Mechanika mechanismů

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Cíl předmětu

- Základy modelování a simulací mechanismů a obecně soustav mnoha těles.
- Textbook: Stejskal, V. - Valasek, M.: Kinematics and Dynamics of Machinery, Marcel Dekker, New York 1996
- Zkouška: 3 příklady a 5 otázek

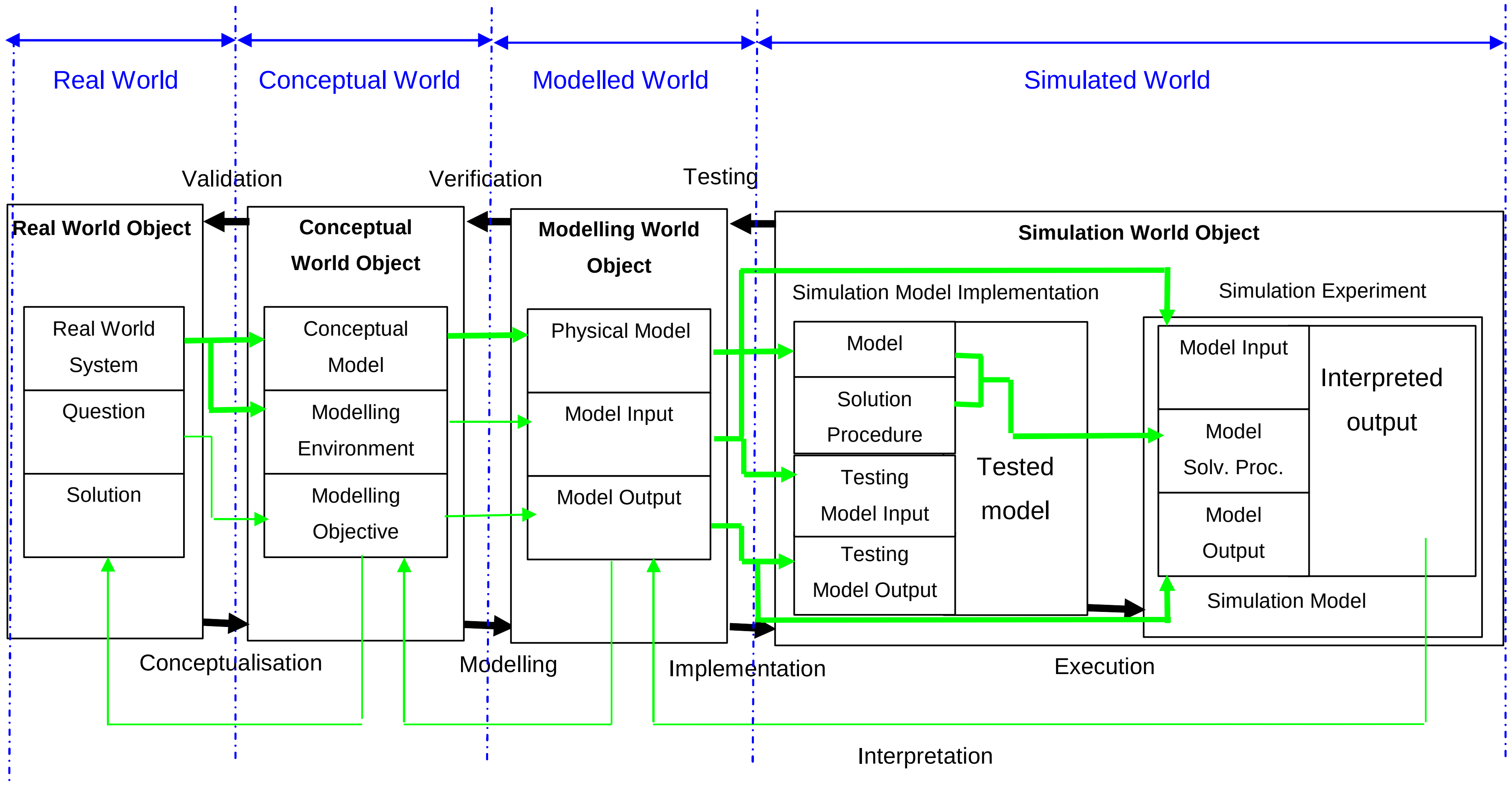
Základní body programu

1) Proces vývoje simulačního modelu
2) Maticová formulace kinematiky
3) Různé souřadnicové systémy pro popis systémů s mnoha tělesy
4) Řešení kinematických smyček
5) Numerické metody řešení kinematiky
6) Kinematická syntéza soustav mnoha těles
7) Dynamika soustav mnoha těles Lagrangeovými rovnicemi smíšeného typu
8) Numerické metody řešení DAE

Proces vývoje simulačního modelu

- Modelování = proces vývoje mechanického modelu
- Mechanický model je později transformován na matematický model nebo simulační model pro další zkoumání (analýza, simulace, návrh řízení atd.)
- Proces modelování je velmi náročný, protože využívá znalostí a zkušeností z mnoha vědních oborů
- Nelze jej popsat úplnou sadou vět a pravidel a zcela systematickým postupem

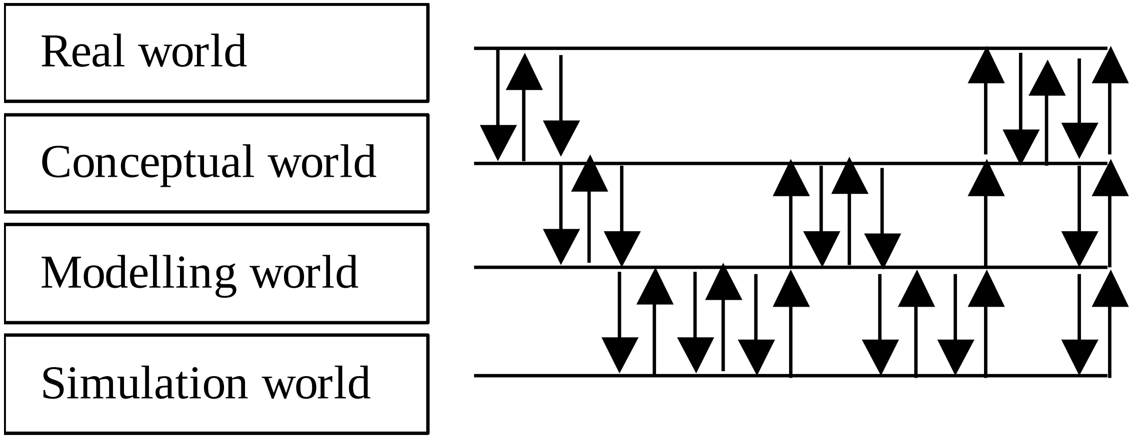
Znalostní životní cyklus vývoje simulačního modelu



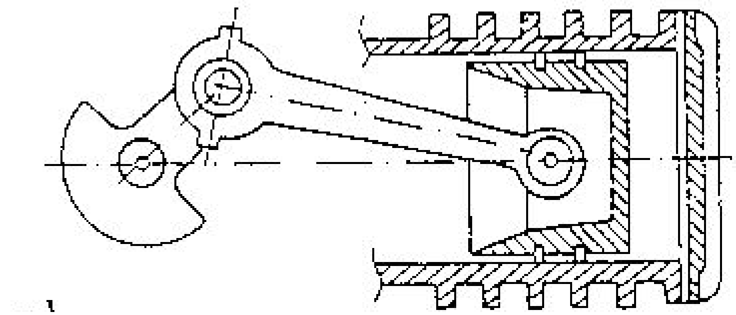
Kroky vývoje simulačního modelu

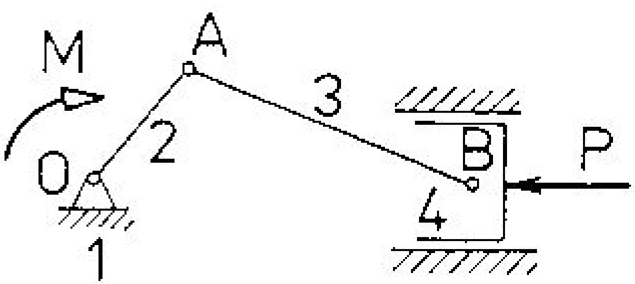
- 1. krok - analýza objektu reálného světa (reálného, předpokládaného) v určitém experimentálním rámci s cílem odpovědět na nějakou otázku
- 2. krok - koncepční úkol (konceptualizace), kdy je objekt reálného světa transformován na objekt konceptuálního světa - jsou vybrány uvažované komponenty
- 3. krok - fyzické modelování, při kterém se objekt konceptuálního světa transformuje na objekt fyzického světa - každá složka je nahrazena jedním nebo více ideálními objekty
- 4. krok - sestava simulačního modelu, kde se objekt fyzického světa transformuje na objekt simulačního světa - implementace simulačního modelu a simulační experimenty - nahrazení modelu počítačem proveditelnou sadou instrukcí - od ideálních objektů po matematické rovnice (model) plus řešení postup a do počítačového kódu

