Is there a trade-off between compactness and communities of interest?

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Abstract

How should electoral districts be drawn? In the U.S., many states attempt to limit gerrymandering by requiring that districts be "reasonably compact", but also require that plans respect "the integrity of communities of interest". Yet mandating compactness may come at the cost of communities of interest. In order to achieve a compact district shape, one may need to disregard communities of interest and assemble highly heterogeneous districts as a result, adversely affecting democratic outcomes like representation and responsiveness. Are compactness and community fundamentally conflicting goals?

I make two contributions in this work. First, I develop a new compactness metric, human compactness, that improves upon previous measures by incorporating a notion of travel times. Second, I use a Markov Chain Monte Carlo (MCMC) approach to generate a large sample of districting plans. I find no trade-off between compactness and homogeneity across all four compactness measures I examine: plans with more compact districts do not tend to have lower levels of homogeneity. I further find that my human compactness measure consistently identifies more homogeneous districts, suggesting that a judicious choice of compactness metric can in fact encourage better electoral outcomes.

Contents

1 Introduction

How should electoral districts be drawn? This question is important because it is inextricably tied together with democratic representation, a concept deeply rooted in the political science literature. Representation (making all citizens' voices "present" in public policymaking processes) is one of the key pillars of democracy, if not its *raison d'etre*. In *The Concept of Representation*, Pitkin [1967] identifies four facets of representation: formalistic, symbolic, descriptive, and substantive. The way that districts are drawn can affect all four facets of respresentation.

Formalistic representation involves both authorisation and accountability. Authorisation means that the representative must have come to power through a legitimate mechanism, and accountability means that constituents must be able to punish their representatives and vote them out of office if they do poorly. Districts that are drawn fairly deliver both authorisation and accountability. However, gerrymandering—drawing districts that "pack" or "crack" voters—ensure safe seats for incumbents, meaning that that representatives can perform badly and yet be assured of a large margin of victory. If representatives can stay in power regardless of their performance, they are unaccountable. More generally, if voters cannot materially affect the outcome of elections due to gerrymandering, this casts doubt on the legitimacy—and thus authorisation—of the representatives.

How districts are drawn also affects descriptive and substantive representation, which in turn affect democratic outcomes. Descriptive representation involves the extent to which a representative "mirrors" his constituents: this could be belonging to the same race or socioeconomic class, sharing common experiences, or being part of the same communities of interest. But in order for a representative to mirror his constituents, his constituents must be somewhat homogeneous. A single representative cannot resemble multiple highly heterogeneous populations at once. If a district is spatially divided between "nonwhite and white, rich and poor, rural and urban", "then it may be very hard for one representative to represent all factions well" [Cain, 1984]. The more homogeneous a district, the better able the elected official is to accurately reflect the views of more of his constituents [Brunell, 2010]. Additionally, districts can either be drawn to ensure minority representation—as in a majority-minority district—or dilute minority votes to the point of irrelevance.

Many states have written their constitutions with representation clearly in mind. In order to protect formalistic representation, thirty-seven states prevent gerrymandering by mandating that districts should be "reasonably compact", because "the diagnostic mark of the gerrymander... is the noncompact district" [Polsby and Popper, 1991]. Twenty-four states also promote descriptive and substantive representation by asking redistricting bodies to respect "communities of interest"—areas with "recognised similarities of interest" in "social, cultural, racial, ethnic and economic interests"—when districting.

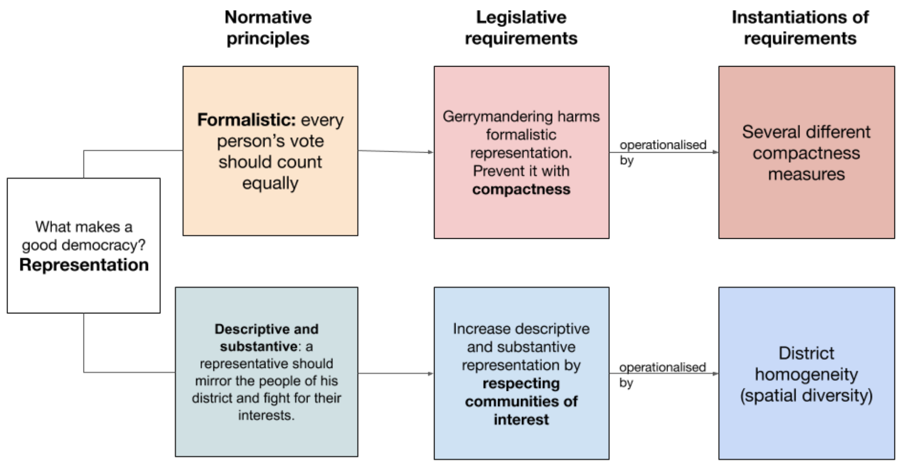

Figure 1: Why representation matters, and how legislators try to promote it

Figure 1 summarises why representation matters, and how legislators try to promote it. In sum, the normative principle of representation guides legislators, which try to protect and promote representation in their constitutions. They do so by mandating that districts should be "reasonably compact" (which protects formalistic representation) and should respect "communities of interest" (which promotes descriptive and substantive representation). Finally, as the constitutions do not specify how compactness and communities of interest should be measured, we must then find a way to operationalise them and ensure that proposed districting plans comply with them.

However, some have argued that compactness and respecting communities of interest are fundamentally conflicting goals. After all, communities of interest do not form neat geometric shapes. For instance—like the Shenandoah Valley—they may follow a river and be long and serpentine. In that case, it may be difficult or impossible to draw a compact-enough plan that does not break up the Valley. Wolf writes that "all [compactness] does is needlessly and unproductively split communities, cities, and counties". If so, then mandating compactness would come at the expense of communities of interest—leaving redistrictors in an impossible situation.

Is this true? Are more compact districts more likely to split communities of interest? While many have argued that compactness may conflict with communities of interest and other desired metrics like minority vote share and electoral competitiveness ([Cain, 1984], Karlan [1989]), no work that I know of has examined the trade-off between compactness and communities of interest.

I thus address this open question in this thesis, which investigates the relationship between two prominent criteria for district design. Using a simulation approach, I generate many districting plans that represent the set of plans a non-partisan districting commission pursuing compactness would possibly generate. I develop a compactness measure of my own which improves upon existing ones, and see if there is any correlation between compactness and communities of interest. My results indicate no trade-off between compactness and homogeneity across all four compactness measures I examine: plans with more compact districts do not tend to have lower levels of homogeneity. I further find that my human compactness measure consistently identifies more homogeneous districts. Rather than a trade-off, the right choice of compactness metric can in fact *encourage* keeping communities of interest together.

1.1 Why compactness is important

Thirty-seven states require their legislative districts be reasonably compact, and eighteen states require congressional districts to be compact as well (Levitt 2019). This is because mandating compactness prevents gerrymandering, a key way in which incumbents can subvert fair elections and evade accountability. As Polsby and Popper [1991] put: "Without the ability to distend district lines... it is not possible to gerrymander. The diagnostic mark of the gerrymander is the noncompact district". This claim is well-supported by the literature: Apollonio et al. [2006] find that "compactness is a good shield against the practice of gerrymandering".

Compactness thus plays a key role in safeguarding formalistic representation. For this reason, the courts have explicitly used compactness as a critical desiderata when challenging unrepresentative plans. Altman [1998] writes:

> In Shaw v. Reno (1993), the Court allowed a challenge to North Carolina's redistricting plan to proceed on the basis that the ill-compactness of the districts indicated a racial gerrymander... Bush v. Vera (1996) declares that violations of compactness and other districting principles are necessary conditions for strict scrutiny to apply.

1.2 Why communities of interest are important

Keeping communities of interest together in a district increases descriptive and substantive representation, which leads to better democratic outcomes. In this thesis, I use district homogeneity as a proxy for communities of interest. This is for two reasons: i) 'communities of interest' are ill-defined and difficult to measure; ii) district homogeneity is regarded as the best proxy for communities of interest. The evidence suggests that more homogeneous districts have higher turnout levels, more responsive elections, and representatives that fight harder for their interests (Stephanopoulos [2012], O'Grady [2019]).

1.2.1 Measuring communities of interest is difficult

Altman writes that communities of interest are important but difficult to pin down:

> The question of how redistricting in general, and compactness in particular, affects 'communities of interest' is important, but ill-defined... the term is often used when we are unable to more conventionally classify the 'interest' involved. In part because of this use of 'communities of interest' as a catch-all, these communities are difficult to quantify. The lack of an objective, quantitative, standard for recognizing such communities makes the subject difficult to examine through either statistics or simulation.

We have seen how difficult defining communities of interest can be. In 2010, the California Redistricting Committee made districting maps that respected "communities of interest" through a year-long, drawn-out process, which involved recruiting unbiased candidates to form the committee, holding dozens of public input hearings, reading through comments and suggestions from over 20,000 individuals and groups, and conducting hundreds of field interviews. It relied on the "active participation" of citizens across California to weigh in on an "open conversation" in which "[the commission] deliberated over the best approach to minimize the splitting of cities, counties, neighbourhoods, and local communities of interest". While this approach did succeed in identifying communities of interest, the Herculean effort involved makes it unlikely to be replicated in other states. More recently, the MGGG Redistricting Lab built a tool inviting members of the public to tag and identify communities of interest—because "communities of interest are notoriously hard to locate" [MGGG Redistricting Lab, 2020].

1.2.2 Using district homogeneity as a proxy

As communities of interest are hard to define and hard to measure, I use district homogeneity—how similar people in a district are, measured on key demographic indicators—as a proxy instead. District homogeneity tracks communities of interest quite closely. The idea is simple: people in the same "communities of interest" are often more alike than not: for instance, they may often be of the same age group, race, or religion. In fact, communities of interest are often viewed through exactly that lens. The Constitution of Colorado defines communities of interest as "ethnic, cultural, economic, trade area, geographic, and demographic factors", and Massachussets defines them based on "trade areas, geographic location, communication and transportation networks... social, cultural and economic interests, or occupations and lifestyles" [Brennan Center for Justice, 2020].

