

How DES Works in Detail

DES is a *block cipher*--meaning it operates on plaintext blocks of a given size (64-bits) and returns ciphertext blocks of the same size. Thus DES results in a permutation among the 2^{64} possible arrangements of 64 bits, each of which may be either 0 or 1.

Example: Let m be the plaintext message in binary format, which is shown as follows:

$m = 0000\ 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111\ 1000\ 1001\ 1010\ 1011\ 1100\ 1101\ 1110\ 1111$

We divide m into two blocks of 32 bits each, a left half block L and a right half R .

$L = 0000\ 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111$

$R = 1000\ 1001\ 1010\ 1011\ 1100\ 1101\ 1110\ 1111$

The first bit of m is "0". The last bit is "1". We read from left to right.

DES operates on the 64-bit blocks using key sizes of 56-bits. The keys are actually stored as being 64 bits long, but every 8th bit in the key is not used (i.e. bits numbered 8, 16, 24, 32, 40, 48, 56, and 64). However, we will nevertheless number the bits from 1 to 64, going left to right, in the following calculations. But, as you will see, the eight bits just mentioned get eliminated when we create per-round keys.

Example: Let k be the key in binary format, which is shown as follows:

$k = 00010011\ 00110100\ 01010111\ 01111001\ 10011011\ 10111100\ 11011111\ 11110001$

The DES algorithm uses the following steps:

Step 1: Create 16 per-round keys, each of which is 48-bits long.

The 64-bit key is permuted according to the following table. Since the first entry in the table is "57", this means that the 57th bit of the original key k becomes the first bit of the permuted key k' . The 49th bit of the original key becomes the second bit of the permuted key. The 4th bit of the original key is the last bit of the permuted key. Note only 56 bits of the original key appear in the permuted key.

57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27

19	11	3	60	52	44	36
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

Example: From the original 64-bit key

$k = 00010011\ 00110100\ 01010111\ 01111001\ 10011011\ 10111100\ 11011111\ 11110001$

we get the 56-bit permutation

$k' = 1111000\ 0110011\ 0010101\ 0101111\ 0101010\ 1011001\ 1001111\ 0001111$

Next, split this key into left and right halves, C_0 and D_0 , where each half has 28 bits.

Example: From the permuted key k' , we get

$C_0 = 1111000\ 0110011\ 0010101\ 0101111$

$D_0 = 0101010\ 1011001\ 1001111\ 0001111$

With C_0 and D_0 defined, we now create sixteen blocks C_n and D_n , $1 \leq n \leq 16$. Each pair of blocks C_n and D_n is formed from the previous pair C_{n-1} and D_{n-1} , respectively, for $n = 1, 2, \dots, 16$, using the following schedule of "left shifts" of the previous block. To do a left shift, move each bit one place to the left, except for the first bit, which is cycled to the end of the block.

Iteration Number	Number of Left Shifts
1	1
2	1
3	2
4	2
5	2

6	2
7	2
8	2
9	1
10	2
11	2
12	2
13	2
14	2
15	2
16	1

This means, for example, C_3 and D_3 are obtained from C_2 and D_2 , respectively, by two left shifts, and C_{16} and D_{16} are obtained from C_{15} and D_{15} , respectively, by one left shift. In all cases, by a single left shift is meant a rotation of the bits one place to the left, so that after one left shift the bits in the 28 positions are the bits that were previously in positions 2, 3,..., 28, 1.

Example: From original pair C_0 and D_0 we obtain:

