TECHNISCHE UNIVERSITÄT DRESDEN

Title of Master thesis

Master-Arbeit
zur Erlangung des Hochschulgrades
Master of Science
im Master-Studiengang Physik

vorgelegt von

Sebastian Liebschner
geboren am 08.05.1990 in Meerane

Institut für Kern- und Teilchenphysik Physik
Fachrichtung Physik
Fakultät Mathematik und Naturwissenschaften
Technische Universität Dresden
2016

Eingereicht am xx. Monat 20xx

1. Gutachter: Prof. Dr. XX
2. Gutachter: Prof. Dr. YY

Summary

Abstract

English: motivation: aesthetic: Coleman-Mandula –> Haag-Lopuszanski-Sohnius-Theorem

Abstract

Deutsch

Contents

1 Introduction **1**

2 The Standard Model **2**
2.1 Symmetries and Transformations . . . 2
2.2 The particles of the SM . . . 4
2.3 Electroweak Symmetry Breaking . . . 6
2.4 Quantization . . . 8
2.5 Lagrangian of the SM . . . 8

3 The Minimal Supersymmetric Standard Model **9**
3.1 Supersymmetry as Extention of Poincaré Symmetry . . . 9
3.2 A Generic Supersymmetric Model in Superspace Formulation . . . 9
3.3 The Minimal Supersymmetric Standard Model in superspace formulation . . . 11

4 R-Symmetry **13**
4.1 R-Symmetry Transformation . . . 13
4.2 The Minimal R-symmetric Supersymmetric Standard Model . . . 14
4.3 The R-symmetric supersymmetric Quantumchromodynacis . . . 15

5 Sparticle Production at Tree Level **18**
5.1 Partonic Processes . . . 18
5.2 Partonic Cross Sections . . . 20
5.3 Hadronic Cross Section . . . 21

6 Renormalization of the MRSSM **22**
6.1 Regularization Schemes . . . 22
6.2 Regularization Scheme Dependences . . . 23
6.3 On-Shell Renormalization . . . 24
6.3.1 The Quark Self-Energy . . . 24
6.3.2 The Squark Self-Energy . . . 25
6.3.3 The Glunio Self-Energy . . . 26
6.4 Renormalization of the Gauge Coupling . . . 28
6.4.1 The Quark Self-Energy Revisited . . . 29
6.4.2 The Gluon Self-Energy . . . 29
6.4.3 The $q\bar{q}G$ Vertex Correction . . . 32
6.4.4 The Beta Function . . . 33
6.5 Supersymmetry Restoring Counterterm . . . 34
6.6 $\overline{\mathrm{MS}}$ - Renormalization . . . 35

7 Squark Production at One-Loop **36**

8 Summary and Outlook **37**

9 Appendix **38**

9.1 System of Units and Metric 38
9.2 Colour Algebra $SU(N)$ 38
9.3 Weyl basis and 2-spinor notation 39
9.4 Anticommuting numbers 41
9.5 Feynman rules for the RSQCD 41
9.6 Passarino-Veltman Integrals 43
9.7 Cross section and Phase Space Integration 43

1 Introduction

motivation: aesthetic: Coleman-Mandula –> Haag-Lopuszanski-Sohnius-Theorem
plots for exclusion of squarks in specific SUSY scenarios (from Michael) –> R-Symmetry could be possible explanation for that because:
MSSM-Lagrangian –> trafo rules for superfields under R-symm –> forbidden terms in MRSSM (write down Lagrangian for R-symmetric SUSYQCD)
suppression of squark production in MRSSM by less diagrams (m_{gluino}^{-4} suppression at low energies in MRSSM and only m_{gluino}^{-2} suppression in MSSM)
R-charges of all fields (show in diagram!) –> only if R-charges of final / initial particles are zero, a diagram is allowed in R-symm. model

2 The Standard Model

The standard model(SM) is a relativistic quantum field theory. It contains different fields, whose quantized excitations are interpreted as particles.

2.1 Symmetries and Transformations

Space-Time Symmetries: The SM is defined on Minkowski space, whose coordinates are label with x^μ $\mu = 0, 1, 2, 4$. As a relativistic theory it is invariant under Poincaré transformations, i.e. it is invariant under Lorentz-transformations (with generators $J^{\mu\nu}$) and translations (with generators P^μ) in spacetime. The set of all Poincaré transformations form the Poincaré group, which is a Lie group. Its generators obey the Poincaré-algebra

$$\begin{aligned}
[P^\mu, P^\nu] &= 0 \\
[P^\mu, J^{\nu\rho}] &= i(g^{\mu\nu} P^\rho - g^{\mu\rho} P^\nu) \\
[J^{\mu\nu}, J^{\rho\sigma}] &= i(g^{\nu\rho} J^{\mu\rho} + g^{\mu\sigma} J^{\nu\rho} - g^{\mu\rho} J^{\nu\sigma} - g^{\nu\sigma} J^{\mu\rho}).
\end{aligned} \tag{2.1}$$

The fields of the SM transform in different representations of the Poincaré-group.

Gauge Symmetries: In order to describe interactions of matter particles one uses gauge theories. In the SM which is such a theory matter fields are described by Dirac spinors. The Lagrangian of a free Dirac field reads

$$\mathcal{L}_{Dirac} = \overline{\Psi}(i\partial\!\!\!/ - m)\Psi. \tag{2.2}$$

To include interactions one imposes a local group symmetry (gauge symmetry) upon this Lagrangian. A spinor transforms under a generic gauge transformation like

$$\psi(x) \to U(x)\psi(x), \tag{2.3}$$

where $U(x)$ is an element of the gauge group in question. Because the gauge group is a unitary matrix Lie group it can be written in the form $U(x) = \exp(-igT^a\theta^a(x))$. Here T^a are the self-adjoint generators of the associated Lie algebra, g is the coupling constant of the gauge group and $\theta^a(x)$ are local parameters. The structure constants f_{abc} of a Lie algebra are defined by $[T^a, T^b] = if_{abc}T^c$.
Because the parameters of the gauge group are local the derivative in 2.2 spoils the gauge invariance. In order to rectify gauge invariance of the Lagrangian one has to introduce a further field for each index a of the generators - the gauge vector $G^{a\mu}$. The matrix valued

gauge vector $G^\mu := G^{a\mu}T^a$ transforms as

$$G^\mu(x) \to U(x)G^\mu(x)U^{-1}(x) + \frac{i}{g}U(x)\partial^\mu U^{-1}(x) \tag{2.4}$$

One then obtains the gauge invariant Lagrangian for a non-abelian gauge theory first found by Yang and Mills [cite original paper]

$$\mathcal{L}_{Yang-Mills} = \overline{\Psi} i \not{D} \Psi - \frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} \tag{2.5}$$

In comparison to the Dirac-Lagrangian 2.2 the partial derivative is replaced with the gauge covariant derivative $D^\mu = \partial^\mu + igT^aG^{a\mu}$ and the gauge invariant kinetic term for the gauge fields is added. This comprises the field strenth-tensor $F^{a\mu\nu}$ which is in its matrix valued form defined by $F^{\mu\nu} = F^{a\mu\nu}T^a = \frac{1}{ig}[D^\mu, D^\nu]$.
The gauge group of the SM is a direct product ??? of the three gauge groups: $U_Y(1)$, $SU_L(2)$ and $SU_C(3)$[1]. The elements $U(x)$ of those and the infinitesimal transformations of the spinors and gauge vectors are given in table 2.1.
These gauge groups give rise to 3 forces: the strong force, the weak force and the electromagnetic force.

$U_Y(1)$	$U(x) = \exp\left(-ig_Y \frac{\hat{Y}}{2}\theta_Y(x)\right)$ $\psi(x) \to \left(1 - ig_Y\frac{\hat{Y}}{2}\theta_Y(x)\right)\psi(x)$ $B^\mu(x) \to B^\mu(x) + \partial^\mu\theta_Y(x)$
$SU_L(2)$	$U(x) = \exp\left(-ig_w\vec{\tau}\cdot\vec{\theta}_w(x)\right)$ $\psi(x) \to \left(\mathbb{1} - ig_w\vec{\tau}\cdot\vec{\theta}_w(x)\right)\psi(x)$ $W^{a\mu}(x) \to W^{a\mu}(x) + \partial^\mu\theta^a_w(x) + g_w\varepsilon^{abc}\theta^b_w(x)W^{c\mu}(x)$
$SU_C(3)$	$U(x) = \exp\left(-ig_sT^a\cdot\theta^a_s(x)\right)$ $\psi(x) \to \left(\mathbb{1} - ig_sT^a\cdot\theta^a_s(x)\right)$ $G^{a\mu}(x) \to G^{a\mu}(x) + \partial^\mu\theta^a_s(x) + g_sf_{abc}\theta^b_s(x)G^{c\mu}$

Table 2.1: The table lists the explicit element $U(x)$ of the gauge groups $U_Y(1)$, $SU_L(2)$ and $SU_C(3)$ and the infinitesimal transformations of spinor and vector fields.
The hypercharge operator $\hat{Y}$ gives the eigenvalue of the hypercharge of the field it is applied to. $\vec{\tau}$ and T^a are the geneators of $SU_L(2)$ and $SU_C(3)$ respectively. In the fundamental representation one has $\vec{\tau} = \frac{\vec{\sigma}}{2}$ where $\vec{\sigma}$ has the 3 Pauli matrices as components and $T^a = \frac{\lambda^a}{2}$ where λ^a are the 8 Gell-Mann matrices. ε_{abc} and f_{abc} are the structure constants of $SU_L(2)$ and $SU_C(3)$ respectively.

[1]The subscript stands for the associated charge of the groups respectively: Y for hypercharge, L for left handedness (weak Isospin I_3) and C for color

2.2 The particles of the SM

In the SM different matter particles take part in different interactions, i.e. their corresponding spinor couples to different gauge vectors.
If a spinor couples to a certain gauge vector it transforms non trivially (like indicated in table 2.1) under the gauge group which is associated with this gauge vector[2]. This means if a particle couples to a certain force its charge which is associated with this force is nonzero.
The charges of particles for a force are defined as the eigenvalues of the generators which correspond to the force.

The Quarks:

Quarks are strongly interacting fermions, which means their spinors transform non trivially under $SU_C(3)$. Because they transform in the fundamental representation of $SU_C(3)$ this means a quark spinor is built up by 3 spinors each carrying another color. This splitting of the quark spinor in colors is often suppressed for the sake of simplicity. This convention is adopted throughout this thesis.
Furthermore the left handed component of quarks interact weakly, which means that their spinors transform (in the fundamental representation) under $SU_L(2)$ meaning that 2 left handed quark spinors are assembled in a doublet.
Finally all quarks carry a hypercharge. In section ??? the mechanism of electroweak symmetry breaking is described. This mechanism explains how electromagnetism arises from the groups $SU_L(2)$ and $U_Y(1)$. All quarks interact electromagnetically.
After all there are 6 quarks which are listed in table2.2. They are categorized in 3 generations because their quantum numbers except (for their mass) is the same in each generation. The two types of quarks which have distinct quantum numbers are the up-type-quarks and the down-type quarks. A up-type-quark and the down-type quark of the same generation built up a doublet.

The Leptons:

Leptons do not interact strongly. They take part in the weak and the electromagnetic interaction, i.e. their spinors transform under the fundamental representation of $SU_L(2)$ and $U_Y(1)$. As for the quarks only the left handed components interact weakly.
As for the quarks there are 6 leptons which are classified into 3 generations (see table 2.2). In each generation is a lepton with a negative electrical charged and an electrically neutral lepton. The latter ones are referred to as neutrinos. Right handed neutrinos have not been observed (yet) and are therefore absent in the SM. The former are called electron, muon and tau. Each left handed leptons with an electric charge is assembled with its neutrino in a doublet.

[2]In the SM all matter particles transform in the fundamental (or trivial) representation of the gauge group in question.

Quarks and Leptons are the matter particles of the SM. They are listed together with their charges for the different forces in table 2.3. One has the color for strong interactions, the third component of the weak isospin I_3 for weak interactions(the eigenvalue of the third generator of the $SU_L(2)$) and the half of the hypercharge $\frac{Y}{2}$ to obtain the electric charge Q via the Gell-Mann–Nishijima formula: $Q = I_3 + \frac{Y}{2}$.
Because the left and right-handed parts of spinors transform differently under the $SU_L(2)$ they are listed separately.
All quarks occur with three different colors.
In the last row the Higgs-boson is listed. Its associated field is responsible for the mass of elementary particles. That is explained in section 2.3.

Particle		1^{st} generation		2^{nd} generation		3^{rd} generation	
u_i	up-type-Quark	u	up-Quark	c	charm-Quark	t	top-Quark
d_i	down-type-Quark	d	down-Quark	s	strange-Quark	b	bottom-Quark
e_i	Charged Lepton	e	Electron	μ	Muon	τ	Tau
ν_i	Neutrino	ν_e	Electron Neutrino	ν_μ	Muon Neutrino	ν_τ	Tau Neutrino

Table 2.2: The matter particles of the SM. Listed are the symbol and the name of the particles.

Particle	Symbol	color	I_3	$\frac{Y}{2}$	Q
Left handed Quarks	$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$	red, green, blue	$\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$+\frac{1}{6}$	$\begin{pmatrix} +\frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
Right-handed Quarks	u_{iR}	red, green, blue	0	$+\frac{2}{3}$	$+\frac{2}{3}$
	d_{iR}	red, green, blue	0	$-\frac{1}{3}$	$-\frac{1}{3}$
Left-handed Leptons	$\ell_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$	-	$\begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Right-handed Leptons	e_{iR}	-	0	$+1$	$+1$
Higgs	H	-	$-\frac{1}{2}$	$+\frac{1}{2}$	0

Table 2.3: This table lists all matter particles in the SM and the Higgs particle with their charges for all forces. This is the color, the weak isospin I_3, the half of their hypercharge and their electrical charge. The index $i = 1, 2, 3$ labels the generation of the matter particles and is written out in table 2.2

The Force Particles

The force particles are described by gauge fields. The gluon field is the gauge field of $SU_C(3)$. Because the $SU_C(3)$ has 8 generators there are 8 gluons. Their coupling constant is g_s.

For the other force particles in the SM - the $W^{\pm}$ bosons, the Z_0 boson and the photon the is situation is slightly more involved. They are obtained as a mixture of the W^b_μ ($b = 1, 2, 3$)and the B_μ field. This mixing procedure is referred to as electroweak symmetry breaking and is explained in section???.
For the moment being the coupling constants and gauge fields before and after EWSB are quoted in table 2.4

before EWSB			after EWSB		
group	coupling constant	gauge field	coupling constant	gauge field	Particle
$SU_C(3)$	g_s	G^a_μ	g_s	G^a_μ	Gluon
$SU_L(2)$	g_w	W^b_μ	$g_W = \sqrt{2}g_w,$	$W^{\pm}_\mu,$	$W^{\pm},$
			$g_Z = \sqrt{g_w^2 + g_Y^2}$	Z^0_μ	Z^0 Boson
$U_Y(1)$	g_Y	B_μ	$e = g_Y \cdot c_w$	A_μ	Photon

Table 2.4: The gauge fields and their coupling constants before and after EWSB. The Gluon field is not affected by EWSB. $a = 1, \dots 8$ and $b = 1, 2, 3$ label the number of gauge fields. c_w is the cosine of the electroweak mixing angle defined in 2.3

The Lagrangian of the SM is built up by qualitatively different terms. Firstly there are the kinetic and minimal coupling terms of the matter fields

$$\mathcal{L}_{matter} = \sum_{i=1}^{3} \left(\overline{\ell}_{iL} i \not{D} l_{iL} + \overline{e}_{iR} i \not{D} e_{iR} + \overline{q}_{iL} i \not{D} q_{iL} + \overline{u}_{iR} i \not{D} u_{iR} \right) \tag{2.6}$$

where $i = 1, 2, 3$ labels the generations of matter. The gauge covariant derivative is given by

$$D_\mu = \partial_\mu + i g_Y \frac{\hat{Y}}{2} + i g_w \vec{\tau} \cdot \vec{W}^\mu + i g_s T^a G^\mu_a \tag{2.7}$$

where for each field the corresponding representation (fundamental or trivial) of the gauge group is to be inserted (see table 2.3). The hyper charge operator $\hat{Y}$ gives the eigenvalue of the hypercharge of the field it is applied to. These can be taken from table 2.3. The kinetic terms of the gauge fields are given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} W^{a\mu\nu} W^a_{\mu\nu} - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} \tag{2.8}$$

2.3 Electroweak Symmetry Breaking

So far no mass terms like in the Dirac Lagrangian 2.2 have been introduced. The reason for that is that they are not gauge invariant. The same argument forbids terms like $-\frac{m^2}{2} A^\mu A_\mu$ for a generic gauge boson. EWSB gives masses to these particles while maintaining gauge invariance. To this end one considers a complex scalar doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.9}$$

which acquires a vacuum expectation value (VEV) $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ by the Higgs potential

$$V(\Phi^\dagger \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \tag{2.10}$$

where $\mu^2, \lambda > 0$. The Higgs sector of the SM reads

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi). \tag{2.11}$$

The Higgs doublet couples to the gauge fields of $SU_L(2)$ and $U_Y(1)$ in the fundamental representation. Inserting an expansion[3] around the VEV $\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + i\sigma(x)) \end{pmatrix}$ one obtains quadratic terms, i.e. mass terms, in the gauge fields in question. In order to get mass eigenstates for the gauge bosons one performs the transformation

$$\begin{pmatrix} A_\mu \\ Z^0_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} \qquad W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp W^2_\mu), \tag{2.12}$$

where the electroweak mixing angle is given by $\cos\theta_w = \frac{g_w}{\sqrt{g_w^2 + g_Y^2}}$ These gauge fields acquire masses:

$$m_W = \frac{g_w}{2} v \qquad m_Z = \frac{\sqrt{g_w^2 + g_Y^2}}{2} v \qquad m_A = 0. \tag{2.13}$$

Apart from the massive $W^\pm_\mu$ and $Z^0_\mu(x)$ bosons one obtains the massless photon A_μ. As the photon is massless it is still associated with a gauge symmetry called $U_{em}(1)$. One therefore often writes EWSB as the braeking of the gauge group $SU_L(2) \times U_Y(1)$ to $U_Y(1)$.
Matter particles acquire mass via Yukawa coulings to the Higgs doublet. For up-type-quarks one uses that the charge conjugate of Φ: $\Phi^C = i\sigma^2 \Phi^*$ transform as Φ.

$$\mathcal{L}_{Yukawa} = \sum_{i,j=1}^{3} \left(y^e_{ij} \bar{\ell}_L \Phi e_R + y^d_{ij} \bar{q}_L \Phi d_R + y^u_{ij} \bar{q}_L \Phi^C u_R \right) + h.c. \tag{2.14}$$

[3]The complex $\phi^+(x)$ and the real $\sigma(x)$ are so called massless Goldstone bosons. These degrees of freedom can be absorbed in the longitudinal polarized degrees of freedom of the arising gauge bosons $W^\pm$ and Z^0. The real $H(x)$ is the Higgs field, whose excitation is the Higgs boson.

where y^e, y^d, y^u are 3×3 matrices in generation space. The fermion mass matrices are therefore:

$$m^e_{ij} = \frac{y^e_{ij}}{\sqrt{2}}v \qquad m^d_{ij} = \frac{y^d_{ij}}{\sqrt{2}}v \qquad m^u_{ij} = \frac{y^u_{ij}}{\sqrt{2}}v. \tag{2.15}$$

The quark mass matrices are not diagonal. One therefore has to distinguish between interaction and mass eigenstates of the quark. The corresponding transformation matrix is the well known CKM-matrix [quote (original paper)].
The upshot of EWSB are masses for all matter particles except for the neutrinos and masses for the gauge bosons $W^\pm$ and Z^0.