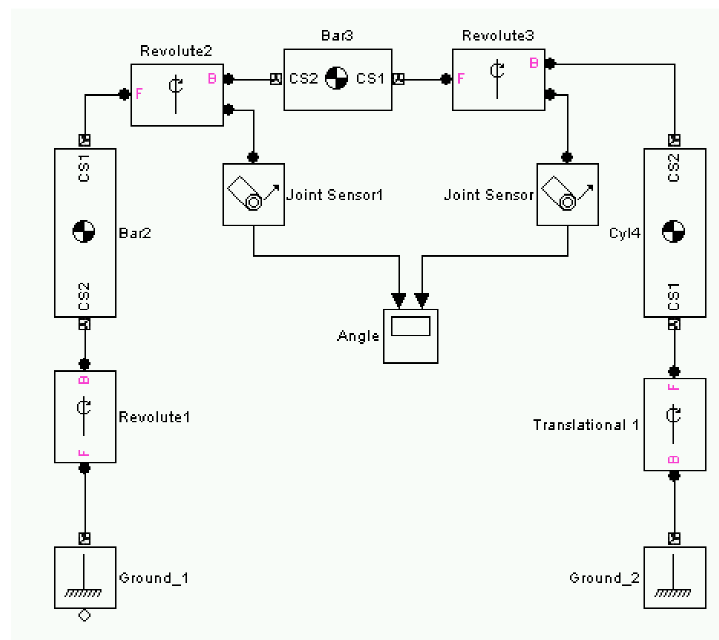
Iterační proces



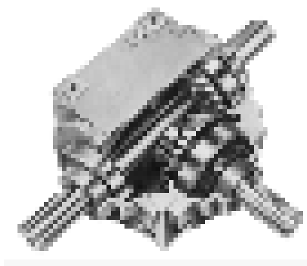
Příklady

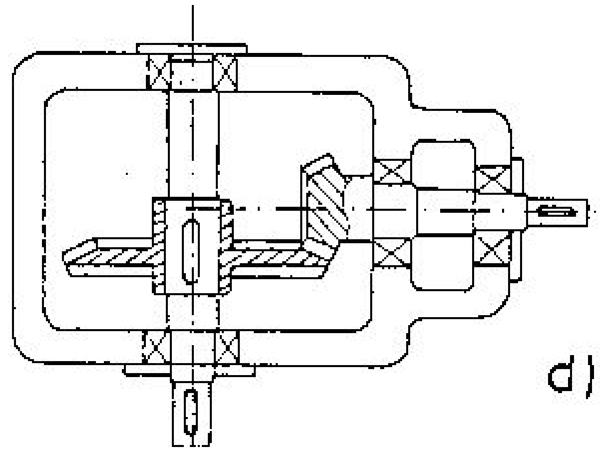



M
A
3
2
O
B
P
4
1

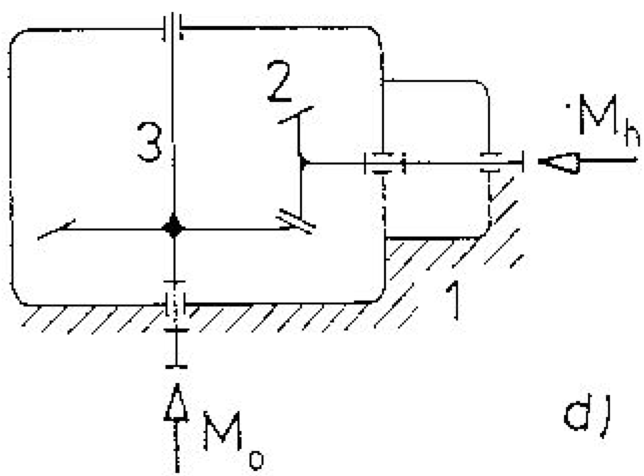

Revolute2
Bar3
Revolute3
CS2
CS1
Joint Sensor1
Joint Sensor
Bar2
Cyl4
CS1
CS2
CS2
CS1
Angle
Revolute1
Translational 1
Ground_1
Ground_2

Příklady



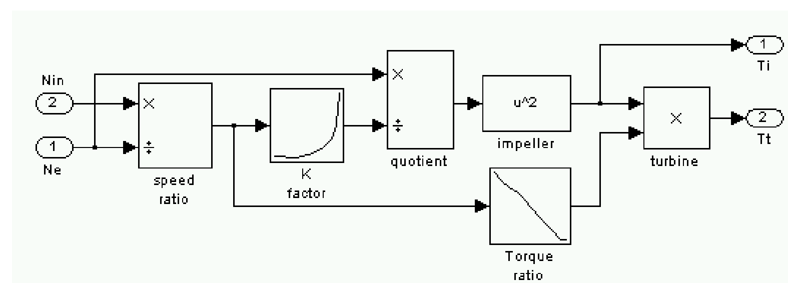


d)



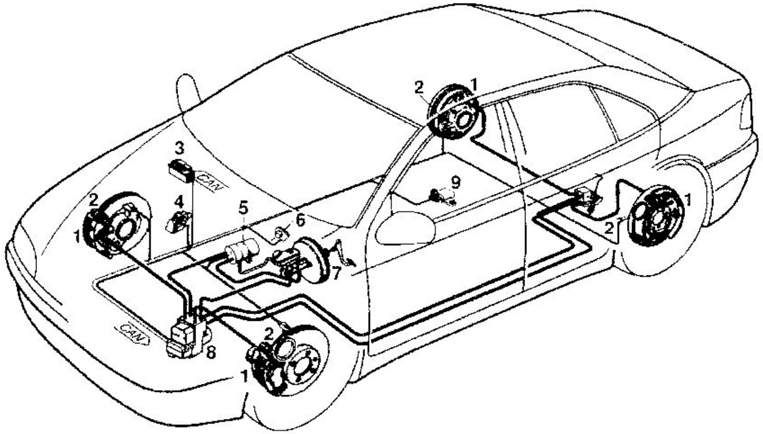
2
3
M_h
1
M_0

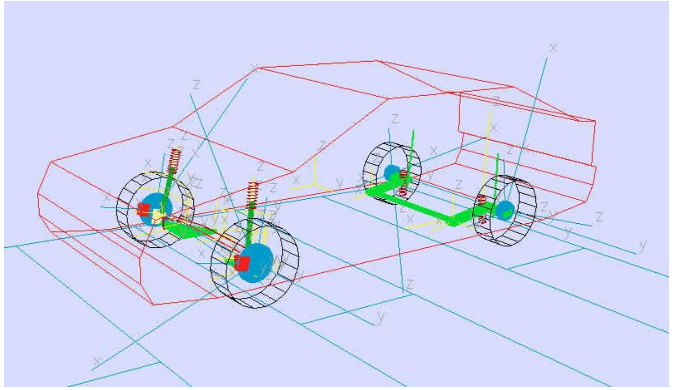
d)

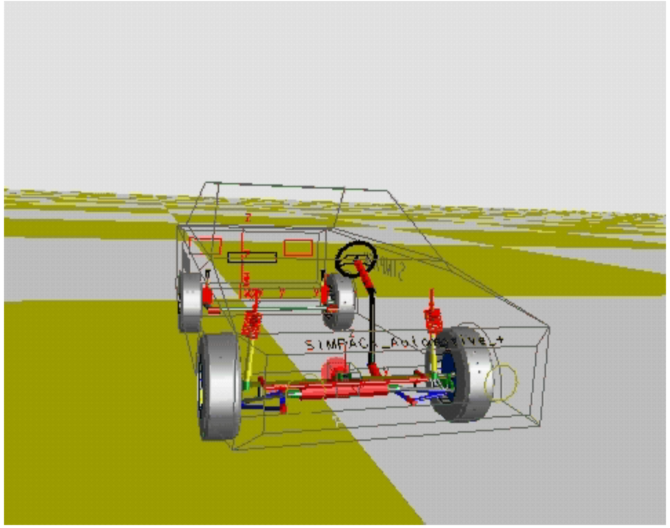


Nin
2
1
Ne
speed
ratio
K
factor
quotient
u^2
impeller
1
Ti
turbine
2
Tt
Torque
ratio