$C_0 = 1111000011001100101010101111$

$D_0 = 0101010101100110011110001111$

$C_1 = 1110000110011001010101011111$

$D_1 = 1010101011001100111100011110$

$C_2 = 1100001100110010101010111111$

$D_2 = 0101010110011001111000111101$

$C_3 = 0000110011001010101011111111$

$D_3 = 0101011001100111100011110101$

$C_4 = 0011001100101010101111111100$

$D_4 = 0101100110011110001111010101$

$C_5 = 1100110010101010111111110000$

$D_5 = 0110011001111000111101010101$

$C_6 = 0011001010101011111111000011$

$D_6 = 1001100111100011110101010101$

$C_7 = 110010101010111111100001100$
 $D_7 = 0110011110001111010101010110$

 $C_8 = 0010101010111111110000110011$
 $D_8 = 1001111000111101010101011001$

 $C_9 = 0101010101111111100001100110$
 $D_9 = 0011110001111010101010110011$

 $C_{10} = 0101010111111110000110011001$
 $D_{10} = 1111000111101010101011001100$

 $C_{11} = 0101011111111000011001100101$
 $D_{11} = 1100011110101010101100110011$

 $C_{12} = 0101111111100001100110010101$
 $D_{12} = 0001111010101010110011001111$

 $C_{13} = 0111111110000110011001010101$
 $D_{13} = 0111101010101011001100111100$

 $C_{14} = 1111111000011001100101010101$
 $D_{14} = 1110101010101100110011110001$

 $C_{15} = 1111100001100110010101010111$
 $D_{15} = 1010101010110011001111000111$

 $C_{16} = 1111000011001100101010101111$
 $D_{16} = 0101010101100110011110001111$

We now form the keys K_n , for $1 \leq n \leq 16$, by applying the following permutation table to each of the concatenated pairs $C_n D_n$. Each pair has 56 bits, but the following table only uses 48 of these.

14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

Therefore, the first bit of K_n is the 14th bit of $C_n D_n$, the second bit the 17th, and so on, ending with the 48th bit of K_n being the 32th bit of $C_n D_n$.

Example: For the first key we have $C_1 D_1 = 1110000\ 1100110\ 0101010\ 1011111\ 1010101\ 0110011\ 0011110\ 0011110$

which, after we apply the permutation table, becomes

$K_1 = 000110\ 110000\ 001011\ 101111\ 111111\ 000111\ 000001\ 110010$

For the other keys we have

$K_2 = 011110\ 011010\ 111011\ 011001\ 110110\ 111100\ 100111\ 100101$

$K_3 = 010101\ 011111\ 110010\ 001010\ 010000\ 101100\ 111110\ 011001$

$K_4 = 011100\ 101010\ 110111\ 010110\ 110110\ 110011\ 010100\ 011101$

$K_5 = 011111\ 001110\ 110000\ 000111\ 111010\ 110101\ 001110\ 101000$

$K_6 = 011000\ 111010\ 010100\ 111110\ 010100\ 000111\ 101100\ 101111$

$K_7 = 111011\ 001000\ 010010\ 110111\ 111101\ 100001\ 100010\ 111100$

$K_8 = 111101\ 111000\ 101000\ 111010\ 110000\ 010011\ 101111\ 111011$

$K_9 = 111000\ 001101\ 101111\ 101011\ 111011\ 011110\ 011110\ 000001$

$K_{10} = 101100\ 011111\ 001101\ 000111\ 101110\ 100100\ 011001\ 001111$

$K_{11} = 001000\ 010101\ 111111\ 010011\ 110111\ 101101\ 001110\ 000110$

$K_{12} = 011101\ 010111\ 000111\ 110101\ 100101\ 000110\ 011111\ 101001$

$K_{13} = 100101\ 111100\ 010111\ 010001\ 111110\ 101011\ 101001\ 000001$

$K_{14} = 010111\ 110100\ 001110\ 110111\ 111100\ 101110\ 011100\ 111010$

$K_{15} = 101111\ 111001\ 000110\ 001101\ 001111\ 010011\ 111100\ 001010$

$K_{16} = 110010\ 110011\ 110110\ 001011\ 000011\ 100001\ 011111\ 110101$

So much for the per-round keys. Now we look at the message itself.

Step 2: Encode each 64-bit block of data.

There is an *initial permutation* IP of the 64 bits of the message data m . This rearranges the bits according to the following table, where the entries in the table show the new arrangement of the bits from their initial order. The 58th bit of m becomes the first bit of IP. The 50th bit of m becomes the second bit of IP. The 7th bit of m is the last bit of IP.

IP

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

Example: Applying the initial permutation to the block of text m , given previously, we get

$m = 0000\ 0001\ 0010\ 0011\ 0100\ 0101\ 0110\ 0111\ 1000\ 1001\ 1010\ 1011\ 1100\ 1101\ 1110\ 1111$

IP = 1100 1100 0000 0000 1100 1100 1111 1111 1111 0000 1010 1010 1111 0000 1010 1010

Here the 58th bit of m is "1", which becomes the first bit of IP. The 50th bit of m is "1", which becomes the second bit of IP. The 7th bit of m is "0", which becomes the last bit of IP.