2.4 Quantization

The Quantization of Spin 0 and Spin $\frac{1}{2}$ fields yield no complication in the Lagrangian formalism. To quantize Spin 1 fields it turns out that the usual gauge invariance needs to be replaced by the so called BRST invariance. This results in 2 extra terms in the Lagrangian. Firstly there is the gauge fixing term

$$\mathcal{L}_{R_\xi} = -\frac{1}{2\xi_A}(\partial^\mu A_\mu)^2 - \frac{1}{2\xi_W}|\partial^\mu W^+_\mu - m_W \xi_W \phi^+|^2 - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu - m_Z \xi_Z \sigma)^2 CHECK! \tag{2.16}$$

Here R_ξ is chosen, where the parameters ξ_i specify the gauge. The last 2 terms are modified with the Goldstone bosons from section 2.3 because of the massiveness of the $W^\pm$ an Z^0 bosons.
Secondly there is a ghost Lagrangian

$$\mathcal{L}_{ghost} = \tag{2.17}$$

see [Denner Gauge Theories S622]

2.5 Lagrangian of the SM

The complete Lagrangian of the SM reads

$$\mathcal{L}_{SM} = \mathcal{L}_{matter} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{R_\xi} + \mathcal{L}_{ghost} \tag{2.18}$$

with the corresponding parts of the previous chapters

3 The Minimal Supersymmetric Standard Model

3.1 Supersymmetry as Extention of Poincaré Symmetry

The superalgebra is defined by

$$\begin{aligned}\{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} &= 0,\\ \{Q_\alpha, \overline{Q}_{\dot\alpha}\} &= 2\sigma^\mu_{\alpha\dot\alpha},\\ [P^\mu, Q_\alpha] = [P^\mu, \overline{Q}_{\dot\alpha}] &= 0,\\ [Q_\alpha, J^{\mu\nu}] &= \frac{1}{2}(\sigma^{\mu\nu})_\alpha^\beta Q_\beta\end{aligned} \tag{3.1}$$

A representation in the form of differential operators is given by

$$\begin{aligned}P^\mu &= i\partial^\mu\\ J^{\mu\nu} &= i(x^\mu\partial^\nu - x^\nu\partial^\mu)\\ Q_\alpha &= i(\partial_\alpha + i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial_\mu\\ \overline{Q}_{\dot\alpha} &= i(-\overline{\partial}_{\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu).\end{aligned} \tag{3.2}$$

INTRODUCE AND MOTIVATE SUSY COVARIANT DERIVATIVES $\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\overline{\theta})_\alpha\partial_\mu$ is the chiral covariant derivative and $\overline{\mathcal{D}\mathcal{D}} = \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}^{\dot\alpha}$

3.2 A Generic Supersymmetric Model in Superspace Formulation

This chapter outlines the generic ingredients and terms of a supersymmetric model. To this end it is practical to work in the language of superspace and superfields.

Superspace is a manifold obtained by enlarging Minkowski space, whose coordinates are label with x^μ, with four anticommuting numbers: θ^α and $\overline{\theta}^{\dot\alpha}$, where $\alpha,\ \dot\alpha = 1, 2$. Superfields are functions on superspace.

The for the MSSM relevant superfields[4] are the chiral superfield $\hat\Phi$, the antichiral superfield $\overline{\hat\Phi}$ and the vector superfield V. Chiral superfields are defined by the restriction $\overline{\mathcal{D}}_{\dot\alpha}\hat\Phi = 0$, antichiral superfields by $\mathcal{D}_\alpha\overline{\hat\Phi} = 0$ and vector superfields by the condition of being real $V^\dagger = V$. Their component decomposition reads[5]

$$\begin{aligned}\hat\Phi(x,\theta,\overline{\theta}) &= A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) - i\theta\sigma^\mu\overline{\theta}\partial_\mu A(x) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial_\mu\partial^\mu A(x) - \frac{i}{\sqrt{2}}\theta\theta\overline{\theta}\overline{\sigma}^\mu\partial_\mu\psi(x)\\ \overline{\hat\Phi}(x,\theta,\overline{\theta}) &= A^\dagger(x) + \sqrt{2}\overline{\theta\psi}(x) + \overline{\theta\theta}F^\dagger(x) + i\theta\sigma^\mu\overline{\theta}\partial_\mu A^\dagger(x) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial_\mu\partial^\mu A^\dagger(x) - \frac{i}{\sqrt{2}}\overline{\theta\theta}\theta\sigma^\mu\partial_\mu\overline{\psi}(x)\\ \hat V(x,\theta,\overline{\theta}) &= \theta\sigma^\mu\overline{\theta}v_\mu + i\theta\theta\overline{\theta\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x),\end{aligned} \tag{3.3}$$

[4]Superfield are throughout this thesis labeled with a hat.

[5]For tho vector superfield Wess-Zumino-gauge is applied.

where $A(x)$ and $F(x)$ are complex scalar fields, $\psi(x)$ and $\lambda(x)$ are left handed Weyl spinors and $D(x)$ being a real scalar field.
The superfields transform under a generic gauge transformation as

$$\begin{aligned}\hat{\Phi} &\to \mathrm{e}^{-2ig\Lambda}\Phi \\ \hat{\overline{\Phi}} &\to \hat{\overline{\Phi}}\mathrm{e}^{2ig\overline{\hat{\Lambda}}} \\ \mathrm{e}^{2g\hat{V}} &\to \mathrm{e}^{-2ig\hat{\overline{\Lambda}}}\mathrm{e}^{2g\hat{V}}\mathrm{e}^{2ig\hat{\Lambda}},\end{aligned} \tag{3.4}$$

where $\hat{\Lambda} = \hat{\Lambda}^a T^a$ and $\hat{V} = \hat{V}^a T^a$. $\hat{\Lambda}^a$ is an arbitrary chiral superfield and the T^a are the generetors of the Lie algebra, which is associated to the gauge group in question. g is the gauge coupling constant of the gauge group.
One can therefore construct the important gauge invariant term $\int \mathrm{d}^4\theta\ \hat{\overline{\Phi}}\mathrm{e}^{2g\hat{V}}\hat{\Phi}$. If one introduces the gauge covariant derivative $D_\mu = \partial + igT^a v^a_\mu$ the component decomposition reads

$$\begin{aligned}\int \mathrm{d}^4\theta\hat{\overline{\Phi}}\mathrm{e}^{2g\hat{V}}\hat{\Phi} &= F^\dagger F + (D_\mu A)^\dagger (D^\mu A) + \overline{\psi}\overline{\sigma}^\mu i D_\mu \psi \\ &- \sqrt{2}g\left(-i(A^\dagger T^a A)\lambda^a + i\overline{\lambda}^a(AT^a A^\dagger)\right) + g(A^\dagger T^a A)D^a.\end{aligned} \tag{3.5}$$

Therefore this term gives rise to the kinetic terms of the components of the chiral and antichiral superfields A, $A^\dagger$, ψ and $\overline{\psi}$, their minimal coupling to the gauge fields v^a_μ and their superpartners λ^a and $\overline{\lambda}^a$ and terms involving the auxiliary fields F, $F^\dagger$ and D.
With the field-strength chiral superfields $\hat{W}_\alpha = -\frac{1}{4}\overline{\mathcal{D}\mathcal{D}}(\mathrm{e}^{-2gV}\mathcal{D}_\alpha \mathrm{e}^{2gV})$ one can write down a gauge invariant term yielding the kinetic terms of the gauge fields and their superpartners:

$$\int \mathrm{d}^2\theta \frac{1}{16g^2}\hat{W}^{\alpha a}\hat{W}^a_\alpha + h.c. = \frac{1}{2}D^a D^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{i}{2}\overline{\lambda}^a\overline{\sigma}^\mu(D_\mu\lambda^a) + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\overline{\lambda}^a). \tag{3.6}$$

A third generic term in a supersymmetric theory arises from the superpotential $W(\hat{\Phi})$ which is a holomorphic function in the chiral superfields:

$$\int \mathrm{d}^2\theta\ W(\hat{\Phi}). \tag{3.7}$$

A renormalizable superpotential is given by $W(\hat{\Phi}) = c_i\hat{\Phi} + \frac{m_{ij}}{2}\hat{\Phi}_i\hat{\Phi}_j + \frac{g_{ijk}}{3!}\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k$. The component decomposition of the corresponding terms is

$$\begin{aligned}\int \mathrm{d}^2\theta\ \hat{\Phi}_1 &= F_1 \\ \int \mathrm{d}^2\theta\ \hat{\Phi}_1\hat{\Phi}_2 &= A_1F_2 + F_1A_2 - \psi_1\psi_2 \\ \int \mathrm{d}^2\theta\ \hat{\Phi}_1\hat{\Phi}_2\hat{\Phi}_3 &= F_1A_2A_3 + A_1F_2A_3 + A_1A_2F_3 - A_1\psi_2\psi_3 - \psi_1A_2\psi_3 - \psi_1\psi_2A_3.\end{aligned} \tag{3.8}$$

The Lagrangian for a supersymmetric theory is therefore given by

$$\begin{aligned}\mathcal{L}_{SUSY} &= \mathcal{L}_{matter} + \mathcal{L}_{gauge} + \mathcal{L}_{superpot} \\ &= \int \mathrm{d}^4\theta \hat{\bar{\Phi}} \mathrm{e}^{2g\hat{V}} \hat{\Phi} + \left(\int \mathrm{d}^2\theta \frac{1}{16g^2} \hat{W}^{\alpha a} \hat{W}^a_\alpha + h.c. \right) + \int \mathrm{d}^2\theta \; W(\hat{\Phi}) \end{aligned} \tag{3.9}$$

Observing the component decomposition 3.5,3.6, 3.8 of the 3 parts of this Lagrangian, one observes that the F and D fields have no kinetic term and are therefor auxiliary fields which can be eliminated by their equation of motion $\frac{\partial \mathcal{L}}{\partial \phi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}$ with $\phi = F, D$. Doing this one obtains

$$\begin{aligned}\mathcal{L}_D &= \frac{1}{2} D^a D^a + g A^\dagger T^a D^a A \qquad \Rightarrow \qquad D^a = -A^\dagger T^a A \\ \mathcal{L}_D &= -\frac{1}{2} \left(A^\dagger T^a A \right)^2 \end{aligned} \tag{3.10}$$

and

$$\begin{aligned}\mathcal{L}_F &= F_i^\dagger F_i + \left(c_i F_i + m_{ij} F_i A_j + \frac{g_{ijk}}{2} F_i A_j A_k + h.c. \right) \qquad \Rightarrow \qquad F_i^\dagger = -\frac{\partial W(A)}{\partial A_i} \\ \mathcal{L}_F &= -\left| \frac{\partial W(A)}{\partial A_i} \right|^2 \end{aligned} \tag{3.11}$$

3.3 The Minimal Supersymmetric Standard Model in superspace formulation

The Lagrangian for the MSSM[6] reads

$$\begin{aligned}\mathcal{L}_{MSSM} = \int \mathrm{d}^4\theta \Big[& \hat{\bar{Q}} \mathrm{e}^{2g'\hat{V}' + 2g\hat{V} + 2g_s\hat{V}_s} \hat{Q} + \hat{\bar{U}} \mathrm{e}^{2g'\hat{V}' + 2g\hat{V} - 2g_s\hat{V}_s^T} \hat{U} + \hat{\bar{D}} \mathrm{e}^{2g'\hat{V}' + 2g\hat{V} - 2g_s\hat{V}_s^T} \hat{D} \\ & + \hat{\bar{L}} \mathrm{e}^{2g'\hat{V}' + 2g\hat{V}} \hat{L} + \hat{\bar{E}} \mathrm{e}^{2g'\hat{V}' + 2g\hat{V}} \hat{E} \\ & + \hat{\bar{H}}_d \mathrm{e}^{2g'\hat{V}' + 2g\hat{V}} \hat{H}_d + \hat{\bar{H}}_u \mathrm{e}^{2g'\hat{V}' + 2g\hat{V}} \hat{H}_u \Big] \\ + \int \mathrm{d}^2\theta & \left[\frac{1}{16g'^2} \hat{W}'^\alpha \hat{W}'_\alpha + \frac{1}{16g^2} \hat{W}^{a\alpha} \hat{W}^a_\alpha + \frac{1}{16g_s^2} \hat{W}_s^{a\alpha} \hat{W}^a_{s\alpha} \right] + h.c. \\ + \int \mathrm{d}^2\theta & \; W_{MSSM} + h.c. \\ + \mathcal{L}_{soft}&. \end{aligned} \tag{3.12}$$

Apart from the already discussed terms in the first 4 lines of 3.12 there is a superpotential W_{MSSM}:

$$W_{MSSM} = y_d \hat{H}_d \hat{Q} \hat{D} + y_u \hat{H}_u \hat{Q} \hat{U} + y_e \hat{H}_d \hat{L} \hat{E} - \mu \hat{H}_d \hat{H}_u \tag{3.13}$$

[6] This is the Lagrangian on the classical level, i.e. there are neither gauge fixing nor ghost terms.

and terms which break supersymmetry softly, i.e. terms with coupling constants with positive mass dimension.

$$\begin{aligned}
\mathcal{L}_{soft} = &-M_{\tilde{Q}}^2|\tilde{q}_L|^2 - M_{\tilde{U}}^2|\tilde{u}_R|^2 - M_{\tilde{D}}^2|\tilde{d}_R|^2 \\
&- M_{\tilde{L}}^2|\tilde{l}_L|^2 - M_{\tilde{E}}^2|\tilde{e}_R|^2 - M_{H_d}^2|H_d|^2 - M_{H_u}^2|H_u|^2 \\
&+ \frac{1}{2}\left(M_1\lambda\lambda + M_2\lambda^a\lambda^a + M_3\lambda_s^a\lambda_s^a\right) + h.c. \\
&- \Big(A_d y_d H_d \tilde{q}_L \tilde{d}_R^\dagger + A_u y_u H_u \tilde{q}_L \tilde{u}_R^\dagger + A_e y_e H_d \tilde{l}_L \tilde{e}_R^\dagger - B\mu H_d H_u\Big) + h.c.
\end{aligned} \tag{3.14}$$

The field content of the MSSM is summarized in 3.1

Superfield	Components	$SU_\mathrm{C}(3) \times SU_\mathrm{L}(2) \times U_\mathrm{Y}(1)$
$\hat{\Phi}$	$A,\, \psi$	
$\hat{V}$	$\lambda,\, v_\mu$	
$\hat{Q}$	$\tilde{q}_L = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix},\, q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(3,\, 2,\, \frac{1}{6})$
$\hat{U}$	$\tilde{u}_R^\dagger,\, u_R$	$(3^*,\, 1,\, -\frac{2}{3})$
$\hat{D}$	$\tilde{d}_R^\dagger,\, d_R$	$(3^*,\, 1,\, +\frac{1}{3})$
$\hat{L}$	$\tilde{l}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix},\, l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(1,\, 2,\, -\frac{1}{2})$
$\hat{E}$	$\tilde{e}_R^\dagger,\, e_R$	$(1,\, 1,\, 1)$
$\hat{H}_d$	$H_d,\, \tilde{H}_d$	$(1,\, 2,\, -\frac{1}{2})$
$\hat{H}_u$	$H_u,\, \tilde{H}_u$	$(1,\, 2,\, +\frac{1}{2})$
$\hat{V}'$	$\lambda',\, B_\mu$	$(1,\, 1,\, 0)$
$\hat{V}^a$	$\lambda^a,\, W_\mu^a$	$(1,\, 3,\, 0)$
$\hat{V}_s^a$	$\lambda_s^a,\, G_\mu^a$	$(8,\, 1,\, 0)$

Table 3.1: The table shows the field content of the MSSM in terms of the superfields and their component decomposition. The first two lines show the decomposition of the generic superfields (cf. 3.3).
The third column shows the representation (for $SU_\mathrm{C}(3)$ and $SU_\mathrm{L}(2)$) in which the fields transform and the charges of the fields for $U_\mathrm{Y}(1)$.

