Algorithmic Smart Factory Layout Planning at Schaeffler AG

Lie Reynaldo Ivander Gunawan
RWTH Aachen University

April 25, 2025

1 Introduction

Schaeffler AG is one of the world's largest family-owned motion technology companies, operating across more than 250 locations with 100 manufacturing plants and over 120,000 employees globally. As the number product families and production processes grow more complex, factory planning at Schaeffler faces increasing challenges. Hence, automated & efficient layout design is essential for minimizing material handling costs and ensuring smooth production flow. However, traditional manual planning methods are time-consuming and often inflexible in the face of shifting demand patterns.

We propose an *Algorithmic Layout Planning* (ALP) framework built on mixed-integer programming (MIP) that:

- Minimizes the weighted horizontal deviation of material flows
- Respects geometric and operational constraints (e.g., pillar clearance, machine-to-machine clearance, input/output zoning)
- Enables integration of real-world production data, such as factory layout size, machine dimension & product routing sequences

2 Model Assumptions & Specifications

The following assumptions & specifications will underlie the mathematical formulation of our problem.

- **Search Space:** The *search area*, defined as the region available for machine placement (will be further sub-divided into input & output zone later), spans a rectangular domain of dimensions $36{,}000 \times 50{,}000\,\text{mm}$ within a larger factory floor measuring $48{,}000 \times 53{,}000\,\text{mm}$. We exclude the rightmost $12{,}000\,\text{mm}$ of the floor ($36{,}000 \leq x \leq 48{,}000$) and the uppermost $3{,}000\,\text{mm}$ ($-3{,}000 \leq y \leq 0$) from the machine placement zone because of the need to reserve space for horizontal and vertical transport movements outside the production area, craning operations, and emergency access routes.

Additionally, this exclusion reduces the search space, which improves convergence without significantly affecting the objective value, as only the horizontal dimension is substantially reduced and the objective is to minimize weighted horizontal distances anyway. Lastly, this search space is also naturally excluded from the rest of the area by a set of structural pillars (see figure 1).

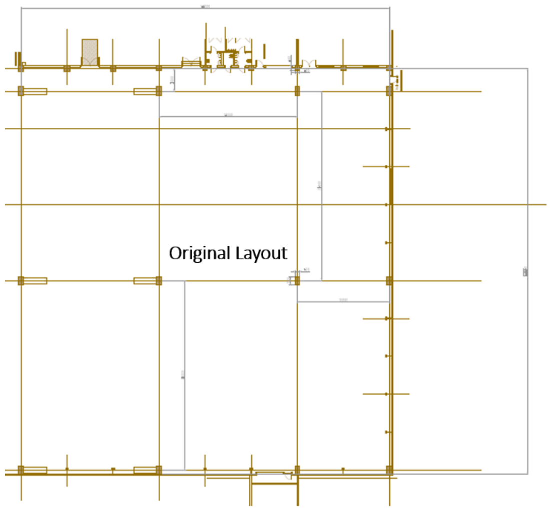

Figure 1: Original layout (left) & its zoning (right)

- **Coordinate System:** Origin point $(0, 0)$ will be at the top-left point of the $36\,000 \times 50\,000$ mm search space; with value of x increases rightward & y increases downward. From now on, units will be omitted and assumed to be in millimeter scale.

Figure 2: Example of machines along with its dimension

- **Machine Abstraction:** Each machine $m \in M$ is abstracted as a rectangle with width w_m and height h_m, with (x_m, y_m) denoting this rectangle's upper-left corner point position in the *search area* that will be optimized later. There are 19 machines in total that will be denoted by M1-M19. For instance, M12 and M13 in figure 2 are modeled as rectangles with dimensions (7081×8500) and (7900×6592) respectively.

Machines M14 and M15 are combined into one composite unit block, denoted M1415, as they always appear together in poduction processes. The width of the composite block is defined as $\max(w_{14}, w_{15})$, while its total height is given by $h_{14} + h_{15} + 1000$, in which 1000 represent the required minimum vertical distance/clearance between any two pair machines (will be explain further in the next section). Machines orientation is assumed to be as it is provided directly in the raw dataset, so machines rotation is not considered.