Next divide the permuted block IP into a left half L_0 of 32 bits, and a right half R_0 of 32 bits.

Example: From IP, we get L_0 and R_0

$L_0 = 1100\ 1100\ 0000\ 0000\ 1100\ 1100\ 1111\ 1111$

$R_0 = 1111\ 0000\ 1010\ 1010\ 1111\ 0000\ 1010\ 1010$

We now proceed through 16 iterations, for $1 \leq n \leq 16$, using a function f which operates on two blocks -- a data block of 32 bits and a key K_n of 48 bits -- to produce a block of 32 bits. Let + denote XOR addition, (bit-by-bit addition modulo 2). Then for n going from 1 to 16 we calculate

$$L_n = R_{n-1}$$

$$R_n = L_{n-1} + f(R_{n-1}, K_n)$$

This results in a final block, for $n = 16$, of $L_{16}R_{16}$. That is, in each iteration, we take the right 32 bits of the previous result and make them the left 32 bits of the current step. For the right 32 bits in the current step, we XOR the left 32 bits of the previous step with the calculation f .

Example: For $n = 1$, we have

$$K_1 = 000110\ 110000\ 001011\ 101111\ 111111\ 000111\ 000001\ 110010$$

$$L_1 = R_0 = 1111\ 0000\ 1010\ 1010\ 1111\ 0000\ 1010\ 1010$$

$$R_1 = L_0 + f(R_0, K_1)$$

It remains to explain how the function f works. To calculate f , we first expand each block R_{n-1} from 32 bits to 48 bits. This is done by using a selection table that repeats some of the bits in R_{n-1} . We'll call the use of this selection table the function E .

Thus $E(R_{n-1})$ has a 32 bit input block, and a 48 bit output block.

Let E be such that the 48 bits of its output, written as 8 blocks of 6 bits each, are obtained by selecting the bits in its inputs in order according to the following table:

E BIT-SELECTION TABLE

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

Thus the first three bits of $E(R_{n-1})$ are the bits in positions 32, 1 and 2 of R_{n-1} while the last 2 bits of $E(R_{n-1})$ are the bits in positions 32 and 1.

Example: We calculate $E(R_0)$ from R_0 as follows:

$$R_0 = 1111\ 0000\ 1010\ 1010\ 1111\ 0000\ 1010\ 1010$$

$$E(R_0) = 011110\ 100001\ 010101\ 010101\ 011110\ 100001\ 010101\ 010101$$

(Note that each block of 4 original bits has been expanded to a block of 6 output bits.)

Next in the f calculation, we XOR the output $E(R_{n-1})$ with the key K_n :

$$K_n + E(R_{n-1}).$$

Example: For $K_1, E(R_0)$, we have

$$K_1 = 000110\ 110000\ 001011\ 101111\ 111111\ 000111\ 000001\ 110010$$

$$E(R_0) = 011110\ 100001\ 010101\ 010101\ 011110\ 100001\ 010101\ 010101$$

$$K_1 + E(R_0) = 011000\ 010001\ 011110\ 111010\ 100001\ 100110\ 010100\ 100111.$$

We have not yet finished calculating the function f . To this point we have expanded R_{n-1} from 32 bits to 48 bits, using the selection table, and XORed the result with the key K_n . We now have 48 bits, or eight groups of six bits. We now do something strange with each group of six bits: we use them as addresses in tables called "S boxes". Each group of six bits will give us an address in a different S box. Located at that address will be a 4 bit number. This 4 bit number will replace the original 6 bits. The net result is that the eight groups of 6 bits are transformed into eight groups of 4 bits (the 4-bit outputs from the S boxes) for 32 bits total.

Write the previous result, which is 48 bits, in the form:

$$K_n + E(R_{n-1}) = B_1B_2B_3B_4B_5B_6B_7B_8,$$

where each B_i is a group of six bits. We now calculate

$$S_1(B_1)S_2(B_2)S_3(B_3)S_4(B_4)S_5(B_5)S_6(B_6)S_7(B_7)S_8(B_8)$$

where $S_i(B_i)$ refers to the output of the i -th S box.