4 R-Symmetry

4.1 R-Symmetry Transformation

R-symmetry is a global $U(1)$ symmetry. R-symmetry should not be confused with R-parity which is a discrete Z_2 symmetry. A continuous global symmetry implies according to Noether's theorem a conserved charge. In the case of R-symmetry this charge is called R-charge and one therefore refers to R-symmetry as $U_R(1)$.
The defining property of $U_R(1)$ is that the anticommuting coordinates θ^α and $\overline{\theta}^{\dot{\alpha}}$ transform like

$$\theta \to \mathrm{e}^{i\alpha}\theta \qquad\qquad \overline{\theta} \to \mathrm{e}^{-i\alpha}\overline{\theta}, \tag{4.1}$$

where α parametrizes the transformation. This in turn implies that R-symmetry does not commute with supersymmetry, meaning that superpartners do not have the same R-charge.
The transformation of chiral and vector superfields reads

$$\begin{aligned} \hat{\Phi}(x,\theta,\overline{\theta}) &\to \mathrm{e}^{ir_{\hat{\Phi}}\alpha}\, \hat{\Phi}(x,\mathrm{e}^{i\alpha}\theta,\mathrm{e}^{-i\alpha}\overline{\theta}) \\ \hat{V}(x,\theta,\overline{\theta}) &\to \hat{V}(x,\mathrm{e}^{i\alpha}\theta,\mathrm{e}^{-i\alpha}\overline{\theta}). \end{aligned} \tag{4.2}$$

If one inserts the component decomposition 3.3 of the superfields one can read off the R-charges of the component fields.

4.2 The Minimal R-symmetric Supersymmetric Standard Model

The MSSM with additional R-symmetry is called minimal R-symmetric supersymmetric standard model (MRSSM). If one imposes R-symmetry upon the MSSM one is faced with a certain arbitrariness, i.e. the choice of the R-charges of the chiral superfields. In this thesis the R-charges are chosen in that way, that every Standard model particle has R-charge zero. Following this one obtains the R-charges of all particles which are summed up in table 4.2. The gauge, matter and H-Higgs fields are the fields of the MSSM. Below the horizontal line one finds the fields which are not present in the MSSM, i.e. the R-Higgs and adjoint chiral fields. These occur for the following reason.
Since in the MSSM the gauginos are Majorana particles their mass terms reads

$$\mathcal{L}_{Majorana\ mass} = -m\lambda\lambda + h.c. \tag{4.3}$$

which is not R-invariant because the Weyl fermion λ has R-charge $+1$. The only other way to account for a fermion mass is to write down a Dirac mass term.

$$L_{Dirac\ mass} = -m\chi\lambda + h.c. \tag{4.4}$$

In order to get a R-symmetric mass term on has to choose the R-charge of the new Weyl-spinor χ to be the opposite of λ.
This explains the necessity of enlarging the field content if one imposes R-symmetry.
Of course the new Weyl-spinor χ must have also a superpartner. One chooses this superpartner to be a scalar, i.e. the additional Weyl fermion comes from a chiral superfield. In order to maintain gauge invariance this chiral superfield has to transform in the adjoint representation, hence the name adjoint chiral in table 4.2. To fix notation the component decomposition of the 8 chiral supermultipletts assosiated to the gluons is given by

$$\hat{O}^a(x,\theta,\overline{\theta}) = \sigma^a + \sqrt{2}\theta i\chi^a + \dots \qquad a = 1,\dots,8. \tag{4.5}$$

superfield		boson		fermion	
$\hat{\Phi}$	$r_{\hat{\Phi}}$	A	$r_{\hat{\Phi}}$	ψ	$r_{\hat{\Phi}} - 1$
$\hat{V}$	0	v^μ	0	λ	$+1$

Table 4.1: This table shows the R-charges of a generic chiral and vector superfield.

Field	Superfield		Boson		Fermion	
Gauge Vector	$\hat{g}$, $\hat{W}$, $\hat{B}$	0	g, W, B	0	$\tilde{g}$, $\tilde{W}$, $\tilde{B}$	+1
Matter	$\hat{L}$, $\hat{E}$	0	$\tilde{l}$, $\tilde{e}_R$	+1	l, e_R	0
	$\hat{Q}$, $\hat{D}$, $\hat{U}$	+1	$\tilde{q}$, $\tilde{d}_R^\dagger$, $\tilde{u}_R^\dagger$	+1	q, d_R, u_R	0
H-Higgs	$\hat{H}_{d,u}$	0	$H_{d,u}$	0	$\tilde{H}_{d,u}$	-1
R-Higgs	$\hat{R}_{d,u}$	+2	$R_{d,u}$	+2	$\tilde{R}_{d,u}$	+1
Adjoint Chiral	$\hat{O}$, $\hat{T}$, $\hat{S}$	0	O, T, S	0	$\tilde{O}$, $\tilde{T}$, $\tilde{S}$	-1

Table 4.2: This table shows the R-charges of all particles in the MRSSM.

The scalar components σ^a are called scalar gluons and the Weyl spinors χ^a are called octinos. The same argument as for the adjoint chiral explains the existence of additional Higgs-superfields which are referred to as R-Higgs fields.
But instead of including more fields in the model R-symmetry also forbids terms which are allowed by supersymmetry. For the above choice of R-charges the μ-term in 3.13 and the A-terms in the last line of 3.14 are excluded. As a consequence terms which allow flavor violating processes like $\mu \to e\gamma$ are allowed in the MSSM but forbidden in the MRSSM [Kribs, Popitz, Weiner].

4.3 The R-symmetric supersymmetric Quantumchromodynacis

The subject of this thesis is the phenomenology of the strongly coupling sector of the MRSSM. The R-symmetric supersymmetric quantumchromodynamics (RSQCD) is therefore considered closer. Its Lagrangian reads

$$\begin{aligned}\mathcal{L}_{RSQCD} = &\int \mathrm{d}^4\theta \, \left(\bar{\hat{Q}}_L \mathrm{e}^{2g_s\hat{V}_s}\hat{Q}_L + \bar{\hat{Q}}_R \mathrm{e}^{-2g_s\hat{V}_s^T}\hat{Q}_R + \bar{\hat{O}}\mathrm{e}^{2g_s\hat{V}_s^{fund}}\hat{O}\right) \\ &+ \left(\int \mathrm{d}^2\theta \frac{1}{16g_s^2}\hat{W}_s^{a\alpha}\hat{W}_{s\alpha}^a + h.c.\right) + \mathcal{L}_{soft} \end{aligned} \tag{4.6}$$

where in terms of component fields the terms are given by

$$\int \mathrm{d}^4\theta \; \hat{\overline{Q}}_L \mathrm{e}^{2g_s\hat{V}_s}\hat{Q}_L = F_L^\dagger F_L + (D_\mu\tilde{q}_L)^\dagger(D^\mu\tilde{q}_L) + \overline{q}_L\overline{\sigma}^\mu iD_\mu q_L$$
$$-\sqrt{2}g_s\left(-i(\tilde{q}_L^\dagger T^a q_L)\lambda^a + i\overline{\lambda}^a(\overline{q}_L T^a\tilde{q}_L)\right) + g_s\tilde{q}_L^\dagger T^a D^a\tilde{q}_L \tag{4.7}$$
$$\int \mathrm{d}^4\theta \; \hat{\overline{Q}}_R \mathrm{e}^{-2g_s\hat{V}_s^T}\hat{Q}_R = F_R^\dagger F_R + (D_\mu\tilde{q}_R)^\dagger(D^\mu\tilde{q}_R) + \overline{q}_R\overline{\sigma}^\mu iD_\mu q_R$$
$$+\sqrt{2}g_s\left(-i(\tilde{q}_R T^{*a} q_R)\lambda^a + i\overline{\lambda}^a(\overline{q}_R T^{*a}\tilde{q}_R^\dagger)\right) - g_s\tilde{q}_R T^{*a} D^a\tilde{q}_R^\dagger \tag{4.8}$$
$$\int \mathrm{d}^4\theta \; \hat{\overline{O}} \mathrm{e}^{2g_s\hat{V}_s^{fund}}\hat{O} = F_O^\dagger F_O + (D_\mu\sigma^a)^\dagger(D^\mu\sigma^a) + \overline{\chi}\overline{\sigma}^\mu iD_\mu\chi$$
$$-\sqrt{2}g_s\left(-i(\sigma_{b\dagger}(-if_{abc})(-i\chi^c))\lambda^a + i\overline{\lambda}^a(i\overline{\chi}_b(+if_{abc})\sigma^{c\dagger})\right)$$
$$-ig_s\sigma^{b^\dagger}f^{abc}D^a\sigma^c \tag{4.9}$$

where in the gauge covariant derivative $D_\mu = \partial_\mu + ig_sT^aG^a_\mu$ the generator T^a needs to be replaced by $-T^{*a}$ or $-if^{abc}$ if applied to a field transforming in the antifundamental or adjoint representation respectively.

The soft breaking Lagrangian accounts for the squark, gaugino and scalar gluon masses. These mass terms arise from a hidden sector spurion. For the gauginos the D-type spurion is given by $\hat{W}'_\alpha = \theta_\alpha D$ and mediates super symmetry breaking at the mediation scale M: $\int \mathrm{d}\theta^2 \frac{\hat{W}'_\alpha}{M} W_s^\alpha \hat{O}$. After integrating out the spurion one obtains

$$\mathcal{L}_{soft} = -\frac{m_{\tilde{q}}^2}{2}(|\tilde{q}_L|^2 + |\tilde{q}_R|^2)$$
$$-\frac{m_{\sigma_1}^2}{2}\sigma_1^2 - \frac{m_{\sigma_2}^2}{2}\sigma_2^2 - m_g(\lambda\chi - \sqrt{2}D^a\sigma^a + h.c.) \tag{4.10}$$

where the complex scalar gluons $\sigma = \frac{\sigma_1 + i\sigma_2}{\sqrt{2}}$ constitutes of two real scalar gluons with different masses. The equations of motion for the auxiliary fields are

$$D^a = -g_s\tilde{q}_L^\dagger T^a\tilde{q}_L + g_s\tilde{q}_R T^a\tilde{q}_R^\dagger + ig_s\sigma^{\dagger b}f^{abc}\sigma^c - \sqrt{2}m_g(\sigma^a + \sigma^{\dagger a}) \tag{4.11}$$
$$F_i = 0 \qquad \text{for} \qquad i = L, R, O \tag{4.12}$$

where D^a is still real as the purely imaginary parts to not contribute by virtue of the antisymmetry of the structure constants. After eliminating the auxiliary fields the complete

Lagrangian in 4 spinor notation[7] reads

$$\begin{aligned}
\mathcal{L}_{RSQCD} =& |D_\mu \sigma|^2 + |D_\mu \tilde{q}_R|^2 + |D_\mu \tilde{q}_L|^2 + \overline{q} i \not{D} q + \overline{\tilde{g}}^a i \not{D} P_L \tilde{g}^a + \overline{\tilde{g}}^a i \not{D} P_R \tilde{g}^a - \frac{1}{4}(F_a^{\mu\nu})^2 \\
&- \sqrt{2} g_s \left(\overline{\tilde{g}}^a P_R (q^C T^a \tilde{q}_L) + (\tilde{q}_L^\dagger T^a \overline{q}^C) P_L \tilde{g}^a \right) \\
&+ \sqrt{2} g_s \left(\overline{\tilde{g}}^a P_R (q T^{*a} \tilde{q}_R^\dagger) + (\tilde{q}_R T^{*a} \overline{q}) P_L \tilde{g}^a \right) \\
&- \sqrt{2} g_s \left(\overline{\tilde{g}}^a P_R (\tilde{g}^b (i f^{abc}) \sigma^c) + (\sigma^{\dagger b} (-i f^{abc}) \overline{\tilde{g}}^c) P_L \tilde{g}^a \right) \\
&- \frac{m_{\tilde{q}}^2}{2} (|\tilde{q}_L|^2 + |\tilde{q}_R|^2) - \frac{m_{\sigma_1}^2}{2} \sigma_1^2 - \frac{m_{\sigma_2}^2}{2} \sigma_2^2 - m_g \overline{\tilde{g}}^a \tilde{g}^a \\
&- \frac{1}{2} \left(g_s \tilde{q}_L^\dagger T^a \tilde{q}_L - g_s \tilde{q}_R T^{*a} \tilde{q}_R^\dagger - i g_s \sigma^{\dagger b} f^{abc} \sigma^c + \sqrt{2} m_g (\sigma^a + \sigma^{\dagger a}) \right)^2
\end{aligned} \tag{4.13}$$

Observe that there is no 3 sgluon vertex, because of the antisymmetry of the structure constants f^{abc}.

[7] How a 4 spinor is composed of Weyl-spinors is given in the Appendix 9.3

5 Sparticle Production at Tree Level

5.1 Partonic Processes

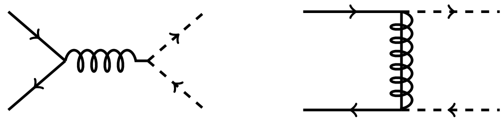


Figure 5.1: Tree level diagrams for $q\overline{q} \to \tilde{q}\tilde{q}^\dagger$

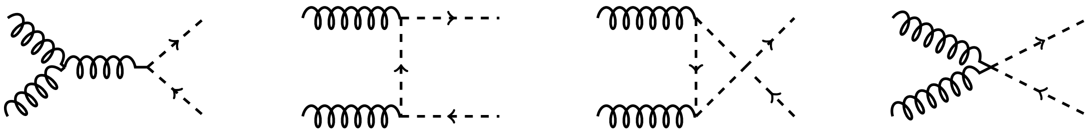


Figure 5.2: Tree level diagrams for $GG \to \tilde{q}\tilde{q}^\dagger$

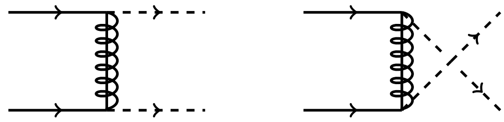


Figure 5.3: Tree level diagrams for $qq \to \tilde{q}\tilde{q}$

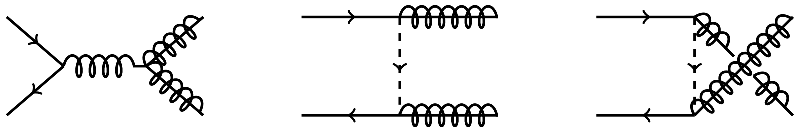


Figure 5.4: Tree level diagrams for $q\overline{q} \to \tilde{g}\overline{\tilde{g}}$

