HOMEWORK 1: NUMBER SYSTEMS AND BINARY ARITHMETICS

Module: Informational technologies FMISB18100

Vilnius Gediminas Technical University

Department of Information Systems

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Abstract

This is the 1st homework for Information Technologies module. In this assignment there are calculations for conversion of numbers of different radix to decimal numbers, conversion of decimal numbers to the numbers of different radix and direct conversion of the integer numbers of different radix. To complete this assignment the recordings of lectures as well as online resources were used. References are provided at the end of the paper.

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Assignment

Homework 1. Student: *LIENE MALKALNE* **group:** *2 group* 1

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Department of Information Systems

Assignment of homework 1: Number systems and binary aritmetics

Part I. Conversion of numbers of different radix to the decimal numbers

Convert the following numbers $X_{r,i}$ of radix $r \in \{2, 3, 4, ..., 16, ..., 32\}$, $i \in \{1, 2, 3, 4\}$ into the decimal numbers $Y_{10,i}$.

1. Binary number $X_2 = 1101110.111101_2$
2. Quaternary $X_4 = 201311321.110323_4$
3. Octal $X_8 = 75012575.023505_8$
4. Hexadecimal $X_{16} = 8EFBAA.ECD108_{16}$

Part II. Conversion of the decimal numbers to the numbers of the different radix

Convert the following decimal numbers $X_{10,i}$ into the numbers $Y_{10,i}$ of the different radix r: $r \in \{2, 3, 4, ..., 16\}$, $i \in \{1, 2, 3, 4\}$. The number of the significant numerals after the radix point (decimal places) should be bigger than 6.

1. Decimal number $X_{10} = 12222.5151_{10}$ into a number Y_2 of radix $r = 2$
2. Decimal number $X_{10} = 1455782.15415_{10}$ into a number Y_4 of radix $r = 4$
3. Decimal number $X_{10} = 2585802.265485_{10}$ into a number Y_8 of radix $r = 8$
4. Decimal number $X_{10} = 235662524665.12324598_{10}$ into a number Y_{16} of radix $r = 16$

Part III. Direct conversion of the integer numbers of different radix

III.1 Convert directly the integer numbers of radix $r \in \{4, 8, 16\}$ into the binary numbers X_2.

1. Quaternary $X_4 = 21301.12133_4$
2. Octal $X_8 = 77564.6255_8$
3. Hexadecimal $X_{16} = E10D.F8ED_{16}$

III.2 Convert directly the integer binary numbers to the numbers of radix $r \in \{4, 8, 16\}$.

1. Binary number $X_{2,1} = 1110.01101_2$ to the quaternary Y_4
2. Binary number $X_{2,2} = 1110011.10110_2$ to the octal Y_8
3. Binary number $X_{2,3} = 1110111011.011001_2$ to the hexadecimal Y_{16}

Remarks

The report of the homework must contain: the title page, assignment, abstract page, table of contents, the main text of the homework whose chapters and numbering of the chapters correspond to the above listed items, the list of the references (if literature were used)

All calculations, derivations of formulas assumptions etc. must be commented in the text of the report; variables of the formulas must be explained when they are used for the first time in the text; figures and tables must be labelled and numbered in the text of the report.

Part I. Conversion of numbers of different radix to the decimal numbers

Convert the following numbers $X_{r,i}$ of radix $r \in \{2, 3, 4, ..., 16, ..., 32\}$, $i \in \{1, 2, 3, 4\}$ into the decimal numbers $Y_{10,i}$.

1. Binary number $X_2 = 1101110.111101_2$

Calculations:

$(1x2^6)+(1x2^5)+(0x2^4)+(1x2^3)+(1x2^2)+(1x2^1)+(0x2^0)+(1x2^{-1})+(1x2^{-2})+(1x2^{-3})+(1x2^{-4})+(0x2^{-5})+(1x2^{-6}) =$
$(1x65)+(1x32)+(0x16)+(1x8)+(1x4)+(1x2)+(0x1)+(1x0.5)+(1x0.25)+(1x0.125)+(1x0.0625)+(0x0.03125)+$
$+(1x0.015625) = 65+32+0+8+4+2+0+0.5+0.25+0.125+0.0625+0+0.015625=111.953125$