Unlike communities of interest, moreover, there is broad agreement on what homogeneity constitutes. I use American Community Survey (ACS) data—

which contains all sorts of demographic data like educational attainment, income, employment, housing, age, race, and so on—at a geographic (Census Tract) level. These are regarded as the "best available proxies for how closely... districts correspond to *geographic communities of interest*" Stephanopoulos [2012, p. 283].

I operationalise district homogeneity using a particular instantiation called *spatial diversity* developed by Stephanopoulos [2012]. It measures the variance in each Census Tract along ACS factors such as race, ethnicity, age, income, education, and so on. The higher the spatial diversity score, the less homogeneous the district.

1.2.3 District homogeneity is associated with better democratic outcomes

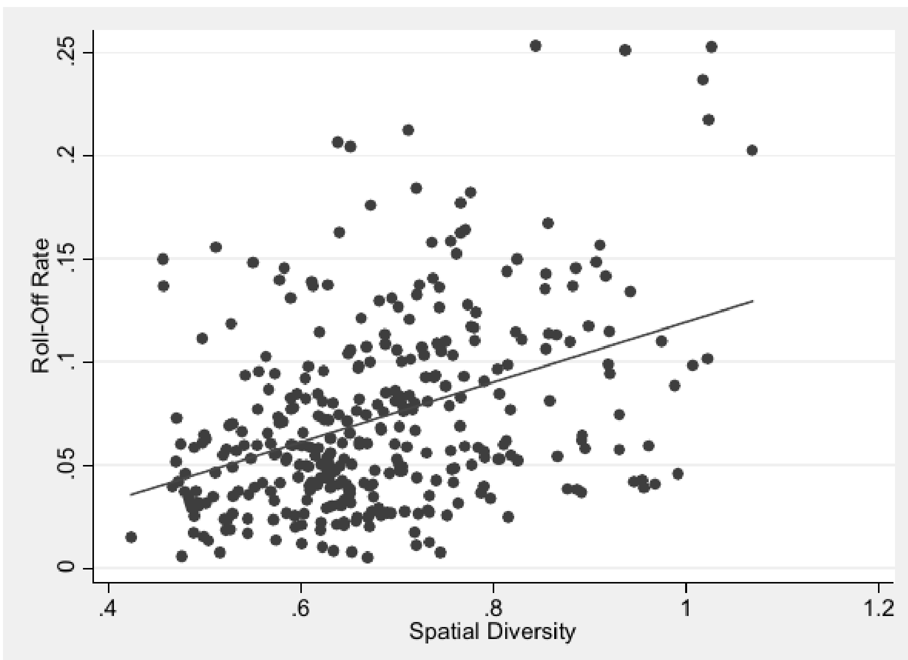

Figure 2: Increased spatial diversity (lower homogeneity) is associated with an increase roll-off rate

In accordance with the evidence presented so far, Stephanopoulos finds that district homogeneity and statewide homogeneity are both strong predictors of democratic outcomes. Figure 2 shows the relationship between spatial diversity and roll-off rate, which is defined as the difference between the proportion of voters who cast a ballot for a presidential race and the proportion who cast a ballot for a lower-ticket (e.g. Congressional) race. Roll-off rates are important indicators of democratic participation, because they zero in on the confusion, lack of knowledge, or apathy that prevents voters from casting their vote in the Congressional race despite having cast a top-ticket vote. Stephanopoulos argues that increasing spatial diversity increases the roll-off rate, which makes sense given what we know so far: homogeneous districts are easier to represent and representatives can better act in their constituents' interests.

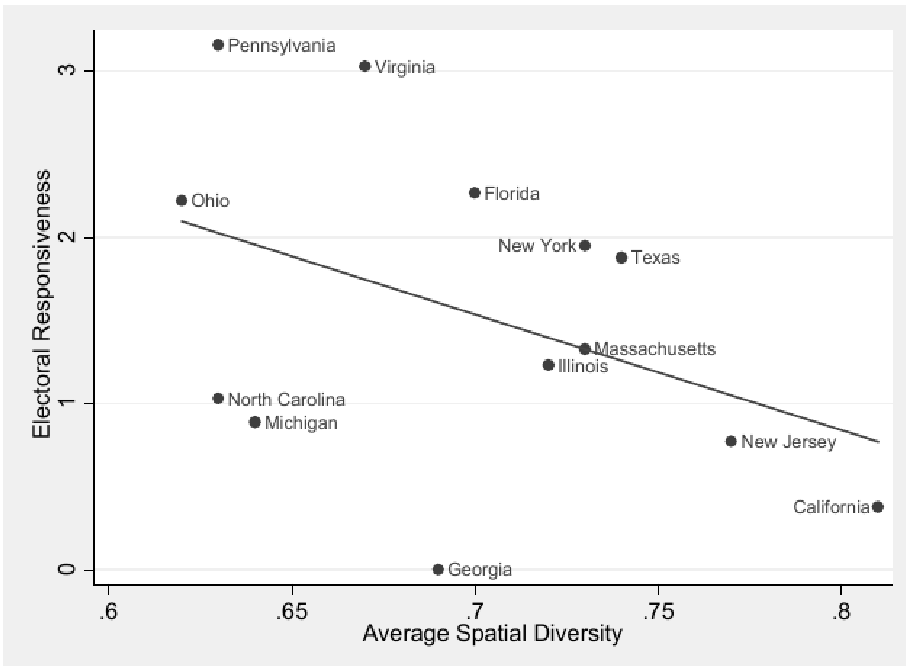

Figure 3: Relationship between spatial diversity and electoral responsiveness

Stephanopulos finds that homogeneous districts also tend to be the ones whose elections are most responsive to changes in public opinion. Figure 3 plots the relationship Stephanopoulos found between spatial diversity and electoral responsiveness. Electoral responsiveness refers to the rate at which a party gains or loses seats given changes in its statewide vote share. For instance, if Democrats would win ten percent more seats if they received five percent more of the vote, then a plan would have a responsiveness of two. The higher a plan's responsiveness, the better it is thought to be. Stephanopoulos finds that more homogeneous districts are more responsive, and writes that "advocates of responsive elections… may push without hesitation for spatially homogeneous districts to be drawn".

In sum, district homogeneity is associated with a variety of positive democratic outcomes.

1.3 A conflict between compactness and communities of interest?

In the previous section, I have established that both compactness and communities of interest are greatly cherished by legislators. However, some have argued that compactness and respecting communities of interest are fundamentally conflicting goals. In order to form districts that are compact enough, a districting commission may have to break apart communities of interest, or agglomerate two communities with nothing in common except that they fit neatly into a neat geometric shape.

Some communities of interest like the Shenandoah Valley may follow a river and be long and serpentine. In that case, it may be difficult or impossible to draw a compact-enough plan that does not break up the Valley. Wolf writes that "all [compactness] does is needlessly and unproductively split communities, cities, and counties". If so, then mandating compactness would come at the expense of communities of interest—leaving redistrictors in an impossible situation.

Previous work has found trade-offs between compactness and other democratic outcomes. DeFord et al. [2019a] show that mandating competitiveness has effects on the partisan lean of the ensuing districting plans. And Schutzman [2020] finds that compactness and partisan symmetry (competitiveness) are somewhat incompatible, suggesting that mandating compactness may have unwanted effects on desired electoral outcomes. It is therefore plausible, as many have suggested, that there is also a trade-off between compactness and communities of interest.

2 Key research questions

Can we have plans that are both very compact and respect communities of interest? Is there a trade-off between community and compactness? Additionally, while legislators have mandated that districting plans be "reasonably compact", they have not specified how compactness should be measured. There are dozens of compactness measures that have been proposed in the literature: if there is indeed a trade-off, might some of them be able to better accommodate both compactness and community?

Along these lines of thought, I pose the following research questions:

2.1 Is there a trade-off between compactness and communities of interest?

Many have claimed that mandating compactness may lead to districts that split communities of interest. But while *some* very compact districts may split communities, there may also be very compact districts that do not. The key question is this. Within the set of plans that an unbiased redistrictor could draw, do more compact plans tend to be more or less homogeneous? Is there a tradeoff between compactness and community?

2.2 Do some compactness metrics better encompass communities of interest than others?

While almost all states mandate that districts are drawn in a "reasonably compact" fashion, they do not specify *how* compactness should be measured. A natural question is to ask which compactness measures we should choose from the dozens proposed in the literature.

While there may be theoretical and methodological reasons to favour one compactness measure over another, there is also a normative consideration. If the plans favoured under one compactness measure are consistently more homogeneous than the others, then this might give us a normative basis for choosing amongst the different compactness measures.

3 Methodology

To answer my research questions, I adopt the following research procedure:

1. Generate a large and representative subset of plausible districting plans
2. Evaluate compactness and spatial diversity scores on that subset of plans
3. Analyse the overall relationship between compactness and communities of interest (operationalised by spatial diversity)

This three-step procedure is used by many previous works, including Chen et al. [2013], DeFord et al. [2019a], and Schutzman [2020]. While the specifics differ, they all follow the same general procedure. I now explain why this procedure (analyzing hypothetical districting plans) has advantages over analyzing enacted or proposed districting plans.

3.1 Why a simulation approach is necessary

I use a simulation approach to generate tens of thousands of plausible districting plans. One might ask: What is the point of using a simulation approach? Why not just use historical districting plans that actually existed in real life? There are two reasons. Firstly, there have not been very many historical districting plans. There may be at most twenty districting plans over the history of a state, but they range from the 1800s to the 2000s. It would be difficult to get geospatial data on these historical plans, and impossible to get any demographic data on district homogeneity/communities of interest.