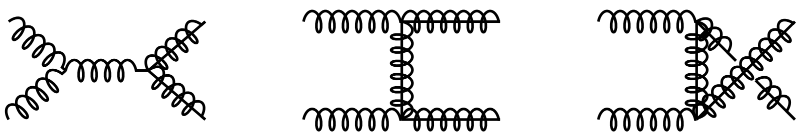


Figure 5.5: Tree level diagrams for $GG \to \tilde{g}\overline{\tilde{g}}$

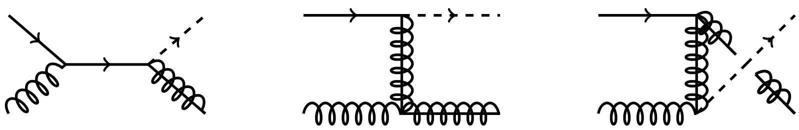


Figure 5.6: Tree level diagrams for $qG \to \tilde{q}\tilde{g}$

The top quark is excluded from the initial states as it is too heavy to be significantly present in hadrons. Therefore its pdf is approximately zero. For consistency reasons also the stop is excluded from the final states. One therefore deals with $n_f - 1 = 5$ quark flavors. Using the Feynman rules in the Appendix one obtains the following sums over absolute squared Feynman amplitudes

$$\begin{aligned}\sum |\mathcal{M}^B|^2(q_i\bar{q}_j \to \tilde{q}\tilde{q}^\dagger) =& \delta_{ij}\left[8N_cC(F)g_s^4\frac{(n_f-1)}{s^2} + 4N_cC(F)\hat{g}_s^4\frac{1}{t_{\tilde{g}}^2} - 8C(F)g_s^2\hat{g}_s^2\frac{1}{t_{\tilde{g}}s}\right](tu - m_{\tilde{q}}^4)\\ &+ (1-\delta_{ij})4N_cC(F)\hat{g}_s^4\frac{tu - m_{\tilde{q}}^4}{t_{\tilde{g}}^2} \end{aligned} \tag{5.1}$$

$$\begin{aligned}\sum |\mathcal{M}^B|^2(GG \to \tilde{q}\tilde{q}^\dagger) =& 4(n_f-1)g_s^4\left[2N_c^2C(F)\left(1 - 2\frac{t_{\tilde{q}}u_{\tilde{q}}}{s^2}\right) - 2C(F)\right]\\ &\left[1 - \epsilon - 2\frac{sm_{\tilde{q}}^2}{t_{\tilde{q}}u_{\tilde{q}}}\left(1 - \frac{sm_{\tilde{q}}^2}{t_{\tilde{q}}u_{\tilde{q}}}\right)\right] \end{aligned} \tag{5.2}$$

$$\begin{aligned}\sum |\mathcal{M}^B|^2(q_iq_j \to \tilde{q}\tilde{q}) =& \delta_{ij}2\hat{g}_s^4N_cC(F)\left[\frac{1}{t_{\tilde{g}}^2} + \frac{1}{u_{\tilde{g}}^2}\right](tu - m_{\tilde{q}}^4)\\ &+ (1-\delta_{ij})4\hat{g}_s^4N_cC(F)\frac{tu - m_{\tilde{q}}^4}{t_{\tilde{g}}^2} \end{aligned} \tag{5.3}$$

$$\begin{aligned}\sum |\mathcal{M}^B|^2(q\bar{q} \to \tilde{g}\bar{\tilde{g}}) =& 8N_c^2C(F)g_s^4\left[\frac{2m_{\tilde{g}}^2s + t_{\tilde{g}}^2 + u_{\tilde{g}}^2}{s^2} - \epsilon\right]\\ &+ 4N_c^2C(F)g_s^2\hat{g}_s^2\left[\frac{m_{\tilde{g}}^2s + t_{\tilde{g}}^2}{st_{\tilde{q}}} + \frac{m_{\tilde{g}}^2s + u_{\tilde{g}}^2}{su_{\tilde{q}}} + \epsilon\left(\frac{t_{\tilde{g}}}{t_{\tilde{q}}} + \frac{u_{\tilde{g}}}{u_{\tilde{q}}}\right)\right]\\ &+ 2C(F)(N_c^2-1)\hat{g}_s^4\left(\frac{t_{\tilde{g}}^2}{t_{\tilde{q}}^2} + \frac{u_{\tilde{g}}^2}{u_{\tilde{q}}^2}\right) \end{aligned} \tag{5.4}$$

$$\begin{aligned}\sum |\mathcal{M}^B|^2(GG \to \tilde{g}\bar{\tilde{g}}) =& 16N_c^3C(F)g_s^4\left(1 - \frac{t_{\tilde{g}}u_{\tilde{g}}}{s^2}\right)\\ &\left[\frac{s^2}{t_{\tilde{g}}u_{\tilde{g}}}(1-\epsilon)^2 - 2(1-\epsilon) + 4\frac{m_{\tilde{g}}^2s}{t_{\tilde{g}}u_{\tilde{g}}}\left(1 - \frac{m_{\tilde{g}}^2s}{t_{\tilde{g}}u_{\tilde{g}}}\right)\right] \end{aligned} \tag{5.5}$$

$$\begin{aligned}\sum |\mathcal{M}^B|^2(qg \to \tilde{q}\tilde{g}) =& 2g_s^2\hat{g}_s^2\left[2N_c^2C(F)\left(1 - 2\frac{su_{\tilde{q}}}{t_{\tilde{g}}^2}\right) - 2C(F)\right]\\ &\left[(-1+\epsilon)\frac{t_{\tilde{g}}}{s} + \frac{2(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)t_{\tilde{g}}}{su_{\tilde{q}}}\left(1 + \frac{m_{\tilde{q}}^2}{u_{\tilde{q}}} + \frac{m_{\tilde{g}}^2}{t_{\tilde{g}}}\right)\right] \end{aligned} \tag{5.6}$$

For NLO-corrections the results are expanded up to $\mathcal{O}(\epsilon)$. Furthermore the usual Mandelstam variables s, t, u and the following modifications of them are used:

$$\begin{aligned} t_{\tilde{g}} &= t - m_{\tilde{g}}^2 & t_{\tilde{q}} &= t - m_{\tilde{q}}^2\\ u_{\tilde{g}} &= u - m_{\tilde{g}}^2 & u_{\tilde{q}} &= u - m_{\tilde{q}}^2 \end{aligned} \tag{5.7}$$

5.2 Partonic Cross Sections

The leading order cross sections are

$$\begin{aligned}
\sigma(q_i\bar{q}_j \to \tilde{q}\tilde{q}^\dagger) &= \delta_{ij}\frac{g_s^4}{16\pi s}(n_f-1)\left[\frac{4}{27}-\frac{16m_{\tilde{q}}^2}{27s}\right] \\
&+ \delta_{ij}\frac{g_s^2\hat{g}_s^2}{16\pi s}\left[\left(\frac{4}{27}+\frac{8m_-^2}{27s}\right)\beta_{\tilde{q}}+\left(\frac{8m_{\tilde{g}}^2}{27s}+\frac{8m_-^4}{27s^2}\right)L_1\right] \\
&+ \frac{\hat{g}_s^4}{16\pi s}\left[-\frac{8}{9}\beta_{\tilde{q}}+\left(-\frac{4}{9}-\frac{8m_-^2}{9s}\right)L_1\right] && (5.8) \\
\sigma(GG \to \tilde{q}\tilde{q}^\dagger) &= \frac{(n_f-1)g_s^4}{16\pi s}\left[\left(\frac{5}{24}+\frac{31m_{\tilde{q}}^2}{12s}\right)\beta_{\tilde{q}}+\left(\frac{4m_{\tilde{q}}^2}{3s}+\frac{m_{\tilde{q}}^4}{3s^2}\right)\ln\frac{1-\beta_{\tilde{q}}}{1+\beta_{\tilde{q}}}\right] && (5.9) \\
\sigma(q_iq_j \to \tilde{q}\tilde{q}) &= \frac{\hat{g}_s^4}{16\pi s}\left[-\frac{8}{9}\beta_{\tilde{q}}+\left(-\frac{4}{9}-\frac{8m_-^2}{9s}\right)L_1\right] && (5.10) \\
\sigma(q\bar{q} \to \tilde{g}\bar{\tilde{g}}) &= \frac{g_s^4}{16\pi s}\left[\frac{16}{9}+\frac{32m_{\tilde{g}}^2}{9s}\right]\beta_{\tilde{g}} \\
&+ \frac{\hat{g}_s^2 g_s^2}{16\pi s}\left[\left(-\frac{4}{3}-\frac{8m_-^2}{3s}\right)\beta_{\tilde{g}}+\left(\frac{8m_{\tilde{g}}^2}{3s}+\frac{8m_-^4}{3s^2}\right)L_2\right] \\
&+ \frac{\hat{g}_s^4}{16\pi s}\left[\left(\frac{32}{27}+\frac{32m_-^4}{m_-^4+m_{\tilde{q}}^2 s}\right)\beta_{\tilde{g}}-\frac{64m_-^2}{27s}L_2\right] && (5.11) \\
\sigma(GG \to \tilde{g}\bar{\tilde{g}}) &= \frac{g_s^4}{16\pi s}\left[\left(-6-\frac{51m_{\tilde{g}}^2}{2s}\right)\beta_{\tilde{g}}+\left(-\frac{9}{2}-\frac{18m_{\tilde{g}}^2}{s}+\frac{18m_{\tilde{g}}^4}{s^2}\right)\ln\frac{1-\beta_{\tilde{g}}}{1+\beta_{\tilde{g}}}\right] && (5.12) \\
\sigma(qG \to \tilde{q}\tilde{g}) &= \frac{g_s^2\hat{g}_s^2}{16\pi s}\left[\frac{\kappa}{s}\left(-\frac{7}{9}-\frac{32m_-^2}{9s}\right)+\left(-\frac{8m_-^2}{9s}+\frac{2m_{\tilde{q}}^2m_-^2}{s^2}+\frac{8m_-^4}{9s^2}\right)L_3\right. \\
&\left.+\left(-1-\frac{2m_-^2}{s}+\frac{2m_{\tilde{q}}m_-^2}{s^2}\right)L_4\right] && (5.13)
\end{aligned}$$

where the abbreviations

$$\begin{aligned}
\beta_{\tilde{q}} &= \sqrt{1-\frac{4m_{\tilde{q}}^2}{s}} & \beta_{\tilde{g}} &= \sqrt{1-\frac{4m_{\tilde{g}}^2}{s}} \\
m_-^2 &= m_{\tilde{g}}^2-m_{\tilde{q}}^2 & \kappa &= \sqrt{(s-m_{\tilde{g}}^2-m_{\tilde{q}}^2)^2-4m_{\tilde{g}}^2m_{\tilde{q}}^2} \\
L_1 &= \ln\frac{s+2m_-^2-s\beta_{\tilde{q}}}{s+2m_-^2+s\beta_{\tilde{q}}} & L_2 &= \ln\frac{s-2m_-^2-s\beta_{\tilde{g}}}{s-2m_-^2+s\beta_{\tilde{g}}} \\
L_3 &= \ln\frac{s-m_-^2-\kappa}{s-m_-^2+\kappa} & L_4 &= \ln\frac{s+m_-^2-\kappa}{s+m_-^2+\kappa} && (5.14)
\end{aligned}$$

are used [?]. In contrast to the MSSM no statistical factor $\frac{1}{2}$ is taken into account for gluino-antigluino production, as these particles are distinguishable in the MRSSM. Still in comparison to the MSSM cross section for the channel $q\bar{q} \to \tilde{g}\bar{\tilde{g}}$ only the first line in 5.11 is doubled up as the part associated with an t (u) channel squark is different. In the MRSSM either $\tilde{q}_L$ or $\tilde{q}_R$

is allowed whilst in the MSSM both squarks contribute to the t (u) channel diagram.

5.3 Hadronic Cross Section

factorization (pictorial explanation in factorization paper chapter 1.4) The hadronic cross section for the production of a final state X, e.g. $X = \tilde{q}, \tilde{q}$, can be obtained by convolving the partonic cross section with the parton density function of the initial partons.

$$\sigma^B(ij \to X) = \int \mathrm{d}x_1 \mathrm{d}x_2 \; f_i(x_1) f_j(x_2) \; \sigma^B(ij \to X, s = x_1 x_2 S). \tag{5.15}$$

As the production of the final state X may proceed with various initial partons one has to sum over all possible possibilities arising from the initial hadrons H_1 and H_2:

$$\sigma^B(H_1 H_2 \to X) = \sum_{i,j} \sigma^B(i, j \to X). \tag{5.16}$$

refer to Kribs, Martin

6 Renormalization of the MRSSM

In order to improve the prediction of the cross section of the previous chapter one has to take quantum corrections into account. These are associated with loops in the corresponding Feynman diagrams. Computing these loop diagrams one might encounter infinities which arise from certain momentum configurations of the unspecified loop momentum. These infinities can be classified due to their origin. Infinities which are associated with loop momenta which tend to infinity are referred to as ultraviolet(UV) divergences. Infinities arising from loop momenta approaching zero can occur in loops with massless particles and are called infrared(IR) singularities. Furthermore there are collinear singularities with occur when a massless particle splits into two massless collinear particles.
These infinities are not physical and must therefore be removed to get sensible predictions. To this end one regularizes them to extract them from the quantity in question. UV-divergences can be removed by means of renormalization, i.e. counterterms are inserted into the Lagrangian to cancel UV-divergences. Infrared and collinear divergenves are removed by adding up all possible contributions which give rise to the considered observable.

6.1 Regularization Schemes

Dimensional Regularization(DREG)

Dimensional regularization(DREG) is a very common procedure for regularizing infinities which was devised by t'Hooft and Veltman [?]. In this scheme loop momenta, gamma- and epsilon-tensors, phase space and fields are defined in D dimensions. As in every regularization scheme a parameter with mass dimension needs to be introduced. In DREG that is the μ parameter which ensures that the loop integrals still have mass dimension 4:

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \to \mu^{4-D} \int \frac{\mathrm{d}^D p}{(2\pi)^D}. \tag{6.1}$$

One often writes $D = 4 - 2\epsilon$. Then the divergences of the loop integral manifest in $\frac{1}{\epsilon}$ poles. However DREG suffers a flaw in supersymmetry. As the degrees of freedom for a massless gauge boson are $D-2$ but the degrees of freedom for its superpartner are 2 there is a mismatch if $D \neq 4$. As a consequence there are 2ϵ degrees of freedom associated with the gluon[8] which do not have a supersymmetric partner. Therefore DREG violates supersymmetry.

[8] These degrees of freedom are identified with scalars and are therefore referred to as ϵ scalars

Dimensional Reduction (DRED)

Dimensional reduction (DRED) was introduced to rectify the imperfections of DREG, i.e. it preserves supersymmetry[9]. DRED promotes only loop momenta to D dimensions. All other quantities which are D dimensional in DREG stay in 4 dimensions.
maybe refer to Collins: Renormalization

6.2 Regularization Scheme Dependences

It is useful to introduce the effective action Γ to discuss the subject of this and ensuing subchapters. A formal introduction of Γ can be found in [?]. In short Γ can be viewed as a modification of the classical action $\Gamma_{cl} = \int \mathcal{L}_{cl}$ by quantum effects:

$$\Gamma = \Gamma_{cl} + \mathcal{O}(\hbar) \tag{6.2}$$

This means that in addition to the vertices in the classical Lagrangian new vertices arise due to loop effects. As already suggested loop corrections might a priori not be finite and then need to be made finite by the addition of counterterms. For $\mathcal{O}(\hbar)$ corrections one writes

$$\Gamma^{(\leq 1)} \to \Gamma^{(\leq 1)} + \Gamma^{(1),ct} \tag{6.3}$$

These counterterms depend on the regularization (and renormalization) scheme. If one chooses to work with DREG supersymmetry will not be preserved at 1-loop order, i.e. $\Gamma^{(\leq 1)}_{DREG}$ is not supersymmetric. To maintain supersymmetry invariance of the renormalized effective action the counterterms will not only consist of supersymmetric counterterms $\Gamma^{(1),ct,sym}_{DREG}$ but also of counterterms restoring supersymmetry $\Gamma^{(1),ct,restore}_{DREG}$.

$$\Gamma^{(1),ct}_{DREG} = \Gamma^{(1),ct,sym}_{DREG} + \Gamma^{(1),ct,restore}_{DREG} \tag{6.4}$$

Fortunately a supersymmetry conserving regularization scheme (at 1-loop level) is given by DRED [?]. One way to acquire supersymmetry restoring counterterms is therefore given by

$$\Gamma^{(\leq 1)}_{DRED} + \Gamma^{(1),ct}_{DRED} \stackrel{!}{=} \Gamma^{(\leq 1)}_{DREG} + \Gamma^{(1),ct}_{DREG}. \tag{6.5}$$

Setting also the finite terms in $\Gamma^{(1),ct,sym}$ equal in DRED and DREG the choice of the supersymmetry restoring counterterms is fixed by:

$$\Gamma^{(1),ct,restore}_{DREG} = \Gamma^{(\leq 1)}_{DRED} - \Gamma^{(\leq 1)}_{DREG}. \tag{6.6}$$

[9] It is not clear if DRED preserves supersymmetry at all orders in perturbation theory but it does preserve supersymmetry at the 1-loop level.

This way supersymmetry is preserved by the renormalization constants.
In the case of the MRSSM it will turn out that the only supersymmetry violation comes from correction associated with the gluon as already alluded to in 6.1. However supersymmetry restoring is always already included in δZ^{DREG}. This is $\delta Z^{\mathrm{DREG}} = \delta Z^{\mathrm{DREG,sym}} + \delta Z^{\mathrm{trans}}$ where $\delta Z^{\mathrm{trans}} = \delta Z^{\mathrm{DREG}} - \delta Z^{\mathrm{DRED}}$ is the supersymmetry restoring renormalization constant. The only point where particularly care is required is the coupling: The gauge coupling g_s and the Yukawa coupling $\hat{g}_s$ receive different supersymmetry restoring counterterms. Therefore one has to make a difference between these couplings at 1-loop level. In order to match g_s to the experimentally measured coupling it is renormalized in $\overline{\mathrm{MS}}$. The Yukawa coupling $\hat{g}_s$ therefore needs to be added with the difference of the supersymmetry restoring counterterms at 1-loop in order to be renormalized the same way.

6.3 On-Shell Renormalization

A part of the computation of NLO processes is the calculation of renormalization constants. The field and mass renormalization constants have been calculated in DREG in the on-shell scheme. This has the advantaged that when turning to the cross section no manipulation of the Green function to the S-matrix element has to be done.

6.3.1 The Quark Self-Energy

The quark self-energy splits into contribution from the SM as well as a supersymmetric analogue which is already present in the MSSM.

