№1

$E\xi = 300$

а) $P(\xi \geq 400)$?

б) $P(\xi \leq 500)$?

Нер-во Маркова

$P(\xi < a) \leq \frac{E\xi}{a}$

а) $P(\xi \geq 400) \leq \frac{300}{400} = 0,75$

т. к. $P(\xi = 400) > 0 \Rightarrow P(\xi > 400) < 0,75$

б) $P(\xi \geq 500) \leq \frac{300}{500} = 0,6 \Rightarrow$

$\Rightarrow P(\xi < 500) \geq 1 - 0,6 = 0,4$

Ответ: $P(\xi > 400) < 0,75$; $P(\xi \leq 500) \geq 0,4$

№2

$n = 1600 \quad p = 0,3$

нер-во Чебышева: $P(\xi \geq a) \leq \frac{E(\xi^2)}{a^2}$

$\exists\, \eta = |\xi - E\xi| \Rightarrow P(\eta \geq a) \leq \frac{E(\eta^2)}{a^2} \Rightarrow$

$$P(|\xi - M\xi| \geq 50) \leq \frac{E((\xi - M\xi)^2)}{2500} = \frac{D(\xi)}{2500} =$$

$$= \frac{Bin(n,p)}{2500} = \frac{np(1-p)}{2500} = \frac{1600 \cdot 0{,}3 \cdot 0{,}7}{2500} \Rightarrow$$

$$\Rightarrow P(|\xi - M\xi| < 50) = 1 - P(|\xi - M\xi| \geq 50) \geq$$

$$\geq 1 - \frac{1600 \cdot 0{,}3 \cdot 0{,}7}{2500} = 0{,}866$$

Ответ: $P(|\xi - M\xi| < 50) \geq 0{,}866$

№3

Выборка $\{9; 5; 7; 7; 4; 10\}$ $\alpha = 0{,}01$

$$\bar{x} = \frac{9+5+7+7+4+10}{6} = 7$$

$D = 1$ $\quad 1 - \frac{\alpha}{2} = 0{,}995$

$x(0{,}995) \approx 2{,}57$ $\quad \Delta = \frac{\sigma \cdot x_\alpha}{\sqrt{n}}$

Довер. интервал:

$$7 - \frac{1}{\sqrt{6}} \cdot 2{,}57 \leq M \leq 7 + \frac{1}{\sqrt{6}} \cdot 2{,}57$$

99% довер. инт.

$$(\bar{x} - \Delta; \bar{x} + \Delta) = (5{,}947; 8{,}063)$$

№4

$x_i \sim N(\mu; \sigma^2)$

ОМП $\hat{\mu}; \hat{\sigma}$

$f(\mu;\sigma^2)(x) = \frac{1}{2\pi\sigma^2} exp\left(\frac{-(y-\mu)^2}{2\sigma^2}\right)$

$L(x) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_i-\mu)^2}{2\sigma^2}} =$

$= \frac{1}{(\sqrt{2\pi\sigma^2})^n} exp\left(-\frac{\sum^n (x_i-\mu)^2}{2\sigma^2}\right)$

$L(x;\mu;\sigma^2) = -\ln(2\pi)^{\frac{n}{2}} - \frac{n}{2}\ln\sigma^2 - \frac{\sum(x_i-\mu)^2}{2\sigma^2}$

$\frac{\partial}{\partial\mu}L = \frac{2\sum(x_i-\mu)}{2\sigma^2} = \frac{n(\bar{x}-\mu)}{\sigma^2} = 0$

$\frac{\partial}{\partial\sigma}L = -\frac{n}{2\sigma^2} + \frac{\sum(x_i-\mu)^2}{2\sigma^4} = 0 \qquad \hat{\mu} = \bar{x}$

$\Rightarrow \qquad \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$