Multivariate Statistics: Due by 11:35 AM Monday, May 27

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1. PCA

   Determine the population principal components Y_1 and Y_2 for the covariance matrix:

   $$\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$$

   Also, calculate the proportion of the total population variance explained by the first principal component.

2. PCA

   Convert the covariance matrix in Question One to a correlation matrix ρ.

   (a)Determine the principal components Y_1 and Y_2 from ρ and compute the proportion of total population variance explained by Y_1.

   (b)Compare the components calculated in Part a with those obtained in Question One. Are they the same? Should they be?

   (c)Compute the correlations ρ_{Y_1,Z_1}, ρ_{Y_1,Z_2} and ρ_{Y_2,Z_1}

3. FA

   The covariance matrix

   $$\rho = \begin{bmatrix} 1.0 & .63 & .45 \\ .63 & 1.0 & .35 \\ .45 & .35 & 1.0 \end{bmatrix}$$

   for the $p = 3$ standardized random variables Z_1, Z_2 and Z_3.

   (a) Show that matrix ρ can be generated by the $m = 1$ factor model

   $$\begin{aligned} Z_1 &= .9F_1 + \varepsilon_1 \\ Z_2 &= .7F_1 + \varepsilon_2 \\ Z_3 &= .5F_1 + \varepsilon_3 \end{aligned}$$

   and

   $$\Psi = Cov(\varepsilon) = \begin{bmatrix} .19 & 0 & 0 \\ 0 & .51 & 0 \\ 0 & 0 & 75 \end{bmatrix}$$

That is, write ρ in the form $\rho = \mathbf{L}\mathbf{L}' + \mathbf{\Psi}$.

(b) Calculate communalities $h_i^2, i = 1, 2, 3$, and interpret these quantities.

(c) Calcualte $Corr\,(Z_i, F_1)$ for $i = 1, 2, 3$. Which variable might carry the greatest weight?

(d) Assuming an $m = 1$ factor model, calculate the loading matrix L using the principal component solution method.