- **Machine Classification:** The machine set $\mathcal{M}$ is partitioned into three distinct categories:
  (a) **Input machines** ($\mathcal{M}_{\text{input}}$): machines used at the beginning of processing chains
  (b) **Output machines** ($\mathcal{M}_{\text{output}}$): machines producing final products/at the end of processing chains
  (c) **Regular machines** ($\mathcal{M}_{\text{regular}} = \mathcal{M} \setminus (\mathcal{M}_{\text{input}} \cup \mathcal{M}_{\text{output}})$): all other non input-output machines

  Machines exhibiting both input and output characteristics are classified as input machines.

- **Input/Output Zoning:** Based on request for distinction between input & output areas, we will do vertical partitioning at $y = 25{,}000\,\text{mm}$ to separate input and output zones, hence input machines ($\mathcal{M}_{\text{input}}$) must satisfy $y_m + h_m \leq 25{,}000$ while output machines ($\mathcal{M}_{\text{output}}$) require $y_m \geq 25{,}001$. This 1 mm is necessary for strict zoning separation since MIP solver used (Gurobi) does not handle strict inequality ($>$ or $<$).

- **Products & Processes:** There are 59 products and sub-products, each will be denoted by $p \in \mathcal{P}$ along with its associated percent-sales s_p. Each product has a process chain—a sequence of machines it visits. For every consecutive machine pair that appears in at least one process chain, we form an unordered pair (M_i, M_j) and include it in the set E. For each $(M_i, M_j) \in E$, we compute the horizontal distance d_{ij} (defined later) and assign the weight

$$s_{ij} = \sum_{\substack{p \in \mathcal{P} \\ (M_i, M_j) \in \text{chain}(p)}} s_p,$$

  by summing over all percent sales of all products whose process chain contains (M_i, M_j). For example, product 16.2 follows

$$M_{18} \to M_4 \to M_5 \to M_9,$$

  hence (M_{18}, M_4), (M_4, M_5), and (M_5, M_9) will be included in E, each weighted by the sum of the percent-sales of product 16.2 denoted by $s_{16.2}$ & any other percent-sales from products traversing that pair. From now on, for simplicity any two pair of machines (M_i, M_j) will simply denoted by its index (i , j).

- **Pillars or Pillar-like Structures:** There are a total of nine pillars or pillar-like structures exist at specified coordinates (though only those within the search area will be used as constraints). Each pillar is modelled as a 600×1100 rectangular block. A minimum clearance of 500 from any pillar edge to any machine is enforced to ensure safe access and leave some room for movement. Since we use p for products, set of all pillars will be denoted by $q \in Q$.

3 Mathematical Formulation

In this section, we present the formal MIP model for our factory layout problem. We first define the decision variables and parameters that capture machine placements. Next, we describe the essential constraints in detail—ensuring non-overlap between each machines, pillar avoidance, zoning separation, and linearization of absolute horizontal deviations. Finally, we specify the objective function that minimizes weighted horizontal deviations of material flows.

3.1 Decision Variables & Parameters

We define the following decision variables to model machine placements and deviation costs:

$$0 \le x_m \le 36000 - w_m, \qquad \forall m \in M, \tag{1}$$

$$0 \le y_m \le 50000 - h_m, \qquad \forall m \in M, \tag{2}$$

$$\delta_{i,j}^k \in \{0,1\}, \qquad \forall (i,j) \in M \times M,\ k \in \{L,R,B,A\}, \tag{3}$$

$$\pi_{m,q}^k \in \{0,1\}, \qquad \forall m \in M,\ q \in Q,\ k \in \{L,R,B,A\}, \tag{4}$$

$$d_{i,j} \ge 0, \qquad \forall (i,j) \in E. \tag{5}$$

The definition for each decision variable & its dependent variable are defined as follows:

- x_m, y_m: upper-left corner point coordinates of rectangular machine m
- $\delta_{i,j}^k$: binary indicator that machine i lies in position k relative to machine j
- $\pi_{m,q}^k$: binary indicator that machine m lies in position k around pillar q
- $k \in \{L,R,B,A\}$ denotes the relative position: Left, Right, Bottom, Above
- $d_{i,j}$: variable capturing absolute horizontal deviation for each pair of machines $(i,j) \in E$