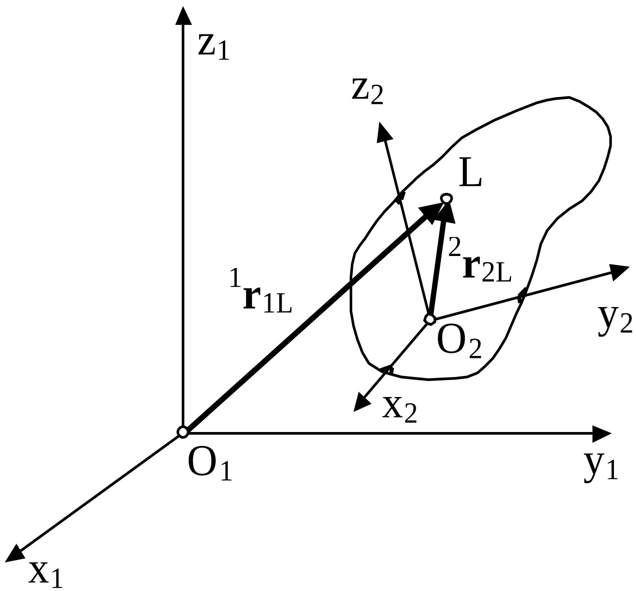
Příklady







Maticová formulace kinematiky



Axes of coordinate system

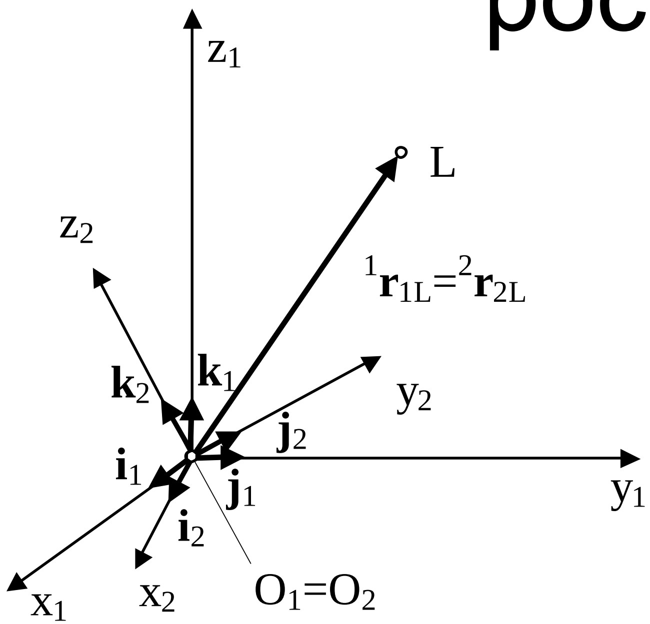
$$ {}^{a}\mathbf{r}_{bL} $$

investigated point

origin of coordinate system

Pohyb tělesa jako transformace mezi souřadnicovými systémy..

Souřadnicové systémy se stejným počátkem



$$^1\mathbf{r}_{1L} = x_{1L}\mathbf{i}_1 + y_{1L}\mathbf{j}_1 + z_{1L}\mathbf{k}_1 = \begin{bmatrix} x_{1L} \\ y_{1L} \\ z_{1L} \end{bmatrix}$$

$$\mathbf{i}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{j}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{k}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{i}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{j}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{k}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$^2\mathbf{r}_{2L} = x_{2L}\mathbf{i}_2 + y_{2L}\mathbf{j}_2 + z_{2L}\mathbf{k}_2 = \begin{bmatrix} x_{2L} \\ y_{2L} \\ z_{2L} \end{bmatrix}$$

$$
{}^{1}\mathbf{i}_2 = \begin{bmatrix} \cos\alpha_x \\ \cos\beta_x \\ \cos\gamma_x \end{bmatrix}, \quad {}^{1}\mathbf{j}_2 = \begin{bmatrix} \cos\alpha_y \\ \cos\beta_y \\ \cos\gamma_y \end{bmatrix}, \quad {}^{1}\mathbf{k}_2 = \begin{bmatrix} \cos\alpha_z \\ \cos\beta_z \\ \cos\gamma_z \end{bmatrix}
$$

$$
{}^{1}\mathbf{r}_{1L} = x_{2L}\,{}^{1}\mathbf{i}_2 + y_{2L}\,{}^{1}\mathbf{j}_2 + z_{2L}\,{}^{1}\mathbf{k}_2
$$

$$
\begin{aligned}
x_{1L} &= x_{2L}\cos\alpha_x + y_{2L}\cos\alpha_y + z_{2L}\cos\alpha_z \\
y_{1L} &= x_{2L}\cos\beta_x + y_{2L}\cos\beta_y + z_{2L}\cos\beta_z \\
x_{1L} &= x_{2L}\cos\gamma_x + y_{2L}\cos\gamma_y + z_{2L}\cos\gamma_z
\end{aligned}
$$

$$
\begin{bmatrix} x_{1L} \\ y_{1L} \\ z_{1L} \end{bmatrix} = \begin{bmatrix} \cos\alpha_x & \cos\alpha_y & \cos\alpha_z \\ \cos\beta_x & \cos\beta_y & \cos\beta_z \\ \cos\gamma_x & \cos\gamma_y & \cos\gamma_z \end{bmatrix} \begin{bmatrix} x_{2L} \\ y_{2L} \\ z_{2L} \end{bmatrix}
$$

$$
{}^{1}\mathbf{r}_{1L} = \mathbf{S}_{12}\,{}^{2}\mathbf{r}_{2L}
$$

Matice směrových kosinů

$$\mathbf{S}_{12} = \begin{bmatrix} \cos\alpha_x & \cos\alpha_y & \cos\alpha_z \\ \cos\beta_x & \cos\beta_y & \cos\beta_z \\ \cos\gamma_x & \cos\gamma_y & \cos\gamma_z \end{bmatrix} = \begin{bmatrix} {}^1\mathbf{i}_2 & {}^1\mathbf{j}_2 & {}^1\mathbf{k}_2 \end{bmatrix}$$

$$\mathbf{S}_{12}^{-1} = \mathbf{S}_{12}^{\mathrm{T}}$$

$$\mathbf{S}_{12}^{-1}\mathbf{S}_{12} = \mathbf{S}_{12}^{\mathrm{T}}\mathbf{S}_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{E}_3$$

$$\mathbf{S}_{12}^{-1}\mathbf{S}_{12} = \mathbf{S}_{12}^{\mathrm{T}}\mathbf{S}_{12} = \begin{bmatrix} {}^1\mathbf{i}_2^{\mathrm{T}} \\ {}^1\mathbf{j}_2^{\mathrm{T}} \\ {}^1\mathbf{k}_2^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} {}^1\mathbf{i}_2 & {}^1\mathbf{j}_2 & {}^1\mathbf{k}_2 \end{bmatrix} =$$

$$\begin{bmatrix} {}^1\mathbf{i}_2^{\mathrm{T}\,1}\mathbf{i}_2 & {}^1\mathbf{i}_2^{\mathrm{T}\,1}\mathbf{j}_2 & {}^1\mathbf{i}_2^{\mathrm{T}\,1}\mathbf{k}_2 \\ {}^1\mathbf{j}_2^{\mathrm{T}\,1}\mathbf{i}_2 & {}^1\mathbf{j}_2^{\mathrm{T}\,1}\mathbf{j}_2 & {}^1\mathbf{j}_2^{\mathrm{T}\,1}\mathbf{k}_2 \\ {}^1\mathbf{k}_2^{\mathrm{T}\,1}\mathbf{i}_2 & {}^1\mathbf{k}_2^{\mathrm{T}\,1}\mathbf{j}_2 & {}^1\mathbf{k}_2^{\mathrm{T}\,1}\mathbf{k}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^T \mathbf{b} = [a_x a_y a_z] \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = a_x b_x + a_y b_y + a_z b_z$$

$$\mathbf{a} \times \mathbf{b} = \mathbf{A}\mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

$${}^2\mathbf{r}_{2L} = \mathbf{S}_{12}^{\mathrm{T}}\, {}^1\mathbf{r}_{1L}$$

Matice úhlové rychlosti

$$\begin{aligned}\dot{\mathbf{S}}_{12} &= \mathbf{S}_{12}\boldsymbol{\Omega}_{12}\\ \mathbf{S}_{12}^{\mathrm{T}}\dot{\mathbf{S}}_{12} &= \boldsymbol{\Omega}_{12}\end{aligned}$$

$$\boldsymbol{\Omega}_{12} = \begin{bmatrix} 0 & -\omega_{12z} & \omega_{12y} \\ \omega_{12z} & 0 & -\omega_{12x} \\ -\omega_{12y} & \omega_{12x} & 0 \end{bmatrix}$$

$$\boldsymbol{\omega}_{12} =^2 \boldsymbol{\omega}_{12} = \begin{bmatrix} \omega_{12x} \\ \omega_{12y} \\ \omega_{12z} \end{bmatrix}$$