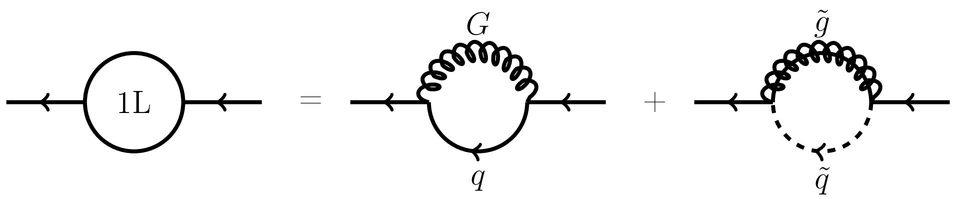
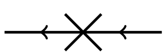


Figure 6.1: diagrammatic contributions to the self-energy of the quark at 1-loop level

The 1-PI diagrams evaluate to

$$i\Gamma^{1L}_{q_i\bar{q}_j} = i\frac{g_s^2}{16\pi^2}\delta_{ij}C(F)\left[2\left(B_0(p^2,0,0)+B_1(p^2,0,0)-\frac{1}{2}\right)\not{p} - 2B_1(p^2,m_{\tilde{g}}^2,m_{\tilde{q}}^2)\not{p}\right]. \qquad (6.7)$$

With the counterterm Feynman rule

$i\Gamma^{1L,ct}_{q_i\bar{q}_j} \mathrel{\hat{=}} i$  $j \mathrel{\hat{=}} i\delta_{ij}\delta Z_q\not{p}$

and the on-shell renormalization condition

$$\frac{\partial}{\partial \not{p}}\left[\Re(\Gamma^{1\mathrm{L}}_{q_i\bar{q}_j}) + \Gamma^{1\mathrm{L,ct}}_{q_i\bar{q}_j}\right]_{p^2=0} = 0 \tag{6.8}$$

where $\Re(\ldots)$ denotes the real part of $\ldots$ one finds

$$\delta Z_q = 2C(F)\frac{g_s^2}{16\pi^2}\Re\left[B_1(p^2, m_{\tilde{g}}^2, m_{\tilde{q}}^2) + \frac{1}{2}\right]. \tag{6.9}$$

Doing the same calculation in DRED one finds that the second term in the squared brackets is absent. Therefore the transition counterterm between DREG and DRED is given by

$$\delta Z_q^{\mathrm{trans}} = \delta Z_q^{\mathrm{DREG}} - \delta Z_q^{\mathrm{DRED}} = C(F)\frac{g_s^2}{16\pi^2}. \tag{6.10}$$

6.3.2 The Squark Self-Energy

The contributions to the self-energy of the left- and right-handed squark are the same. Therefore to avoid unnecessary labeling $\Gamma_{\tilde{q}\tilde{q}^\dagger}$ stands in the following for $\Gamma_{\tilde{q}_L\tilde{q}_L^\dagger} = \Gamma_{\tilde{q}_R\tilde{q}_R^\dagger}$.

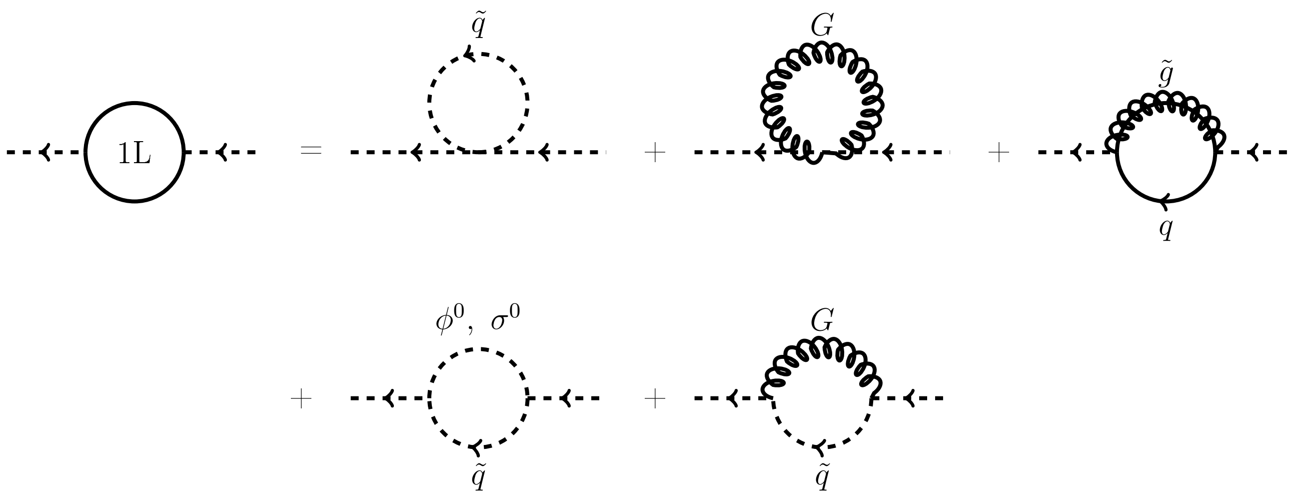
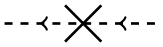


Figure 6.2: diagrammatic contributions to the self-energy of the squark at 1-loop level

$$\begin{aligned} i\Gamma^{1L}_{\tilde{q}_i\tilde{q}_j^\dagger} = i\frac{g_s^2}{16\pi^2}\delta_{ij}C(F)\big[&A_0(m_{\tilde{q}}^2) + 0 - \big(4A_0(m_{\tilde{g}}^2) + 4B_1(p^2, 0, m_{\tilde{g}}^2)p^2\big) \\ &+4m_{\tilde{g}}^2 B_0(p^2, m_{\phi^0}^2, m_{\tilde{q}}^2) - \big(2B_1(p^2, 0, m_{\tilde{q}}^2)p^2 + B_0(p^2, 0, m_{\tilde{q}}^2)(m_{\tilde{q}}^2 + 3p^2)\big)\big]. \end{aligned} \tag{6.11}$$

Suppressing δ_{AB} with $A, B = L, R$ which is present in the tree level propagator the counterterm Feynman rule is given by

$$i\Gamma^{1L,ct}_{q_i\bar{q}_j} \hat{=}\ i$$  $$j \hat{=}\ i\delta_{ij}\left[\delta Z_{\tilde{q}}(p^2 - m_{\tilde{q}}^2) - \delta m_{\tilde{q}}^2\right].$$

The on-shell renormalization conditions read

$$\frac{\partial}{\partial p^2}\left[\Re(\Gamma^{1\mathrm{L}}_{\tilde{q}_i\tilde{q}_j^\dagger}) + \Gamma^{1\mathrm{L,ct}}_{\tilde{q}_i\tilde{q}_j^\dagger}\right]_{p^2=m_{\tilde{q}}^2} = 0 \qquad \left[\Re(\Gamma^{1\mathrm{L}}_{\tilde{q}_i\tilde{q}_j^\dagger}) + \Gamma^{1\mathrm{L,ct}}_{\tilde{q}_i\tilde{q}_j^\dagger}\right]_{p^2=m_{\tilde{q}}^2} = 0. \tag{6.12}$$

This results in the following renormalization constants

$$\begin{aligned}\delta Z_{\tilde{q}} = \frac{g_s^2}{16\pi^2}C(F)\Re\Big[&4B_1(p^2,0,m_{\tilde{g}}^2) + 2B_1(B_1(p^2,0,m_{\tilde{q}}^2)) + 3B_0(p^2,0,m_{\tilde{q}}^2)\\ &+ 4m_{\tilde{q}}^2\frac{\partial}{\partial p^2}B_1(p^2,0,m_{\tilde{g}}^2) - 4m_{\tilde{g}}^2\frac{\partial}{\partial p^2}B_0(p^2,m_{\phi^0}^2,m_{\tilde{q}}^2) + 2m_{\tilde{q}}^2\frac{\partial}{\partial p^2}B_1(p^2,0,m_{\tilde{q}}^2)\\ &+4m_{\tilde{q}}^2\frac{\partial}{\partial p^2}B_0(p^2,0,m_{\tilde{q}}^2)\Big]_{p^2=m_{\tilde{q}}^2}\end{aligned} \tag{6.13}$$

$$\begin{aligned}\delta m_{\tilde{q}}^2 = \frac{g_s^2}{16\pi^2}C(F)\Big[&A_0(m_{\tilde{q}}^2) - (4A_0(m_{\tilde{g}}^2) + 4B_1(m_{\tilde{q}}^2,0,m_{\tilde{g}}^2)m_{\tilde{q}}^2) + 4m_{\tilde{g}}^2B_0(m_{\tilde{q}}^2,m_{\phi^0}^2,m_{\tilde{q}}^2)\\ &-(2B_1(m_{\tilde{q}}^2,0,m_{\tilde{q}}^2)m_{\tilde{q}}^2 + 4B_0(m_{\tilde{q}}^2,0,m_{\tilde{q}}^2)m_{\tilde{q}}^2)\Big]\end{aligned} \tag{6.14}$$

The squark self-energy exhibits no regularization dependence. The transition counterterms are therefore

$$\delta Z_{\tilde{q}}^{\mathrm{trans}} = \delta m_{\tilde{q}}^{2\ \mathrm{trans}} = 0. \tag{6.15}$$

6.3.3 The Glunio Self-Energy

The 4-spinor of the gluino comprises two Weyl spinors which describe very different particles.

$$\tilde{g}^a = \begin{pmatrix}\lambda^a\\ \overline{\chi}^a\end{pmatrix} \tag{6.16}$$

The left-handed part λ^a is associated with the superpartner of the gluon and therefore the "actual" gluino whereas the right-handed part $\overline{\chi}^a$ was introduced to assign a Dirac-mass to the gluino and is referred to as the octino.

From the Lagrangian 4.13 one can see that the couplings of the two particles are quite distinct. This is reflected by different field renormalization constants of the left- and right-handed part of the gluino. As for the quarks the fermion (and momentum) flow is from the right to the

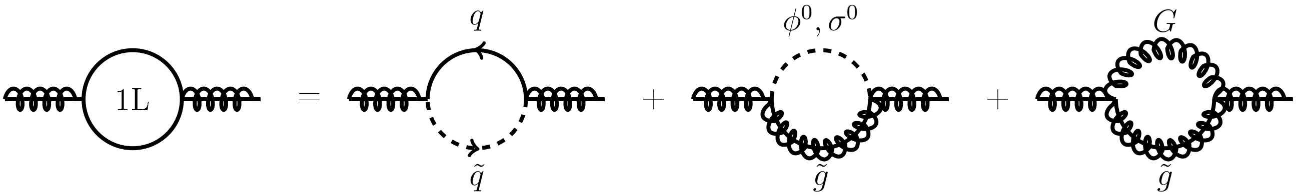


Figure 6.3: diagrammatic contributions to the self-energy of the squark at 1-loop level

left.

$$\begin{aligned}
i\Gamma^{1L}_{\tilde{g}^a\bar{\tilde{g}}^b} = i\frac{g_s^2}{16\pi^2}\delta_{ab}\big[&-4T(F)\left((n_f-1)B_1(p^2,0,m_{\tilde{q}}^2)+B_1(p^2,m_t^2,m_{\tilde{q}}^2)\right)P_L\not{p} \\
&+C(A)\left((B_0(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)-B_0(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2))m_{\tilde{g}}-(B_1(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)+B_1(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2))\not{p}\right) \\
&+C(A)\left((2-4B_0(p^2,0,m_{\tilde{g}}^2))m_{\tilde{g}}-(1-2(B_0(p^2,0,m_{\tilde{g}}^2)+B_1(p^2,0,m_{\tilde{g}}^2)))\not{p}\right)\big] \qquad (6.17)
\end{aligned}$$

Where $n_f = 6$ is the number of quark flavors.
The counterterm Feynman rule reads

$$i\Gamma^{1L,ct}_{\tilde{g}^a\bar{\tilde{g}}^b} \hat{=}\ a \quad b \ \hat{=}\ i\delta_{ab}\left[(\delta Z^L_{\tilde{g}}P_L+\delta Z^R_{\tilde{g}}P_R)\not{p}-\left(\tfrac{\delta Z^L_{\tilde{g}}+\delta Z^R_{\tilde{g}}}{2}m_{\tilde{g}}+\delta m_{\tilde{g}}\right)\right].$$

The on-shell renormalization conditions for the fields are

$$\frac{\partial}{\partial(P_L\not{p})}\left[\Re(\Gamma^{1L}_{\tilde{g}^a\bar{\tilde{g}}^b})+\Gamma^{1L,ct}_{\tilde{g}^a\bar{\tilde{g}}^b}\right]_{\not{p}=m_{\tilde{g}}}=0 \qquad \frac{\partial}{\partial(P_R\not{p})}\left[\Re(\Gamma^{1L}_{\tilde{g}^a\bar{\tilde{g}}^b})+\Gamma^{1L,ct}_{\tilde{g}^a\bar{\tilde{g}}^b}\right]_{\not{p}=m_{\tilde{g}}}=0 \qquad (6.18)$$

where the derivative of $\Sigma = \Sigma^{VL}P_L\not{p}+\Sigma^{VR}P_R\not{p}+\Sigma^{SL}P_L+\Sigma^{SR}P_R$ with respect to $P_A\not{p}$ $(A = L, R)$ is defined by

$$\frac{\partial}{\partial(P_A\not{p})}\Sigma\bigg|_{\not{p}=m} = \Sigma^{VA}+\frac{\partial}{\partial p^2}\left(m^2\Sigma^{VL}+m^2\Sigma^{VR}+m\Sigma^{SL}+m\Sigma^{SR}\right). \qquad (6.19)$$

This leads to the following renormalization constants

$$\begin{aligned}
\delta Z^L_{\tilde{g}} = \frac{g_s^2}{16\pi^2}\Re\big[&4T(F)\left((n_f-1)B_1(m_{\tilde{g}}^2,0,m_{\tilde{q}}^2)+B_1(m_{\tilde{g}}^2,m_t^2,m_{\tilde{q}}^2)\right) \\
&+C(A)(B_1(m_{\tilde{g}}^2,m_{\tilde{g}}^2,m_{\phi^0}^2)+B_1(m_{\tilde{g}}^2,m_{\tilde{g}}^2,m_{\sigma^0}^2)) \\
&+C(A)(1-2(B_0(m_{\tilde{g}}^2,0,m_{\tilde{g}}^2)+B_1(m_{\tilde{g}}^2,0,m_{\tilde{g}}^2))) \\
&+4T(F)m_{\tilde{g}}^2\frac{\partial}{\partial p^2}\left(((n_f-1))B_1(p^2,0,m_{\tilde{q}}^2)+B_1(p^2,m_t^2,m_{\tilde{q}}^2)\right) \\
&-2C(A)m_{\tilde{g}}\frac{\partial}{\partial p^2}\left(B_0(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)-B_0(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2)-B_1(p^2,m_{\tilde{g}}^2,m_{\phi^0}^2)-B_1(p^2,m_{\tilde{g}}^2,m_{\sigma^0}^2)\right) \\
&-4C(A)m_{\tilde{g}}^2\frac{\partial}{\partial p^2}\left(-B_0(p^2,0,m_{\tilde{g}}^2)+B_0(p^2,0,m_{\tilde{g}}^2)\right)\big]_{p^2=m_{\tilde{g}}^2} \qquad (6.20)
\end{aligned}$$

and

$$\begin{aligned}\delta Z^R_{\tilde g} = &\frac{g_s^2}{16\pi^2}\Re\left[C(A)(B_1(m^2_{\tilde g},m^2_{\tilde g},m^2_{\phi^0})+B_1(m^2_{\tilde g},m^2_{\tilde g},m^2_{\sigma^0}))\right.\\ &+C(A)(1-2(B_0(m^2_{\tilde g},0,m^2_{\tilde g})+B_1(m^2_{\tilde g},0,m^2_{\tilde g})))\\ &+4T(F)m^2_{\tilde g}\frac{\partial}{\partial p^2}\left(((n_f-1))B_1(p^2,0,m^2_{\tilde q})+B_1(p^2,m_t^2,m^2_{\tilde q})\right)\\ &-2C(A)m_{\tilde g}\frac{\partial}{\partial p^2}\left(B_0(p^2,m^2_{\tilde g},m^2_{\phi^0})-B_0(p^2,m^2_{\tilde g},m^2_{\sigma^0})-B_1(p^2,m^2_{\tilde g},m^2_{\phi^0})-B_1(p^2,m^2_{\tilde g},m^2_{\sigma^0})\right)\\ &\left.-4C(A)m^2_{\tilde g}\frac{\partial}{\partial p^2}\left(-B_0(p^2,0,m^2_{\tilde g})+B_0(p^2,0,m^2_{\tilde g})\right)\right]_{p^2=m^2_{\tilde g}}. \end{aligned} \tag{6.21}$$

As for the quark there are constant terms amid the Passarino-Veltman integrals. These arise only in DREG and not in DRED. The transition counterterms are

$$\delta Z^{A\ \text{trans}}_{\tilde g} = \delta Z^{A\ \text{DREG}}_{\tilde g} - \delta Z^{A\ \text{DRED}}_{\tilde g} = C(A)\frac{g_s^2}{16\pi^2} \tag{6.22}$$

for $A = L, R$. The gluino mass counterterm is ascertained by the condition

$$\left[\Re(\Gamma^{1\text{L}}_{\tilde g^a\bar{\tilde g}^b}) + \Gamma^{1\text{L,ct}}_{\tilde g^a\bar{\tilde g}^b}\right]_{\not p = m_{\tilde g}} = 0 \tag{6.23}$$

which is equivalent to

$$\delta m_{\tilde g} = \Re\left(m_{\tilde g}\frac{\Sigma^{VL}+\Sigma^{VR}}{2} + \frac{\Sigma^{SL}+\Sigma^{SR}}{2}\right) \tag{6.24}$$

and yields

$$\begin{aligned}\delta m_{\tilde g} = &\frac{g_s^2}{16\pi^2}m^2_{\tilde g}\ \Re\left[-2T(F)\left((n_f-1)B_1(m^2_{\tilde g},0,m^2_{\tilde q})+B_1(m^2_{\tilde g},m_t^2,m^2_{\tilde q})\right)\right.\\ &+C(A)\left(B_0(m^2_{\tilde g},m^2_{\tilde g},m^2_{\phi^0})-B_0(m^2_{\tilde g},m^2_{\tilde g},m^2_{\sigma^0})-B_1(m^2_{\tilde g},m^2_{\tilde g},m^2_{\phi^0})-B_1(m^2_{\tilde g},m^2_{\tilde g},m^2_{\sigma^0})\right)\\ &\left.+C(A)\left(1-2B_0(m^2_{\tilde g},0,m^2_{\tilde g})+2B_1(m^2_{\tilde g},0,m^2_{\tilde g})\right)\right]. \end{aligned} \tag{6.25}$$

Again there is a transition counterterm

$$\delta m^{\text{trans}}_{\tilde g} = \delta m^{\text{DREG}}_{\tilde g} - \delta m^{\text{DRED}}_{\tilde g} = C(A)\frac{g_s^2}{16\pi^2}m_{\tilde g}. \tag{6.26}$$

6.4 Renormalization of the Gauge Coupling

The gauge coupling g_s is renormalized in the $\overline{\text{MS}}$-scheme with the modification that additional logarithms are substracted. This is to decouple heavy particles from the running of $\alpha_s = \frac{g_s^2}{4\pi}$. This renormalization procedure allows to adopt the experimental values of α_s from the PDF's. The running due to effects of heavy particles is then encoded in the logarithms of δg_s.

