σ^0 are running in the loop

in the zero-momentum subtraction

$$
\begin{aligned}
i\Gamma^{\text{1L,heavy}}_{G^a_\mu G^b_\nu}(p^2=0) = {} & i\frac{g_s^2}{16\pi^2}\delta_{ab}\Bigg[-4T(F)n_f\left(\Delta_\epsilon-\ln\frac{m_{\tilde q}^2}{\mu^2}\right)m_{\tilde q}^2 g^{\mu\nu} - C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)m_{\phi^0}^2 g^{\mu\nu} \\
& - C(A)\left(\Delta_\epsilon-\ln\frac{m_{\sigma^0}^2}{\mu^2}\right)m_{\sigma^0}^2 g^{\mu\nu} - \frac{4}{3}T(F)\left(\Delta_\epsilon-\ln\frac{m_t^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) \\
& - \frac{4}{3}C(A)\left(\Delta_\epsilon-\ln\frac{m_{\tilde g}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) \\
& - \frac{2}{3}T(F)n_f\left(\Delta_\epsilon-\ln\frac{m_{\tilde q}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) + 4T(F)n_f\left(\Delta_\epsilon-\ln\frac{m_{\tilde q}^2}{\mu^2}\right)m_{\tilde q}^2 g^{\mu\nu} \\
& - \frac{1}{6}C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) + C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)m_{\phi^0}^2 g^{\mu\nu} \\
& -\frac{1}{6}C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) + C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)m_{\phi^0}^2 g^{\mu\nu}\Bigg] \\
= {} & i\frac{g_s^2}{16\pi^2}\delta_{ab}\Bigg[-\frac{4}{3}T(F)\left(\Delta_\epsilon-\ln\frac{m_t^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) \\
& - \frac{4}{3}C(A)\left(\Delta_\epsilon-\ln\frac{m_{\tilde g}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) \\
& - \frac{2}{3}T(F)n_f\left(\Delta_\epsilon-\ln\frac{m_{\tilde q}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) \\
& - \frac{1}{6}C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu) \\
& -\frac{1}{6}C(A)\left(\Delta_\epsilon-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\Bigg] \qquad (7.32)
\end{aligned}
$$

The counterterm Feynman rule for the gluon propagator is

$i\Gamma^{\text{1L,ct}}_{G^a_\mu G^b_\nu} \mathrel{\hat=} a,\mu$  $b,\nu \mathrel{\hat=} -i\delta Z_G\left(p^2g^{\mu\nu}-p^\mu p^\nu\right)\delta_{ab}.$

The renormalization condition for δZ_G^{aux} reads

$$
\Gamma^{\text{1L,light}}_{G^a_\mu G^b_\nu}\Big|_{\text{UV-div}} + \Gamma^{\text{1L,heavy}}_{G^a_\mu G^b_\nu}(p^2=0) - \delta Z_G^{\text{aux}}\left(p^2g^{\mu\nu}-p^\mu p^\nu\right)\delta_{ab} = 0 \qquad (7.33)
$$

and yields

$$\delta Z_G^{aux} = \frac{g_s^2}{16\pi^2}\left\{\left[-\frac{4}{3}T(F)(n_f-1)+\frac{5}{3}C(A)\right]\Delta_\epsilon + \left[-\frac{4}{3}T(F)\left(\Delta_\epsilon - \ln\frac{m_t^2}{\mu^2}\right)\right.\right.$$
$$-\frac{4}{3}C(A)\left(\Delta_\epsilon - \ln\frac{m_{\tilde{g}}^2}{\mu^2}\right) - \frac{2}{3}T(F)n_f\left(\Delta_\epsilon - \ln\frac{m_{\tilde{q}}^2}{\mu^2}\right)$$
$$-\frac{1}{6}C(A)\left(\Delta_\epsilon - \ln\frac{m_{\phi^0}^2}{\mu^2}\right) - \frac{1}{6}C(A)\left(\Delta_\epsilon - \ln\frac{m_{\sigma^0}^2}{\mu^2}\right). \tag{7.34}$$

7.4.3 The $q\bar{q}G$ Vertex Correction

As for the self energies the vertex correction constitutes of loops from light particles which correspond to the Standard Model and additional heavy particle loops.

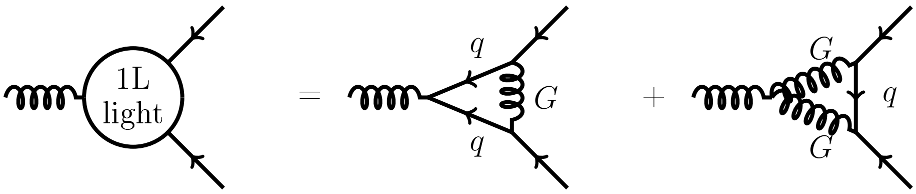


Figure 7.6: Contribution from light (Standard Model) particles to the $q\bar{q}G$ vertex. correction

$$i\Gamma^{\text{1L,light}}_{q_i\bar{q}_jG^a_\mu}\Big|_{\text{UV-div}} = -ig_sT^a_{ij}\gamma^\mu\frac{g_s^2}{16\pi^2}\left[\left(C(F)-\frac{C(A)}{2}\right)2(B_0(0,0,0)-2C_{00}(0,0,0,0,0))\right.$$
$$\left. +C(A)(B_0(0,0,0)+2C_{00}(0,0,0,0,0))\right]$$
$$= -ig_sT^a_{ij}\gamma^\mu\frac{g_s^2}{16\pi^2}\Delta_\epsilon\left[\left(C(F)-\frac{C(A)}{2}\right)+\frac{3}{2}C(A)\right] \tag{7.35}$$

The term in the curved brackets of eq. 7.35 corresponds to the first diagram in fig. 7.6 whereas the last term in the squared brackets corresponds to the second diagram.