$${}^1\boldsymbol{\omega}_{12} = \mathbf{S}_{12}{}^2\boldsymbol{\omega}_{12}$$

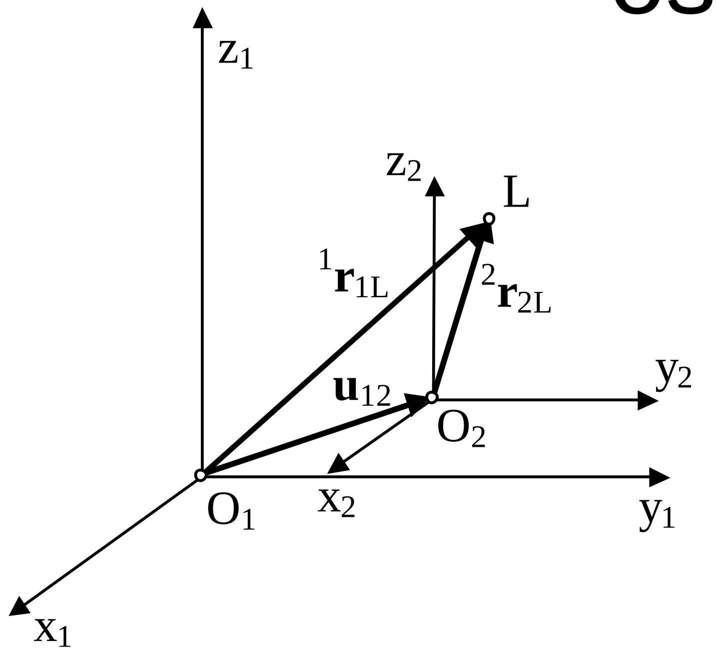
Matice úhlového zrychlení

$$\mathcal{A}_{12} = \dot{\mathbf{\Omega}}_{12} = \begin{bmatrix} 0 & -\alpha_{12z} & \alpha_{12y} \\ \alpha_{12z} & 0 & -\alpha_{12x} \\ -\alpha_{12y} & \alpha_{12x} & 0 \end{bmatrix}$$

$$\boldsymbol{\alpha}_{12} =^2 \boldsymbol{\alpha}_{12} = \begin{bmatrix} \alpha_{12x} \\ \alpha_{12y} \\ \alpha_{12z} \end{bmatrix}$$

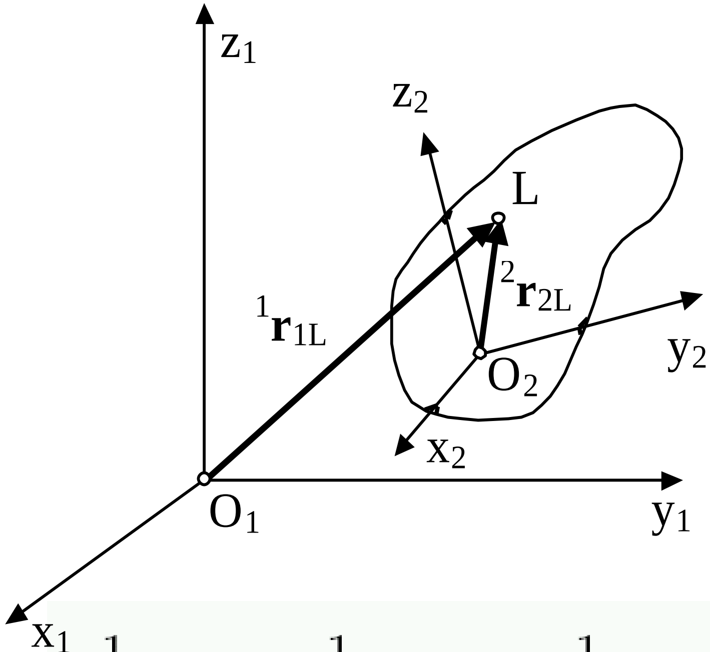
$${}^1\boldsymbol{\alpha}_{12} = \mathbf{S}_{12}{}^2\boldsymbol{\alpha}_{12}$$

Souřadnicové systémy s paralelními osami



$$
\begin{aligned}
{}^1\mathbf{r}_{1L} &= \mathbf{r}_{1O_2} + {}^1\mathbf{r}_{2L} \\
{}^1\mathbf{r}_{2L} &= {}^2\mathbf{r}_{2L} \\
\mathbf{r}_{1O_2} &= {}^1\mathbf{r}_{1O_2}
\end{aligned}
$$

Obecné souřadnicové systémy



$$ {}^{1}\mathbf{r}_{1L} = {}^{1}\mathbf{r}_{1O_2} + {}^{1}\mathbf{r}_{2L} $$

$$ {}^{1}\mathbf{r}_{2L} = \mathbf{S}_{12}\,{}^{2}\mathbf{r}_{2L} $$

$$ \mathbf{r}_{1O_2} = {}^{1}\mathbf{r}_{1O_2} $$

$$ {}^{1}\mathbf{r}_{1L} = \mathbf{S}_{12}\,{}^{2}\mathbf{r}_{2L} + \mathbf{r}_{1O_2} $$

$$
\begin{aligned}
{}^{1}\mathbf{r}_{1L} &= \mathbf{S}_{12}\,{}^{2}\mathbf{r}_{2L} + \mathbf{u}_{12} \\
1 &= 1
\end{aligned}
$$

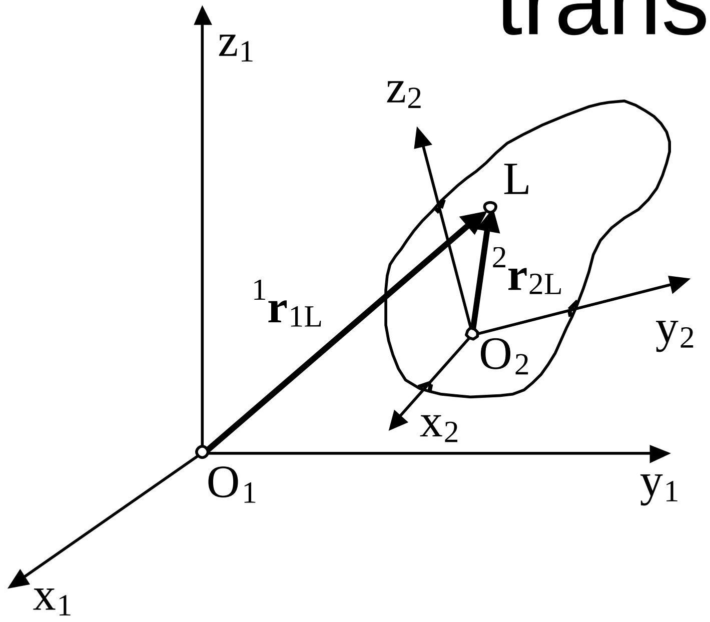
$$
\begin{bmatrix} {}^{1}\mathbf{r}_{1L} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{12} & \mathbf{r}_{1O_2} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{2}\mathbf{r}_{2L} \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} {}^{1}r_{1Lx} \\ {}^{1}r_{1Ly} \\ {}^{1}r_{1Lz} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha_x & \cos\alpha_y & \cos\alpha_z & u_{12x} \\ \cos\beta_x & \cos\beta_y & \cos\beta_z & u_{12y} \\ \cos\gamma_x & \cos\gamma_y & \cos\gamma_z & u_{12z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{2}r_{2Lx} \\ {}^{2}r_{2Ly} \\ {}^{2}r_{2Lz} \\ 1 \end{bmatrix}
$$