Answer: $(111.953125)_{10}$

2. Quaternary $X_4 = 201311321.110323_4$

Calculations:

$(2x4^8)+(0x4^7)+(1x4^6)+(3x4^5)+(1x4^4)+(1x4^3)+(3x4^2)+(2x4^1)+(1x4^0)+(1x4^{-1})+(1x4^{-2})+(0x4^{-3})+(3x4^{-4})+$
$+(2x4^{-5})+(3x4^{-6}) =$
$(2x65536)+(0x16384)+(1x4096)+(3x1024)+(1x256)+(1x64)+(3x16)+(2x4)+(1x1)+(1x0.25)+(1x0.0625)+$
$+(0x0.015625)+(3x0.00390625)+(2x0.0009765625)+(3x0.000244140625) =$
$131072+0+4096+3072+256+64+48+8+1+0.25+0.0625+0+0.01171875+0.001953125+0.000732421875=$
138617.326904296875

Answer: $(138617.326904296875)_{10}$

3. Octal $X_8 = 75012575.023505_8$

Calculations:

$(7x8^7)+(5x8^6)+(0x8^5)+(1x8^4)+(2x8^3)+(5x8^2)+(7x8^1)+(5x8^0)+(0x8^{-1})+(2x8^{-2})+(3x8^{-3})+(5x8^{-4})+(0x8^{-5})+(5x8^{-6})=$
$(7x2097152)+(5x262144)+(0x32768)+(1x4096)+(2x512)+(5x64)+(7x8)+(5x1)+(0x0.125)+(2x0.015625)+(3x$
$0.001953125)+(5x0.000244140625)+(0x0.000030517578125)+ +(5x0.000003814697265625)=$
$14680064+1310720+0+4096+1024+320+56+5+0+0.03125+0.005859375+0.001220703125+0+$
$+0.000019073486328125=15996285.038349151611328125$

Answer: $(15996285.038349151611328125)_{10}$

4. Hexadecimal $X_{16} = 8EFBAA.ECD108_{16}$

Calculations:

$(8x16^5)+(14x16^4)+(15x16^3)+(11x16^2)+(10x16^1)+(10x16^0)+(14x16^{-1})+(12x16^{-2})+(13x16^{-3})+(1x16^{-4})+$
$+(0x16^{-5})+(8x16^{-6}) =$
$(8x1048576)+(14x65536)+(15x4096)+(11x256)+(10x16)+(10x1)+(14x0.0625)+(12x0.00390625)+$

$+(13x0.000244140625)+(1x0.000015258789063)+(0/1048576)+(8x0.00000005960464477539062 5) =$
$8388608+917504+61440+2816+160+10+0.875+0.046875+$
$+0.003173828125+0.000015258789063+0+0.000000476837158203125 =$
$9370538.925064563751221203125$

Answer: $(9370538.925064563751220703 13)_{10}$

Comment: *There is a discrepancy in last 6 numerals*

Part II. Conversion of the decimal numbers to the numbers of the different radix

Convert the following decimal numbers $X_{10,i}$ into the numbers $Y_{10,i}$ of the different radix r:
$r \in \{2, 3, 4, ..., 16\}$, $i \in \{1, 2, 3, 4\}$. The number of the significant numerals after the radix point (decimal places) should be bigger than 6

1. Decimal number $X_{10} = 12222.5151_{10}$ into a number Y_2 of radix r = 2

Calculations:

12222		**5151**	
12222/2 = 6111	Rem: 0 (last ↓)	0.5151 x 2 = 1.0302	1 (first ↓)
6111/2 = 3055.5	Rem: 1	0.0302 x 2 = 0.0604	0
3055/2 = 1527.5	Rem: 1	0.0604 x 2 = 0.1208	0
1527/2 = 763.5	Rem: 1	0.1208 x 2 = 0.2416	0
763/2 = 381.5	Rem: 1	0.2416 x 2 = 0.4832	0
381/2 = 190.5	Rem: 1	0.4832 x 2 = 0.9664	0