To repeat, each of the functions S_1, S_2, \dots, S_8 , takes a 6-bit block as input and yields a 4-bit block as output. The table to determine S_i is shown and explained below:

S1

14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

If S_i is the function defined in this table and B is a block of 6 bits, then $S_i(B)$ is determined as follows: The first and last bits of B represent in base 2 a number in the decimal range 0 to 3 (or binary 00 to 11). Let that number be i . The middle 4 bits of B represent in base 2 a number in the decimal range 0 to 15 (binary 0000 to 1111). Let that number be j . Look up in the table the number in the i -th row and j -th column. It is a number in the range 0 to 15 and is uniquely represented by a 4 bit block. That block is the output $S_i(B)$ of S_i for the input B . For example, for input block $B = 011011$ the first bit is "0" and the last bit "1" giving 01 as the row. This is row 1. The middle four bits are "1101". This is the binary equivalent of decimal 13, so the column is column number 13. In row 1, column 13 appears 5. This determines the output; 5 is binary 0101, so that the output is 0101. Hence $S_i(011011) = 0101$.

The tables defining the functions S_1, \dots, S_8 are the following:

S1

14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

S2

15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9

S3

10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
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13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12

S4

7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14

S5

2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3

S6

12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13

S7

4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12

S8

13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

Example: For the first round, we obtain as the output of the eight S boxes:

$$K_1 + E(R_0) = 011000\ 010001\ 011110\ 111010\ 100001\ 100110\ 010100\ 100111.$$

$$S_1(B_1)S_2(B_2)S_3(B_3)S_4(B_4)S_5(B_5)S_6(B_6)S_7(B_7)S_8(B_8) = 0101\ 1100\ 1000\ 0010\ 1011\ 0101\ 1001\ 0111$$

The final stage in the calculation of f is to do a permutation P of the S-box output to obtain the final value of f :

$$f = P(S_1(B_1)S_2(B_2)...S_8(B_8))$$

The permutation P is defined in the following table. P yields a 32-bit output from a 32-bit input by permuting the bits of the input block.

P

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

Example: From the output of the eight S boxes:

$$S_1(B_1)S_2(B_2)S_3(B_3)S_4(B_4)S_5(B_5)S_6(B_6)S_7(B_7)S_8(B_8) = 0101\ 1100\ 1000\ 0010\ 1011\ 0101\ 1001\ 0111$$

we get

$$f = 0010\ 0011\ 0100\ 1010\ 1010\ 1001\ 1011\ 1011$$

$$R_1 = L_0 + f(R_0, K_1)$$

$$= 1100\ 1100\ 0000\ 0000\ 1100\ 1100\ 1111\ 1111$$

$$+ 0010\ 0011\ 0100\ 1010\ 1010\ 1001\ 1011\ 1011$$

$$= 1110\ 1111\ 0100\ 1010\ 0110\ 0101\ 0100\ 0100$$

In the next round, we will have $L_2 = R_1$, which is the block we just calculated, and then we must calculate $R_2 = L_1 + f(R_1, K_2)$, and so on for 16 rounds. At the end of the sixteenth round we have the blocks L_{16} and R_{16} . We then *reverse* the order of the two blocks into the 64-bit block $R_{16}L_{16}$ and apply a final permutation IP^{-1} as defined by the following table:

IP^{-1}

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

That is, the output of the algorithm has bit 40 of the input block as its first bit, bit 8 as its second bit, and so on, until bit 25 of the input block is the last bit of the output.

Example: If we process all 16 blocks using the method defined previously, we get, on the 16th round,

$$L_{16} = 0100\ 0011\ 0100\ 0010\ 0011\ 0010\ 0011\ 0100$$

$$R_{16} = 0000\ 1010\ 0100\ 1100\ 1101\ 1001\ 1001\ 0101$$

We reverse the order of these two blocks and apply the final permutation to

$$R_{16}L_{16} = 00001010\ 01001100\ 11011001\ 10010101\ 01000011\ 01000010\ 00110010\ 00110100$$

We obtain the ciphertext

$$c = 10000101\ 11101000\ 00010011\ 01010100\ 00001111\ 00001010\ 10110100\ 00000101$$