Lineární systém

$$\dot{\boldsymbol{z}} = \boldsymbol{f}(z,\boldsymbol{w},t) = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{w} = \begin{bmatrix}\boldsymbol{z}_2\\ \boldsymbol{w}\end{bmatrix}$$

s n souřadnicemi a $r = 2n$ stavovými veličinami

$$\boldsymbol{z} = \begin{bmatrix}\boldsymbol{z}_1\\ \boldsymbol{z}_2\end{bmatrix}, \quad \begin{matrix}\boldsymbol{z}_1 = \begin{bmatrix}z_1 & \dots & z_n\end{bmatrix}^T\\ \boldsymbol{z}_2 = \begin{bmatrix}z_{n+1} & \dots & z_r\end{bmatrix}^T\end{matrix}$$

kde

$$\boldsymbol{A} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{E}_n\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{E}_n\end{bmatrix}$$

Kritérium optimality

$$J = \int_0^T L(\boldsymbol{z},\boldsymbol{w},t)\,dt\,, \quad L = \boldsymbol{w}^T\boldsymbol{R}\boldsymbol{w}$$

Pontrjaginův princip maxima

Hamlitonián

$$H = -L(\boldsymbol{z},\boldsymbol{w},t) + \boldsymbol{f}(\boldsymbol{z},\boldsymbol{w},t)^T\boldsymbol{p}$$

kde $\boldsymbol{p}$ jsou konjugované proměnné k $\boldsymbol{z}$:

$$\boldsymbol{p} = \begin{bmatrix}\boldsymbol{p}_1\\ \boldsymbol{p}_2\end{bmatrix}, \quad \begin{matrix}\boldsymbol{p}_1 = \begin{bmatrix}p_1 & \dots & p_n\end{bmatrix}^T\\ \boldsymbol{p}_2 = \begin{bmatrix}p_{n+1} & \dots & p_r\end{bmatrix}^T\end{matrix}, \quad \dot{\boldsymbol{p}} = \begin{bmatrix}\boldsymbol{p}_2\\ \boldsymbol{\rho}(t)\end{bmatrix}$$

Konjugované rovnice

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{z}} = -\boldsymbol{A}^T\boldsymbol{p} = -\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{p}_1\end{bmatrix}$$

Podmínka optimality řízení

$$\frac{\partial H}{\partial \boldsymbol{w}} = -2\boldsymbol{R}\boldsymbol{w} + \boldsymbol{B}^T\boldsymbol{p} = \boldsymbol{0}$$

Tvar konjugovaných proměnných

Posazenim rovnosti rovnic [][] zíkáme obecný tvar

$$\boldsymbol{p}_1 = -\dot{\boldsymbol{p}}_2 = -\boldsymbol{\rho}(t)$$
$$\boldsymbol{p}_2 = \int \boldsymbol{\rho}(t)\,dt$$

s podmínkou

$$\dot{\boldsymbol{p}}_1 = -\dot{\boldsymbol{\rho}}(t) = \boldsymbol{0}\,, \ \forall\, t > 0$$

která je splněna pokud

$$\boldsymbol{\rho}(t) = \boldsymbol{\alpha}\,, \ \boldsymbol{\alpha} \in \mathbb{R}^n$$

následně

$$\boldsymbol{p} = \begin{bmatrix}\boldsymbol{p}_1\\ \boldsymbol{p}_2\end{bmatrix} = \begin{bmatrix}-\boldsymbol{\alpha}\\ \boldsymbol{\alpha}t + \boldsymbol{\beta}\end{bmatrix}, \ \boldsymbol{\beta} \in \mathbb{R}^n$$

Tvar optimálního řízení

Úpravou vztahu [] získáme optimální akční zásah

$$\boldsymbol{w} = \frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{p} = \frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{p}_2$$

po jehož dosazení získáme postupnou integrací stav systému $\boldsymbol{z}$ jako funkci $\boldsymbol{p}$

$$\boldsymbol{z}_2 = \int \boldsymbol{w}\,dt = \frac{1}{2}\int \boldsymbol{R}^{-1}\boldsymbol{p}_2\,dt = \frac{1}{2}\boldsymbol{R}^{-1}\int \boldsymbol{p}_2\,dt$$
$$\boldsymbol{z}_1 = \int \boldsymbol{z}_2\,dt = \frac{1}{2}\boldsymbol{R}^{-1}\iint \boldsymbol{p}_2\,dt^2$$

kde

$$\int \boldsymbol{p}_2\,dt = \frac{1}{2}\boldsymbol{\alpha}t^2 + \boldsymbol{\beta}t + \boldsymbol{\gamma} \qquad \boldsymbol{\gamma} \in \mathbb{R}^n$$
$$\iint \boldsymbol{p}_2\,dt^2 = \frac{1}{6}\boldsymbol{\alpha}t^3 + \frac{1}{2}\boldsymbol{\beta}t^2 + \boldsymbol{\gamma}t + \boldsymbol{\delta} \qquad \boldsymbol{\delta} \in \mathbb{R}^n$$

Dosazeno a upraveno

$$\boldsymbol{z}(t) = \frac{1}{2}\begin{bmatrix}\boldsymbol{R}^{-1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{R}^{-1}\end{bmatrix}\begin{bmatrix}\iint \boldsymbol{p}_2\,dt^2\\ \int \boldsymbol{p}_2\,dt\end{bmatrix}$$
$$2\begin{bmatrix}\boldsymbol{R} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{R}\end{bmatrix}\boldsymbol{z}(t) = \begin{bmatrix}\frac{1}{6}\boldsymbol{\alpha}t^3 + \frac{1}{2}\boldsymbol{\beta}t^2 + \boldsymbol{\gamma}t + \boldsymbol{\delta}\\ \frac{1}{2}\boldsymbol{\alpha}t^2 + \boldsymbol{\beta}t + \boldsymbol{\gamma}\end{bmatrix}$$

Řešení úlohy optimálního řízení lin. systému

Při okrajovych podminkach $\boldsymbol{z}(0) = \boldsymbol{z}_0$ a $\boldsymbol{z}(T) = \boldsymbol{z}_T$ lze vytvořit soustavu lineárních rovnic, jejichž řešením jsou konstanty optimálního řízení

$$\begin{bmatrix}\boldsymbol{\kappa}(0)\\ \boldsymbol{\kappa}(T)\end{bmatrix}\boldsymbol{\lambda} = 2\boldsymbol{\Omega}\begin{bmatrix}\boldsymbol{z}_0\\ \boldsymbol{z}_T\end{bmatrix}, \ \boldsymbol{\lambda} = \begin{bmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\beta}\\ \boldsymbol{\gamma}\\ \boldsymbol{\delta}\end{bmatrix}, \ \boldsymbol{\kappa}(t) = \begin{bmatrix}\frac{1}{6}\boldsymbol{E}_n\,t^3 & \frac{1}{2}\boldsymbol{E}_n\,t^2 & \boldsymbol{E}_n\,t & \boldsymbol{E}_n\\ \frac{1}{2}\boldsymbol{E}_n\,t^2 & \boldsymbol{E}_n\,t & \boldsymbol{E}_n & 0\end{bmatrix}, \ \boldsymbol{\Omega} = \boldsymbol{E}_r \otimes \boldsymbol{R}$$