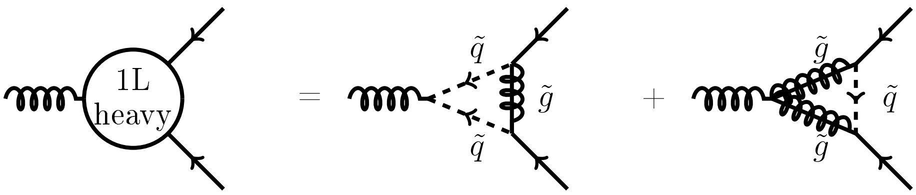


Figure 7.7: Contribution from heavy particles to the $q\bar{q}G$ vertex correction.

$$\begin{aligned}
i\Gamma^{\text{1L,heavy}}_{q_i\bar{q}_j G^a_\mu}(p_i^2=0) &= -ig_s T^a_{ij}\gamma^\mu \frac{g_s^2}{16\pi^2}\left[\left(C(F)-\frac{C(A)}{2}\right)4C_{00}(0,0,m_{\tilde{g}}^2,m_{\tilde{q}}^2,m_{\tilde{q}}^2)\right.\\
&\qquad \left.+\,C(A)\left(B_0(0,m_{\tilde{g}}^2,m_{\tilde{q}}^2)-2C_{00}(0,0,m_{\tilde{g}}^2,m_{\tilde{g}}^2,m_{\tilde{q}}^2)\right)\right]\\
&= -ig_s T^a_{ij}\gamma^\mu \frac{g_s^2}{16\pi^2}\left[\left(C(F)-\frac{C(A)}{2}\right)\left(\Delta_\epsilon-\ln\frac{m_{\tilde{g}}^2}{\mu^2}\right)\right.\\
&\qquad \left.+\,\frac{C(A)}{2}\left(\Delta_\epsilon-\ln\frac{m_{\tilde{g}}^2}{\mu^2}\right)\right]
\end{aligned} \tag{7.36}$$

The argument $p_i^2=0$ of the vertex function denotes, that all external particles are taken on shell, i.e. at zero momentum. In the second line identities from 11.6 have been used. The renormalization condition for the counterterm of the gauge coupling δg_s reads

$$i\Gamma^{\text{1L,light}}_{q_i\bar{q}_j G^a_\mu}\Big|_{\text{UV-div}} + i\Gamma^{\text{1L,heavy}}_{q_i\bar{q}_j G^a_\mu}(p_i^2=0) + \left[-ig_s T^a_{ij}\gamma^\mu\left(\frac{\delta g_s}{g_s}+\delta Z_q^{\text{aux}}+\frac{\delta Z_G^{\text{aux}}}{2}\right)\right] = 0. \tag{7.37}$$

Finally one can read off the $\frac{\delta g_s}{g_s}$

$$\begin{aligned}
\frac{\delta g_s}{g_s} = \frac{g_s^2}{16\pi^2}&\left[\left(\frac{2}{3}T(F)(n_f-1)-\frac{11}{6}C(A)\right)\Delta_\epsilon + \left(\frac{5}{6}C(A)+\frac{2}{3}T(F)+\frac{1}{3}T(F)n_f\right)\Delta_\epsilon\right.\\
&\left. -\frac{2}{3}C(A)\ln\frac{m_{\tilde{g}}^2}{\mu^2}-\frac{1}{3}T(F)n_f\ln\frac{m_{\tilde{q}}^2}{\mu^2}-\frac{2}{3}T(F)\ln\frac{m_t^2}{\mu^2}-\frac{1}{12}C(A)\left(\ln\frac{m_{\phi^0}^2}{\mu^2}+\ln\frac{m_{\sigma^0}^2}{\mu^2}\right)\right]
\end{aligned} \tag{7.38}$$

7.4.4 The Beta Function

The beta function describes the dependence of the gauge coupling g_s upon the energy scale μ. Writing down the action of a theory in D dimensions one needs to introduce an energy scale μ in order to keep the action dimensionless. But μ is no physical parameter and can be absorbed into the fields and parameters. To this end one defines

$$g_{sB} = \mu^\epsilon g_s\left(1+\frac{\delta g_s}{g_s}\right) \tag{7.39}$$

which must not depend upon the unphysical scale μ, ergo

$$0 = \frac{\mathrm{d}g_{sB}}{\mathrm{d}\ln\mu} = \frac{\partial g_{sB}}{\partial\ln\mu} + \beta\frac{\partial g_{sB}}{\partial g_s} \tag{7.40}$$

where the definition of the beta function $\frac{\partial g_s}{\partial \ln \mu}$ has been inserted. Equation 7.40 serves to calculate $\beta(g_s, \epsilon)$. By equating coefficients and using the shortcuts

$$\begin{aligned}
\frac{\beta_0^L}{2} &= \frac{2}{3}T(F)(n_f - 1) - \frac{11}{6}C(A) \\
\frac{\beta_0^H}{2} &= \frac{5}{6}C(A) + \frac{2}{3}T(F) + \frac{1}{3}T(F)n_f \\
L &= -\frac{2}{3}C(A)\ln\frac{m_{\tilde{g}}^2}{\mu^2} - \frac{1}{3}T(F)n_f \ln\frac{m_{\tilde{q}}^2}{\mu^2} - \frac{2}{3}T(F)\ln\frac{m_t^2}{\mu^2} - \frac{1}{12}C(A)\left(\ln\frac{m_{\phi^0}^2}{\mu^2} + \ln\frac{m_{\sigma^0}^2}{\mu^2}\right)
\end{aligned}$$

so that

$$\frac{\delta g_s}{g_s} = \frac{g_s^2}{16\pi^2}\left(\frac{\beta_0^L}{2\epsilon_{\mathrm{UV}}} + \frac{\beta_0^H}{2\epsilon_{\mathrm{UV}}} + L\right) \tag{7.41}$$

one finds

$$\beta(g_s, \epsilon) = -\epsilon g_s\left(1 + \frac{g_s^2}{16\pi^2}L\right) + \beta(g_s) + \mathcal{O}(2-\mathrm{loop}) \tag{7.42}$$

$$\beta(g_s) = \frac{g_s^3}{16\pi^2}\beta_0^L + \mathcal{O}(2-\mathrm{loop}). \tag{7.43}$$

This is the beta function from QCD first found by [Gross, Politzer, Wil]

7.5 Supersymmetry Restoring Counterterm

As already discussed in section 7.5 care is required in terms of supersymmetry restoring when renormalizing the gauge coupling g_s and the Yukawa coupling $\hat{g}_s$. In doing so one needs the already calculated supersymmetry restoring counterterms of the quark, squark and gluino from 7.10, 7.15 and 7.22 as well as the supersymmetry restoring counterterm of the gluon.