Extracting δg_s from the quark-quark-gluon vertex requires not only the computation of $i\Gamma^{1L}_{q_i\bar{q}_j G^a}$ but also the (re)evaluation of the auxiliary field renormalization constants δZ_q^{aux} and δZ_G^{aux}. These will not be the same as the on-shell field renormalization.

6.4.1 The Quark Self-Energy Revisited

The quark self-energy has two contribution which are shown in figure 6.1. The first one corresponds to light particles the second one to heavy particles: $i\Gamma^{\text{1L}}_{q_i\bar{q}_j} = i\Gamma^{\text{1L,light}}_{q_i\bar{q}_j} + i\Gamma^{\text{1L,heavy}}_{q_i\bar{q}_j}$. For light particles only the UV-divergent part is kept. For heavy particles also the μ-dependent terms are kept:

$$i\Gamma^{\text{1L,light}}_{q_i\bar{q}_j}|_{\text{UV-div}} = i\frac{g_s^2}{16\pi^2}\delta_{ij}\not{p}C(F)\frac{1}{\epsilon_{\text{UV}}} \tag{6.27}$$

$$i\Gamma^{\text{1L,heavy}}_{q_i\bar{q}_j}|_{\text{UV-div},\mu\text{-dep}} = i\frac{g_s^2}{16\pi^2}\delta_{ij}\not{p}C(F)\left(\frac{1}{\epsilon_{\text{UV}}} - \ln\frac{m_{\tilde{q}}^2}{\mu^2}\right) \tag{6.28}$$

The renormalization constant for the evaluation of δg_s is determined by the condition

$$\frac{\partial}{\partial\not{p}}\left[\Gamma^{\text{1L,light}}_{q_i\bar{q}_j}|_{\text{UV-div}} + \Gamma^{\text{1L,heavy}}_{q_i\bar{q}_j}|_{\text{UV-div},\mu\text{-dep}} + \Gamma^{\text{1L,ct}}_{q_i\bar{q}_j}\right]_{p^2=0} = 0 \tag{6.29}$$

and computes to

$$\delta Z_q^{\text{aux}} = -\frac{g_s^2}{16\pi^2}C(F)\left[\left(\frac{1}{\epsilon_{\text{UV}}}\right) + \left(\frac{1}{\epsilon_{\text{UV}}} - \ln\frac{m_{\tilde{q}}^2}{\mu^2}\right)\right] \tag{6.30}$$

The first curved bracket corresponds to contributions from light particles in the loop whereas the second curved bracket corresponds to contributions from heavy particles in the loop.

6.4.2 The Gluon Self-Energy

As for the quark self-energy there are again contributions to the self-energy originating from light and heavy particles. As for the quark only specific terms are kept in calculating the auxiliary renormalization constants.

$$\begin{aligned} i\Gamma^{\text{1L,light}}_{G^a_\mu G^b_\nu}|_{\text{UV-div}} &= i\frac{g_s^2}{16\pi^2\epsilon_{\text{UV}}}\delta_{ab}\left[0 - \frac{4(n_f-1)}{3}T(F)(p^2g^{\mu\nu} - p^\mu p^\nu) + \frac{C(A)}{12}(p^2g^{\mu\nu} + 2p^\mu p^\nu)\right. \\ &\quad \left. + \frac{C(A)}{12}(19p^2g^{\mu\nu} - 22p^\mu p^\nu)\right] \\ &= i\frac{g_s^2}{16\pi^2\epsilon_{\text{UV}}}\delta_{ab}\left[-\frac{4(n_f-1)}{3}T(F) + \frac{5}{3}C(A)\right](p^2g^{\mu\nu} - p^\mu p^\nu) \end{aligned} \tag{6.31}$$

The heavy particle contributions are given by

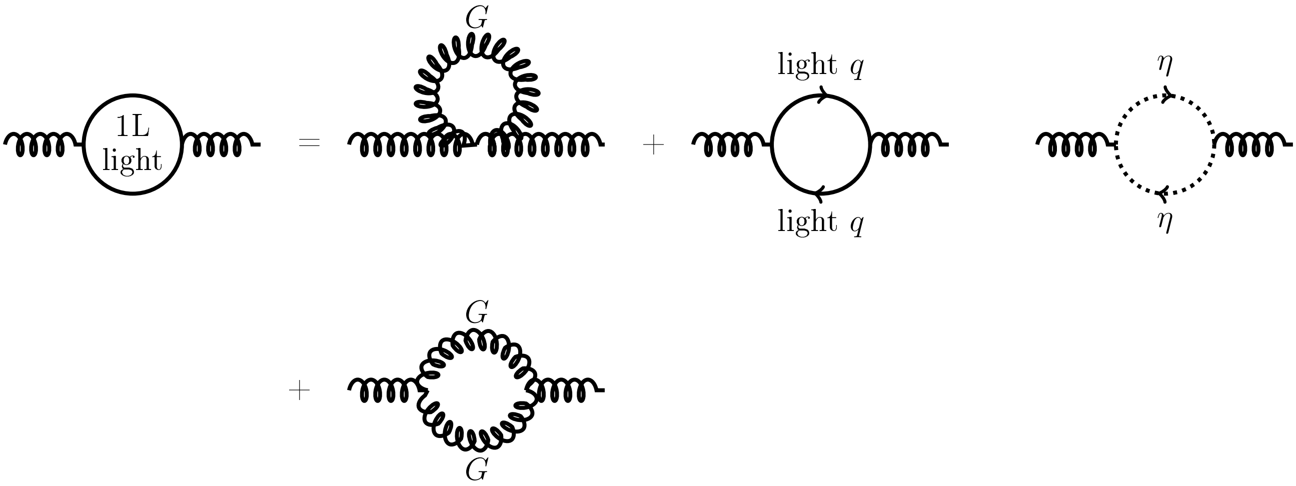


Figure 6.4: contribution to the self-energy of the gluon originating from light particles

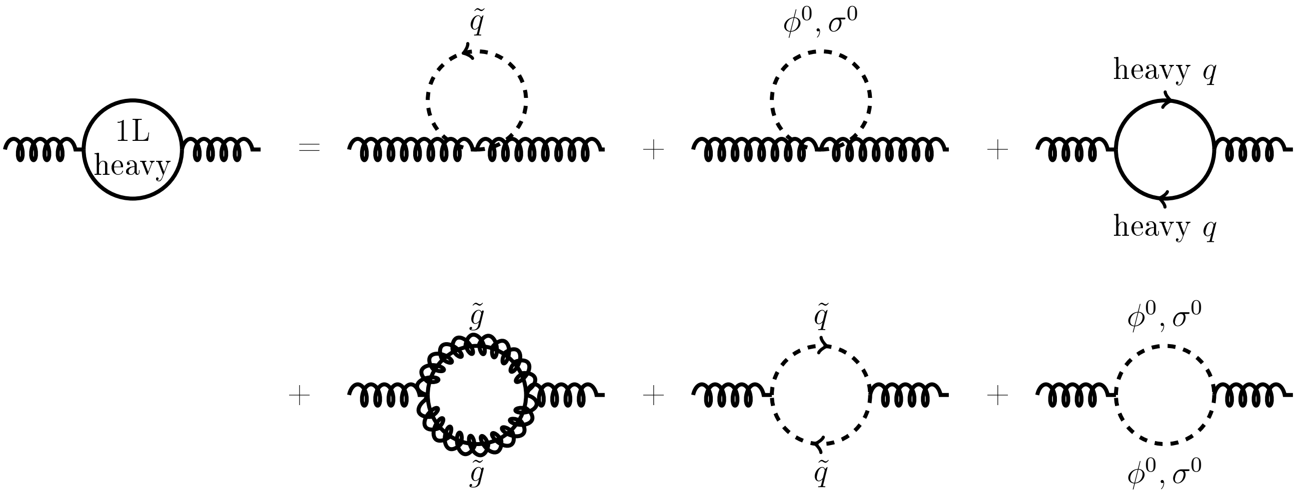


Figure 6.5: contribution to the self-energy of the gluon originating from heavy particles, in the last diagram either ϕ^0 or σ^0 are running in the loop

$$
\begin{aligned}
i\Gamma^{1\mathrm{L,heavy}}_{G^a_\mu G^b_\nu}|_{\mathrm{UV-div},\mu-\mathrm{dep}} = {} & i\frac{g_s^2}{16\pi^2}\delta_{ab}\Bigg[-4T(F)n_f\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\tilde q}}{\mu 2}\right)m^2_{\tilde q}g^{\mu\nu}-C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)m^2_{\phi^0}g^{\mu\nu}\\
& -C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\sigma^0}}{\mu 2}\right)m^2_{\sigma^0}g^{\mu\nu}-\frac{4}{3}T(F)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_t}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\\
& -\frac{4}{3}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\tilde g}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\\
& -\frac{2}{3}T(F)n_f\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\tilde q}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)+4T(F)n_f\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\tilde q}}{\mu 2}\right)m^2_{\tilde q}g^{\mu\nu}\\
& -\frac{1}{6}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)+C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)m^2_{\phi^0}g^{\mu\nu}\\
& -\frac{1}{6}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)+C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)m^2_{\phi^0}g^{\mu\nu}\Bigg]\\
= {} & i\frac{g_s^2}{16\pi^2}\delta_{ab}\Bigg[-\frac{4}{3}T(F)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_t}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\\
& -\frac{4}{3}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\tilde g}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\\
& -\frac{2}{3}T(F)n_f\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\tilde q}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\\
& -\frac{1}{6}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\\
& -\frac{1}{6}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m^2_{\phi^0}}{\mu 2}\right)(p^2g^{\mu\nu}-p^\mu p^\nu)\Bigg]
\end{aligned}
\tag{6.32}
$$

The counterterm Feynman rule for the gluon propagator is

$i\Gamma^{1\mathrm{L,ct}}_{G^a_\mu G^b_\nu} \hat{=}\ a,\mu$  $b,\nu\ \hat{=}\ -i\delta Z_G\left(p^2g^{\mu\nu}-p^\mu p^\nu\right)\delta_{ab}.$

The renormalization condition for $\delta Z^{\mathrm{aux}}_G$ reads

$$
\Gamma^{1\mathrm{L,light}}_{G^a_\mu G^b_\nu}|_{\mathrm{UV-div}}+\Gamma^{1\mathrm{L,heavy}}_{G^a_\mu G^b_\nu}|_{\mathrm{UV-div},\mu-\mathrm{dep}}-\delta Z^{\mathrm{aux}}_G\left(p^2g^{\mu\nu}-p^\mu p^\nu\right)\delta_{ab}=0 \tag{6.33}
$$

and yields

$$\delta Z_G^{aux} = \frac{g_s^2}{16\pi^2}\left\{\left[-\frac{4}{3}T(F)(n_f-1)+\frac{5}{3}C(A)\right]\frac{1}{\epsilon_{\mathrm{UV}}}+\left[-\frac{4}{3}T(F)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_t^2}{\mu^2}\right)\right.\right.$$
$$-\frac{4}{3}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_{\tilde{g}}^2}{\mu^2}\right)-\frac{2}{3}T(F)n_f\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_{\tilde{q}}^2}{\mu^2}\right)$$
$$-\frac{1}{6}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_{\phi^0}^2}{\mu^2}\right)-\frac{1}{6}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_{\sigma^0}^2}{\mu^2}\right). \tag{6.34}$$

6.4.3 The $q\overline{q}G$ Vertex Correction

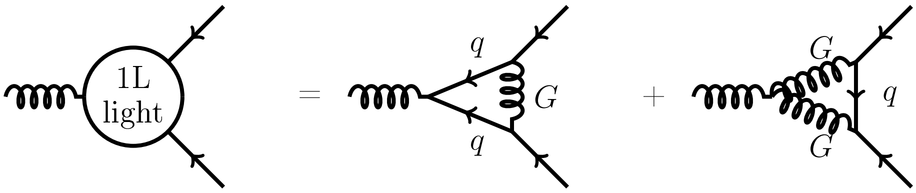


Figure 6.6: contribution from light particles to the $q\overline{q}G$ vertex correction

$$i\Gamma^{\mathrm{1L,light}}_{q_i\overline{q}_jG^a_\mu}|_{\mathrm{UV-div}} = -ig_sT^a_{ij}\gamma^\mu\frac{g_s^2}{16\pi^2\epsilon_{\mathrm{UV}}}\left[\left(C(F)-\frac{C(A)}{2}\right)+\frac{3}{2}C(A)\right] \tag{6.35}$$

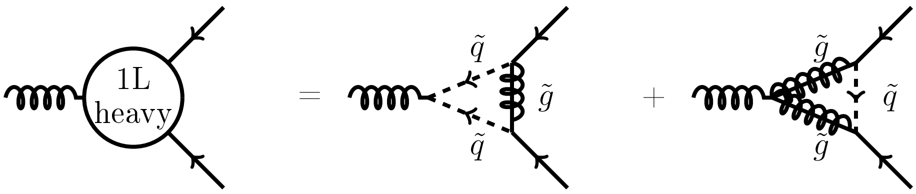


Figure 6.7: contribution from heavy particles to the $q\overline{q}G$ vertex correction

$$i\Gamma^{\mathrm{1L,heavy}}_{q_i\overline{q}_jG^a_\mu}|_{\mathrm{UV-div},\mu-\mathrm{dep}} = -ig_sT^a_{ij}\gamma^\mu\frac{g_s^2}{16\pi^2}\left[\left(C(F)-\frac{C(A)}{2}\right)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_{\tilde{g}}^2}{\mu^2}\right)\right.$$
$$\left.+\frac{1}{2}C(A)\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\ln\frac{m_{\tilde{g}}^2}{\mu^2}\right)\right] \tag{6.36}$$

The sum of the 1-loop corrections and the counterterm should be set to zero:

$$i\Gamma^{\mathrm{1L,light}}_{q_i\overline{q}_jG^a_\mu}|_{\mathrm{UV-div}}+i\Gamma^{\mathrm{1L,heavy}}_{q_i\overline{q}_jG^a_\mu}|_{\mathrm{UV-div},\mu-\mathrm{dep}}+\left[-ig_sT^a_{ij}\gamma^\mu\left(\frac{\delta g_s}{g_s}+\delta Z_q^{\mathrm{aux}}+\frac{\delta Z_G^{\mathrm{aux}}}{2}\right)\right]=0. \tag{6.37}$$

Finally one can read off the $\frac{\delta g_s}{g_s}$

$$\frac{\delta g_s}{g_s} = \frac{g_s^2}{16\pi^2}\Bigg[\left(\frac{2}{3}T(F)(n_f-1) - \frac{11}{6}C(A)\right)\frac{1}{\epsilon_{\text{UV}}} + \left(\frac{5}{6}C(A) + \frac{2}{3}T(F) + \frac{1}{3}T(F)n_f\right)\frac{1}{\epsilon_{\text{UV}}}$$
$$-\frac{2}{3}C(A)\ln\frac{m_{\tilde{g}}^2}{\mu^2} - \frac{1}{3}T(F)n_f\ln\frac{m_{\tilde{q}}^2}{\mu^2} - \frac{2}{3}T(F)\ln\frac{m_t^2}{\mu^2} - \frac{1}{12}C(A)\left(\ln\frac{m_{\phi^0}^2}{\mu^2} + \ln\frac{m_{\sigma^0}^2}{\mu^2}\right) \quad (6.38)$$

6.4.4 The Beta Function

The beta function describes the dependence of the gauge coupling g_s upon the energy scale μ. Writing down the action of a theory in D dimensions one needs to introduce an energy scale μ in order to keep the action dimensionless. But μ is no physical parameter and can be absorbed into the fields and parameters. To this end one defines

$$g_{sB} = \mu^\epsilon g_s\left(1 + \frac{\delta g_s}{g_s}\right) \quad (6.39)$$

which must not depend upon the unphysical scale μ, ergo

$$0 = \frac{\mathrm{d}g_{sB}}{\mathrm{d}\ln\mu} = \frac{\partial g_{sB}}{\partial\ln\mu} + \beta\frac{\partial g_{sB}}{\partial g_s} \quad (6.40)$$

where the definition of the beta function $\frac{\partial g_s}{\partial \ln\mu}$ has been inserted. Equation 6.40 serves to calculate $\beta(g_s,\epsilon)$. By equating coefficients and using the shortcuts

$$\frac{\beta_0^L}{2} = \frac{2}{3}T(F)(n_f-1) - \frac{11}{6}C(A)$$
$$\frac{\beta_0^H}{2} = \frac{5}{6}C(A) + \frac{2}{3}T(F) + \frac{1}{3}T(F)n_f$$
$$L = -\frac{2}{3}C(A)\ln\frac{m_{\tilde{g}}^2}{\mu^2} - \frac{1}{3}T(F)n_f\ln\frac{m_{\tilde{q}}^2}{\mu^2} - \frac{2}{3}T(F)\ln\frac{m_t^2}{\mu^2} - \frac{1}{12}C(A)\left(\ln\frac{m_{\phi^0}^2}{\mu^2} + \ln\frac{m_{\sigma^0}^2}{\mu^2}\right)$$

so that

$$\frac{\delta g_s}{g_s} = \frac{g_s^2}{16\pi^2}\left(\frac{\beta_0^L}{2\epsilon_{\text{UV}}} + \frac{\beta_0^H}{2\epsilon_{\text{UV}}} + L\right) \quad (6.41)$$

one finds

$$\beta(g_s,\epsilon) = -\epsilon g_s\left(1 + \frac{g_s^2}{16\pi^2}L\right) + \beta(g_s) + \mathcal{O}(2-\text{loop}) \quad (6.42)$$

$$\beta(g_s) = \frac{g_s^3}{16\pi^2}\beta_0^L + \mathcal{O}(2-\text{loop}). \quad (6.43)$$

This is the beta function from QCD first found by [Gross, Politzer, Wil]

6.5 Supersymmetry Restoring Counterterm

As already discussed in section 6.5 care is required in terms of supersymmetry restoring when renormalizing the gauge coupling g_s and the Yukawa coupling $\hat{g}_s$. In doing so one needs the already calculated supersymmetry restoring counterterms of the quark, squark and gluino from 6.10, 6.15 and 6.22 as well as the supersymmetry restoring counterterm of the gluon.

