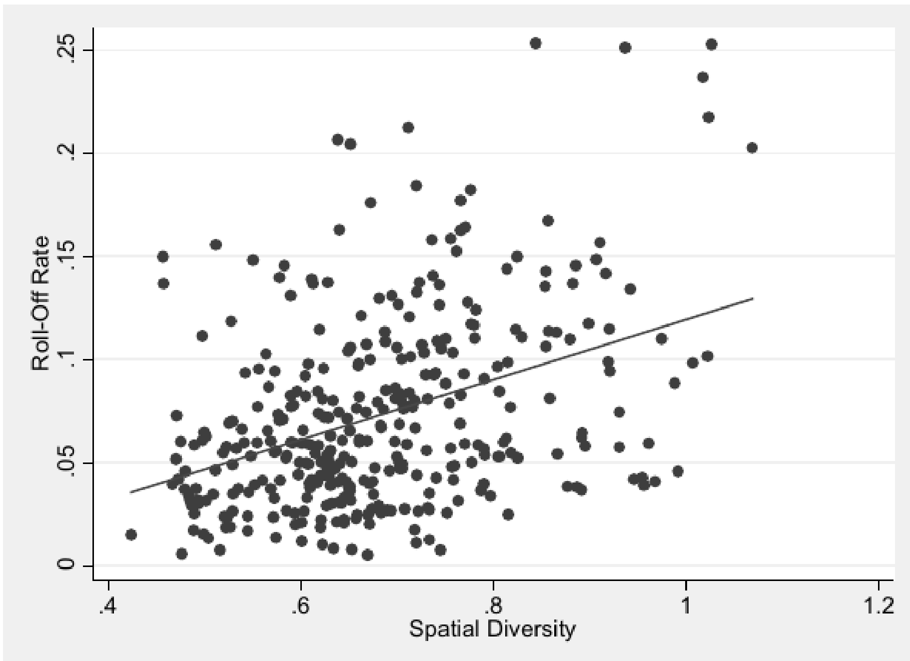
spatial diversity remained a statistically significant predictor of roll-off rate. With these variables held constant at their means, a House district's shift from the tenth to the ninetieth percentile in spatial diversity was associated with an increase in roll-off rate of about six percentage points



Stephanopoulos [2012]

Overview of research strategy

Two key research questions:

1. Do more compact districts have better, equal, or worse spatial diversity scores?
2. Is there an inherent trade-off between compactness and homogeneity?
3. Does spatial diversity give us a normative basis to select one compactness metric over another?

The research procedure:

1. Generate a large and representative subset of plausible districting plans

2.

Overview of compactness measures

To empirically evaluate a trade-off between compactness and homogeneity, we must first define the metrics by which we evaluate a proposed districting plan over each of these dimensions. Here, I introduce many different compactness measures. I give a brief overview of the different types of measures, explain the pros and cons of each, present a compactness measure that I develop, and support my decision to use an ensemble of measures to increase robustness.

Geometric compactness metrics

Shape-based versus dispersion-based

The most

Polsby-Popper

The Polsby-Popper measure, introduced by Polsby and Popper in 1991, is a ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district.

$$4\pi \times \frac{A}{P^2}$$

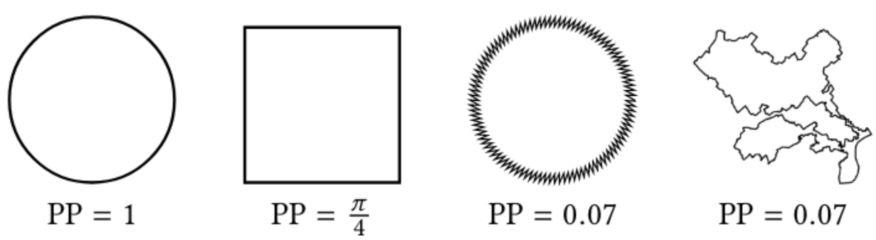


Figure 1: Polsby-Popper scores of four example regions: a perfect circle, a square, a circle with a ragged boundary, an an example district from a Pennsylvania plan. Taken from Schutzman [2020].