$$
{}^{1}\mathbf{r}_{1L} = \mathbf{T}_{12}\,{}^{2}\mathbf{r}_{2L}
$$

$$
\mathbf{r}_{1L} = \mathbf{T}_{12}\mathbf{r}_{2L}
$$

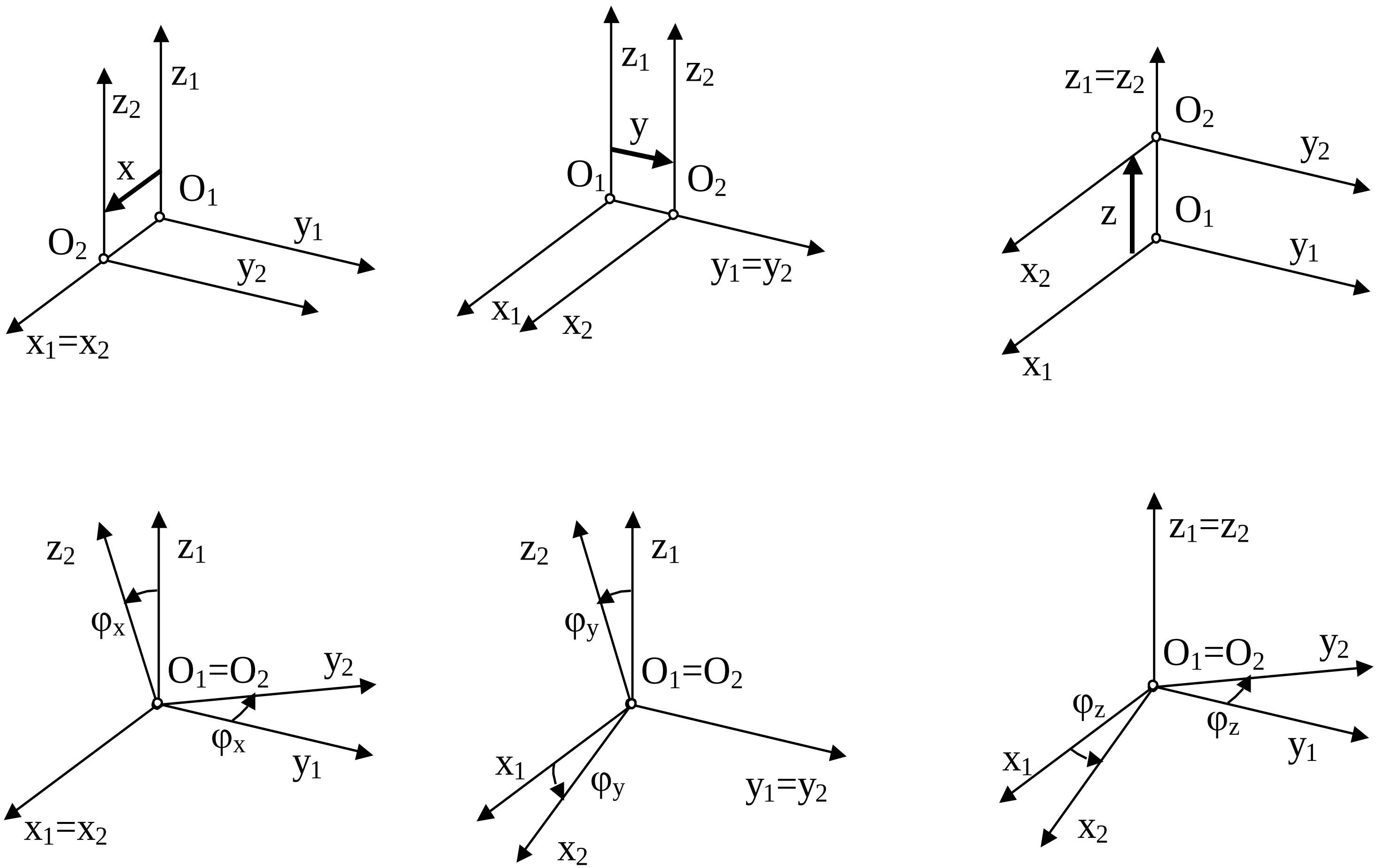
Pohyb jako časově proměnná transformace



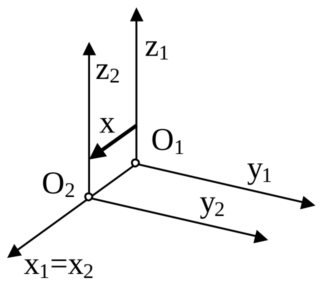
$$\mathbf{r}_{1L}(t) = \mathbf{T}_{12}(t)\mathbf{r}_{2L}$$

$$\mathbf{T}_{12}(t) = \begin{bmatrix} \mathbf{S}_{12}(t) & \mathbf{u}_{12}(t) \\ \mathbf{0} & 1 \end{bmatrix}$$

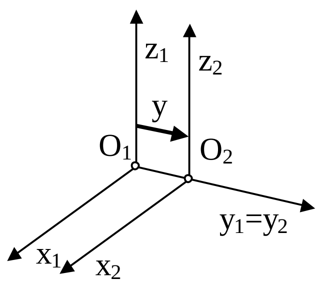
Základní pohyby



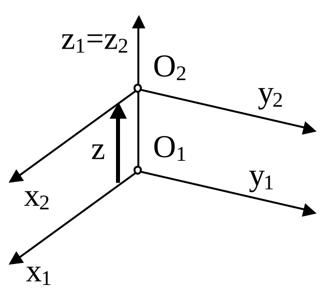
Základní pohyby posuvné



$$\mathbf{T}_x(x) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

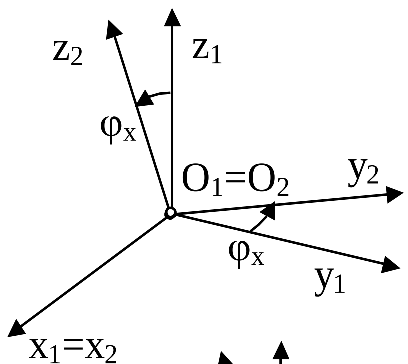


$$\mathbf{T}_y(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

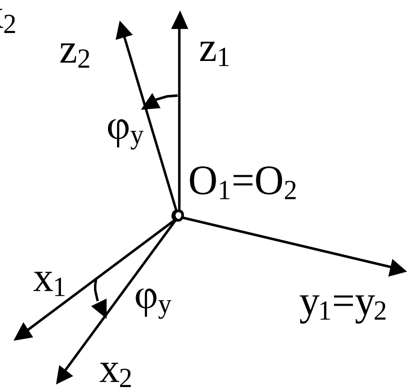


$$\mathbf{T}_z(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

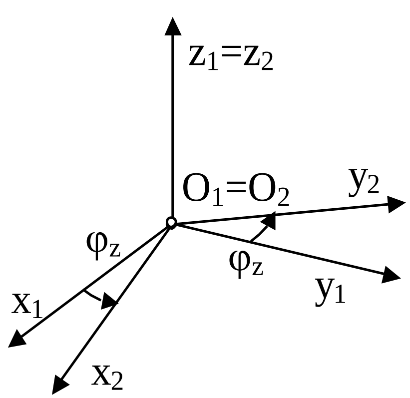
Základní pohyby rotační



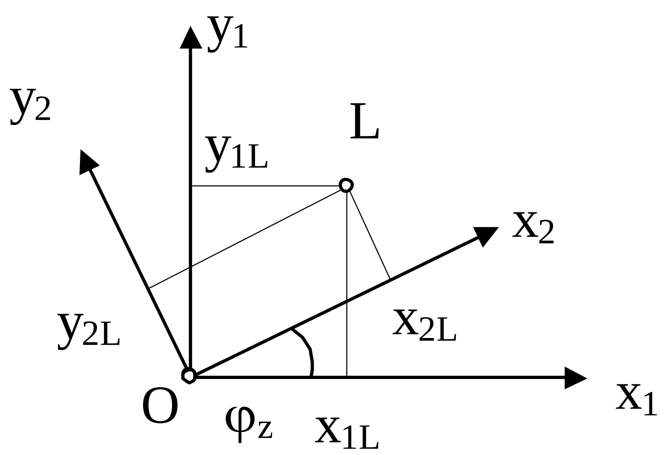
$$\mathbf{T}_{\varphi x}(\varphi_x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_x & -\sin\varphi_x & 0 \\ 0 & \sin\varphi_x & \cos\varphi_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\mathbf{T}_{\varphi y}(\varphi_y) = \begin{bmatrix} \cos\varphi_y & 0 & \sin\varphi_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi_y & 0 & \cos\varphi_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

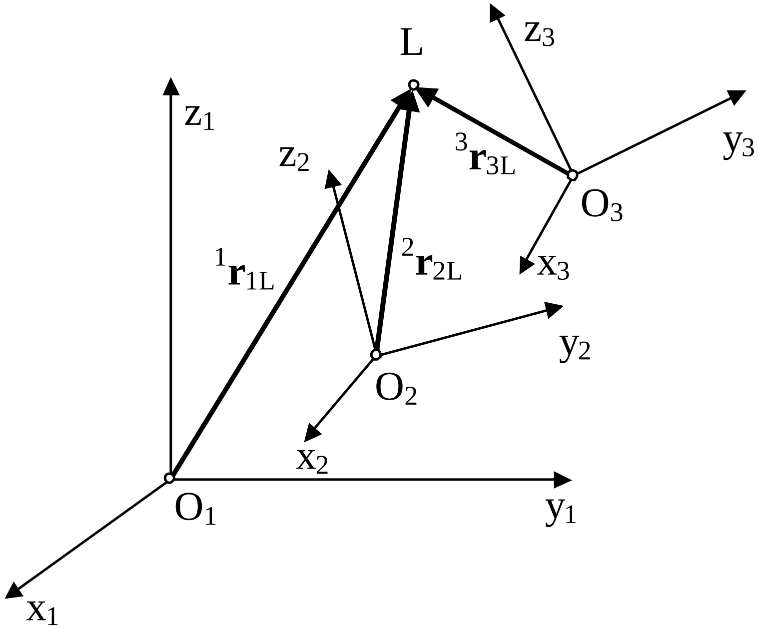


$$\mathbf{T}_{\varphi z}(\varphi_z) = \begin{bmatrix} \cos\varphi_z & -\sin\varphi_z & 0 & 0 \\ \sin\varphi_z & \cos\varphi_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$
\begin{aligned}
x_{1L} &= x_{2L}\cos\varphi_z y_{2L}\sin\varphi_z \\
y_{1L} &= x_{2L}\sin\varphi_z + y_{2L}\cos\varphi_z \\
z_{1L} &= z_{2L}
\end{aligned}
$$