The Gluon Self-Energy Revisited

The only regularization dependence of the gluon self-energy arises from the gluon loop, i.e. the last diagram in figure 6.4. With the definition of $\Gamma^{(1),\text{ct,restore}}_{\text{DREG}}$ in 6.6 one obtains

$$i\Gamma^{(1),\text{ct,restore}}_{\text{DREG},G^a_\mu G^b_\nu} = -i\frac{1}{3}C(A)\frac{g_s^2}{16\pi^2}(p^2 g^{\mu\nu} - p^\mu p^\nu)\delta_{ab} \tag{6.44}$$

which translates to the transition counterterm

$$\delta Z^{\text{trans}}_G = \frac{C(A)}{3}\frac{g_s^2}{16\pi^2}. \tag{6.45}$$

The $q\bar{q}G$ Vertex Correction Revisited

The supersymmetry restoring contributions to the gauge coupling correction are shown in figure 6.6 and evaluate to

$$\begin{aligned} i\Gamma^{(1),\text{ct,restore}}_{\text{DREG},q_i\bar{q}_j G^a_\mu} &= -ig_s T^a_{ij}\gamma^\mu \frac{g_s^2}{16\pi^2}\left[\left(C(F) - \frac{C(A)}{2}\right) + \frac{C(A)}{2}\right] && (6.46)\\ &= -ig_s T^a_{ij}\gamma^\mu\left[\frac{\delta g_s^{\text{trans}}}{g_s} + \delta Z_q^{\text{trans}} + \frac{\delta Z_G^{\text{trans}}}{2}\right] && (6.47)\end{aligned}$$

where in the second line the equation with the supersymmetry restoring counterterms has been performed. This yields

$$\frac{\delta g_s^{\text{trans}}}{g_s} = -\frac{C(A)}{6}\frac{g_s^2}{16\pi^2}. \tag{6.48}$$

The $q\tilde{q}^\dagger\tilde{g}$ Vertex Correction

The supersymmetry restoring corrections to the Yukawa coupling origin from the below diagram The supersymmetry restoring part is

$$\begin{aligned} i\Gamma^{(1),\text{ct,restore}}_{\text{DREG},q_i\tilde{q}_j\tilde{g}^a} &= -ig_s\sqrt{2}P_L T^a_{ij}\frac{g_s^2}{16\pi^2}C(A) && (6.49)\\ &= -ig_s\sqrt{2}P_L T^a_{ij}\left[\frac{\delta\hat{g}_s^{\text{trans}}}{g_s} + \frac{\delta Z_q^{\text{trans}} + \delta Z_{\tilde{q}}^{\text{trans}} + \delta Z_{\tilde{g}}^{\text{trans}}}{2}\right]. && (6.50)\end{aligned}$$

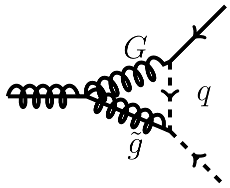


Figure 6.8: diagram of the supersymmetry restoring correction of the $q\tilde{q}\tilde{g}$ vertex

The supersymmetry restoring part of the Yukawa renormalization constants is therefore

$$\frac{\delta\hat{g}_s^{\text{trans}}}{g_s} = -\frac{C(F) - C(A)}{2}\frac{g_s^2}{16\pi^2}. \tag{6.51}$$

As a consequence of the two different supersymmetry restoring parts of the coupling renormalization constants an additional renormalization constant $\delta g_s^{\text{restore}}$ needs to be introduced. As described in section it is given by

$$\frac{\delta g_s^{\text{restore}}}{g_s} = \frac{\delta\hat{g}_s^{\text{trans}}}{g_s} - \frac{\delta g_s^{\text{trans}}}{g_s} = \frac{g_s^2}{16\pi^2}\left(\frac{2C(A)}{3} - \frac{C(F)}{2}\right). \tag{6.52}$$

In short this means that the gauge coupling g_s is renormalized with δg_s given in 6.38 and the Yukawa coupling $\hat{g}_s$ is renormilzed with $\delta\hat{g}_s = \delta g_s + \delta g_s^{\text{restore}}$.
The finite correction g_s^{restore} is the same as in supersymmetric QCD which should not surprise too much as all its contributions origin from loops with gluons. So there are no new contributions in RSQCD with respect to SQCD.

6.6 $\overline{\text{MS}}$ - Renormalization

To check for UV-finiteness it might prove useful to summarize the UV-divergent part of all renormalization constants. In order to obtain these the Passarino-Veltman integrals need to be substituted by their $\frac{1}{\epsilon}$ coefficient. These had been taken from [?] and checked with `FeynArts` and `FormCalc` [?], [?], [?].

7 Squark Production at One-Loop

8 Summary and Outlook

9 Appendix

9.1 System of Units and Metric

In this thesis the natural units are used, i.e. $c = \hbar (= k_B) = 1$. Furthermore the Minkowski metric is chosen to be

$$g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \tag{9.1}$$

9.2 Colour Algebra $SU(N)$

[Marina von Steinkirch]
Tha Casimir operator $C(R)\mathbb{1}$ of a semi-simple Lie algebra in the irreducible representation R is given by

$$g^{ab} T^a(R) T^b(R) = C(R)\mathbb{1} \tag{9.2}$$

where $T^a(R)$ is the a-th generator of the matrix valued representation R, g^{ab} is the metric of group, $C(R)$ is the Quadratic Casimir invariant of the representation R and $\mathbb{1}$ is the identity in the representation space.
Apart from $C(R)$ it is common to define the Dynkin-Index $T(R)$:

$$\mathrm{Tr}\left[T^a(R) T^b(R)\right] = T(R)\delta^{ab}. \tag{9.3}$$

The two constants are connected by

$$C(R) \cdot \dim(R) = T(R) \cdot \dim(G) \tag{9.4}$$

where $\dim(G)$ is the dimension of the group and $\dim(R)$ is the dimension of the irreducible representation R.
In the case of $SU(N)$ one has a diagonal metric $g^{ab} = \delta^{ab}$ and therefore 9.2 turns to

$$\sum_a (T^a(R))^2 = C(R)\mathbb{1}_{\dim(R)\times\dim(R)} \tag{9.5}$$

and one can write down the following useful formulae for the fundamental representation $R = F$: $T^a_{ij} = \frac{\lambda^a_{ij}}{2}$ and the adjoint representation $R = A$: $(T^a_{ij})^{adj} = -if_{aij}$

$$\begin{aligned}
T^a_{ik}T^a_{kj} &= C(F)\mathbb{1}_{ij} && \text{with } C(F) = \frac{N^2-1}{2N} = \frac{4}{3} \\
f^{abc}f^{dbc} &= C(A)\delta^{ad} && \text{with } C(A) = N = 3 \\
\mathrm{Tr}\left[T^aT^b\right] &= T(F)\delta^{ab} && \text{with } T(F) = \frac{1}{2}
\end{aligned} \tag{9.6}$$

where λ^a_{ij} are for $N_c = 3$ the Gell-Mann matrices and f_{abc} are the structure constants of $SU(N_c)$.

9.3 Weyl basis and 2-spinor notation

As representation of the γ-matrices the Weyl or chiral representation is chosen:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} -\mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix} \tag{9.7}$$

with

$$\sigma^\mu = \begin{pmatrix} \mathbb{1}_2 & \sigma^i \end{pmatrix}, \qquad \overline{\sigma}^\mu = \begin{pmatrix} \mathbb{1}_2 & -\sigma^i \end{pmatrix}, \tag{9.8}$$

where σ^i are the Pauli matrices and $\mathbb{1}_n$ is the nxn unit matrix. The left and right handed projectors are then given by

$$P_L = \frac{1}{2}(\mathbb{1}_4 - \gamma_5)\begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix} \qquad P_R = \frac{1}{2}(\mathbb{1}_4 + \gamma_5)\begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix} \tag{9.9}$$

The generator of the Lorentz group on 4-spinor space is composed of the above matrices. Because of the block form of those it is not surprising that the representation on 4-spinor space is reducible to two representations on 2-spinor (Weyl spinor) spaces. It is therefore sensible to decompose a 4 spinor into a left and a right handed Weyl spinor[10]

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot\alpha} \end{pmatrix} \tag{9.10}$$

where $\alpha, \dot\alpha = 1, 2$. Left handed Weyl spinors are labeled with undotted and right handed Weyl spinors with dotted indices. One distinguishes 4 different Weyl spinors:

$$\psi^\alpha, \qquad \overline{\psi}^{\dot\alpha} = (\psi^\alpha)^*, \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \qquad \text{and} \qquad \psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\psi^{\dot\beta} = (\psi_\alpha)^*, \tag{9.11}$$

[10]The projectors in the chiral basis 9.9 explain the names left and right handed Weyl spinors.

where * denotes complex conjugation and indices are lowered with the antisymmetric $\epsilon_{\alpha\beta}$ ($\epsilon_{\dot\alpha\dot\beta}$), which obeys

$$\epsilon^{\alpha\beta} = \epsilon_{\beta\alpha},\ \epsilon^{\dot\alpha\dot\beta} = \epsilon_{\dot\beta\dot\alpha} \qquad \text{and} \qquad \epsilon_{12} = \epsilon_{\dot1\dot2} = 1. \tag{9.12}$$

By virtue of the antisymmetry of ϵ one has for the Lorentz invariant product:

$$\begin{aligned} \psi\chi &:= \psi^\alpha\chi_\alpha = -\chi_\alpha\psi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi, \\ \overline{\psi}\overline{\chi} &:= \overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha} = -\overline{\chi}^{\dot\alpha}\overline{\psi}_{\dot\alpha} = \overline{\chi}_{\dot\alpha}\overline{\psi}^{\dot\alpha} = \overline{\chi}\overline{\psi} \end{aligned} \tag{9.13}$$

To make the index structure of the Pauli matrices explicit one writes $\sigma^\mu_{\alpha\dot\alpha}$ and $\sigma^{\mu\dot\alpha\alpha}$ for the formulae in 9.8. For the definition of the super algebra in ??? the generators of the Lorentz group on the left and right handed Weyl spinor space are introduced:

$$\begin{aligned} \frac{1}{2}(\sigma^{\mu\nu})_\alpha^{\ \beta} &:= \frac{i}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)_\alpha^{\ \beta} \\ \frac{1}{2}(\overline{\sigma}^{\mu\nu})^{\dot\alpha}_{\ \dot\beta} &:= \frac{i}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu)^{\dot\alpha}_{\ \dot\beta} \end{aligned} \tag{9.14}$$

With the definition of bared and charge conjugated 4-spinors[11]

$$\overline{\Psi} := \Psi^\dagger\gamma^0, \qquad \Psi^C := i\gamma^2\gamma^0\overline{\Psi}^T \tag{9.15}$$

one obtains:

$$\begin{aligned} \Psi &= \begin{pmatrix} \psi_\alpha \\ \overline{\chi}^{\dot\alpha} \end{pmatrix}, & \overline{\Psi} &= \begin{pmatrix} \chi^\alpha & \overline{\psi}_{\dot\alpha} \end{pmatrix}, \\ \Psi^C &= \begin{pmatrix} \chi_\alpha \\ \overline{\psi}^{\dot\alpha} \end{pmatrix}, & \overline{\Psi}^C &= \begin{pmatrix} \psi^\alpha & \overline{\chi}_{\dot\alpha} \end{pmatrix}. \end{aligned} \tag{9.16}$$

The 4-spinor of an arbitrary quark q is given in terms of Weyl spinors q_L and $\overline{q}_R$ by

$$q = \begin{pmatrix} q_L \\ \overline{q}_R \end{pmatrix} \tag{9.17}$$

whereas the 4-spinor of the Dirac gauginos is given in terms of the Weyl spinors λ and $\overline{\chi}$[12]

$$\tilde{g}^a = \begin{pmatrix} -i\lambda^a \\ i\overline{\chi}^a \end{pmatrix} \tag{9.18}$$

[11] Ψ^T denotes the transpose of the spinor Ψ and $\Psi^\dagger$ is the Hermitian adjoint of Ψ.

[12] λ is the superpartner of the gluon, called the gluino and $\overline{\chi}$ is the Weyl spinor of the chiral superfield which is associated with the gluon, called the octino.

9.4 Anticommuting numbers

Anticommuting numbers θ^α are also referred to as Grassmann numbers and are defined by $\theta^\alpha\theta^\beta = -\theta^\beta\theta^\alpha$ and commute with ordinary numbers.
They occur in superspace formalism in the form of 2 tuples, i.e. θ^α with $\alpha = 1, 2$. The complex conjugate of this tuple is denoted with $\overline{\theta}^{\dot{\alpha}}$. Derivatives are defined by

$$\begin{aligned} \partial^\alpha\theta_\beta &:= \frac{\partial}{\partial\theta_\alpha}\theta_\beta := \delta^\alpha_\beta & \partial_\alpha\theta^\beta &:= \frac{\partial}{\partial\theta^\alpha}\theta^\beta := \delta^\beta_\alpha \\ \overline{\partial}^{\dot{\alpha}}\overline{\theta}_{\dot{\beta}} &:= \frac{\partial}{\partial\overline{\theta}_{\dot{\alpha}}}\overline{\theta}_{\dot{\beta}} := \delta^{\dot{\alpha}}_{\dot{\beta}} & \overline{\partial}_{\dot{\alpha}}\overline{\theta}^{\dot{\beta}} &:= \frac{\partial}{\partial\overline{\theta}^{\dot{\alpha}}}\overline{\theta}^{\dot{\beta}} := \delta^{\dot{\beta}}_{\dot{\alpha}} \end{aligned} \tag{9.19}$$

whereby one needs to be cautious as these definitions imply

$$\partial_\alpha = -\epsilon_{\alpha\beta}\partial^\beta \qquad \partial_{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}}\partial^{\dot{\beta}}. \tag{9.20}$$

Integrals are defined by:

$$\int \mathrm{d}\theta_\alpha\ (a + b\theta^\beta + c\theta^\beta\theta^\gamma) := b\delta^\beta_\alpha + c(\delta^\beta_\alpha\theta^\gamma - \delta^\gamma_\alpha\theta^\beta) \qquad \text{and} \qquad \int \mathrm{d}\theta_\alpha\ (a\overline{\theta}^{\dot{\beta}}) := (a\overline{\theta}^{\dot{\beta}}) \int \mathrm{d}\theta_\alpha \tag{9.21}$$

where the first line mirrows the claim of translation invariance. One furthermore introduces the shortcuts

$$\int \mathrm{d}\theta^2 = \int \frac{1}{4}\epsilon_{\alpha\beta}\mathrm{d}\theta^\alpha\mathrm{d}\theta^\beta, \qquad \int \mathrm{d}\overline{\theta}^2 = \int \frac{1}{4}\epsilon_{\dot{\alpha}\dot{\beta}}\mathrm{d}\theta^{\dot{\alpha}}\mathrm{d}\theta^{\dot{\beta}}, \qquad \text{and} \qquad \int \mathrm{d}^4\theta := \int \mathrm{d}\theta^2\mathrm{d}\overline{\theta}^2 \tag{9.22}$$

9.5 Feynman rules for the RSQCD

The following Feynman rules are derived from 4.13. When compared with the Feynman rules of the supersymmetric QCD the diagrams involving scalar gluons are new. In addition the gluon-quark-squark vertex is different in RSQCD for the gauginos are Dirac fermions.
refer to paper cited in Beenakker(Appendix: Feynmanrules)

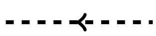
(1)

$$(\sigma^0)^a \; (\sigma^0)^b \;\hat{=}\; \frac{i}{p^2 - m_{\sigma^0}^2 + i\varepsilon}\delta_{ab}$$

(2)

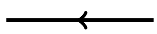
$$(\phi^0)^a \; (\phi^0)^b \;\hat{=}\; \frac{i}{p^2 - m_{\phi^0}^2 + i\varepsilon}\delta_{ab}$$