Reock

The Reock score is a measure of the ratio of the district to the area of the minimum bounding circle that encloses the district's geometry.

$$\frac{Area}{AreaOfMinimumBoundingCircle}$$

Convex Hull

The Convex Hull metric is a ratio of the area of the district to the area of the minimum convex polygon that can enclose the district's geometry.

$$\frac{Area}{AreaOfMinimumConvexPolygon}$$

Choosing a compactness metric

Which compactness measure should we choose? All three compactness measures are well-cited in the literature and enjoy widespread use. They have been cited in U.S. Supreme Court cases, *amici* briefs, and redistricting commissions [Moncrief, 2011]. Despite their widespread acceptance, however, the problems with compactness measures are many, and well-covered in the literature. No compactness measure is perfect: as an example, the most popular compactness measure in the literature—Polsby-Popper—is sensitive to small perturbations in data resolution (the coastline problem). [1] It is therefore important to use an *ensemble* of compactness measures to make sure that one's data and conclusions are robust.

But even this is not enough. Because most compactness measures are purely geometric, they are all vulnerable to a specific family of geographic perturbations. Indeed, Barnes and Solomon [Forthcoming] show that minimally tweaking the geometric features of states is enough for the four most popular compactness measures (Polsby-Popper, Convex Hull, Reock, Schwartzberg) to give very different conclusions on nominally identical data.

Thus, it is important to include a non-geometric compactness measure in the ensemble to guard against the possibility that the results are driven by a specific quirk in geography. Many such measures have been proposed. For instance, Dube and Clark [2016] bring in a discipline of mathematics—graph theory—to formulate a new metric of compactness.

However, one particular class of metrics I term *point-wise distance metrics* stands out for its ease of understanding (critical if it is to be persuasive to Supreme Court judges), theoretical attractiveness, and academic consensus. This class of compactness metrics tries to measure the distance between two voters in a district, and assigns higher scores the lower that distance is.

This class of metrics enjoy strong theoretical grounding. Paramount to the idea of single-member districts is that there is some value in voters who live in the same area being put into the same district. Eubank and Rodden [2019]:

[1] The Polsby-Popper metric measures the ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district. But depending on the resolution of the map, the perimeter can be effectively infinite. Barnes and Solomon find that the choice of resolution has "a substantial impact on compactness scores, with the Polsby-Popper score especially affected."

> “Voters in the same area are likely to share political interests; voters in the same area are better able to communicate and coordinate with one another; politicians can better maintain connections with voters in the same area; voters in the same area are especially likely to belong to the same social communities — all suggest the importance of voters being located in districts with their geographic peers.”

In contrast, districts that carve voters out of their natural communities and pool them with unrelated, distant voters are bad ones. Therefore, we should be sensitive not just to geometric shape, but rather whether or not voters live close to one another. This class of metrics is more readily understandable to laymen and possesses a normative bent that more abstract mathematical compactness measures lack. It has therefore been an active area of development in the literature. Chambers and Miller [2010] present a measure of “bizarreness”, which is the “expected relative difficulty in traveling between two points within the district”. And Fryer Jr and Holden [2011] measures “the distance between voters within the same district relative to the minimum distance achievable”.

Fryer Jr and Holden’s approach is however an NP-hard problem

I make two key improvements to existing metrics. First, I use driving durations rather than Euclidean (as-the-crow-flies) distances between voters. This keeps the metric robust to quirks in political geography like mountains and lakes, and better represents the notion of natural communities. This idea is not new and has been discussed in the literature. In fact, while Fryer Jr and Holden [2011] used Euclidean distance in his metric, he points out its shortcomings:

> Suppose there is a city on a hill. On the West side is [a] mild, long incline toward the rest of the city, which is in a plane. On the East side is a steep cliff, either impassable or with just a narrow, winding road that very few people use. While the next residential center to the East is much closer to the hilltop on a horizontal plane, it is much further on all sorts of distances that we think might matter: transportation time, intensity of social interactions, sets of shared local public goods and common interests, etc. Thus, for all practical purposes, one probably wants to include the hilltop in a Western district rather than an Eastern one. More general notions of distance can handle this.

In this case, driving durations would better reflect this quirk in political geography. The “impassable” region on the East would have a short Euclidean distance, and any districting plan that put the hilltop with the Eastern district would be unfairly penalised by these point-wise distance metrics. On the other hand, the impassable region would have a long driving duration, accurately reflecting the political geography. In fact, Fryer Jr and Holden specifically suggest using driving durations to improve their metric: “one can extend much of [our analysis] by using driving distance or what legal scholars refer to as ‘communities of interest’ ”.