But the biggest problem in trying to draw a link between districting plans and any outcome of interest is that of endogeneity. Suppose we believe that less compact plans lead to less political participation:

$$Compactness \rightarrow Participation$$

To identify whether this relationship is true, we could look at several enacted districting plans and measure their compactness and political participation. Then we would be able to run an OLS regression and retrieve the coefficients. But these coefficients would not have a causal interpretation. We know that compactness is a result of districting procedures that are political in nature. Political participation affects who wins the state, and the winning party then has outsize influence on the next districting plan. The districting plans affect the outcome of the election, which in turn affects future districting plans. This makes it difficult to find the marginal effect of an increase in compactness on participation.

Even finding natural experiments may not be enough to remove the endogeneity. The Supreme Court has often struck down proposed districting plans and forced parties to propose a new one. We can think of this as an exogenous shock and calculate compactness and political participation in both plans. But even this has knock-on effects. When the Supreme Court strikes down a plan, it's safe to say that there will be significantly increased media coverage on the proceedings—which will surely affect interest and participation in the subsequent elections.

It would be useful to vary compactness unilaterally while knowing that that variation was not due to a previous change in political participation. But this is precisely what simulation approaches allow us to do. If we could simulate plans that represent the set of plans that a non-partisan committee pursuing compactness might generate, then we would solve the problems of small sample size, lack of data, and endogeneity in one fell swoop.

A simulation approach is therefore advantageous due to the limitations of our data. But the simulation procedure introduces several new considerations. We need to choose two things in the procedure: a method to generate districting plans, and a compactness metric to score these districting plans. This choice highly consequential: different generating functions and the choice of compactness metric can give very different results. I now explain how I chose both of these.

3.2 Choosing which compactness measures to evaluate

To empirically evaluate a trade-off between compactness and homogeneity, we must first figure out how to measure compactness. I give a brief overview of the different types of measures and explain the pros and cons of each. I present a compactness measure that I develop and finally explain my decision to analyse four different compactness measures to increase the robustness of my results.[1]

Over a hundred compactness measures have been proposed in the literature. Here, I focus on two main families: *geometric* compactness metrics and *point-wise distance* metrics.

3.2.1 Geometric compactness metrics

Geometric compactness metrics are by far the largest class of compactness measures. They look at some geometric properties of proposed districts. These properties are most often shapes, area or perimeter—although more esoteric measures do exist. Here, I explain the three most popular compactness measures, although other popular compactness measures e.g. Schwartzberg are qualitatively similar.

3.2.1.1 Polsby-Popper

The Polsby-Popper measure is by far the most popular measure used in the literature. It is the ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district [Polsby and Popper, 1991]. A perfect circle has a Polsby-Popper score of 1.

$$4\pi \times \frac{A}{P^2}$$

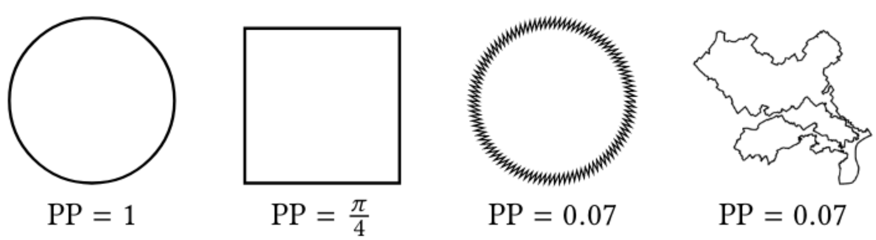

Figure 4: Polsby-Popper scores of four example districts: a perfect circle, a square, a circle with a ragged boundary, an an example district from a Pennsylvania plan. Taken from Schutzman [2020].

[1] I use the phrases “compactness metric” and “compactness measure” interchangeably.

3.2.1.2 Reock

The Reock score is the ratio of the district's area to the area of the minimum bounding circle that encloses the district's geometry [Reock, 1961].

$$\frac{Area}{AreaOfMinimumBoundingCircle}$$

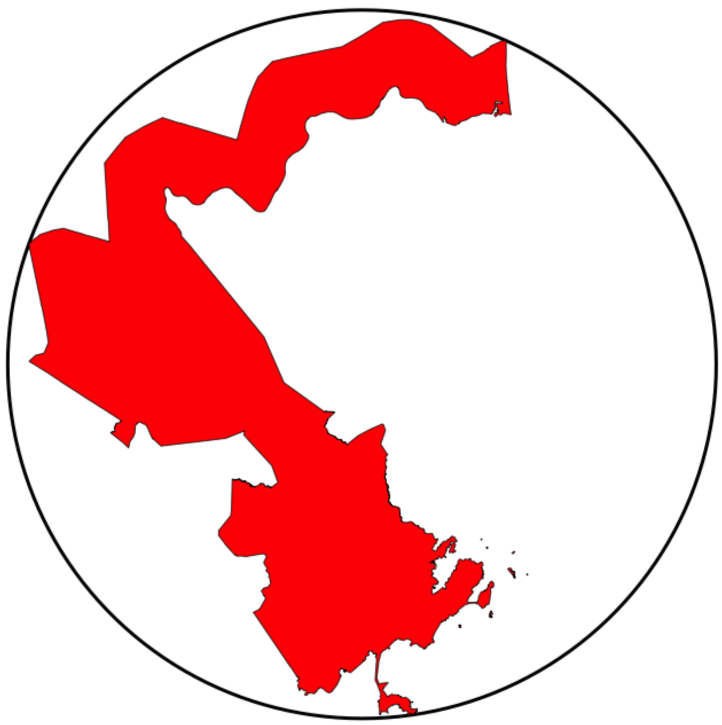

Figure 5: A visualisation of the Reock metric.

3.2.1.3 Convex Hull

The Convex Hull metric is a ratio of the area of the district to the area of the minimum convex polygon that can enclose the district's geometry. A circle, square, or any other convex polygon has the maximum Convex Hull score of 1.

$$\frac{Area}{AreaOfMinimumConvexPolygon}$$

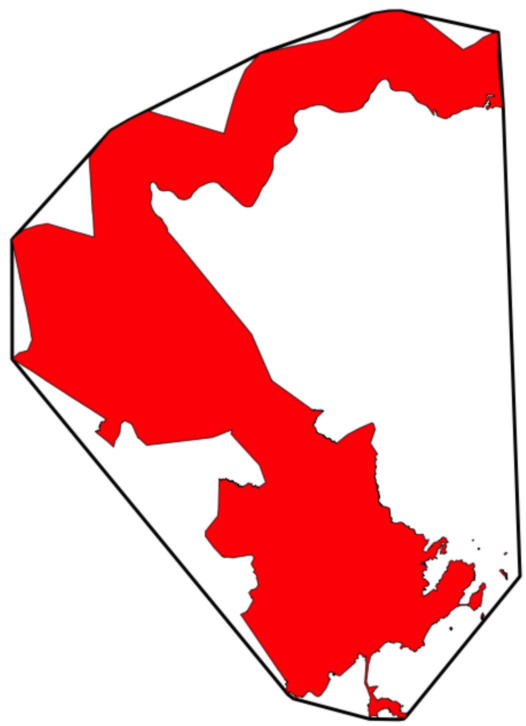

Figure 6: A visualisation of the Convex Hull metric.

3.2.2 Point-wise distance compactness measures

Another large family of compactness measures are *non-geometric measures*, which do not take into account the geometric properties (e.g. area, perimeter) of the district explicitly. Many such non-geometric measures have been proposed. For instance, Dube and Clark [2016] bring in a discipline of mathematics—graph theory—to formulate a new metric of compactness. And Kaufman et al.

[Forthcoming] use a machine learning model to try and ape human intuition—quantifying the intuitive metric of "I know it when I see it".[2]

But one particular class of metrics I term *point-wise distance* compactness stands out for its ease of understanding (critical if it is to be persuasive to Supreme Court judges), theoretical attractiveness, and academic consensus. Roughly speaking, this class of compactness metrics tries to measure the distance between voters in a district, and assigns higher scores the lower that distance is.

This class of metrics enjoys strong theoretical grounding. Paramount to the idea of single-member districts is that there is some value in voters who live in the same area being put into the same district. Eubank and Rodden [2019] write:

> "Voters in the same area are likely to share political interests; voters in the same area are better able to communicate and coordinate with one another; politicians can better maintain connections with voters in the same area; voters in the same area are especially likely to belong to the same social communities — all suggest the importance of voters being located in districts with their geographic peers."

A wealth of empirical evidence supports the above statement. Arzheimer and Evans [2012] find that constituents support less strongly candidates that live far from them, even controlling for strong predictors of vote choice like party feeling and socio-economic distance. Similarly, Dyck and Gimpel [2005] find that voters living further away from a voting site are less likely to turn out to vote. In part, voters strongly support proximate candidates because they think that these candidates better represent their interests. If voters prefer a representative who lives close to them, then we can satisfy the most voters by drawing districts where everyone lives close to everyone else—only then can that district have a representative who lives close to everybody.

In contrast, districts that put people with unrelated, faraway others carve voters out of their natural communities and are thus to be avoided. We care about whether co-districtors live in the same area and belong to the same communities of interest, not just the compactness of their electoral district. And point-wise distance metrics deliver exactly that.

Therefore, point-wise distance metrics are more intuitive to laymen and possess a normative bent that more abstract mathematical compactness measures lack. It has therefore been an active area of development in the literature. Chambers and Miller [2010] present a measure of "bizarreness", which is the "expected relative difficulty in traveling between two points within the district". And Fryer Jr and Holden [2011] measures "the distance between voters within the same district relative to the minimum distance achievable".

[2] This penalises districting plans that have a large difference between districts e.g. one very good district and one very bad one.