The Gluon Self-Energy Revisited

The only regularization dependence of the gluon self-energy arises from the gluon loop, i.e. the last diagram in figure 7.4. With the definition of $\Gamma^{(1),\mathrm{ct,restore}}_{\mathrm{DREG}}$ in 7.6 one obtains

$$i\Gamma^{(1),\mathrm{ct,restore}}_{\mathrm{DREG},G_\mu^a G_\nu^b} = -i\frac{1}{3}C(A)\frac{g_s^2}{16\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu)\delta_{ab} \tag{7.44}$$

which translates to the transition counterterm

$$\delta Z_G^{\mathrm{trans}} = \frac{C(A)}{3}\frac{g_s^2}{16\pi^2}. \tag{7.45}$$

The $q\bar{q}G$ Vertex Correction Revisited

The supersymmetry restoring contributions to the gauge coupling correction are shown in figure 7.6 and evaluate to

$$i\Gamma^{(1),\text{ct,restore}}_{\text{DREG},q_i\bar{q}_jG^a_\mu} = -ig_sT^a_{ij}\gamma^\mu\frac{g_s^2}{16\pi^2}\left[\left(C(F)-\frac{C(A)}{2}\right)+\frac{C(A)}{2}\right] \tag{7.46}$$

$$= -ig_sT^a_{ij}\gamma^\mu\left[\frac{\delta g_s^{\text{trans}}}{g_s}+\delta Z_q^{\text{trans}}+\frac{\delta Z_G^{\text{trans}}}{2}\right] \tag{7.47}$$

where in the second line the equation with the supersymmetry restoring counterterms has been performed. This yields

$$\frac{\delta g_s^{\text{trans}}}{g_s} = -\frac{C(A)}{6}\frac{g_s^2}{16\pi^2}. \tag{7.48}$$

The $q\tilde{q}^\dagger\tilde{g}$ Vertex Correction

The supersymmetry restoring corrections to the Yukawa coupling origin from the below diagram The supersymmetry restoring part is

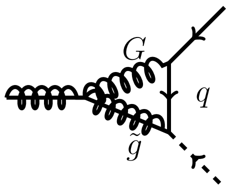


Figure 7.8: diagram of the supersymmetry restoring correction of the $q\tilde{q}\tilde{g}$ vertex

$$i\Gamma^{(1),\text{ct,restore}}_{\text{DREG},q_i\tilde{q}_j\tilde{g}^a} = -ig_s\sqrt{2}P_LT^a_{ij}\frac{g_s^2}{16\pi^2}C(A) \tag{7.49}$$

$$= -ig_s\sqrt{2}P_LT^a_{ij}\left[\frac{\delta\hat{g}_s^{\text{trans}}}{g_s}+\frac{\delta Z_q^{\text{trans}}+\delta Z_{\tilde{q}}^{\text{trans}}+\delta Z_{\tilde{g}}^{\text{trans}}}{2}\right]. \tag{7.50}$$

The supersymmetry restoring part of the Yukawa renormalization constants is therefore

$$\frac{\delta\hat{g}_s^{\text{trans}}}{g_s} = -\frac{C(F)-C(A)}{2}\frac{g_s^2}{16\pi^2}. \tag{7.51}$$

As a consequence of the two different supersymmetry restoring parts of the coupling renormalization constants an additional renormalization constant $\delta g_s^{\text{restore}}$ needs to be introduced. As described in section it is given by

$$\frac{\delta g_s^{\text{restore}}}{g_s} = \frac{\delta\hat{g}_s^{\text{trans}}}{g_s} - \frac{\delta g_s^{\text{trans}}}{g_s} = \frac{g_s^2}{16\pi^2}\left(\frac{2C(A)}{3}-\frac{C(F)}{2}\right). \tag{7.52}$$

In short this means that the gauge coupling g_s is renormalized with δg_s given in 7.38 and the Yukawa coupling $\hat{g}_s$ is renormilzed with $\delta \hat{g}_s = \delta g_s + \delta g_s^{\text{restore}}$.
The finite correction g_s^{restore} is the same as in supersymmetric QCD which should not surprise too much as all its contributions origin from loops with gluons. So there are no new contributions in RSQCD with respect to SQCD.

7.6 $\overline{\text{MS}}$ - Renormalization

To quickly check for UV-finiteness of a Greenfunction or a physical observable it is useful to have the extracted UV-divergences of the renormalization constants at hand. How these are obtained is described in 11.6.
(In order to obtain these the Passarino-Veltman integrals need to be substituted by their $\frac{1}{\epsilon}$ coefficient. These had been taken from [?] and checked with `FeynArts` and `FormCalc` [?], [?], [?].)

$$
\begin{aligned}
\delta Z_{\tilde{g}}^L &= -\frac{g_s^2}{16\pi^2 \epsilon_{\text{UV}}} 2\left[T(F) n_f + C(A)\right] & \delta Z_{\tilde{g}}^R &= -\frac{g_s^2}{16\pi^2 \epsilon_{\text{UV}}} 2C(A) \\
\delta m_{\tilde{g}} &= \frac{g_s^2}{16\pi^2 \epsilon_{\text{UV}}} \left[T(F) n_f - 2C(A)\right] m_{\tilde{g}} & & \\
\delta Z_{\tilde{q}} &= 0 & \delta m_{\tilde{q}}^2 &= 0 \\
\frac{\delta g_s}{g_s} &= \frac{g_s^2}{16\pi^2 \epsilon_{\text{UV}}} \left[T(F) n_f - C(A)\right] & & \\
\delta Z_{\phi^0} &= 0 & \delta m_{\phi^0}^2 &= -\frac{g_s^2}{16\pi^2 \epsilon_{\text{UV}}} \left[8T(F) n_f - 16C(A)\right] m_{\tilde{g}}^2 \\
\delta Z_{\sigma^0} &= 0 & \delta m_{\sigma^0}^2 &= 0 \\
\delta Z_G &= -\frac{g_s^2}{16\pi^2 \epsilon_{\text{UV}}} 2T(F) n_f & &
\end{aligned}
\tag{7.53}
$$

8 Infrared and Collinear Singularities

Looptools p.22, Kinoshita–Lee–Nauenberg theorem

9 Squark Production at One-Loop

The cross section for squark production does not exist in the limit of an infinitely large sgluon mass, instead it was found that it diverges logarithmically.

$$\lim_{m_{\sigma^0}\to\infty} \sigma(qq \to \tilde{q}\tilde{q}) \sim \ln \frac{m_{\sigma^0}^2}{\mu^2} \tag{9.1}$$

This is actually expected as an effective field theory of the MRSSM where the sgluon is integrated out is no longer supersymmetric. This is because the sgluon is together with the octino part of a supermultiplet. Integrating out only the sgluon means that the octino misses its superpartner in the effective field theory. In this case the decoupling theorem [**?**] does no longer hold.
Refer to super oblique correction and quantify difference of g and $\hat{g}$ from eq 4 in [**?**]