The parameters are defined as follows:

- w_m, h_m: width and height of machine m
- (q_x, q_y): coordinates of center point of rectangular pillar q
- $s_{i,j}$: cumulative sales weight for each pair of machine $(i,j) \in E$
- $\delta_{\min}^{MM} = 1000$ mm: minimum clearance between any two machines
- $\delta_{\min}^{q} = 500$ mm: minimum machine-to-pillar clearance
- M: a sufficiently large constant ("big-M")

3.2 Constraints

We enforce a set of constraints to ensure a valid layout. Each group is described below:

- **Non-overlap.** To prevent any two machines from overlapping, for every unordered pair $(i, j) \in M \times M$ we require that machine i either lies entirely to the left, right, below, or above machine j, with at least $\delta_{\min}^{MM}$ clearance:

$$x_i + w_i + \delta_{\min}^{MM} \le x_j + M\,(1 - \delta_{i,j}^{L}), \tag{6}$$
$$x_j + w_j + \delta_{\min}^{MM} \le x_i + M\,(1 - \delta_{i,j}^{R}), \tag{7}$$
$$y_i + h_i + \delta_{\min}^{MM} \le y_j + M\,(1 - \delta_{i,j}^{B}), \tag{8}$$
$$y_j + h_j + \delta_{\min}^{MM} \le y_i + M\,(1 - \delta_{i,j}^{A}), \tag{9}$$
$$\delta_{i,j}^{L} + \delta_{i,j}^{R} + \delta_{i,j}^{B} + \delta_{i,j}^{A} \ge 1. \tag{10}$$

  Here, each binary variable $\delta_{i,j}^{k}$ activates the corresponding separation constraint when equal to 1, guaranteeing that machines do not encroach upon one another. For example $\delta_{3,5}^{L} = 1$ implies that M3 is located at least $\delta_{\min}^{MM}$ to the left of M5.

- **Pillar Clearance.** Each machine must lie entirely on one side of every pillar, accounting for a safety clearance of $\delta_{\min}^{q} = 500$ mm. Each pillar is modeled as a rectangle of width 600 mm and height 1100 mm centered at (q_x, q_y), giving horizontal and vertical extents:

$$q_x^{\min} = q_x - 300 \qquad\qquad q_x^{\max} = q_x + 300$$
$$q_y^{\min} = q_y - 550 \qquad\qquad q_y^{\max} = q_y + 550$$

  To maintain the clearance from pillar q, machine m must lie entirely to the left, right, below, or above the pillar rectangle:

$$x_m + w_m + \delta_{\min}^{q} \le q_x^{\min} + M\,(1 - \pi_{m,q}^{L}), \tag{11}$$
$$q_x^{\max} + \delta_{\min}^{q} \le x_m + M\,(1 - \pi_{m,q}^{R}), \tag{12}$$
$$y_m + h_m + \delta_{\min}^{q} \le q_y^{\min} + M\,(1 - \pi_{m,q}^{B}), \tag{13}$$
$$q_y^{\max} + \delta_{\min}^{q} \le y_m + M\,(1 - \pi_{m,q}^{A}), \tag{14}$$
$$\pi_{m,q}^{L} + \pi_{m,q}^{R} + \pi_{m,q}^{B} + \pi_{m,q}^{A} \ge 1. \tag{15}$$

  Here, each binary variable $\pi_{m,q}^{k}$ ensures that machine m is positioned entirely to side k of pillar q, respecting the full pillar dimensions and buffer. For example, $\pi_{4,q2}^{R} = 1$ implies machine 4 is placed at least $\delta_{\min}^{q}$ to the right of pillar $q2$.

- **Input/Output Zoning.** To respect functional zoning, we split the search area horizontally at $y = 25000$ mm. Input machines must lie entirely below this line, and output machines entirely above it:

$$y_m + h_m \le 25000, \qquad \forall m \in M_{\text{in}}, \tag{16}$$
$$y_m \ge 25001, \qquad \forall m \in M_{\text{out}}. \tag{17}$$

  The 1 mm gap ensures strict separation in the MIP model.