Simultánní (složené) pohyby



$$
\begin{aligned}
{}^1\mathbf{r}_{1L} &= \mathbf{T}_{12}\,{}^2\mathbf{r}_{2L} \\
{}^2\mathbf{r}_{2L} &= \mathbf{T}_{23}\,{}^3\mathbf{r}_{3L}
\end{aligned}
$$

$$
{}^1\mathbf{r}_{1L} = \mathbf{T}_{13}\,{}^3\mathbf{r}_{3L}
$$

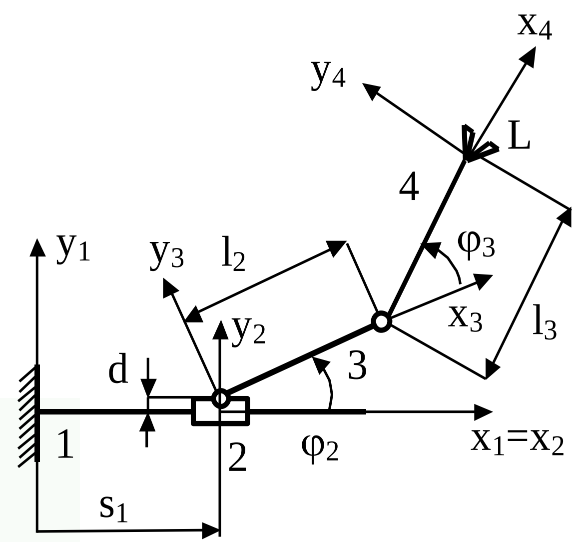
$$
{}^1\mathbf{r}_{1L} = \mathbf{T}_{12}\mathbf{T}_{23}\,{}^3\mathbf{r}_{3L}
$$

$$
\mathbf{T}_{13} = \mathbf{T}_{12}\mathbf{T}_{23}
$$

$$
\mathbf{T}_{1,n+1} = \mathbf{T}_{12}\mathbf{T}_{23}\mathbf{T}_{34}\cdots\mathbf{T}_{n-1,n}\mathbf{T}_{n,n+1}
$$

Příklad: rovinný manipulátor

$$^{1}\mathbf{r}_{1L} = \mathbf{T}_{14}{}^{4}\mathbf{r}_{4L}$$



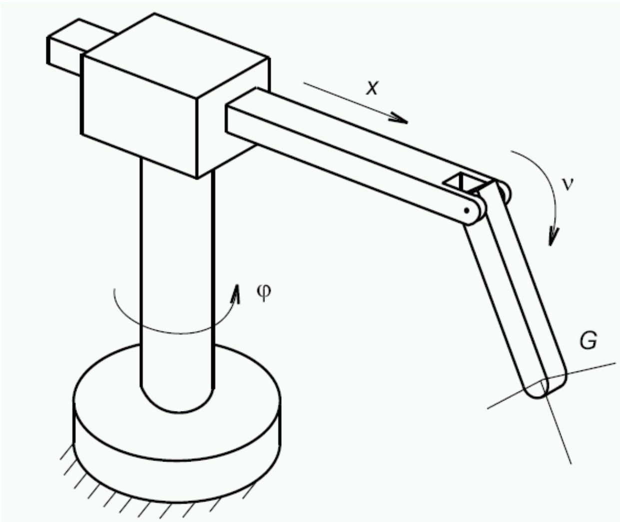
$$\mathbf{T}_{14} = \mathbf{T}_{12}\mathbf{T}_{23}\mathbf{T}_{34}$$

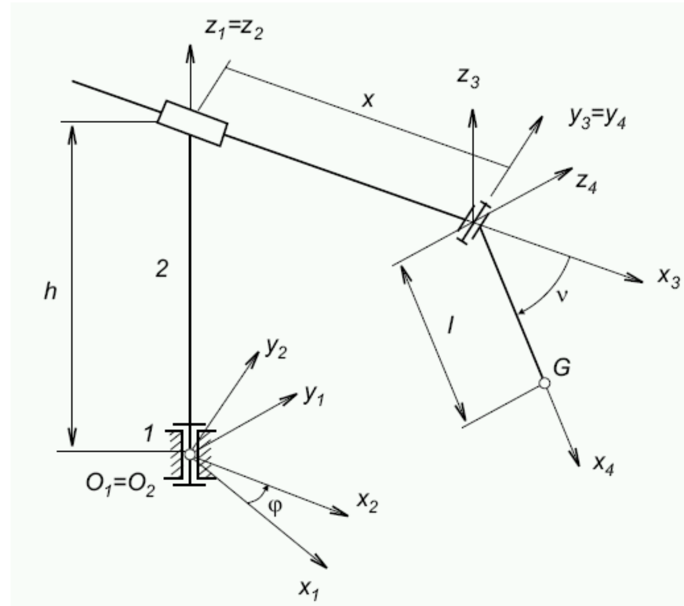
$$\begin{aligned}
\mathbf{T}_{12} &= \mathbf{T}_x(s_1) \\
\mathbf{T}_{23} &= \mathbf{T}_y(d)\mathbf{T}_\varphi(\varphi_2) \\
\mathbf{T}_{34} &= \mathbf{T}_x(l_2)\mathbf{T}_\varphi(\varphi_3)\mathbf{T}_x(l_3)
\end{aligned}$$

$$\mathbf{r}_{1L} = \mathbf{T}_x(s_1)\mathbf{T}_y(d)\mathbf{T}_\varphi(\varphi_2)\mathbf{T}_x(l_2)\mathbf{T}_\varphi(\varphi_3)\mathbf{T}_x(l_3)\mathbf{r}_{2L}$$

$$\begin{aligned}
x_{1L} &= s_1 + (l_2 + l_3\cos\varphi_3)\cos\varphi_2 - l_3\sin\varphi_2\sin\varphi_3 \\
y_{1L} &= d + (l_2 + l_3\cos\varphi_3)\sin\varphi_2 + l_3\cos\varphi_2\sin\varphi_3
\end{aligned}$$

Příklad: prostorový manipulátor





$$\boldsymbol{T}_{14} \;=\; \boldsymbol{T}_{12}\,\boldsymbol{T}_{23}\,\boldsymbol{T}_{34}$$

$$\boldsymbol{T}_{12} = \boldsymbol{T}_{\varphi_\mathrm{z}}(\varphi)\,, \quad \boldsymbol{T}_{23} = \boldsymbol{T}_\mathrm{z}(h)\,\boldsymbol{T}_\mathrm{x}(x)\,, \quad \boldsymbol{T}_{34} = \boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta)$$

$$\boldsymbol{T}_{14} \;=\; \boldsymbol{T}_{\varphi_\mathrm{z}}(\varphi)\,\boldsymbol{T}_\mathrm{z}(h)\,\boldsymbol{T}_\mathrm{x}(x)\,\boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta)$$

$$\boldsymbol{r}_{1\mathrm{G}} = \boldsymbol{T}_{14}\,\boldsymbol{r}_{4\mathrm{G}},$$

$$\boldsymbol{r}_{4\mathrm{G}} = \begin{bmatrix} l \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Rychlosti

$$\begin{aligned} {}^1\mathbf{r}_{1L} &= \mathbf{T}_{12}{}^2\mathbf{r}_{2L} \\ {}^1\mathbf{v}_{1L} &= \dot{\mathbf{T}}_{12}{}^2\mathbf{r}_{2L} \end{aligned}$$

$${}^1\mathbf{r}_{1L} = \begin{bmatrix} x_{1L} \\ y_{1L} \\ z_{1L} \\ 1 \end{bmatrix} \qquad {}^1\mathbf{v}_{1L} = \begin{bmatrix} \dot{x}_{1L} \\ \dot{y}_{1L} \\ \dot{z}_{1L} \\ 0 \end{bmatrix}$$

$$\begin{aligned} \dot{\mathbf{T}}_{12} &= \mathbf{T}_{12}\mathbf{V}_{12} \\ \mathbf{V}_{12} &= \mathbf{T}_{12}^{-1}\dot{\mathbf{T}}_{12} \\ \mathbf{V}_{12} &= {}^2\mathbf{V}_{12} \end{aligned}$$

$$\mathbf{T}_{12} = \begin{bmatrix} \mathbf{S}_{12} & \mathbf{u}_{12} \\ \mathbf{0} & 1 \end{bmatrix} \qquad \mathbf{T}_{12}^{-1} = \begin{bmatrix} \mathbf{S}_{12}^T & -\mathbf{S}_{12}^T\mathbf{u}_{12} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$\mathbf{T}_{12}\mathbf{T}_{12}^{-1} = \left[\begin{array}{cc} \mathbf{S}_{12}\mathbf{S}_{12}^T & -\mathbf{S}_{12}\mathbf{S}_{12}^T\mathbf{u}_{12} + \mathbf{u}_{12} \\ \mathbf{0} & 1 \end{array} \right] = \left[\begin{array}{cc} \mathbf{E}_3 & \mathbf{0} \\ \mathbf{0} & 1 \end{array} \right] = \mathbf{E}_4$$

$$\dot{\mathbf{T}}_{12} = \left[\begin{array}{cc} \dot{\mathbf{S}}_{12} & \dot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{array} \right]$$