10 Summary and Outlook

11 Appendix

11.1 System of Units and Metric

In this thesis the natural units are used, i.e. $c = \hbar (= k_B) = 1$. Furthermore the Minkowski metric is chosen to be

$$g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \tag{11.1}$$

11.2 Constants of the Colour Algebra $SU(N)$

[Marina von Steinkirch]
Tha Casimir operator $C(R)\mathbb{1}$ of a semi-simple Lie algebra in the irreducible representation R is given by

$$g^{ab} T^a(R) T^b(R) = C(R)\mathbb{1} \tag{11.2}$$

where $T^a(R)$ is the a-th generator of the matrix valued representation R, g^{ab} is the metric of group, $C(R)$ is the Quadratic Casimir invariant of the representation R and $\mathbb{1}$ is the identity in the representation space.
Apart from $C(R)$ it is common to define the Dynkin-Index $T(R)$:

$$\mathrm{Tr}\left[T^a(R) T^b(R)\right] = T(R)\delta^{ab}. \tag{11.3}$$

The two constants are connected by

$$C(R) \cdot \dim(R) = T(R) \cdot \dim(G) \tag{11.4}$$

where $\dim(G)$ is the dimension of the group and $\dim(R)$ is the dimension of the irreducible representation R.
In the case of $SU(N)$ one has a diagonal metric $g^{ab} = \delta^{ab}$ and therefore 11.2 turns to

$$\sum_a (T^a(R))^2 = C(R)\mathbb{1}_{\dim(R)\times\dim(R)} \tag{11.5}$$

and one can write down the following useful formulae for the fundamental representation $R = F$: $T^a_{ij} = \frac{\lambda^a_{ij}}{2}$ and the adjoint representation $R = A$: $(T^a_{ij})^{adj} = -if_{aij}$

$$\begin{aligned}
T^a_{ik}T^a_{kj} &= C(F)\mathbb{1}_{ij} && \text{with } C(F) = \frac{N^2-1}{2N} = \frac{4}{3} \\
f^{abc}f^{dbc} &= C(A)\delta^{ad} && \text{with } C(A) = N = 3 \\
\mathrm{Tr}\left[T^aT^b\right] &= T(F)\delta^{ab} && \text{with } T(F) = \frac{1}{2}
\end{aligned} \tag{11.6}$$

where λ^a_{ij} are for $N_c = 3$ the Gell-Mann matrices and f_{abc} are the structure constants of $SU(N_c)$.

11.3 Weyl basis and 2-spinor notation

As representation of the γ-matrices the Weyl or chiral representation is chosen:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} -\mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix} \tag{11.7}$$

with

$$\sigma^\mu = \begin{pmatrix} \mathbb{1}_2 & \sigma^i \end{pmatrix}, \qquad \overline{\sigma}^\mu = \begin{pmatrix} \mathbb{1}_2 & -\sigma^i \end{pmatrix}, \tag{11.8}$$

where σ^i are the Pauli matrices and $\mathbb{1}_n$ is the nxn unit matrix. The left and right handed projectors are then given by

$$P_L = \frac{1}{2}(\mathbb{1}_4 - \gamma_5)\begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix} \qquad P_R = \frac{1}{2}(\mathbb{1}_4 + \gamma_5)\begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix} \tag{11.9}$$

The generator of the Lorentz group on 4-spinor space is composed of the above matrices. Because of the block form of those it is not surprising that the representation on 4-spinor space is reducible to two representations on 2-spinor (Weyl spinor) spaces. It is therefore sensible to decompose a 4 spinor into a left and a right handed Weyl spinor[16]

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{11.10}$$

where $\alpha, \dot{\alpha} = 1, 2$. Left handed Weyl spinors are labeled with undotted and right handed Weyl spinors with dotted indices. One distinguishes 4 different Weyl spinors:

$$\psi^\alpha, \qquad \overline{\psi}^{\dot{\alpha}} = (\psi^\alpha)^*, \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \qquad \text{and} \qquad \psi_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dot{\beta}} = (\psi_\alpha)^*, \tag{11.11}$$

[16] The projectors in the chiral basis 11.9 explain the names left and right handed Weyl spinors.

where * denotes complex conjugation and indices are lowered with the antisymmetric $\epsilon_{\alpha\beta}$ ($\epsilon_{\dot{\alpha}\dot{\beta}}$), which obeys

$$\epsilon^{\alpha\beta} = \epsilon_{\beta\alpha},\ \epsilon^{\dot{\alpha}\dot{\beta}} = \epsilon_{\dot{\beta}\dot{\alpha}} \qquad \text{and} \qquad \epsilon_{12} = \epsilon_{\dot{1}\dot{2}} = 1. \tag{11.12}$$

By virtue of the antisymmetry of ϵ one has for the Lorentz invariant product:

$$\begin{aligned} \psi\chi &:= \psi^\alpha\chi_\alpha = -\chi_\alpha\psi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi, \\ \overline{\psi}\overline{\chi} &:= \overline{\psi}_{\dot{\alpha}}\overline{\chi}^{\dot{\alpha}} = -\overline{\chi}^{\dot{\alpha}}\overline{\psi}_{\dot{\alpha}} = \overline{\chi}_{\dot{\alpha}}\overline{\psi}^{\dot{\alpha}} = \overline{\chi}\overline{\psi} \end{aligned} \tag{11.13}$$

To make the index structure of the Pauli matrices explicit one writes $\sigma^\mu_{\alpha\dot{\alpha}}$ and $\sigma^{\mu\dot{\alpha}\alpha}$ for the formulae in 11.8. For the definition of the super algebra in ??? the generators of the Lorentz group on the left and right handed Weyl spinor space are introduced:

$$\begin{aligned} \frac{1}{2}(\sigma^{\mu\nu})_\alpha^{\ \beta} &:= \frac{i}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)_\alpha^{\ \beta} \\ \frac{1}{2}(\overline{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} &:= \frac{i}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu)^{\dot{\alpha}}_{\ \dot{\beta}} \end{aligned} \tag{11.14}$$

With the definition of bared and charge conjugated 4-spinors[17]

$$\overline{\Psi} := \Psi^\dagger\gamma^0, \qquad \Psi^C := i\gamma^2\gamma^0\overline{\Psi}^T \tag{11.15}$$

one obtains:

$$\begin{aligned} \Psi &= \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix}, & \overline{\Psi} &= \begin{pmatrix} \chi^\alpha & \overline{\psi}_{\dot{\alpha}} \end{pmatrix}, \\ \Psi^C &= \begin{pmatrix} \chi_\alpha \\ \overline{\psi}^{\dot{\alpha}} \end{pmatrix}, & \overline{\Psi}^C &= \begin{pmatrix} \psi^\alpha & \overline{\chi}_{\dot{\alpha}} \end{pmatrix}. \end{aligned} \tag{11.16}$$

The 4-spinor of an arbitrary quark q is given in terms of Weyl spinors q_L and $\overline{q}_R$ by

$$q = \begin{pmatrix} q_L \\ \overline{q}_R \end{pmatrix} \tag{11.17}$$

whereas the 4-spinor of the Dirac gauginos is given in terms of the Weyl spinors λ and $\overline{\chi}$[18]

$$\tilde{g}^a = \begin{pmatrix} -i\lambda^a \\ i\overline{\chi}^a \end{pmatrix} \tag{11.18}$$

[17] Ψ^T denotes the transpose of the spinor Ψ and $\Psi^\dagger$ is the Hermitian adjoint of Ψ.
[18] λ is the superpartner of the gluon, called the gluino and $\overline{\chi}$ is the Weyl spinor of the chiral superfield which is associated with the gluon, called the octino.

