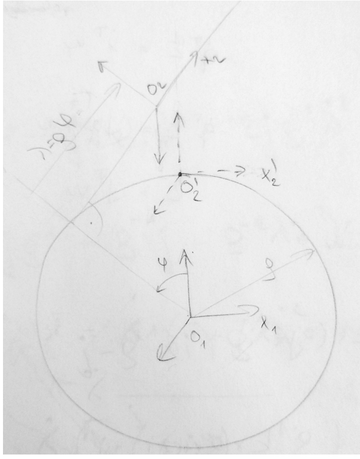

$$\begin{bmatrix}\hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}}\end{bmatrix} = \begin{bmatrix}{}^i\hat{\boldsymbol{\imath}}_i & {}^i\hat{\boldsymbol{\jmath}}_i & {}^i\hat{\boldsymbol{k}}_i\end{bmatrix} = \boldsymbol{E}_3 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \;, \quad \boldsymbol{S}_{12} = \begin{bmatrix}{}^1\hat{\boldsymbol{\imath}}_2 & {}^1\hat{\boldsymbol{\jmath}}_2 & {}^1\hat{\boldsymbol{k}}_2\end{bmatrix} \;, \quad \boldsymbol{R}_z(\phi) = \begin{bmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix} \tag{1}$$

$$\boldsymbol{r}_{12} = {}^1\boldsymbol{r}_{12} = \overrightarrow{O_1\,O_2} \;, \quad ({}^1\boldsymbol{r}_{12} = \boldsymbol{S}_{12}{}^2\boldsymbol{r}_{12} = -\boldsymbol{S}_{12}{}^2\boldsymbol{r}_{21} = -\boldsymbol{S}_{12}\boldsymbol{r}_{21}) \;, \quad \boldsymbol{t}_{12} = \begin{bmatrix}\boldsymbol{v}_{12}\\ \boldsymbol{\omega}_{12}\end{bmatrix} \;, \tag{2}$$

Involute

Transformation

$$\boldsymbol{r}_{12} = \boldsymbol{R}_z(\phi)(\rho\hat{\boldsymbol{\jmath}} + \lambda\hat{\boldsymbol{\imath}}) \tag{3}$$
$$\boldsymbol{S}_{12} = \boldsymbol{R}_z(\phi) \tag{4}$$

λ parameter form

$$\boldsymbol{r}_{12} = \boldsymbol{R}_z(\rho^{-1}\lambda)(\rho\hat{\boldsymbol{\jmath}} + \lambda\hat{\boldsymbol{\imath}}) \tag{5}$$
$$\boldsymbol{S}_{12} = \boldsymbol{R}_z(\rho^{-1}\lambda) \tag{6}$$

Twist $\boldsymbol{t}$

$$\boldsymbol{t}_{12}(\dot\lambda) = \begin{bmatrix}\boldsymbol{R}_z(\rho^{-1}\lambda)\left(\rho^{-1}\hat{\boldsymbol{k}} \times (\rho\hat{\boldsymbol{\jmath}} + \lambda\hat{\boldsymbol{\imath}}) + \dot\lambda\hat{\boldsymbol{\imath}}\right)\\ \rho^{-1}\hat{\boldsymbol{k}}\end{bmatrix}\dot\lambda = \begin{bmatrix}\boldsymbol{R}_z(\rho^{-1}\lambda)\,\rho^{-1}\lambda\hat{\boldsymbol{\jmath}}\\ \rho^{-1}\hat{\boldsymbol{k}}\end{bmatrix}\dot\lambda = \boldsymbol{J}(\lambda)\dot\lambda \tag{7}$$

Acceleration $\dot{\boldsymbol{t}}$

$$\dot{\boldsymbol{t}}_{12} = \boldsymbol{J}(\lambda)\ddot\lambda + \begin{bmatrix}\boldsymbol{R}_z(\rho^{-1}\lambda)\hat{\boldsymbol{\jmath}}\rho^{-1}\dot\lambda^2 - \boldsymbol{R}_z(\rho^{-1}\lambda)\,\hat{\boldsymbol{\imath}}\rho^{-2}\lambda\dot\lambda^2\\ \boldsymbol{0}\end{bmatrix} = \boldsymbol{J}(\lambda)\ddot\lambda + \begin{bmatrix}\boldsymbol{R}_z(\rho^{-1}\lambda)(\hat{\boldsymbol{\jmath}} - \rho^{-1}\lambda\hat{\boldsymbol{\imath}})\rho^{-1}\dot\lambda^2\\ \boldsymbol{0}\end{bmatrix} \tag{8}$$