3.2.3 Flaws with existing compactness metrics

In this section, I show existing compactness metrics are useful but somewhat inadequate. Geometric compactness measures have several well-known problems, and while point-wise distance metrics fix many of these problems, they have issues of their own. I thus develop a new compactness metric which improves upon existing point-wise distance metrics.

All three geometric compactness measures are well-cited in the literature and enjoy widespread use. They have been cited in U.S. Supreme Court cases, *amici* briefs, and redistricting commissions [Moncrief, 2011]. Despite their widespread use, however, the problems with compactness measures are many, and well-covered in the literature. As an example, the most popular compactness measure in the literature—Polsby-Popper—is sensitive to small perturbations in data resolution (the coastline problem).[3] The same is true for other geometric compactness measures: no single metric is perfect.

Because all three of these compactness measures are purely geometric, they are all vulnerable to geographic perturbations. Indeed, Barnes and Solomon [Forthcoming] show that minimal changes in the geometric features of states are enough for the four most popular compactness measures (Polsby-Popper, Convex Hull, Reock, Schwartzberg) to give very different conclusions on nominally identical data. These changes do not have to be made on purpose: small changes in the way the data is collected or processed can suffice to affect the conclusions we draw.

And despite the relative merits of point-wise distance metrics, there are two areas of improvement—one theoretical, the other empirical. Firstly, all point-wise distance metrics suggested in the literature use Euclidean distances. But many have rightly suggested that we should consider travel times/driving durations instead. For instance, while Fryer Jr and Holden [2011] used Euclidean distance in their metric, they point out its shortcomings:

> Suppose there is a city on a hill. On the West side is [a] mild, long incline toward the rest of the city, which is in a plane. On the East side is a steep cliff, either impassable or with just a narrow, winding road that very few people use. While the next residential center to the East is much closer to the hilltop on a horizontal plane, it is much further on all sorts of distances that we think might matter: transportation time, intensity of social interactions, sets of shared local public goods and common interests, etc. Thus, for all practical purposes, one probably wants to include the hilltop in a Western

[3]The Polsby-Popper metric measures the ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district. But depending on the resolution of the map, the perimeter can be effectively infinite. Barnes and Solomon find that the choice of resolution has "a substantial impact on compactness scores, with the Polsby-Popper score especially affected."

district rather than an Eastern one. More general notions of distance can handle this.

Here we see the key problem with using Euclidean distances in point-wise distance metrics. The "impassable" region on the East would have a short Euclidean distance, and any districting plan that put the hilltop with the Eastern district would be unfairly penalised by these point-wise distance metrics. Evidently, using driving durations instead would give us more accurate scores. Using driving durations, the impassable region would have a long driving duration, accurately reflecting the political geography. In this and many other cases like it (e.g. large bodies of water), driving durations better reflect a state's unique political geographies.

After acknowledging the shortcomings of Euclidean distance, Fryer Jr and Holden specifically suggest using driving durations to improve their metric: "one can extend much of [our analysis] by using driving distance or what legal scholars refer to as 'communities of interest' ".

There are thus strong theoretical grounds for using driving durations in point-wise distance metrics. Why then have scholars not adopted it, seeing as they agree on its superiority? This brings me to my empirical criticism: the point-wise distance metrics scholars have proposed are either far too computationally complex to compute at scale, or have restrictions that make using travel times difficult, if not impossible. For instance, the metric that Fryer Jr and Holden [2011] propose requires solving an *NP-complete* problem. A term used in computer science, an NP-complete problem scales exponentially with the size of the input. This makes it prohibitively expensive on larger states. And while they have an approximation that runs much quicker, they provide no bounds on the correctness of this approximation.

Similarly, Olson has a metric that minimises the average distance from each voter to the center of their district. He says of travel times "that it might be the right kind of thing to measure, but it would take too long... The large amount of map data and extra computer time to calculate all those travel times would slow the process down horribly. It would then require a room filling supercomputer to get an answer in a reasonable amount of time." [Olson, 2010]. And finally, Chambers and Miller's measure cannot feasibly be improved with driving durations due to the difficulty of finding point-to-point travel distances without passing through another district. This is because most routing engines allow you only to specify a route between two (or more) points. They do not further allow you to specify regions through which the route cannot pass.

3.2.4 Building a new compactness metric: Human compactness

Given the difficulties of adapting existing point-based distance metrics to use driving durations, I develop a new measure called *human compactness*. This metric incorporates driving durations at the very outset, and builds in optimisations to run quickly. The human compactness metric measures the ratio of driving durations between one's nearest neighbours and one's fellow districtors. This ratio ranges from 0 to 1. The higher this ratio is, the more compact the district. Intuitively, it encourages drawing districts that put one's next-door neighbours together in the same district.

The human compactness metric works at three levels: at the voter-level, the district-level, and the overall plan-level. At the voter level, human compactness of a voter is the ratio of: the sum of driving durations to one's K nearest neighbours, to the sum of driving durations to one's co-districtors, where K is the number of voters in that voter's district.

A simple example will be illuminating. The following figures give a simple demonstration of how the human compactness metric is calculated both on the voter- and district- level. The example works for both Euclidean distances and driving durations: only a simple swap is required.

Figure 7 shows a highly simplified state assignment, with two districts, Red and Blue, and three voters in each district. We label each point from top-left to bottom-right. Note here that Red and Blue are not partisan affliations: R1, R2 and R3 are red voters simply because they happen to fall in the Red district.

We will first calculate the individual human compactness score for each voter in the Red district. Figure 8 illustrates this for the top-left voter, R1. First, we find the sum of distances between R1 and his fellow co-districtors R2 and R3. This sum, $5 + 6$, forms the denominator of the human compactness score.

Next, we find the sum of driving durations between R1 and his nearest neighbours. Because there are two other voters in his district, we will find his two nearest neighbours. To find the two nearest neighbours, here I have drawn a circle centered upon R1, and expanded the circle on all sides until it touches two other voters.[4]. We can see that R1's nearest neighbours are the points B1 and R2, with a distance of 1.5 and 5 respectively. The human compactness score of R1 is thus

$$HC_{R1} = \frac{d_{B1} + d_{R2}}{d_{R2} + d_{R3}} = \frac{1.5 + 5}{5 + 6} = 0.59$$

This is how we calculate an individual human compactness score. We repeat the same procedure with R2 and R3, and obtain $HC_{R2} = \frac{4+4.5}{5+4} = 0.94$ and $HC_{R3} = \frac{2+2.5}{4+6} = 0.45$. The compactness score for point R3 is particularly low. We can see why this is the case in Figure 10. Because point R3 is so close to B2

[4]The method of drawing an ever-expanding circle to get one's K-nearest neighbours only works for Euclidean distances. In reality, the "circle of K-nearest neighbours" will not be a circle, but rather be what is called an *isochrone*: a line drawn on a map that connects points that have the same travel duration. The shape of the isochrone will vary with geographic features like cliffs or man-made features like highways. My implementation of the human compactness algorithm precomputes all the K-nearest neighbours for every single point, negating the need to calculate isochrones.