190/2 = 95	Rem: 0	0.9664 x 2 = 1.9328	1
95/2 = 47.5	Rem: 1	0.9328 x 2 = 1.8656	1
47/2 = 23.5	Rem: 1		
23/2 = 11.5	Rem: 1		
11/2 = 5.5	Rem: 1		
5/2 = 2.5	Rem: 1		
2/2 = 1	Rem: 0		
½ = 0.5	Rem: 1 (first ↑)		

Answer: $(10111110111110.10000011)_2$

2. Decimal number $X_{10} = 1455782.15415_{10}$ into a number Y_4 of radix r = 4

Calculations:

145578		**15415**	
145578/4 = 36394.5	Rem: 2 (last ↓)	0.15415 x 4 = 0.6166	0 (first ↓)
36394/4 = 9098.5	Rem: 2	0.6166 x 4 = 2.4664	2
9,098/4 = 2274.5	Rem: 2	0.4664 x 4 = 1.8656	1
2274/4 = 568.5	Rem: 2	0.8656 x 4 = 3.4624	3
568.5/4 = 142	Rem: 0	0.4624 x 4 = 1.8496	1
142/4 = 35.5	Rem: 2	0.8496 x 4 = 3.3984	3
35/4 = 8.75	Rem: 3	0.3984 x 4 = 1.5936	1
8/4 = 2	Rem: 0	0.5936 x 4 = 2.3744	2
2/4 = 0.5	Rem: 2 (first ↑)	0.3744 x 4 = 1.4976	1

Answer: $(203202222.021313121)_4$

3. Decimal number $X_{10} = 2585802.265485_{10}$ into a number Y_8 of radix r = 8

Calculations:

2585802	**265485**

2585802/8 = 323225.25	Rem: 2 (last ↓)	0.265485 x 8 = 2.12388	2 (first ↓)
323225/8 = 40403.125	Rem: 1	0.12388 x 8 = 0.99104	0
40403/8 = 5050.375	Rem: 3	0.99104 x 8 = 7.92832	7
5050/8 = 631.25	Rem: 2	0.92832 x 8 = 7.42656	7
631/8 = 78.875	Rem: 7	0.42656 x 8 = 3.41248	3
78/8 = 9.75	Rem: 6	0.41248 x 8 = 3.29984	3
9/8 = 1.125	Rem: 1	0.29984 x 8 = 2.39872	2
1/8 = 0.125	Rem: 1(first ↑)	0.39872 x 8 = 3.18976	3
		0.18976 x 8 = 1.51808	1

(0.125=1; 0.25=2; 0.375=3; 0.5=4; 0.625=5; 0.75=6; 0.875=7)

Answer: $(11672312.207733231)_8$

4. Decimal number X_{10} = $235662524665.12324598_{10}$ into a number Y_{16} of radix r = 16

Calculations:

235662524665		**12324598**	
235662524665/16 = 14728907791.5625	Rem: 9	0.12324598 x 16 = 1.97193568	1 (1st)
14728907791/16 = 920556736.9375	Rem: F	0.97193568 x 16 = 15.55097088	F
920556736/16 = 57534796	Rem: 0	0.55097088 x 16 = 8.81553408	8
57534796/16 = 3595924.75	Rem: C	0.81553408 x 16 = 13.04854528	D
3595924/16 = 224745.25	Rem: 4	0.04854528 x 16 = 0.77672448	0
224745/16 = 14046.5625	Rem: 9	0.77672448 x 16 = 12.42759168	C
14046/16 = 877.875	Rem: E	0.42759168 x 16 = 6.84146688	6
877/16 = 54.8125	Rem: D	0.84146688 x 16 = 13.46347008	D
54/16 = 3.375	Rem: 6	0.46347008 x 16 = 7.41552128	7
3/16 = 0.1875	Rem: 3 (1st)	0.41552128 x 16 = 6.64834048	6

(0=0; 1=0.0625; 2=0.125; 3=0.1875; 4=0.25; 5=0.3125; 6=0.375; 7=0.4375; 8=0.5; 9=0.5625; 10(A)=0.625; 11(B)=0.6875; 12(C)=0.75; 13(D)=0.8125; 14(E)=0.875; 15(F)=0.9375)

Answer: $(36DE94C0F9.1F8D0C6D76)_{16}$

Part III. Direct conversion of the integer numbers of different radix

III.1 Convert directly the integer numbers of radix r ∈ {4, 8, 16} into the binary numbers X_2.