Joint torque

$$\tau = \boldsymbol{J}^T(\lambda)\,\boldsymbol{w}_{12} = \begin{bmatrix}\rho^{-1}\lambda\hat{\boldsymbol{\jmath}}^T\boldsymbol{R}_z(-\rho^{-1}\lambda) & \rho^{-1}\hat{\boldsymbol{k}}^T\end{bmatrix}\boldsymbol{w}_{12} \tag{9}$$

Motion sub-space

$$\boldsymbol{\Omega}_{ang} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \hat{\boldsymbol{k}}\end{bmatrix} \qquad \boldsymbol{\Omega}_{lin} = \begin{bmatrix}\boldsymbol{0} & \hat{\boldsymbol{\jmath}} & \boldsymbol{0}\end{bmatrix} \tag{10}$$

Constraint wrench sub-space

$$\boldsymbol{\Phi}_{ang} = \begin{bmatrix}\hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \boldsymbol{0}\end{bmatrix} \qquad \boldsymbol{\Phi}_{lin} = \begin{bmatrix}\hat{\boldsymbol{\imath}} & \boldsymbol{0} & \hat{\boldsymbol{k}}\end{bmatrix} \tag{11}$$

Evolute

Transformations

$$\boldsymbol{r}_{21} = -\lambda\hat{\boldsymbol{\imath}} - \rho\hat{\boldsymbol{\jmath}} \tag{12}$$
$$\boldsymbol{S}_{21} = \boldsymbol{R}_z(-\phi) \tag{13}$$

λ parameter form

$$\boldsymbol{r}_{21} = -\lambda\hat{\boldsymbol{\imath}} - \rho\hat{\boldsymbol{\jmath}} \tag{14}$$
$$\boldsymbol{S}_{21} = \boldsymbol{R}_z(-\rho^{-1}\lambda) \tag{15}$$

Twist $\boldsymbol{t}$

$$\boldsymbol{t}_{21} = \begin{bmatrix} \boldsymbol{v}_{21} \\ \boldsymbol{w}_{21} \end{bmatrix} = \begin{bmatrix} -\dot{\lambda}\hat{\boldsymbol{\imath}} - \dot{\phi}\hat{\boldsymbol{k}} \times \rho\hat{\boldsymbol{\jmath}} \\ -\dot{\phi}\hat{\boldsymbol{k}} \end{bmatrix} = \begin{bmatrix} -2\dot{\lambda}\hat{\boldsymbol{\imath}} \\ -\dot{\phi}\hat{\boldsymbol{k}} \end{bmatrix} \tag{16}$$

λ parameter form

$$\boldsymbol{t}_{21}(\dot{\lambda}) = \begin{bmatrix} -2\hat{\boldsymbol{\imath}} \\ -\rho^{-1}\hat{\boldsymbol{k}} \end{bmatrix} \dot{\lambda} \tag{17}$$

Acceleration $\dot{\boldsymbol{t}}$

$$\dot{\boldsymbol{t}}_{21} = \begin{bmatrix} \boldsymbol{v}_{21} \\ \boldsymbol{w}_{21} \end{bmatrix} = \begin{bmatrix} -2\ddot{\lambda}\hat{\boldsymbol{\imath}} \\ -\ddot{\phi}\hat{\boldsymbol{k}} \end{bmatrix} \tag{18}$$

Single variable form

$$\dot{\boldsymbol{t}}_{21}(\ddot{\lambda}) = \begin{bmatrix} -2\hat{\boldsymbol{\imath}} \\ -\rho^{-1}\hat{\boldsymbol{k}} \end{bmatrix} \ddot{\lambda} \tag{19}$$