$$\mathbf{V}_{12} = \left[\begin{array}{cc} \mathbf{S}_{12}^T\dot{\mathbf{S}}_{12} & \mathbf{S}_{12}^T\dot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{array} \right] = \left[\begin{array}{cc} \mathbf{\Omega}_{12} & \mathbf{S}_{12}^T\dot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{array} \right]$$

$$\mathbf{\Omega}_{12} = {}^2\mathbf{\Omega}_{12}$$

$$\mathbf{\Omega}_{12}\mathbf{r} = \boldsymbol{\omega}_{12} \times \mathbf{r}$$

$${}^1\mathbf{V}_{12} = \mathbf{T}_{12}{}^2\mathbf{V}_{12}\mathbf{T}_{12}^{-1}$$

Zrychlení

$$\begin{aligned} {}^1\mathbf{v}_{1L} &= \dot{\mathbf{T}}_{12}{}^2\mathbf{r}_{2L} \\ {}^1\mathbf{a}_{1L} &= \ddot{\mathbf{T}}_{12}{}^2\mathbf{r}_{2L} \end{aligned}$$

$${}^1\mathbf{a}_{1L} = \begin{bmatrix} \ddot{x}_{1L} \\ \ddot{y}_{1L} \\ \ddot{z}_{1L} \\ 0 \end{bmatrix}$$

$$\begin{aligned} \ddot{\mathbf{T}}_{12} &= \dot{\mathbf{T}}_{12}\mathbf{V}_{12} + \mathbf{T}_{12}\dot{\mathbf{V}}_{12} \\ \mathbf{A}_{12} &= \dot{\mathbf{V}}_{12} \\ \ddot{\mathbf{T}}_{12} &= \mathbf{T}_{12}(\mathbf{A}_{12} + \mathbf{V}_{12}^2) = \mathbf{T}_{12}\mathbf{B}_{12} \\ \mathbf{B}_{12} &= \mathbf{T}_{12}^{-1}\ddot{\mathbf{T}}_{12} = \mathbf{A}_{12} + \mathbf{V}_{12}^2 \end{aligned}$$

$$\dot{\mathbf{V}}_{12} = \begin{bmatrix} \dot{\mathbf{\Omega}}_{12} & \dot{\mathbf{S}}_{12}^T\dot{\mathbf{u}}_{12} + \mathbf{S}_{12}^T\ddot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{bmatrix}$$

$$\dot{\mathbf{V}}_{12} = \begin{bmatrix} \mathcal{A}_{12} & \dot{\mathbf{S}}_{12}^T\dot{\mathbf{u}}_{12} + \mathbf{S}_{12}^T\ddot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{bmatrix}$$

$$\dot{\mathbf{V}}_{12}^2 = \left[\begin{array}{cc} \mathbf{\Omega}_{12}^2 & \mathbf{\Omega}_{12}\mathbf{S}_{12}^T\dot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{array} \right]$$

$$\mathbf{B}_{12} = \dot{\mathbf{V}}_{12} + \mathbf{V}_{12}^2 = \left[\begin{array}{cc} \mathcal{A}_{12} + \mathbf{\Omega}_{12}^2 & \mathbf{S}_{12}^T\ddot{\mathbf{u}}_{12} \\ \mathbf{0} & 0 \end{array} \right]$$

$${}^1\ddot{\mathbf{r}}_{1L} = {}^1\mathbf{a}_{1L} = \mathbf{S}_{12}\mathcal{A}_{12}{}^2\mathbf{r}_{2L} + \mathbf{S}_{12}\mathbf{\Omega}_{12}^2{}^2\mathbf{r}_{2L} + \ddot{\mathbf{u}}_{12} = \mathbf{S}_{12}({}^2\boldsymbol{\alpha}_{12} \times {}^2\mathbf{r}_{2L} + {}^2\boldsymbol{\omega}_{12} \times ({}^2\boldsymbol{\omega}_{12} \times {}^2\mathbf{r}_{2L})) + \ddot{\mathbf{u}}_{12}$$

Simultánní (složené) pohyby

$$\mathbf{T}_{13} = \mathbf{T}_{12}\mathbf{T}_{23}$$

$$^1\mathbf{r}_{1L} = \mathbf{T}_{13}{}^3\mathbf{r}_{3L}$$

$$^1\mathbf{v}_{1L} = \dot{\mathbf{T}}_{13}{}^3\mathbf{r}_{3L}$$

$$\dot{\mathbf{T}}_{13} = \dot{\mathbf{T}}_{12}\mathbf{T}_{23} + \mathbf{T}_{12}\dot{\mathbf{T}}_{23} = \mathbf{T}_{13}\mathbf{V}_{13}$$

$$\begin{aligned}
\mathbf{V}_{13} &= \mathbf{T}_{13}^{-1}\dot{\mathbf{T}}_{13} = (\mathbf{T}_{12}\mathbf{T}_{23})^{-1}(\dot{\mathbf{T}}_{12}\mathbf{T}_{23} + \mathbf{T}_{12}\dot{\mathbf{T}}_{23}) = \\
&= \mathbf{T}_{23}^{-1}\mathbf{T}_{12}^{-1}(\dot{\mathbf{T}}_{12}\mathbf{T}_{23} + \mathbf{T}_{12}\dot{\mathbf{T}}_{23}) = \\
&= \mathbf{T}_{23}^{-1}\mathbf{T}_{12}^{-1}\dot{\mathbf{T}}_{12}\mathbf{T}_{23} + \mathbf{T}_{23}^{-1}\mathbf{T}_{12}^{-1}\mathbf{T}_{12}\dot{\mathbf{T}}_{23} = \mathbf{T}_{23}^{-1}\mathbf{V}_{12}\mathbf{T}_{23} + \mathbf{V}_{23}
\end{aligned}$$

$$\mathbf{V}_{13} = \mathbf{T}_{23}^{-1}\mathbf{V}_{12}\mathbf{T}_{23} + \mathbf{V}_{23}$$

$$\mathbf{\Omega}_{13} = \mathbf{S}_{23}^{-1}\mathbf{\Omega}_{12}\mathbf{S}_{23} + \mathbf{\Omega}_{23}$$

$$^1\mathbf{a}_{1L} = \ddot{\mathbf{T}}_{13}{}^3\mathbf{r}_{3L} = (\ddot{\mathbf{T}}_{12}\mathbf{T}_{23} + 2\dot{\mathbf{T}}_{12}\dot{\mathbf{T}}_{23} + \mathbf{T}_{12}\ddot{\mathbf{T}}_{23})^3\mathbf{r}_{3L}$$

Základní pohyby

$$\mathbf{T}_x(x) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\mathbf{T}}_x(\dot{x}) = \begin{bmatrix} 1 & 0 & 0 & \dot{x} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{T}_x \mathbf{D}_x(\dot{x}) = \mathbf{T}_x \mathbf{D}_x(1) \mathbf{E}_4(\dot{x})$$

$$\mathbf{D}_x(\dot{x}) = \begin{bmatrix} 0 & 0 & 0 & \dot{x} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{T}_y(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\mathbf{T}}_y(\dot{y}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dot{y} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{T}_y\mathbf{D}_y(\dot{y}) = \mathbf{T}_y\mathbf{D}_y(1)\mathbf{E}_4(\dot{y})$$

$$\mathbf{D}_y(\dot{y}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dot{y} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{T}_z(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\mathbf{T}}_z(\dot{z}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \dot{z} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathbf{T}_z\mathbf{D}_z(\dot{z}) = \mathbf{T}_z\mathbf{D}_z(1)\mathbf{E}_4(\dot{z})$$

$$\mathbf{D}_z(\dot{z}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dot{z} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{T}_{\varphi x}(\varphi_x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_x & -\sin\varphi_x & 0 \\ 0 & \sin\varphi_x & \cos\varphi_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\mathbf{T}}_{\varphi x}(\dot\varphi_x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\sin\varphi_x\dot\varphi_x & -\cos\varphi_x\dot\varphi_x & 0 \\ 0 & \cos\varphi_x\dot\varphi_x & -\sin\varphi_x\dot\varphi_x & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathbf{T}_{\varphi x}(\varphi_x)\mathbf{D}_{\varphi x}(\dot\varphi_x) = \mathbf{T}_{\varphi x}(\varphi_x)\mathbf{D}_{\varphi x}(1)\mathbf{E}_4(\dot\varphi_x)$$

$$\mathbf{D}_{\varphi x}(\dot\varphi_x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\dot\varphi_x & 0 \\ 0 & \dot\varphi_x & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{V}_{\varphi_x} = \mathbf{D}_{\varphi_x}(\dot\varphi_x) = \mathbf{D}_{\varphi_x}(1)\mathbf{E}_4(\dot\varphi_x)$$

$$\mathbf{A}_{\varphi_x} = \mathbf{D}_{\varphi_x}(\ddot\varphi_x) = \mathbf{D}_{\varphi_x}(1)\mathbf{E}_4(\ddot\varphi_x)$$