The use of driving durations seems strictly superior in many cases involving human-scale distances. Working with Nicholas Eubank and Jonathan Rodden, I update their gerrymandering-detection metric to use driving durations instead [Eubank and Rodden, 2019]. We find a consistently different picture of the social context of American suburban voters, raising the possibility of false positives under the Euclidean distance measure [Eubank, Lieu, and Rodden, Forthcoming].

Why then have

Similar concerns were echoed by Brian Olson, the creator of BDistricting, who also chose to use Euclidean distances rather than driving durations because: "It might be the right kind of thing to measure, but it would take too long... the large amount of map data and extra computer time to calculate all those travel times would slow the process down horribly. It would then require a room filling supercomputer to get an answer in a reasonable amount of time".

Therefore, while many have recognised the theoretical advantages of using travel times over Euclidean distances / geographic compactness, no one has of yet come up with a computationally feasible way to use it. By adopting techniques like downsampling, memoisation, and the use of data structures from computer science, I can make the calculation of travel times computationally feasible, and (hopefully) competitive with existing compactness algorithms like Convex Hull or Polsby-Popper.

that builds upon all these approaches. In Eubank et al. [Forthcoming], we

Secondly,

Further details on the metric can be found in Appendix A.

As a result,

As the Schwartzberg and Polsby-Popper measure are mathematically equivalent, I include only Polsby-Popper in the ensemble.

I therefore have what I believe to be the most robust ensemble of compactness measures in the literature

Human compactness

Overview of automated districting algorithms

In order to find out whether compactness measures track spatial diversity, we have to generate many counterfactual plausible plans that span the entirety of possible districting plans and measure the correlation between compactness and spatial diversity. This requires using a computer to draw a large number of plans according to some minimal criteria.

The idea of drawing a large number of districting plans with a computer has a long and storied history, starting in the 60s and 70s. The approach has almost always been used to identify gerrymandering; for instance Cirincione et al. [2000] build an algorithm to "quantitatively [assess] whether the [1990 South Carolina] plan is a racial gerrymander". More recently, Chen et al. [2013] "generat[e] a large number of hypothetical alternative districting plans that are blind as to party and race, relying only on criteria of geographic contiguity and compactness." They do this using a Markov Chain simulation algorithm, a procedure that makes iterative changes for a large number of steps until a unique districting plan emerges. At each step of Cirincione et al.'s algorithm, they randomly select a Census Block Group to serve as a "seed" of the district, then randomly add its neighbouring block groups to it until a district with the desired population is formed. Similarly, Chen et al. begin by initialising all precincts as an individual, separate district, then randomly agglomerating neighbouring precincts until the desired number of districts is reached.

While this "standard simulation algorithm" enjoys a certain degree of success, it has one crippling weakness. The way in which this class of algorithms operates necessarily explores only a tiny subset of all possible districting plans. Subsequent work pointed out this flaw: Magleby and Mosesson wrote that automated processes "may take a biased sample of all possible legislative maps... and fail to efficiently produce a meaningful distribution of all alternative maps". And Fifield et al. contend that "[standard Monte Carlo algorithms] are unlikely to yield a representative sample of redistricting plans for a target population." [2] This poses a huge issue for the validity of any statistical analysis, because any correlation that we discover on a biased subset of plans may be spurious when measured over the actual distribution of plans. [3]

Thankfully, scholars have developed an improvement over the standard algorithm with stronger theoretical guarantees. This second class of algorithms reframe the districting problem as a *graph partition* problem (borrowing insights from graph theory and computer science), and use a *Markov Chain Monte Carlo* (MCMC) approach to sample possible districting plans. This approach is best laid out in Fifield et al. [Working Paper]. Broadly speaking, the approach initialises a specific graph partition as a step in the Markov Chain, then *flips* a random node of the graph to get another valid partition. This process is repeated until the Markov Chain approaches its steady state distribution: when this happens, the Markov chain is called "well-mixed".

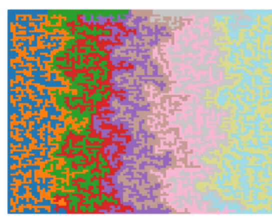
[2] See Fifield et al. [Working Paper], pg. 16, for a technical explanation of why these algorithms don't produce uniform redistricting plans: "For example ..., the creation of earlier districts may make it impossible to yield contiguous districts. These algorithms rely on rejection sampling to incorporate constraints, which is an inefficient strategy. More importantly, the algorithms come with no theoretical result and are not even designed to uniformly sample redistricting plans."