11.4 Anticommuting numbers

Anticommuting numbers θ^α are also referred to as Grassmann numbers and are defined by $\theta^\alpha\theta^\beta = -\theta^\beta\theta^\alpha$ and commute with ordinary numbers.
They occur in superspace formalism in the form of 2 tuples, i.e. θ^α with $\alpha = 1, 2$. The complex conjugate of this tuple is denoted with $\overline{\theta}^{\dot{\alpha}}$. Derivatives are defined by

$$\begin{aligned} \partial^\alpha\theta_\beta &:= \frac{\partial}{\partial\theta_\alpha}\theta_\beta := \delta^\alpha_\beta & \qquad \partial_\alpha\theta^\beta &:= \frac{\partial}{\partial\theta^\alpha}\theta^\beta := \delta^\beta_\alpha \\ \overline{\partial}^{\dot{\alpha}}\overline{\theta}_{\dot{\beta}} &:= \frac{\partial}{\partial\overline{\theta}_{\dot{\alpha}}}\overline{\theta}_{\dot{\beta}} := \delta^{\dot{\alpha}}_{\dot{\beta}} & \qquad \overline{\partial}_{\dot{\alpha}}\overline{\theta}^{\dot{\beta}} &:= \frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}}\overline{\theta}^{\dot{\beta}} := \delta^{\dot{\beta}}_{\dot{\alpha}} \end{aligned} \tag{11.19}$$

whereby one needs to be cautious as these definitions imply

$$\partial_\alpha = -\epsilon_{\alpha\beta}\partial^\beta \qquad\qquad \partial_{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}}\partial^{\dot{\beta}}. \tag{11.20}$$

Integrals are defined by:

$$\int \mathrm{d}\theta_\alpha \; (a + b\theta^\beta + c\theta^\beta\theta^\gamma) := b\delta^\beta_\alpha + c(\delta^\beta_\alpha\theta^\gamma - \delta^\gamma_\alpha\theta^\beta) \qquad \text{and} \qquad \int \mathrm{d}\theta_\alpha \; (a\overline{\theta}^{\dot{\beta}}) := (a\overline{\theta}^{\dot{\beta}}) \int \mathrm{d}\theta_\alpha \tag{11.21}$$

where the first line mirrows the claim of translation invariance. One furthermore introduces the shortcuts

$$\int \mathrm{d}\theta^2 = \int \frac{1}{4}\epsilon_{\alpha\beta}\mathrm{d}\theta^\alpha\mathrm{d}\theta^\beta, \qquad \int \mathrm{d}\overline{\theta}^2 = \int \frac{1}{4}\epsilon_{\dot{\alpha}\dot{\beta}}\mathrm{d}\theta^{\dot{\alpha}}\mathrm{d}\theta^{\dot{\beta}}, \qquad \text{and} \qquad \int \mathrm{d}^4\theta := \int \mathrm{d}\theta^2\mathrm{d}\overline{\theta}^2 \tag{11.22}$$

11.5 Feynman rules for the RSQCD

The following Feynman rules are derived from 4.13. When compared with the Feynman rules of the supersymmetric QCD the diagrams involving scalar gluons are new. In addition the gluon-quark-squark vertex is different in RSQCD for the gauginos are Dirac fermions.
refer to paper cited in Beenakker(Appendix: Feynmanrules)

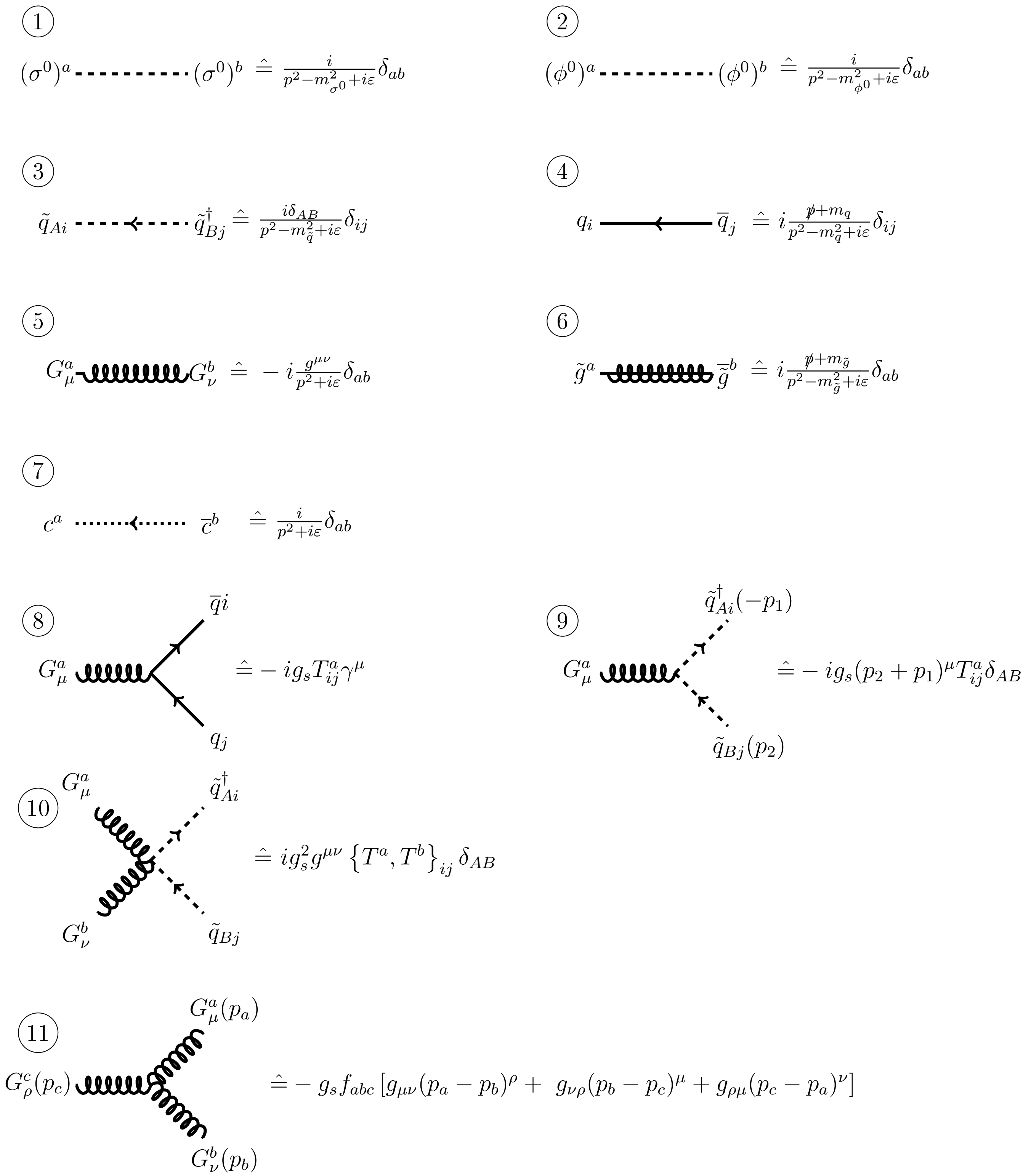
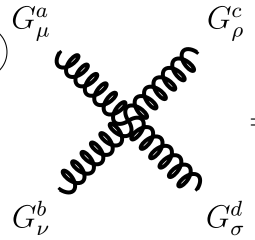