(3)



$$\tilde{q}_{Ai} \; \tilde{q}^\dagger_{Bj} \;\hat{=}\; \frac{i\delta_{AB}}{p^2 - m_{\tilde{q}}^2 + i\varepsilon}\delta_{ij}$$

(4)



$$q_i \; \overline{q}_j \;\hat{=}\; i\frac{\not{p} + m_q}{p^2 - m_q^2 + i\varepsilon}\delta_{ij}$$

(5)

$$G^a_\mu \; G^b_\nu \;\hat{=}\; -i\frac{g^{\mu\nu}}{p^2 + i\varepsilon}\delta_{ab}$$

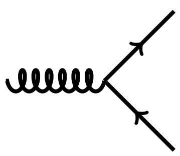
(6)

$$\tilde{g}^a \; \overline{\tilde{g}}^b \;\hat{=}\; i\frac{\not{p} + m_{\tilde{g}}}{p^2 - m_{\tilde{g}}^2 + i\varepsilon}\delta_{ab}$$

(7)

$$c^a \; \overline{c}^b \;\hat{=}\; \frac{i}{p^2 + i\varepsilon}\delta_{ab}$$

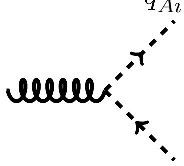
(8)



G^a_μ, $\overline{q}i$, q_j

$$\hat{=} - ig_s T^a_{ij}\gamma^\mu$$

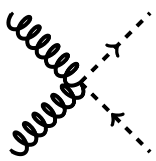
(9)



G^a_μ, $\tilde{q}^\dagger_{Ai}(-p_1)$, $\tilde{q}_{Bj}(p_2)$

$$\hat{=} - ig_s(p_2 + p_1)^\mu T^a_{ij}\delta_{AB}$$

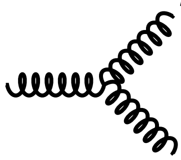
(10)



G^a_μ, G^b_ν, $\tilde{q}^\dagger_{Ai}$, $\tilde{q}_{Bj}$

$$\hat{=}\; ig_s^2 g^{\mu\nu}\left\{T^a, T^b\right\}_{ij}\delta_{AB}$$

(11)



$G^c_\rho(p_c)$, $G^a_\mu(p_a)$, $G^b_\nu(p_b)$

$$\hat{=} - g_s f_{abc}\left[g_{\mu\nu}(p_a - p_b)^\rho + \; g_{\nu\rho}(p_b - p_c)^\mu + g_{\rho\mu}(p_c - p_a)^\nu\right]$$

Figure 9.1: In the Feynman diagrams of the propagators the momentum is flowing from the right to the left hand side.
In the Feynman diagrams of the vertices all momenta flow towards the vertex.
The indices $A, B = L, R$ label the right/left-"handedness" of the squarks. The indices $i, j = 1, 2, 3$ are the color indices in the (anti)fundamental representation where $a, b, c, \ldots = 1, \ldots, 8$ are the color indices of the adjoint representation.

9.6 Passarino-Veltman Integrals

$$\begin{aligned}
A_0(m^2) &= \frac{i}{16\pi^2}\int_l \frac{1}{l^2-m^2} \\
B_{0,\mu,\mu\nu}(p^2,m_1^2,m_2^2) &= \frac{i}{16\pi^2}\int_l \frac{\{1,l_\mu,l_\mu l_\nu\}}{[l^2-m_1^2][(l+p)^2-m_2^2]} \\
C_{0,\mu,\mu\nu}(p_1^2,p_2^2,m_1^2,m_2^2,m_3^2) &= \frac{i}{16\pi^2}\int_l \frac{\{1,l_\mu,l_\mu l_\nu\}}{[l^2-m_1^2][(l+p_1)^2-m_2^2][(l+p_2)^2-m_3^2]}
\end{aligned} \tag{9.23}$$

with the shortcut $\int_l = \mu^{2\epsilon}\int \frac{\mathrm{d}^D l}{(2\pi)^D}$. Furthermore there are supressed ε's which prescripe how the poles in the complex plane are avoided. They are hidden in the infinitesimal shift of the masses: $m_i^2 \to m_i^2 - i\varepsilon$.
The tensor integrals can be decomposed as

$$\begin{aligned}
B_\mu &= p_\mu B_1 \\
B_{\mu\nu} &= g_{\mu\nu}B_{00} + p_\mu p_\nu B_{11} \\
C_\mu &= p_{1\mu}C_1 + p_{2\mu}C_2 \\
C_{\mu\nu} &= g_{\mu\nu}C_{00} + p_{1\mu}p_{1\nu}C_{11} + p_{2\mu}p_{2\nu}C_{22} + (p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu})C_{12}
\end{aligned} \tag{9.24}$$

$$A_0(m^2) = m^2\left(\Delta - \ln\frac{m^2}{\mu^2} + 1\right) + \mathcal{O}(\epsilon) \tag{9.25}$$

$\Delta = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$

$$B_0(0,m_1^2,m_2^2) = \frac{A_0(m_1^2) - A_0(m_2^2)}{m_1^2-m_2^2} = \Delta + 1 - \frac{m_1^2\ln\frac{m_1^2}{\mu^2} - m_2^2\frac{m_2^2}{\mu^2}}{m_1^2-m_2^2} + \mathcal{O}(\epsilon) \tag{9.26}$$

$$B_0(0,m^2,m^2) = \frac{\partial}{\partial m^2}A_0(m^2) = \Delta - \ln\frac{m^2}{\mu^2} + 1 + \mathcal{O}(\epsilon) \tag{9.27}$$

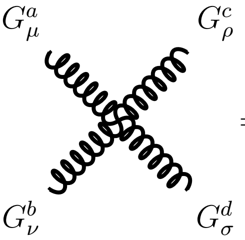
9.7 Cross section and Phase Space Integration

Once the Feynman amplitude $\mathcal{M}$ for a $2 \to N$ body scattering[13] is computed one can calculate physical observables with it. The differential cross section for $2 \to N$ scattering is given by

$$\mathrm{d}\sigma = \frac{1}{F}\mathrm{d}\Phi_N|\mathcal{M}|^2. \tag{9.28}$$

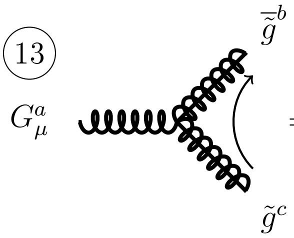
[13]with kinematics $k_a + k_b \to p_1 + \ldots p_N$

12



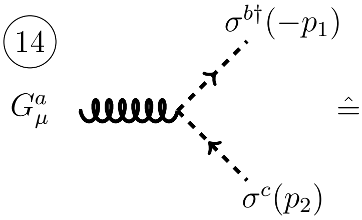
$$\hat{=} \ -ig_s^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})+f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}) \\ +f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})]$$

13



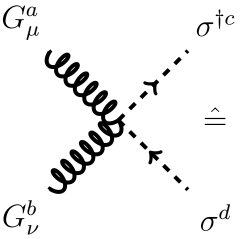
$$\hat{=} -g_s f_{abc}\gamma^\mu$$

14



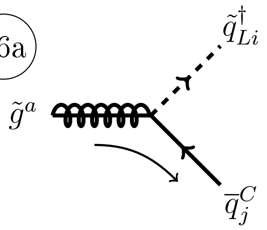
$$\hat{=} -g_s(p_1+p_2)^\mu f_{abc}$$

15



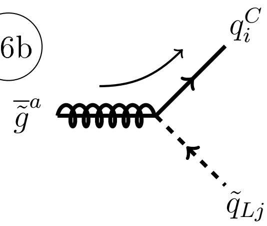
$$\hat{=} \ +ig_s^2 g^{\mu\nu}[f^{aec}f^{bed}+f^{bec}f^{aed}]$$

16a



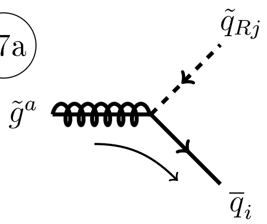
$$\hat{=} -i\sqrt{2}g_s T^a_{ij}P_L$$

16b



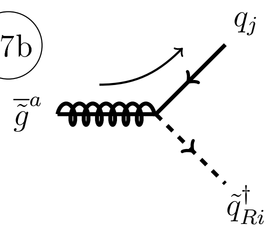
$$\hat{=} -i\sqrt{2}g_s T^a_{ij}P_R$$

17a



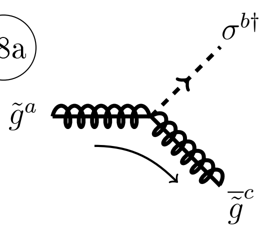
$$\hat{=} +i\sqrt{2}g_s T^a_{ij}P_L$$

17b



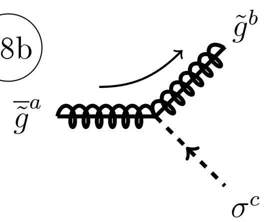
$$\hat{=} +i\sqrt{2}g_s T^a_{ij}P_R$$

18a



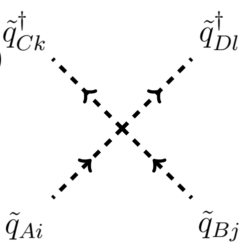
$$\hat{=} -\sqrt{2}g_s f^{abc}P_L$$

18b



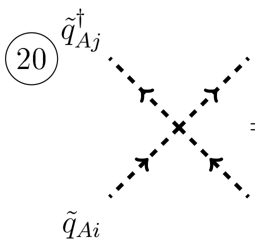
$$\hat{=} +\sqrt{2}g_s f^{abc}P_R$$

19

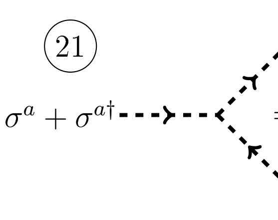


$$\hat{=} \ -ig_s^2[T^a_{ki}T^a_{lj}(\delta_{AL}\delta_{CL}-\delta_{AR}\delta_{CR})(\delta_{BL}\delta_{DL}-\delta_{BR}\delta_{DR}) \\ T^a_{kj}T^a_{li}(\delta_{BL}\delta_{CL}-\delta_{BR}\delta_{CR})(\delta_{AL}\delta_{DL}-\delta_{AR}\delta_{DR})]$$

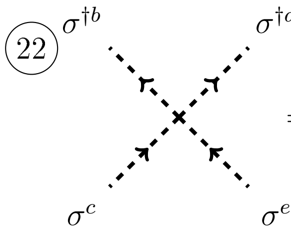
Figure 9.2: The curved arrows indicate the fermion flow. The Feynman rules 16b, 17b and 18b are the complex conjugates of 16a, 17a and 18a respectively. Applying a flipping rule to a vertex one has to reverse the curved arrow, i.e. the fermion flow and replace Ψ with $\bar{\Psi}^C$. In addition one has to add a minus sign for Feynman rule 13.



$$\hat{=} \; -g_s^2 T^a_{ij} f^{abc}(\delta_{AL}\delta_{CL} - \delta_{AR}\delta_{BR})$$



$$\hat{=} -i\sqrt{2} g_s m_{\tilde{g}} T^a_{ij}(\delta_{AL}\delta_{BL} - \delta_{AR}\delta_{BR})$$



$$\hat{=} \; -g_s^2(f^{abc}f^{ade} + f^{abc}f^{adc})$$

The flux factor is defined by $F = 4\sqrt{(k_a \cdot k_b)^2 - (m_a m_b)^2}$ which equals $F = 2s$ for massless initial state particles. The differential for the N body phase space in D dimensions is given by

$$\mathrm{d}\Phi_N = \left(\mu^{2\epsilon}\right)^{N-1} \left(\prod_{f=1}^{N} \frac{\mathrm{d}^{D-1} p_f}{(2\pi)^{D-1}} \frac{1}{2E_f} \right) (2\pi)^D \delta^{(D)}(k_a + k_b - \sum_{f=1}^{N} p_f). \tag{9.29}$$

The factor $\mu^{2\epsilon}$ is included to maintain the mass dimension of the cross section. In this thesis the sum of $|\mathcal{M}|^2$ over helicities and colors $\sum |\mathcal{M}|^2$ has been calculated. Furthermore initial state particles are considered as massless. This gives

$$\mathrm{d}\sigma = \frac{1}{2s} \mathrm{d}\Phi_2 K_{ab} \sum |\mathcal{M}|^2 \tag{9.30}$$

where K_{ab} encodes the averaging over initial state helicities and colors. Specifying to the center-of-mass frame and assuming that $\sum |\mathcal{M}|^2$ is only a function of the modulus of one of the final state particle's 3-momentum $|\vec{p}_i|$ and the angle θ between $\vec{k}_a$ and $\vec{p}_1$ one can write

$$\begin{aligned} \int \mathrm{d}\Phi_2 &= \frac{1}{s}\mu^{2\epsilon} \int \frac{\mathrm{d}|\vec{p}_1| \mathrm{d}\Omega_1^{D-1}}{(2\pi)^{D-2}} |\vec{p}_1|^{D-2} \delta\left(k_a^0 + k_b^0 - \sqrt{m_1^2 + |\vec{p}_1|^2} - \sqrt{m_2^2 + |\vec{p}_2|^2} \right) \\ &= \frac{1}{s} \frac{1}{(2\pi)^{D-2}} \frac{2\pi^{\frac{D}{2}-1}}{\Gamma(\frac{D}{2}-1)} \mu^{2\epsilon} \int_0^\infty \mathrm{d}|\vec{p}_1| \int_0^\pi \mathrm{d}\cos\theta p_1^{D-2} \sin^{D-4}\theta \\ &\quad \delta\left(k_a^0 + k_b^0 - \sqrt{m_1^2 + |\vec{p}_1|^2} - \sqrt{m_2^2 + |\vec{p}_2|^2} \right). \end{aligned} \tag{9.31}$$

In the second line the integral over the D-dimensional hypersphere

$$\int \mathrm{d}\Omega^D = \int_0^{2\pi} \mathrm{d}\phi \prod_{i=1}^{D-2} \int_0^{\pi} \sin^i \theta_i \mathrm{d}\theta_i = \frac{2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \tag{9.32}$$

was used. Because $\sum |\mathcal{M}|^2$ is calculated in terms of Mandelstam variables

$$\begin{aligned} t &= (k_a - p_1)^2 \\ t &= -2\left(|\vec{k}_a|\sqrt{m_1^2 + |\vec{p}_1|^2} - |\vec{k}_a||\vec{p}_1|\cos\theta\right) + m_1^2 \end{aligned} \tag{9.33}$$

$$\begin{aligned} u &= (k_b - p_1)^2 \\ u &= -2\left(|\vec{k}_a|\sqrt{m_1^2 + |\vec{p}_1|^2} + |\vec{k}_b||\vec{p}_1|\cos\theta\right) + m_1^2 \end{aligned} \tag{9.34}$$

it is useful to perform a change of coordinates yielding

$$\mathrm{d}|\vec{p}_1|\ \mathrm{d}\cos\theta = -\frac{\sqrt{m_1^2 + |\vec{p}_1|^2}}{8|\vec{k}_a|^2|\vec{p}_1|^2}\mathrm{d}u\ \mathrm{d}t. \tag{9.35}$$

Inserting 9.35 into 9.31 and using $|\vec{k}_a| = \frac{\sqrt{s}}{2}$ gives

$$\begin{aligned} \int \mathrm{d}\Phi_2 =& \frac{1}{s}\frac{\pi^{-\frac{D}{2}+1}}{2^{D-3}\Gamma(\frac{D}{2}-1)} \int \mathrm{d}u\ \mathrm{d}t\ \left(\frac{tu - m_1^2 m_2^2}{\mu^{2\epsilon} s}\right)^{\frac{D-4}{2}} \\ & \frac{1}{4}\Theta(tu - 4m_1^2 m_2^2)\delta\left(s + t + u - m_1^2 - m_2^2\right) \end{aligned} \tag{9.36}$$

where the Θ-function comes from the bounds of $|\vec{p}_1|$ and θ visible in 9.31 and the combination of 9.33 and 9.34. Working in $D = 4 - 2\epsilon$ dimensions and inserting $\Theta(s - 4m^2)$ with $m = \frac{m_1 + m_2}{2}$ to account for the production threshold one finds

$$\begin{aligned} \frac{\mathrm{d}^2\sigma}{\mathrm{d}t\mathrm{d}u} =& \frac{K_{ab}}{s^2}\frac{\pi S_\epsilon}{\Gamma(1-\epsilon)}\left[\frac{tu - m_1^2 m_2^2}{\mu^2 s}\right]^{-\epsilon} \Theta(tu - m_1^2 m_2^2) \\ & \Theta(s - 4m^2)\delta(s + t + u - m_1^2 - m_2^2)\sum |\mathcal{M}|^2 \end{aligned} \tag{9.37}$$

where $S_\epsilon = (4\pi)^{-2+\epsilon}$ as defined in [?]. The averaging factors K_{ab} are given by

$$K_{qq} = \frac{1}{4N_c^2} \qquad K_{GG} = \frac{1}{4(1-\epsilon)^2(N_c^2-1)^2} \qquad K_{qG} = \frac{1}{4(1-\epsilon)N_c(N_c^2-1)}. \tag{9.38}$$

Erklärung

Hiermit erkläre ich, dass ich diese Arbeit im Rahmen der Betreuung am Institut für ??? Physik ohne unzulässige Hilfe Dritter verfasst und alle Quellen als solche gekennzeichnet habe.
Vorname Nachname

Dresden, Monat 2012