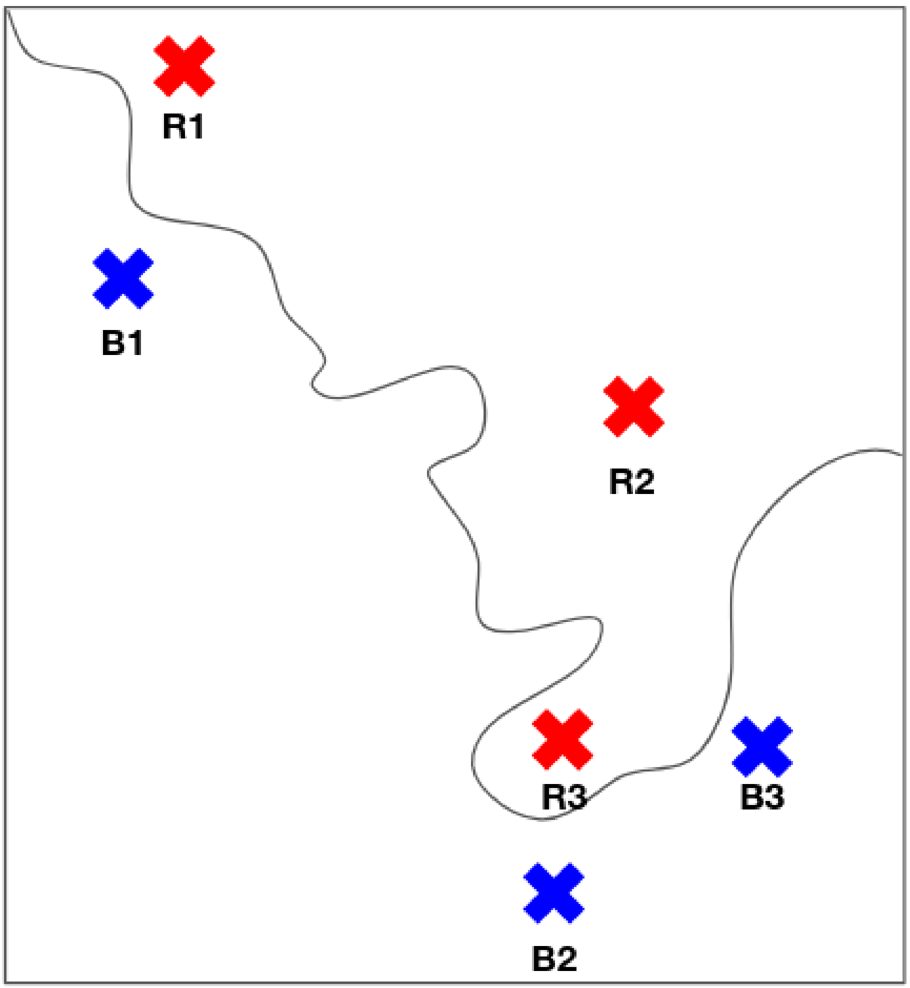

Figure 7: A simplified state assignment with two districts and six voters

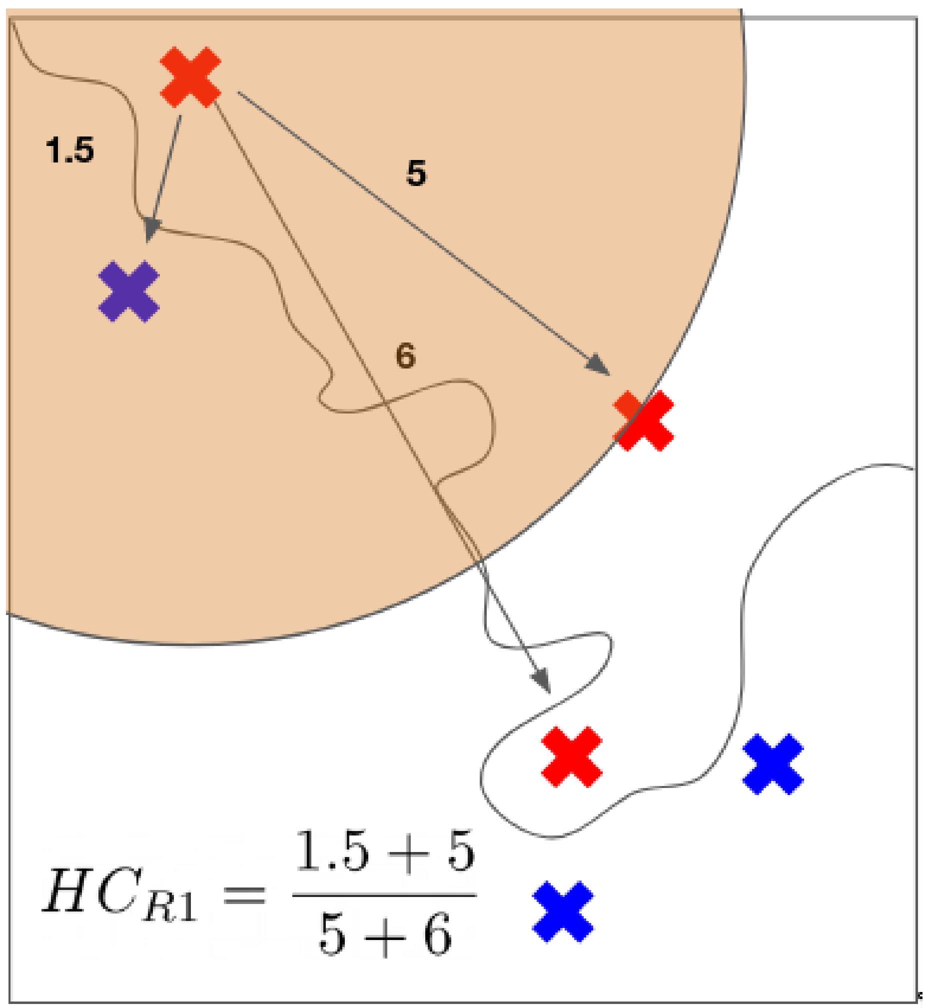

Figure 8: Human compactness measure for voter R1

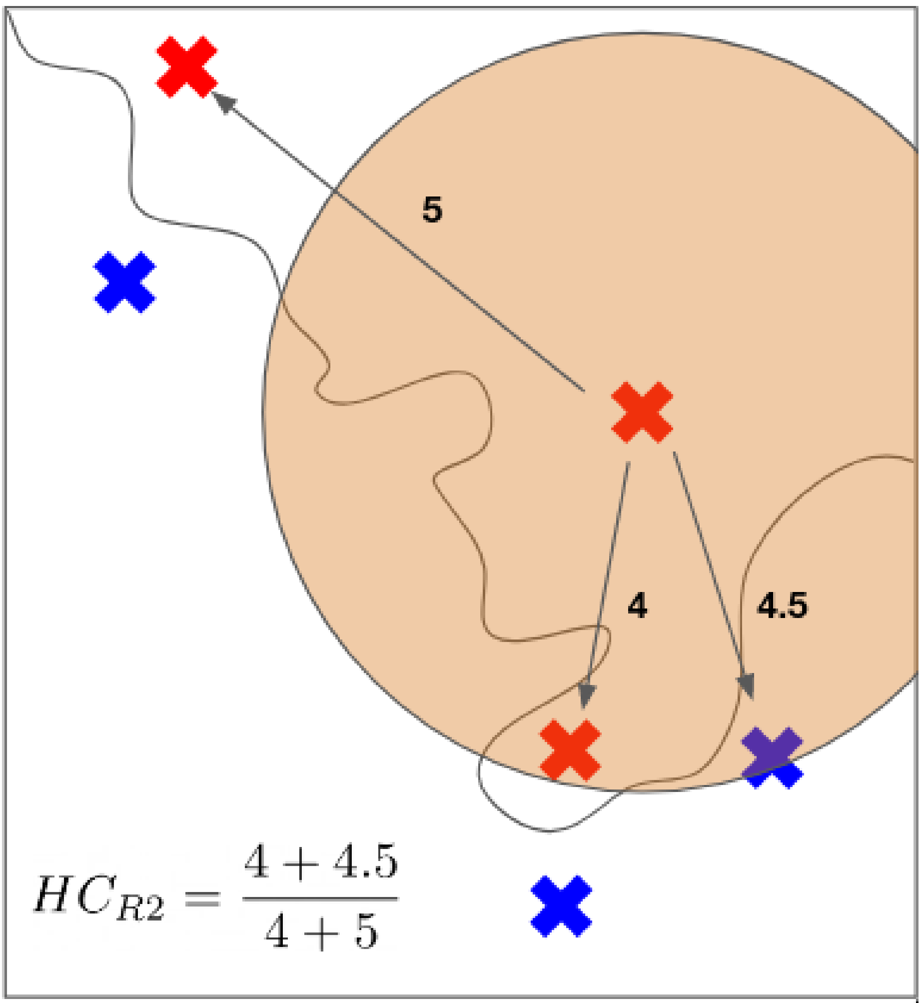

Figure 9: Human compactness measure for voter R2

and B3, it really should be put in the same district with them—R3 likely lives in the same neighbourhood and/or community as B2 and B3. This is why the human compactness metric gives it a very low score.

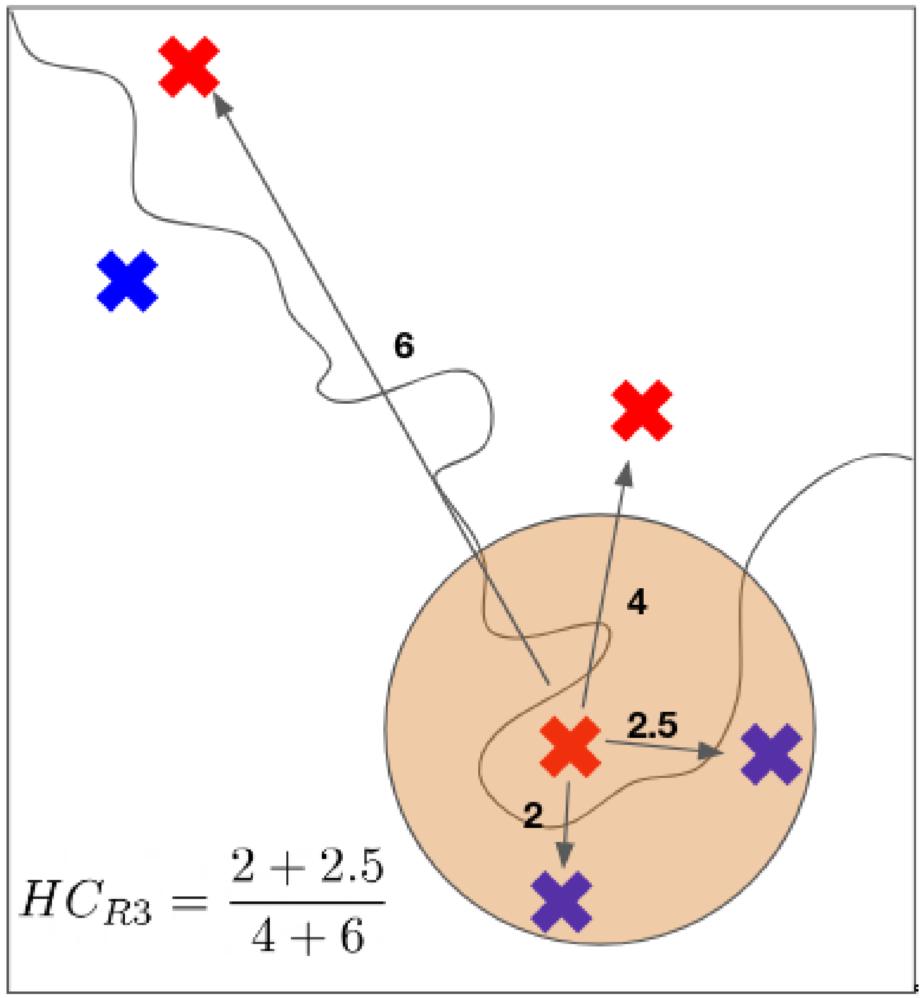

Figure 10: Human compactness measure for voter R3

The *district's* human compactness measure, HC_R, simply takes the ratio of all the sum of durations, as follows:[5]

[5]Another reasonable approach might be take the arithmetic mean of all individual human compactness scores. In that case the district-level human compactness score would be $0.59 + 0.94 + 0.45/3 = 0.66$, basically identical to the value we obtained.

$$HC_R = \frac{(1.5 + 5) + (4 + 4.5) + (2.5 + 2)}{(5 + 6) + (5 + 4) + (4 + 6)} = 0.65$$

Finally, we obtain the districting plan's *plan-level* compactness score by taking the simple arithmetic mean of all district-level compactness scores. Other aggregation functions are plausible: for instance, taking the median, or the root-mean-squared value. In the Results section, I run robustness checks with the root-mean-squared aggregation function and find qualitatively similar results.