[3] Generating a biased sample is not necessarily a problem if all you want to do is *optimise*, e.g. draw the most compact plan possible. Recent work builds upon this standard algorithm, using Voronoi diagrams or iterative flood fill procedures rather than random chance, to assign the precincts to be agglomerated. See Levin and Friedler [2019] for a technical overview.

This class of algorithms inherit desirable well-known theoretical guarantees of the Markov Chain.[4] They are therefore much less likely (both theoretically and empirically) to generate a biased subset of plans. Conducting a small-scale validation study on a 25-precinct set, Fifield et al. compare the distribution of plans generated by their algorithm to those generated by the standard redistricting algorithm. They prove that their algorithm produces plans that hew much more closely to the *actual* distribution of all possible districting plans.

Due to the many advantages of the MCMC approach, I use it in all my analyses. I use an superior proposal distribution called Recombination (Recom) by DeFord et al., which uses a spanning tree method to bipartition pairs of adjacent districts at each step [DeFord et al., 2019a]. This proposal distribution improves upon the `Flip` proposal in Fifield et al. in two significant ways: it generates plans in much fewer steps [5], and it generates plans that are much more realistic. The `Flip` proposal tends to generate very uncompact, snakelike districts, as can be seen in the figure.

(a) Initial Partition



(b) 1,000,000 Flip steps

(c) 100 ReCom steps

Figure 2: The `Recom` proposal generates more realistic plans in much fewer steps. Taken from DeFord et al. [2019b].

Data generation

1. Generate 100,000 districting plans

I download Census Tract level. These can be downloaded from the United States Census Bureau website.

I use the open-source software library GerryChain to generate the ensembles. Replication code and data are included in the Supplementary Information. I obtain the ReCom Markov chain procedure from one of the co-authors of the DeFord et al. [2019b] paper, and generate 10,000 districting plans for 10 states (Connecticut, Georgia, Idaho, Louisiana, Maine, Maryland, New Hampshire, Rhode Island, Utah, and Wisconsin) for a total of 100,000 plans.

[4]See DeFord et al. [2019b] for a technical overview

[5]the "mixing time" of the Markov Chain—that is, the number of steps it takes for the Markov Chain to be "close enough" to the stationary distribution—is order of magnitudes smaller in `Recom` compared to `Flip`.

2. Calculate spatial diversity and compactness scores for each of the 100,000 districting plans

I obtain data on spatial diversity from Professor Nicholas Stephanopoulos

In order to keep the calculation of human compactness to a reasonable time, I first precalculate a *duration matrix* for every state: this gives the point-to-point driving durations from each voter to every other voter in the state. In order to

I obtain voter data from Eubank and Rodden,

In order to do this, I have to

3. Analyse

Discussion and future work

Appendix A: Calculation of human compactness

Very similar paper

We posit that this is due to thepolitical geographies of the two states, and examining this effect isan important thread for understanding what kinds of reforms mightor might not be effective in various jurisdictions. Future work coulduse more sophisticated mathematical and statistical techniques todescribe a relationship between political geography and the trade-offs we consider here. Our analysis suggests that a one-size-fits-allapproach to drawing 'fair' districts is inappropriate and that indi-vidual states and localities should carefully consider the relevanttrade-offs when redistricting or implementing redistricting reformini-tiatives. One factor ignored in this analysis, which is critical to theprocess of drawing districts, isrespecting communities-of-interest.Even defining and locat-ing geographically such communities is avery difficult problem, let alone the determination of whether ornot to preserve that group in a single district. We therefore pro-pose our analysis as a framework for discussion about trade-offs inredistricting rather than as a policy recommendation.In this work, we have demonstrated with a simple model thatdemanding districts be drawn to be as compact as possible anddemanding that they satisfy a notion of partisan symmetry areincompatible, but to different degrees depending on the particu-larfeatures of the geographic region in question. Since existing propos-als and methodologies for automated and algorithmic redistrictinginvolve finding an approximate solution to an optimization problem,it is important to understand how changing the objective functionof these procedures can affect the outcome. As more jurisdictionsconsider redistricting reforms, they should be cautious about abdi-cating the line drawing process to algorithms which encode valuesdifferent from those of the voters who use the districts to elect theirrepresentatives.

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