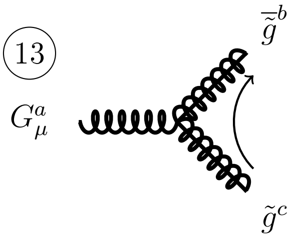
Figure 11.1: In the Feynman diagrams of the propagators the momentum is flowing from the right to the left hand side.
In the Feynman diagrams of the vertices all momenta flow towards the vertex.
The indices $A, B = L, R$ label the right/left-"handedness" of the squarks. The indices $i, j = 1, 2, 3$ are the color indices in the (anti)fundamental representation where $a, b, c, \ldots = 1, \ldots, 8$ are the color indices of the adjoint representation.

(12)



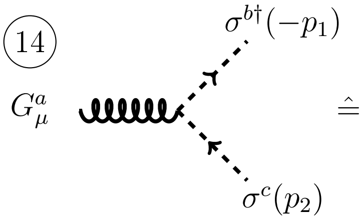
$$\hat{=}\ -ig_s^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})+f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho})$$
$$+f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})]$$

(13)



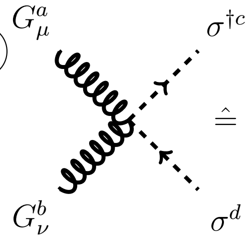
$$\hat{=}-g_s f_{abc}\gamma^\mu$$

(14)



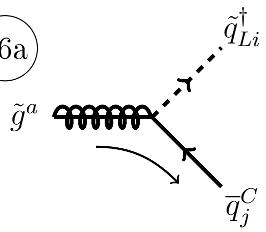
$$\hat{=}-g_s(p_1+p_2)^\mu f_{abc}$$

(15)



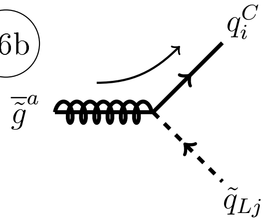
$$\hat{=}\ +ig_s^2g^{\mu\nu}[f^{aec}f^{bed}+f^{bec}f^{aed}]$$

(16a)



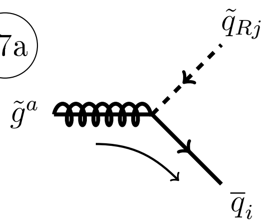
$$\hat{=}-i\sqrt{2}g_sT^a_{ij}P_L$$

(16b)



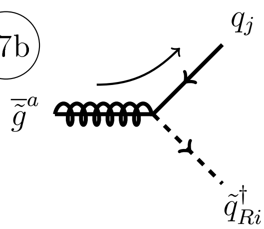
$$\hat{=}-i\sqrt{2}g_sT^a_{ij}P_R$$

(17a)



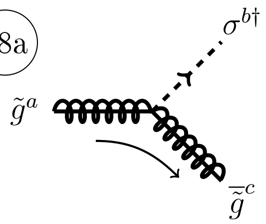
$$\hat{=}+i\sqrt{2}g_sT^a_{ij}P_L$$

(17b)



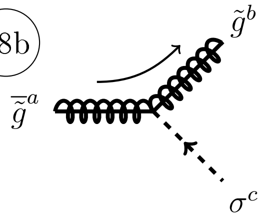
$$\hat{=}+i\sqrt{2}g_sT^a_{ij}P_R$$

(18a)



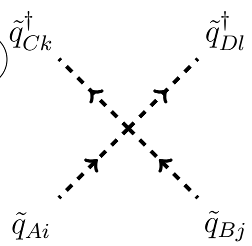
$$\hat{=}-\sqrt{2}g_sf^{abc}P_L$$

(18b)



$$\hat{=}+\sqrt{2}g_sf^{abc}P_R$$

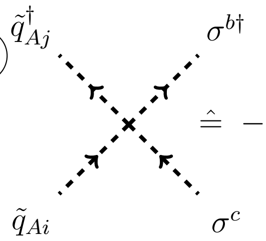
(19)



$$\hat{=}\ -ig_s^2[T^a_{ki}T^a_{lj}(\delta_{AL}\delta_{CL}-\delta_{AR}\delta_{CR})(\delta_{BL}\delta_{DL}-\delta_{BR}\delta_{DR})$$
$$T^a_{kj}T^a_{li}(\delta_{BL}\delta_{CL}-\delta_{BR}\delta_{CR})(\delta_{AL}\delta_{DL}-\delta_{AR}\delta_{DR})]$$

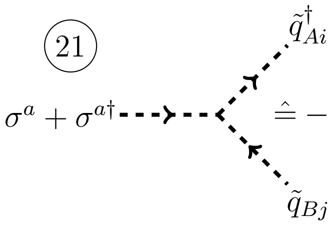
Figure 11.2: The curved arrows indicate the fermion flow. The Feynman rules 16b, 17b and 18b are the complex conjugates of 16a, 17a and 18a respectively. Applying a flipping rule to a vertex one has to reverse the curved arrow, i.e. the fermion flow and replace Ψ with $\bar{\Psi}^C$. In addition one has to add a minus sign for Feynman rule 13.