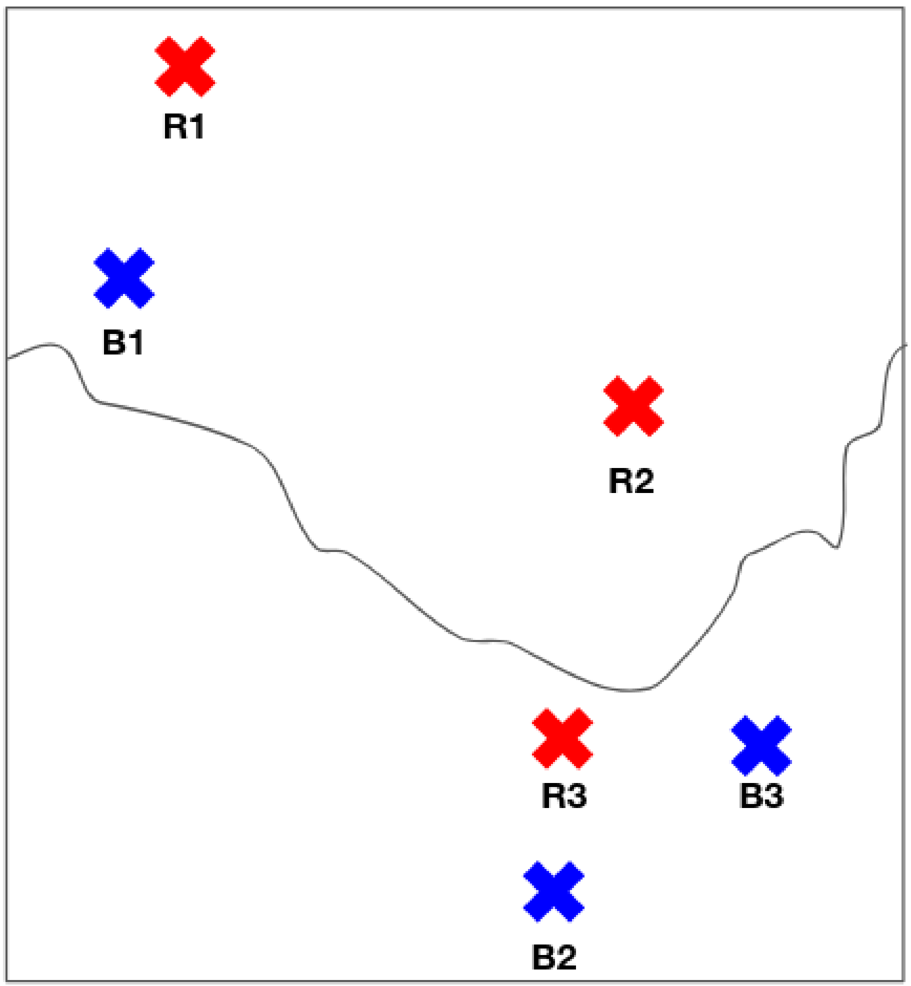

Figure 11: An alternative, more humanly compact proposed districting plan

We have seen how to calculate the human compactness score for a proposed districting plan. Now we demonstrate the conditions under which human compactness score will assign better scores.

Figure 11 shows a proposed alternative districting plan. Only the boundary has changed—the points have not. We can see intuitively that this plan is more compact. Rather than being “carved out” of his natural community in a snakelike fashion, R3 is now put in a reasonably-shaped district with B2 and B3. We can calculate the spatial diversity of this new district by imputing reasonable distance values for R1–B1 and R2–B1. We thus get

$$HC_{R*} = \frac{(1.5+5)+(1.5+4.5)+(4+4.5)}{(1.5+5)+(1.5+4.5)+(5+4.5)} = 0.95$$

As we can see, the new district (and by extension districting plan) is given a much higher score under the human compactness metric, which largely accords with our intuitions. The human compactness measure enjoys two significant advantages over existing approaches. First, the human compactness metric improves upon the algorithmic complexity of Fryer Jr and Holden’s algorithm from an NP-hard problem to one with a $O(n^2)$ polynomial runtime. This is an exponential decrease in algorithmic complexity. I also use programming techniques like precomputation and memoisation to decrease the time taken to compute the metric greatly. My implementation is competitive with geometry-based compactness measures like Reock: on my machine, both metrics took roughly the same amount of time (~0.20s per step). This greatly increases the capability of political science researchers to conduct ensemble analysis without requiring “room-filling supercomputers”. Further details on these algorithmic optimisations can be found in Technical Appendix B.

Because of these algorithmic improvements and the way I have designed the metric, I am able to use driving durations rather than Euclidean (as-the-crow-flies) distances between voters. This is a large improvement with strong theoretical and empirical support. Many previous scholars have suggested exactly this, giving it strong theoretical support. It keeps the metric robust to quirks in political geography like mountains and lakes, and better represents the notion of natural communities.

Figure 12 shows how driving durations is able to get the right answer despite quirks in political geography. It represents the situation that Fryer Jr and Holden [2011] point out: the voters in red live atop a cliff, and the valley below (inhabited by the voters in blue) is impassible. In this case, it would be better to put the voters in red together, as they are “closer” together on all sorts of metrics that would matter: shared communities, public services, and so on. A compactness measure that used Euclidean distances would not be able to accommodate this.

Empirically, too, the use of driving durations seems strictly superior in many cases involving human-scale distances. Working with Eubank and Rodden, I update their gerrymandering-detection metric to use driving durations instead [Eubank and Rodden, 2019]. We find a consistently different picture of the social

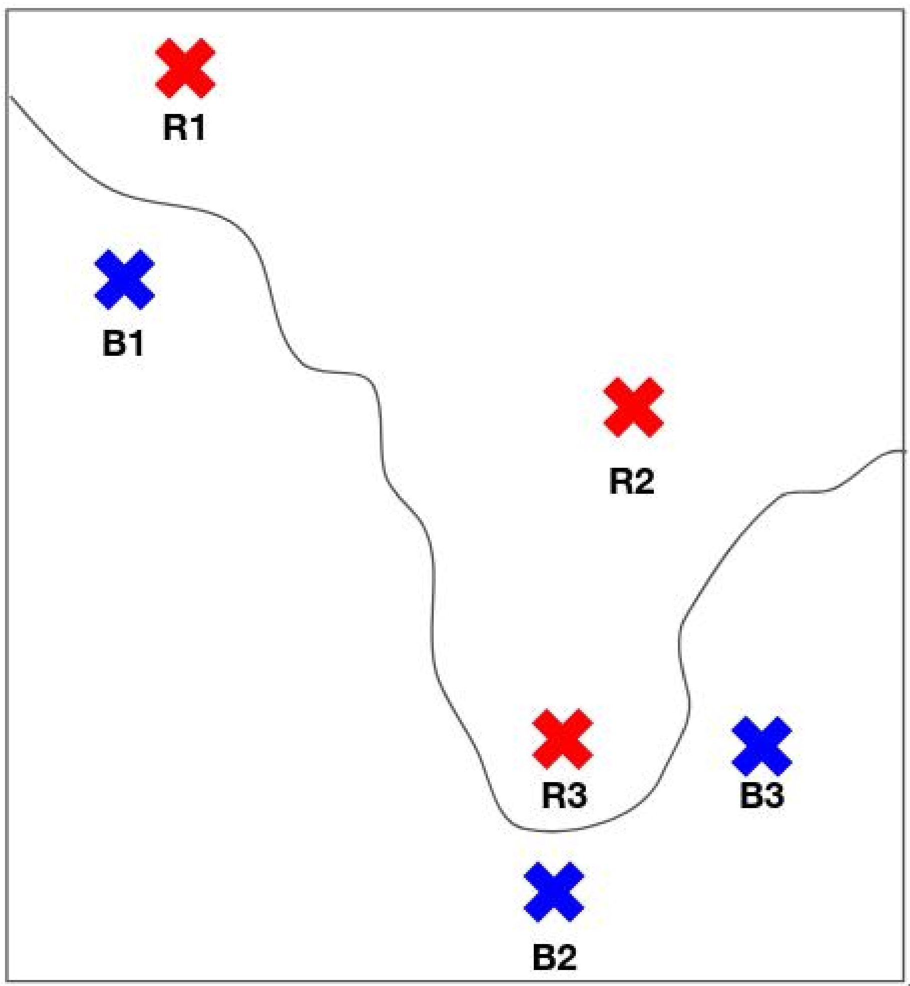

Figure 12: With an impassible cliff face, Euclidean distance gives the wrong answer

context of American suburban voters, raising the possibility of false positives under the Euclidean distance measure [Eubank, Lieu, and Rodden, Forthcoming].

3.2.5 Choosing which compactness metrics to evaluate

As each compactness measure has its advantages and disadvantages, we should include the broadest subset of measures possible in my analyses. This will also maximise the generalisability of my results: in order to claim that there is or isn't a tradeoff between compactness and homogeneity, we should make sure that the relationship holds for as many compactness measures as possible. At the same time, though, the inclusion of each additional compactness measure incurs time, effort and computational costs. Given finite time and resources, there is thus a trade-off between the number of compactness measures and the number of states/districting plans I can analyse. I therefore made a judgement call to include the most-used and most representative compactness measures.

Without question, we must include at least one of the geometric compactness measures. This is because these geometric compactness measures are by far the most widely used, both inside academic political science and out. As mentioned, they have been cited in U.S. Supreme Court cases, *amici* briefs, and redistricting commissions [Moncrief, 2011]. I therefore include the three most popular geometric compactness measures (Polsby-Popper, Convex Hull, and Reock).