$$\mathbf{B}_{\varphi_x} = \mathbf{D}_{\varphi_x}(\ddot\varphi_x) + \mathbf{D}^2_{\varphi_x}(\dot\varphi_x) = \mathbf{D}_{\varphi_x}(1)\mathbf{E}_4(\ddot\varphi_x) + \mathbf{D}^2_{\varphi_x}(1)\mathbf{E}_4(\dot\varphi^2_x)$$

$$\mathbf{T}_{\varphi y}(\varphi_y) = \begin{bmatrix} \cos\varphi_y & 0 & \sin\varphi_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varphi_y & 0 & \cos\varphi_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\mathbf{T}}_{\varphi y}(\dot{\varphi}_y) = \begin{bmatrix} -\sin\varphi_y\dot{\varphi}_y & 0 & \cos\varphi_y\dot{\varphi}_y & 0 \\ 0 & 0 & 0 & 0 \\ -\cos\varphi_y\dot{\varphi}_y & 0 & -\sin\varphi_y\dot{\varphi}_y & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathbf{T}_{\varphi y}(\varphi_y)\mathbf{D}_{\varphi y}(\dot{\varphi}_y) = \mathbf{T}_{\varphi y}(\varphi_y)\mathbf{D}_{\varphi y}(1)\mathbf{E}_4(\dot{\varphi}_y)$$

$$\mathbf{D}_{\varphi y}(\dot{\varphi}_y) = \begin{bmatrix} 0 & 0 & \dot{\varphi}_y & 0 \\ 0 & 0 & 0 & 0 \\ -\dot{\varphi}_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{V}_{\varphi_y} = \mathbf{D}_{\varphi_y}(\dot{\varphi}_y) = \mathbf{D}_{\varphi_y}(1)\mathbf{E}_4(\dot{\varphi}_y)$$

$$\mathbf{A}_{\varphi_y} = \mathbf{D}_{\varphi_y}(\ddot{\varphi}_y) = \mathbf{D}_{\varphi_y}(1)\mathbf{E}_4(\ddot{\varphi}_y)$$

$$\mathbf{B}_{\varphi_y} = \mathbf{D}_{\varphi_y}(\ddot{\varphi}_y) + \mathbf{D}^2_{\varphi_y}(\dot{\varphi}_y) = \mathbf{D}_{\varphi_y}(1)\mathbf{E}_4(\ddot{\varphi}_y) + \mathbf{D}^2_{\varphi_y}(1)\mathbf{E}_4(\dot{\varphi}^2_y)$$

$$\mathbf{T}_{\varphi z}(\varphi_z) = \begin{bmatrix} \cos\varphi_z & -\sin\varphi_z & 0 & 0 \\ \sin\varphi_z & \cos\varphi_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\mathbf{T}}_{\varphi z}(\dot{\varphi}_z) = \begin{bmatrix} -\sin\varphi_z\dot{\varphi}_z & -\cos\varphi_z\dot{\varphi}_z & 0 & 0 \\ \cos\varphi_z\dot{\varphi}_z & -\sin\varphi_z\dot{\varphi}_z & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathbf{T}_{\varphi z}(\varphi_z)\mathbf{D}_{\varphi z}(\dot{\varphi}_z) = \mathbf{T}_{\varphi z}(\varphi_z)\mathbf{D}_{\varphi z}(1)\mathbf{E}_4(\dot{\varphi}_z)$$

$$\mathbf{D}_{\varphi z}(\dot{\varphi}_z) = \begin{bmatrix} 0 & -\dot{\varphi}_z & 0 & 0 \\ \dot{\varphi}_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{V}_{\varphi_z} = \mathbf{D}_{\varphi_z}(\dot{\varphi}_z) = \mathbf{D}_{\varphi_z}(1)\mathbf{E}_4(\dot{\varphi}_z)$$

$$\mathbf{A}_{\varphi_z} = \mathbf{D}_{\varphi_z}(\ddot{\varphi}_z) = \mathbf{D}_{\varphi_z}(1)\mathbf{E}_4(\ddot{\varphi}_z)$$

$$\mathbf{B}_{\varphi_z} = \mathbf{D}_{\varphi_z}(\ddot{\varphi}_z) + \mathbf{D}^2_{\varphi_z}(\dot{\varphi}_z) = \mathbf{D}_{\varphi_z}(1)\mathbf{E}_4(\ddot{\varphi}_z) + \mathbf{D}^2_{\varphi_z}(1)\mathbf{E}_4(\dot{\varphi}^2_z)$$

Inverze, komutativnost, diferencovatelnost

$$\mathbf{T}_Z^{-1}(arg) = \mathbf{T}_Z(-arg), \quad Z: \; x, y, z, \varphi_x, \varphi_y, \varphi_z$$

$$\mathbf{T}_x\mathbf{T}_y\mathbf{T}_z = \mathbf{T}_y\mathbf{T}_x\mathbf{T}_z = \mathbf{T}_z\mathbf{T}_y\mathbf{T}_x$$

$$\mathbf{T}_x\mathbf{T}_{\varphi_x} = \mathbf{T}_{\varphi_x}\mathbf{T}_x$$

$$\mathbf{T}_y\mathbf{T}_{\varphi_y} = \mathbf{T}_{\varphi_y}\mathbf{T}_y$$

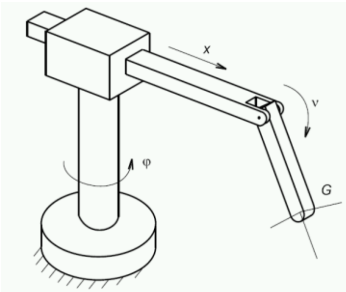
$$\mathbf{T}_z\mathbf{T}_{\varphi_z} = \mathbf{T}_{\varphi_z}\mathbf{T}_z$$

$$\mathbf{TET} = \mathbf{TTE}$$

$$\frac{\partial}{\partial p}\mathbf{T}_Z(g(p)) = \mathbf{T}_Z(g(p))\mathbf{D}_Z(\frac{\partial g}{\partial p}) = \mathbf{T}_Z(g(p))\mathbf{D}_Z(1)\mathbf{E}_4(\frac{\partial g}{\partial p})$$

$$\frac{\partial}{\partial p}\mathbf{D}_Z(g(p)) = \mathbf{D}_Z(\frac{\partial g}{\partial p}) = \mathbf{D}_Z(1)\mathbf{E}_4(\frac{\partial g}{\partial p})$$

Příklad: prostorový manipulátor - rychlosti



$$\boldsymbol{v}_{1\mathrm{G}} = \dot{\boldsymbol{T}}_{14}\boldsymbol{r}_{4\mathrm{G}} = [\boldsymbol{T}_{\varphi_\mathrm{z}}(\varphi)\boldsymbol{D}_{\varphi_\mathrm{z}}(\omega_{12})\boldsymbol{T}_\mathrm{z}(h)\boldsymbol{T}_\mathrm{x}(x)\boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta)+ \\ + \boldsymbol{T}_{\varphi_\mathrm{z}}(\varphi)\boldsymbol{T}_\mathrm{z}(h)\boldsymbol{T}_\mathrm{x}(x)\boldsymbol{D}_\mathrm{x}(v_{23})\boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta)+ \\ + \boldsymbol{T}_{\varphi_\mathrm{z}}(\varphi)\boldsymbol{T}_\mathrm{z}(h)\boldsymbol{T}_\mathrm{x}(x)\boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta)\boldsymbol{D}_{\varphi_\mathrm{y}}(\omega_{34})]\,\boldsymbol{r}_{4\mathrm{G}}$$

$${}^4\boldsymbol{V}_{14} = \boldsymbol{T}_{14}^{-1}\,\dot{\boldsymbol{T}}_{14}$$

$$\boldsymbol{T}_{14}^{-1} = \boldsymbol{T}_{\varphi_\mathrm{y}}(-\vartheta)\boldsymbol{T}_\mathrm{x}(-x)\boldsymbol{T}_\mathrm{z}(-h)\boldsymbol{T}_{\varphi_\mathrm{z}}(-\varphi)$$

$${}^4\boldsymbol{V}_{14} = \boldsymbol{T}_{\varphi_\mathrm{y}}(-\vartheta)\boldsymbol{T}_\mathrm{x}(-x)\boldsymbol{T}_\mathrm{z}(-h)\boldsymbol{D}_{\varphi_\mathrm{z}}(\omega_{12})\boldsymbol{T}_\mathrm{z}(h)\boldsymbol{T}_\mathrm{x}(x)\boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta)+ \\ + \boldsymbol{T}_{\varphi_\mathrm{y}}(-\vartheta)\boldsymbol{D}_\mathrm{x}(v_{23})\boldsymbol{T}_{\varphi_\mathrm{y}}(\vartheta) + \boldsymbol{D}_{\varphi_\mathrm{y}}(\omega_{34}).$$