20



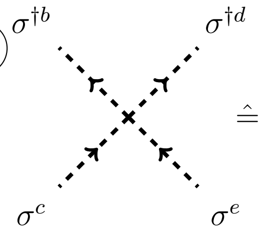
$$\hat{=} \; - g_s^2 T^a_{ij} f^{abc}(\delta_{AL}\delta_{CL} - \delta_{AR}\delta_{BR})$$

21



$$\hat{=} - i\sqrt{2} g_s m_{\tilde{g}} T^a_{ij}(\delta_{AL}\delta_{BL} - \delta_{AR}\delta_{BR})$$

22



$$\hat{=} \; - g_s^2(f^{abc}f^{ade} + f^{abc}f^{adc})$$

11.6 Passarino-Veltman Integrals

The definition of the Passarino-Veltman integrals in this thesis agrees with the one from `Looptools` [?] convention. The original paper [?] uses slightly different conventions. A pedagogical introduction to the evaluation of one-loop integrals can be found in [?]

$$\begin{aligned}
\frac{i}{16\pi^2} A_0(m^2) &= \int_l \frac{1}{l^2 - m^2} \\
\frac{i}{16\pi^2} B_{0,\mu,\mu\nu}(p^2, m_1^2, m_2^2) &= \int_l \frac{\{1, l_\mu, l_\mu l_\nu\}}{[l^2 - m_1^2][(l+p)^2 - m_2^2]} \\
\frac{i}{16\pi^2} C_{0,\mu,\mu\nu}(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2) &= \int_l \frac{\{1, l_\mu, l_\mu l_\nu\}}{[l^2 - m_1^2][(l+p_1)^2 - m_2^2][(l+p_1+p_2)^2 - m_3^2]}
\end{aligned} \tag{11.23}$$

with the shortcut $\int_l = \mu^{2\epsilon} \int \frac{\mathrm{d}^D l}{(2\pi)^D}$. Furthermore there are supressed ε's which prescripe how the poles in the complex plane are avoided. They are hidden in the infinitesimal shift of the masses: $m_i^2 \to m_i^2 - i\varepsilon$.

The tensor integrals can be decomposed as

$$\begin{aligned}
B_\mu &:= p_\mu B_1 \\
B_{\mu\nu} &:= g_{\mu\nu} B_{00} + p_\mu p_\nu B_{11} \\
C_\mu &= p_{1\mu} C_1 + p_{2\mu} C_2 \\
C_{\mu\nu} &:= g_{\mu\nu} C_{00} + p_{1\mu} p_{1\nu} C_{11} + p_{2\mu} p_{2\nu} C_{22} + (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu}) C_{12}
\end{aligned} \tag{11.24}$$

In the the special case of vanishing momenta the integrals take a succinct form.

$$A_0(m^2) = m^2 \left(\Delta_\epsilon - \ln \frac{m^2}{\mu^2} + 1 \right) + \mathcal{O}(\epsilon) \tag{11.25}$$

where the typical UV-divergent constant $\Delta_\epsilon = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$ is defined. It comprises the Euler-Mascheroni constant γ_E.

$$B_0(0, m_1^2, m_2^2) = \frac{A_0(m_1^2) - A_0(m_2^2)}{m_1^2 - m_2^2} = \Delta_\epsilon + 1 - \frac{m_1^2 \ln \frac{m_1^2}{\mu^2} - m_2^2 \ln \frac{m_2^2}{\mu^2}}{m_1^2 - m_2^2} + \mathcal{O}(\epsilon) \tag{11.26}$$

$$B_0(0, m^2, m^2) = \frac{\partial}{\partial m^2} A_0(m^2) = \Delta_\epsilon - \ln \frac{m^2}{\mu^2} + \mathcal{O}(\epsilon) \tag{11.27}$$

$$B_0(0,0,0) = 0 \tag{11.28}$$

$$B_1(0, m_1^2, m_2^2) = -\frac{\Delta_\epsilon}{2} + \frac{1}{2} \ln \frac{m_1^2}{\mu^2} + \frac{-3 + 4t - t^2 - 4t \ln t + 2t^2 \ln t}{4(t-1)^2} + \mathcal{O}(\epsilon) \tag{11.29}$$

$$B_1(0,0,0) = 0 \tag{11.30}$$

The scaleless integrals are defined to be zero. The parameter t is given by $\frac{m_2^2}{m_1^2}$. As can be seen from 11.29 B_1 is in contrast to B_0 not symmetric in its masses but it can be shown [Romao] that

$$B_1(p^2, m_1^2, m_2^2) = -(B_0(p^2, m_2^2, m_1^2) + B_1(p^2, m_2^2, m_1^2)) \tag{11.31}$$

It can further be shown that

$$C_{00}(0,0,0, m_1^2, m_1^2, m_2^2) = -\frac{1}{2} B_1(0, m_1^2, m_2^2) \tag{11.32}$$

and that $C_{00}(0,0,0, m_1^2, m_1^2, m_2^2)$ is a symmetric function of its masses.
From the generic ϵ-expansion of the B_0 integral [**?**]

$$B_0(p^2, m_1^2, m_2^2) = \Delta_\epsilon - \int_0^1 \mathrm{d}x \;\ln \frac{-x(1-x)p^2 + x m_2^2 + (1-x) m_1^2}{\mu^2} \tag{11.33}$$

and Passarino-Veltman decomposition [**?**] one can determine the UV-divergent part of all B and C integrals. In chapter 7.6 this was necessary in order to obtain the renormalization constants in the $\overline{\mathrm{MS}}$-scheme. Infrared and collinear singularities arise from the special case where one or multiple masses tend to zero. These poles are either regularized in terms of a small mass cutoff Λ or also dimensionally as ϵ-poles. In the later case the integral first needs to undergo the limit to zero masses and than being evaluated.
The following list shows all necessary integrals needed to determine the renormalization con-

stants in 7.6.

$$A_0(m^2)\big|_{\mathrm{UV-div}} = m^2\Delta_\epsilon \tag{11.34}$$

$$B_0(p^2, m_1^2, m_2^2)\big|_{\mathrm{UV-div}} = \Delta_\epsilon \tag{11.35}$$

$$B_1(p^2, m_1^2, m_2^2)\big|_{\mathrm{UV-div}} = -\frac{1}{2}\Delta_\epsilon \tag{11.36}$$

$$C_{00}(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2)\big|_{\mathrm{UV-div}} = \frac{1}{4}\Delta_\epsilon \tag{11.37}$$

The UV-divergent part of C_{11}, C_{22}, C_{12} equals zero. As can be seen already from the superficial degree of divergence also $C_i|_{\mathrm{UV-div}} = 0$ for $i \in \{0, 1, 2\}$.