It is also important to include a non-geometric compactness measure as the geometric compactness measures are all sensitive to small changes in the way the geospatial data are collected and processed.

I use my human compactness measure as a representative of non-geometric compactness measures, for two reasons. Firstly, many of the compactness measures have a formal mathematical definition but have no code available online. It would have taken too much time for me to re-implement the compactness measure and calculate it for 100,000 districting plans. Secondly, human compactness is the only measure that incorporates travel durations, which has strong theoretical/normative backing and some tentative empirical support. Nonetheless, I would have liked to include another non-geometric compactness measure.

Given these considerations, I settle on using four different compactness measures: Polsby-Popper, Reock, Convex Hull, and Human Compactness.

3.3 Choosing an algorithm to generate districting plans

In order to find out whether compactness measures track spatial diversity, we have to generate many districting plans that span the set of plans a nonpartisan districtor would draw. We would then measure the correlation between compactness and spatial diversity. This requires using a computer to draw a large number of plans, and I use a simulation approach to do so.

How should we draw the representative plans? Many approaches have been suggested in the computational districting literature (Cirincione et al. [2000], Chen et al. [2013], Fifield et al. [Working Paper], DeFord et al. [2019b]). I have chosen a simulation approach known as Markov Chain Monte Carlo (MCMC). This approach is a stochastic process that generates plans by performing something like a random walk over plausible districting plans. I chose this approach because it generates the largest and most representative subset of plans while making the least assumptions about what makes a district "good". Additionally, my chosen approach is regarded as the state-of-the-art in the computational redistricting literature [DeFord et al., 2019b].

The details of how I chose the algorithm are in Technical Appendix A. I start with a literature review of seminal work in computational districting and evaluate three different state-of-the-art MCMC approaches before choosing my preferred one (neutral ensemble + `ReCom` proposal).

4 Research procedure

Now that we have chosen both the compactness metric and the simulation procedure, we can refine the previous three-step procedure into something more specific:

1. Use the MCMC simulation algorithm to generate 10,000 districting plans for every state
2. Calculate spatial diversity and four compactness scores (Polsby-Popper, Reock, Convex Hull, and Human Compactness) for each districting plan
3. Perform data analysis (OLS regressions, difference-in-means test) and analyse the results

I now describe each step in detail.

4.1 Generating 100,000 districting plans with the MCMC algorithm

I download Census Tract data from the United States Census Bureau website. I use Census Tracts rather than Census Blocks because Census Tracts are the smallest (highest-resolution) units that have spatial diversity data.

I use the open-source software library GerryChain to generate the ensembles. Replication code and data are included in my GitHub repository. I obtain the ReCom Markov chain procedure from one of the co-authors (Daryl Deford) of the DeFord et al. [2019b] paper. I then fed the Census Tract data into the

GerryChain library. Using the Recom Markov chain procedure, I generated 10,000 districting plans for 10 states (Connecticut, Georgia, Idaho, Louisiana, Maine, Maryland, New Hampshire, Rhode Island, Utah, and Wisconsin) for a total of 100,000 plans.

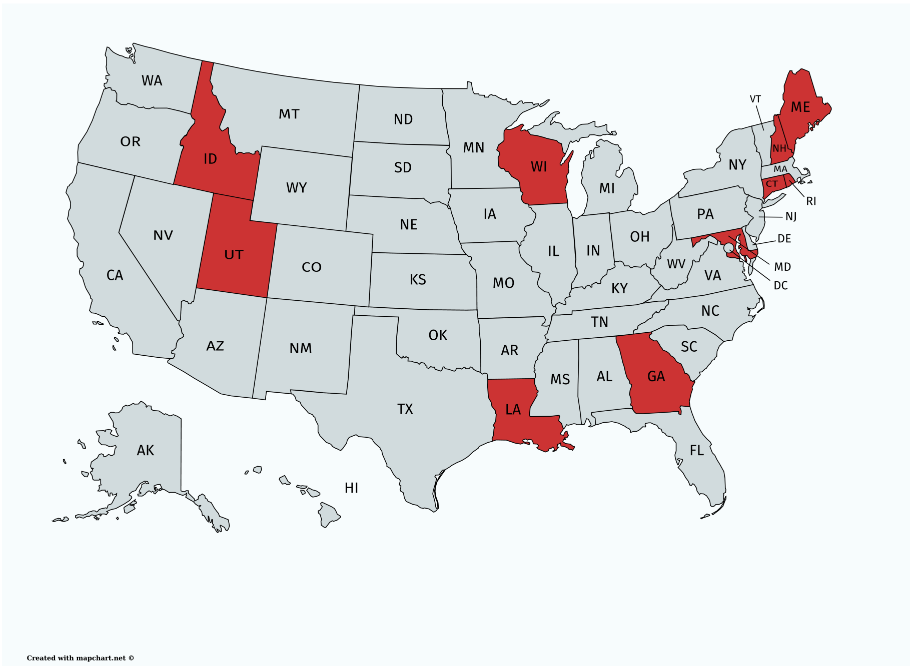

Figure 13: States I analysed, marked in red

Figure 13 marks the states I analysed in red. I chose these states mainly due to size considerations. All of these states are small-to-medium sized (in terms of the number of Congressional districts): the largest states like California, Texas and Florida are absent. This is because my algorithm scales in both time and memory with the *square* of the size of the state ($O(n^2)$). The analysis is achievable with larger desktop machines. Unfortunately, my own laptop had only 8GB of RAM and not very much free disk space, making it infeasible to examine larger states. Nonetheless, I was still able to analyse medium-sized states like Louisiana, Maryland and Georgia (14 districts).

Size aside, I tried to get states that spanned the entire country, including Western states (Idaho, Utah), Southern states (Louisiana, Georgia), and Northeastern states (Maine, Rhode Island, New Hampshire, Connecticut). I would also have liked to generate plans from a Pacific state like Oregon and a Midwestern state like Kansas, but time constraints prevented me from doing so.

Nonetheless, the number of states that I analyse exceeds most other similar analyses. For instance, the seminal and heavily-cited work Chen et al. [2013]

only analyse the state of Florida, and even very recent work by DeFord et al. [2019b] and Schutzman [2020] analyse only five and two states respectively.

4.2 Calculating spatial diversity and compactness scores for 100,000 plans

After generating the plans, I calculate spatial diversity and compactness scores for all of the plans. As mentioned, spatial diversity is an operationalisation of district homogeneity: the higher spatial diversity is, the less homogeneous (more heterogeneous) the district is. I obtain data on spatial diversity from Professor Nicholas Stephanopoulos. The dataset he gave me has eight *factor scores* for each Census Tract in the country, where a factor score is a combined variable that covers vital areas like race, education, profession, marital status, and housing. A district's spatial diversity score is calculated by the sum of the standard deviation of each factor score, normalised by the proportion of the variance each factor score explains. As an example, consider a district made up of three Census Tracts (A, B, C), and let each Tract have three factor scores (1, 2, 3). Let the proportion of the variance explained by each factor score be 50%, 30% and 20% respectively. Then the total spatial diversity score would be:

$$\sigma(A_1, B_1, C_1) \times 0.5 + \sigma(A_2, B_2, C_2) \times 0.3 + \sigma(A_3, B_3, C_3) \times 0.2$$

I calculate spatial diversity score for every district, and, following Stephanopoulos, take the arithmetic mean of all districts in a districting plan to get the overall spatial diversity score for that plan.

Next, I calculate compactness scores. As the Polsby-Popper metric is so well-known and widely used, there was already an existing implementation in the GerryChain library which I made use of. Similarly, existing libraries like SciPy already had a Convex Hull method. Finally, I wrote my own implementation of Reock, making use of the Smallest Enclosing Circle code written by Project Nayuki [Project Nayuki, 2020].

In order to calculate human compactness scores, I have to know where voters live (to calculate driving durations between them). I therefore obtain a dataset of "voter representative points" (VRPs) from Eubank and Rodden [2019]. These points aggregate many actual voters, downsampling the data into a size that can be worked with. While this down-sampling and placements of points randomly does introduce some noise, "the variability contributed... is empirically very small" [Eubank and Rodden, 2019]. I sample 1,000 VRPs for each Congressional District in a state. That means that a state like Maine with two districts will have 2,000 VRPs, and a state like Louisiana—with seven districts before the new redistricting plan—will have 7,000.

I then calculate all pairwise driving durations between all VRPs using an open-source routing engine called Open Source Routing Machine (OSRM) built by

Luxen and Vetter [2011]. The routing engine is able to calculate driving durations between any two points—very similar to Google Maps—but the number of queries it can process is orders of magnitude larger than the limits imposed by the Google Maps API. For these ten states, I calculate about 400 million point-to-point driving durations in total. As point of comparison, using Google Map's Distance Matrix API for that number of requests would cost $1,480,000[6]. And if I had tried to analyse California (with 53 Congressional districts), this would require almost 3 billion point-to-point driving durations.

Because my analysis is on the tract level, I map VRPs to Census Tracts using a spatial join. I sum the pairwise point-to-point distances to get a matrix of pairwise *tract-to-tract* driving durations. I then sum the driving durations from each point in the district to another and calculate the human compactness score for each district.

Finally, I aggregate the individual district scores into a plan-level score by simply taking the arithmetic mean. For instance, if a districting plan has three districts with Polsby-Popper scores of 0.25, 0.5, and 1, the Polsby-Popper score for that plan would be $(0.25 + 0.5 + 1)/3 = 0.5833$. As a robustness check, I also use the sum of square roots as an aggregation function: that is, $\sqrt{0.25} + \sqrt{0.5} + \sqrt{1} = 0.736$[7], obtaining qualitatively similar results.