- **Linearization of Absolute Deviations.** The absolute horizontal deviation $d_{i,j}$ captures how far apart the horizontal *midpoints* of machines i and j are. Since x_i and x_j represent the upper-left coordinates of rectangular machine m, we compute the midpoint of each machine as $x_i + \frac{w_i}{2}$ and $x_j + \frac{w_j}{2}$. Therefore:

$$d_{i,j} \geq \left(x_i + \tfrac{w_i}{2}\right) - \left(x_j + \tfrac{w_j}{2}\right), \tag{18}$$
$$d_{i,j} \geq \left(x_j + \tfrac{w_j}{2}\right) - \left(x_i + \tfrac{w_i}{2}\right), \tag{19}$$

which together enforce $d_{i,j} = \left|\left(x_i + \frac{w_i}{2}\right) - \left(x_j + \frac{w_j}{2}\right)\right|$ at optimality.

3.3 Objective Function

Our aim is to place machines so that high-flow pair of machiness have minimum horizontal deviation & hence minimize horizontal movement needed in production processes:

$$\min \sum_{(i,j)\in E} s_{i,j}\, d_{i,j}. \tag{20}$$

Each term $s_{i,j}d_{i,j}$ weighs the deviation $d_{i,j}$ by the importance $s_{i,j}$ (see section 2) of the flow between machines i and j, encouraging the solver to prioritize alignment of high-traffic pairs while minimizing horizontal deviation.

4 Model Optimization Result

We perform the optimization model defined in previous section using the Gurobi Optimizer & fine-tune the core settings to boost performance; the parameters for auto hyper-parameter tuning are listed in Table 1.

Table 1: Automated Parameter Tuning Settings

Parameter	**Purpose**
`TuneTimeLimit`	Sets a maximum duration (set to 10 minutes) for the tuning process to prevent excessive computation time.
`TuneTrials`	Specifies the number of different parameter combinations (set to 5) to test during tuning.
`MIPFocus`	Adjusts the solver's focus: 1 for finding feasible solutions quickly, 2 for balancing between feasible and optimal solutions, and 3 for prioritizing optimality.
`Cuts`	Controls the aggressiveness of adding cutting planes to reduce the solution space and potentially speed up the solving process.
`Heuristics`	Determines the fraction of time the solver spends on heuristic methods to find good solutions early.
`VarBranch`	Sets the strategy for choosing which variable to branch on during the search process.
`NodeMethod`	Chooses the LP algorithm (primal or dual simplex) for solving node relaxations.

The final model is quite complex, with a total of 764 variables, 277 linear constraints, and 684 indicator constraints. We chose to terminate the solver at an optimality gap of approximately 4%, as improvements to the upper bound showed very significant diminishing returns—the upper bound is plateauing from around 40% to 4% optimality gap even after hundreds of thousands of seconds—while the lower bound increased only marginally & very slowly. Moreover, based on a study by Daniel Ahlbom (2017), on average, the difference between objective value solutions proven to be optimal and those stopped at a 5% optimality gap using the same solver (Gurobi) is only about ∼1%. Thus, even in the worst-case scenario, our solution is most likely already very close—if not equal—to optimality.

Lastly, we have got quite satisfactory result for all practical purposes. The resulting layout can be seen on the figure below. Note how the machines are positioned very closely horizontally in alignment with the primary objective function to minimize weighted horizontal movements, while still maintaining sufficient distance between one another and also from the pillars. Further evaluation will be detailed in the next section.