11.7 Cross section and Phase Space Integration

Once the Feynman amplitude $\mathcal{M}$ for a $2 \to N$ body scattering[19] is computed one can calculate physical observables with it. The differential cross section for $2 \to N$ scattering is given by

$$\mathrm{d}\sigma = \frac{1}{F}\mathrm{d}\Phi_N|\mathcal{M}|^2. \tag{11.38}$$

The flux factor is defined by $F = 4\sqrt{(k_a \cdot k_b)^2 - (m_a m_b)^2}$ which equals $F = 2s$ for massless initial state particles. The differential for the N body phase space in D dimensions is given by

$$\mathrm{d}\Phi_N = \left(\mu^{2\epsilon}\right)^{N-1}\left(\prod_{f=1}^{N}\frac{\mathrm{d}^{D-1}p_f}{(2\pi)^{D-1}}\frac{1}{2E_f}\right)(2\pi)^D\delta^{(D)}(k_a + k_b - \sum_{f=1}^{N}p_f). \tag{11.39}$$

The factor $\mu^{2\epsilon}$ is included to maintain the mass dimension of the cross section. As in this thesis the sum of $|\mathcal{M}|^2$ over helicities and colors $\sum|\mathcal{M}|^2$ has been calculated one can write

$$\mathrm{d}\sigma = \frac{1}{2s}\mathrm{d}\Phi_2 K_{ab}\sum|\mathcal{M}|^2 \tag{11.40}$$

where K_{ab} encodes the averaging over initial state helicities and colors. Specifying to the center-of-mass frame and assuming that $\sum|\mathcal{M}|^2$ is only a function of the modulus of one of the final state particle's 3-momentum $|\vec{p}_i|$ and the angle θ between $\vec{k}_a$ and $\vec{p}_1$ one can write

$$\begin{aligned}\int \mathrm{d}\Phi_2 &= \mu^{2\epsilon}\int\frac{\mathrm{d}|\vec{p}_1|\mathrm{d}\Omega_1^{D-1}}{(2\pi)^{D-2}4E_1E_2}|\vec{p}_1|^{D-2}\delta\left(k_a^0 + k_b^0 - \sqrt{m_1^2 + |\vec{p}_1|^2} - \sqrt{m_2^2 + |\vec{p}_1|^2}\right)\\ &= \frac{1}{(2\pi)^{D-2}}\frac{2\pi^{\frac{D}{2}-1}}{\Gamma(\frac{D}{2}-1)}\mu^{2\epsilon}\int_0^\infty \mathrm{d}|\vec{p}_1|\int_0^\pi \mathrm{d}\cos\theta\frac{1}{4E_1E_2}p_1^{D-2}\sin^{D-4}\theta\\ &\quad\delta\left(k_a^0 + k_b^0 - \sqrt{m_1^2 + |\vec{p}_1|^2} - \sqrt{m_2^2 + |\vec{p}_1|^2}\right).\end{aligned} \tag{11.41}$$

[19] with kinematics $k_a + k_b \to p_1 + \dots p_N$

In the second line the integral over the D-dimensional hypersphere

$$\int \mathrm{d}\Omega^D = \int_0^{2\pi} \mathrm{d}\phi \prod_{i=1}^{D-2} \int_0^{\pi} \sin^i \theta_i \mathrm{d}\theta_i = \frac{2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \tag{11.42}$$

has been used. Because $\sum |\mathcal{M}|^2$ is calculated in terms of Mandelstam variables

$$t = (k_a - p_1)^2$$

$$t = -2\left(|\vec{k}_a|\sqrt{m_1^2 + |\vec{p}_1|^2} - |\vec{k}_a||\vec{p}_1|\cos\theta\right) + m_1^2 \tag{11.43}$$

$$u = (k_a - p_2)^2$$

$$u = -2\left(|\vec{k}_a|\sqrt{m_2^2 + |\vec{p}_1|^2} + |\vec{k}_a||\vec{p}_1|\cos\theta\right) + m_2^2 \tag{11.44}$$

it is useful to perform a change of coordinates yielding

$$\mathrm{d}|\vec{p}_1| \ \mathrm{d}\cos\theta = -\frac{E_1 E_2}{4|\vec{k}_a|^2|\vec{p}_1|^2(E_1+E_2)}\mathrm{d}u \ \mathrm{d}t. \tag{11.45}$$

Inserting 11.45 into 11.41 and using $2|\vec{k}_a| = \sqrt{s} = E_1 + E_2$ gives

$$\begin{aligned}\int \mathrm{d}\Phi_2 =& \frac{1}{s}\frac{\pi^{-\frac{D}{2}+1}}{2^{D-3}\Gamma(\frac{D}{2}-1)}\int \mathrm{d}u \ \mathrm{d}t \ \left(\frac{tu - m_1^2 m_2^2}{\mu^{2\epsilon} s}\right)^{\frac{D-4}{2}} \\ & \frac{1}{4}\Theta(tu - 4m_1^2m_2^2)\delta\left(s + t + u - m_1^2 - m_2^2\right)\end{aligned} \tag{11.46}$$

where the Θ-function comes from the bounds of $|\vec{p}_1|$ and θ visible in 11.41 and the combination of 11.43 and 11.44. Working in $D = 4-2\epsilon$ dimensions and inserting $\Theta(s-4m^2)$ with $m = \frac{m_1+m_2}{2}$ to account for the production threshold one finds

$$\begin{aligned}\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\mathrm{d}u} =& \frac{K_{ab}}{s^2}\frac{\pi S_\epsilon}{\Gamma(1-\epsilon)}\left[\frac{tu - m_1^2m_2^2}{\mu^2 s}\right]^{-\epsilon} \Theta(tu - m_1^2m_2^2) \\ & \Theta(s-4m^2)\delta(s+t+u-m_1^2-m_2^2)\sum|\mathcal{M}|^2\end{aligned} \tag{11.47}$$

where $S_\epsilon = (4\pi)^{-2+\epsilon}$ as defined in [?]. The averaging factors K_{ab} are given by

$$K_{qq} = \frac{1}{4N_c^2} \qquad K_{GG} = \frac{1}{4(1-\epsilon)^2(N_c^2-1)^2} \qquad K_{qG} = \frac{1}{4(1-\epsilon)N_c(N_c^2-1)}. \tag{11.48}$$

Erklärung

Hiermit erkläre ich, dass ich diese Arbeit im Rahmen der Betreuung am Institut für ??? Physik ohne unzulässige Hilfe Dritter verfasst und alle Quellen als solche gekennzeichnet habe.
Vorname Nachname

Dresden, Monat 2012