1. Quaternary $X_4 = 21301.12133_4$

Calculations:

2	1	3	0	1.	1	2	1	3	3
21	21	21	21	21	21	21	21	21	21
10	01	11	00	01.	01	10	01	11	11

Answer: $(1001110001.0110011111)_2$

2. Octal $X_8 = 77564.6255_8$

Calculations:

7	7	5	6	4.	6	2	5	5
421	421	421	421	421	421	421	421	421
111	111	101	110	100.	110	010	101	101

Answer: $(111111101110100.110010101101)_2$

3. Hexadecimal $X_{16} = E10D.F8ED_{16}$

Calculations:

14	1	0	13.	15	8	14	13
8421	8421	8421	8421	8421	8421	8421	8421
1110	0001	0000	1101.	1111	1000	1110	1101

Answer: $(1110000100001101.1111100011101101)_2$

III.2 Convert directly the integer binary numbers to the numbers of radix r ∈{4, 8, 16}.

1. Binary number $X_{2,1} = 1110.01101_2$ to the quaternary Y_4

Calculations:

11	10.	01	10	10
21	21.	21	21	21
3	2.	1	2	2

Answer: $(32.122)_4$

2. Binary number $X_{2,2} = 1110011.10110_2$ to the octal Y_8

Calculations:

001	110	011.	101	100
421	421	421	421	421
1	6	3.	5	4

Answer: $(163.54)_8$

3. Binary number $X_{2,3} = 1110111011.011001_2$ to the hexadecimal Y_{16}

Calculations:

0011	1011	1011.	0110	0100
8421	8421	8421	8421	8421
3	11(B)	11(B).	6	4

Answer: $(3BB.64)_{16}$

Tables

Table 1.

2 Power	4 Power	8 Power	16 Power
$2^1 = 2$	$4^1 = 4$	$8^1 = 8$	$16^1 = 16$
$2^2 = 4$	$4^2 = 16$	$8^2 = 64$	$16^2 = 256$
$2^3 = 8$	$4^3 = 64$	$8^3 = 512$	$16^3 = 4096$
$2^4 = 16$	$4^4 = 256$	$8^4 = 4096$	$16^4 = 65536$
$2^5 = 32$	$4^5 = 1024$	$8^5 = 32768$	$16^5 = 1048576$
$2^6 = 64$	$4^6 = 4096$	$8^6 = 262144$	$16^6 = 16777216$
$2^7 = 128$	$4^7 = 16384$	$8^7 = 2097152$	$16^7 = 268435456$
$2^8 = 256$	$4^8 = 65536$	$8^8 = 16777216$	$16^8 = 4294967296$
$2^9 = 512$	$4^9 = 262144$	$8^9 = 134217728$	$16^9 = 68719476736$
$2^{10} = 1024$	$4^{10} = 1048576$	$8^{10} = 1073741824$	$16^{10} = 1099511627776$

Table 2.

Neg 2 Power	Neg 4 Power	Neg 16 Power
$2^{-1} = 0.5$	$4^{-1} = 0.25$	$16^{-1} = 0.0625$
$2^{-2} = 0.25$	$4^{-2} = 0.0625$	$16^{-2} = 0.00390625$
$2^{-3} = 0.125$	$4^{-3} = 0.015625$	$16^{-3} = 0.000244140625$
$2^{-4} = 0.0625$	$4^{-4} = 0.00390625$	$16^{-4} = 0.000015258789063$
$2^{-5} = 0.03125$	$4^{-5} = 0.0009765625$	$16^{-5} = 9.5367431641e-7$
$2^{-6} = 0.015625$	$4^{-6} = 0.000244140625$	$16^{-6} = 5.9604644775e-8$
$2^{-7} = 0.0078125$	$4^{-7} = 0.00006103515625$	$16^{-7} = 3.7252902985e-9$

References

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