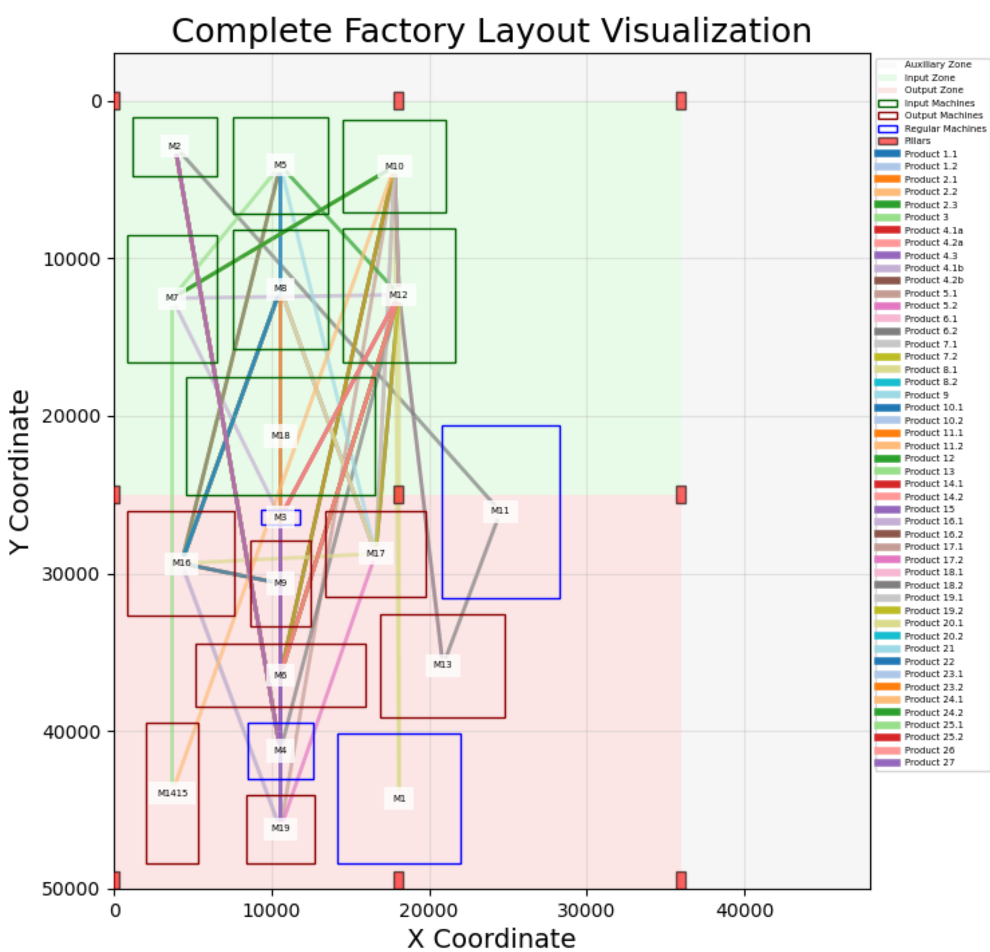

Figure 3: Factory Layout based on Optimization Model

5 Evaluation & Conclusion

Now that we have the optimization results, we evaluate it against benchmark criterias set by our clients. Table 2 below summarizes each request from Schaeffler ordered by priority and provides details on its fulfillment.

Table 2: Constraint Fulfillment Checklist

Priority	**Schaeffler Request**	**Modeling Results**
1	All required machines must be placed	All specified machines have been positioned within the hall ✓
2	Distance between two machines at least 1 meter	Minimum inter-machine spacing of 1 m enforced by placement constraints (see figure 3) ✓
3	Ways only run horizontally or vertically with products weighted based on their overall utilization	This is incorporated in the objective function with resulting pathways follow more or less minimal horizontal deviation (see figure 3) ✓
4	Each machine must be adjacent to a way	All machines is positioned at least to one horizontal or vertical corridor, ensuring direct access to the pathway network ✓
5	Distance to pillars is at least 0.5 meters	All machine footprints maintain a 0.5 m clearance from structural pillars (see figure 3) ✓
6	Distinction between input and output areas	Input and output zones are clearly separated and machines are oriented accordingly (see figure 3) ✓
7	Consider overcraning	Assuming that the crane operations can be done efficiently at the peripheral corridor then there should be more than enough space in auxiliary zone for this

Based on the evaluation table above, we conclude that the model has successfully fulfilled most, if not all, requirements using the exact methodology.

References

[1] Gurobi Optimization, LLC, *Gurobi Optimizer Reference Manual*, 2025.

[2] B. D. Persson, *Quadratic Programming Models in Strategic Sourcing Optimization*, Master's thesis, Uppsala University, 2017.