4.3 Performing data analysis on the 100,000 plans

After calculating the overall spatial diversity and compactness scores on all the plans, I start running exploratory data analysis and statistical tests. The results are detailed below.

5 Results

[TODO] take andys comments

My key results are as follows:

1. Political geography largely pins down the spatial diversity of each individual district[8].
2. Different compactness measures have different ideas of what "good" plans look like.
3. Different compactness measures are correlated with one another[9].

[6]Volume discounts do exist, but you have to contact the Sales Team, and I doubt I could afford it anyway...

[7]This penalises districting plans that have a large difference between districts e.g. one very good district and one very bad one.

[8]Small urban districts have high SD, large rural ones have low SD.

[9]The geometric compactness measures agree most with one another, the human compactness measure not as much.

4. Only human compactness is negatively correlated with spatial diversity: geometric/dispersion-based measures have either no or a positive (bad) effect on spatial diversity[10].

Overall, the evidence suggests that optimising over compactness will give you less spatially diverse districts, and human compactness will do the best job of it.

5.1 Initial analysis

Before proceeding to more quantitative statistical tests, I want to show what the generated plans look like and what the *distribution* of those plans looks like.

5.1.1 The best and worst plans according to different compactness measures

After having obtained all the plans and their corresponding scores, I plot the plans with the best and worst spatial diversity and compactness scores to get an understanding for the types of plans that each metric encourages. This will give us valuable intuition for understanding the subsequent results.

For ease of exposition I show states with only two districts, but the analysis extends to states with any number of districts. (Plots of the other eight states are available in my GitHub repository). I also use Polsby-Popper to represent the other two dispersion-based compactness metrics as my explanations are similarly applicable to those metrics.

Figure 14 plots the best and worst plans according to several metrics. Let us begin with the middle row (Polsby-Popper), as its interpretation is the most straightforward. The Polsby-Popper (and other dispersion-based) metric penalises districts that are very "snakelike" and prefers districts that have regular shapes like squares or circles. This is clearly reflected in the plot. The best plan has a district with a very regular shape, and the worst plan has a snakelike district that contorts through half the state.

On the top row is human compactness. A good plan under human compactness minimises the total travel times between every member of the district. This encourages small, compact districts that avoid splitting urban centers.

We can see that the top plan under human compactness corresponds well to the actual population density of New Hampshire as seen in Figure 15. The top plan puts the two most populous and urban counties in New Hampshire—Rockingham and Hillsborough—together in the same district. The worst plan under human

[10]OLS regressions with state dummies show that only human compactness has a significantly negative coefficient on spatial diversity. Difference-in-means tests show that only the most compact plans under human compactness are less spatially diverse than average, and are less spatially diverse than the most compact plans under geometric/dispersion-based measures.

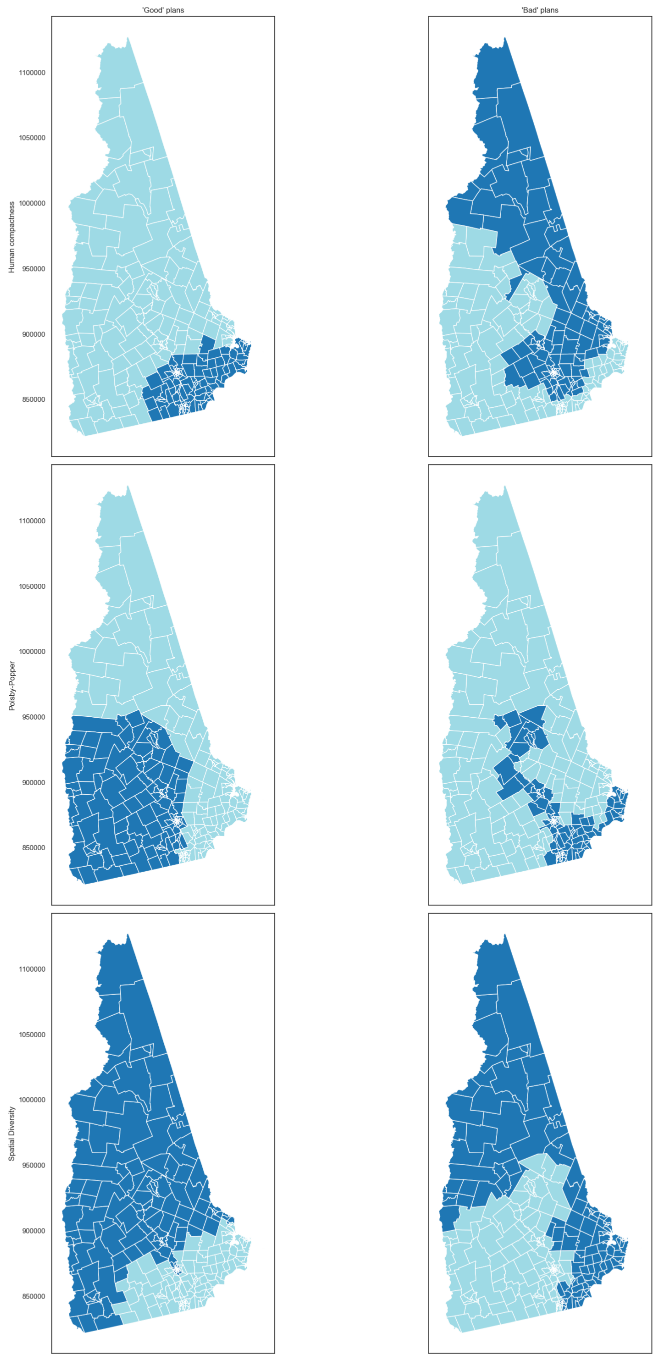

Figure 14: Best and worst districting plans of New Hampshire under different metrics

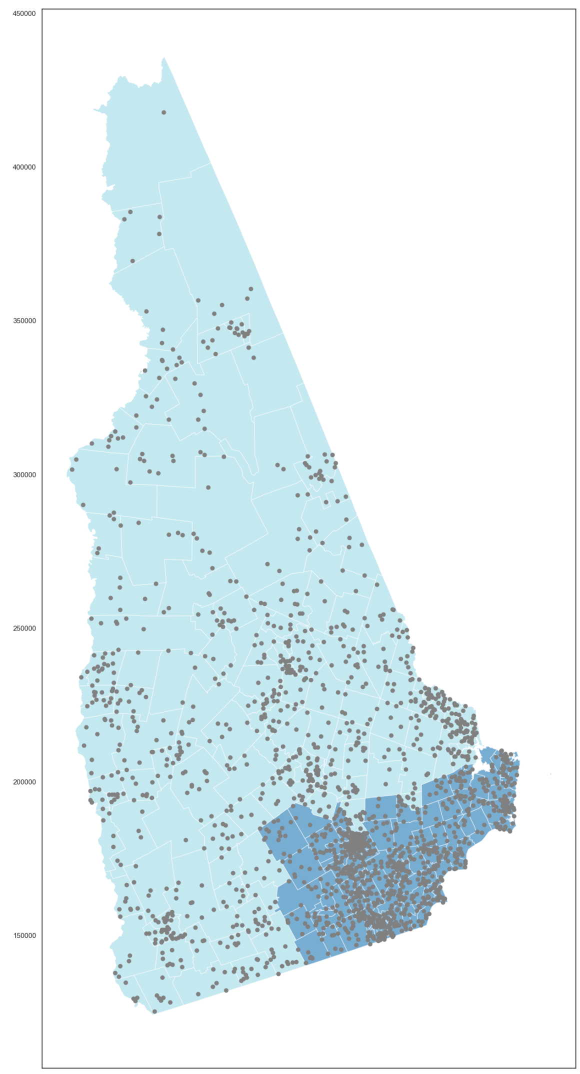

Figure 15: Population density plot of New Hampshire. Each dot represents roughly 600 people.

compactness splits the counties in such a way that one's co-districtors are far away, and one's nearest neighbours are in a separate district.

As expected, the top plan under spatial diversity (bottom row) closely resembles the top plan under human compactness. In relatively homogeneous New Hampshire, the main source of spatial diversity is the urban-rural divide. A plan that keeps urbanites together in one district is favoured under spatial diversity.

And while the worst plan under spatial diversity looks different from that under human compactness at first glance, they are actually quite similar. Both plans split up the two populous urban counties, having a "fish-hook" shaped district that starts from the rural north of the state and swoops down to the south to carve out a large part of the counties.

This case study shows that dispersion-based measures may not always reflect existing communities of interest. This seems to fuel criticism of dispersion-based measures on exactly that basis ("it makes no sense to combine areas that have nothing in common except that they fit neatly into a square" [Wolf, 2015]). In this example, human compactness and spatial diversity agree neatly on what the best districting plans should look like.

While human compactness generally tracks spatial diversity better than other compactness metrics (I provide evidence for this later), it does not always do so. Figure 16 gives the population of Idaho. We can see that a large proportion of the population is concentrated in a U-shaped "belt" spanning the southern half of the state. A good plan under spatial diversity will attempt to put this relatively urban "belt" in the same district, and this is indeed what we observe in Figure 17. But due to its great distance and jagged perimeter, such a plan is penalised under both human compactness and dispersion-based measures, both of which prefer a relatively compact square-shaped district.

As we can see, compactness measures need not always agree with spatial diversity, particularly in the case study of Idaho. Intuitively, this seems to make sense: spatial diversity tries to put similar people together, and people who live in the same area are often, but not always, similar.

5.1.2 How district homogeneity varies within districts and states (really needs work!)

In this section, I show that spatial diversity varies enormously between districts, but this is to a large extent dependent on the state's political geography. I find that small urban districts have high spatial diversity, while large rural ones have low spatial diversity—regardless of districting plan. This also extends to the level of the state: while the spatial diversity of districting plans can range from 0.4 to 0.9, the spatial diversity of a state's districting plan usually lies within a small range of ~0.05.

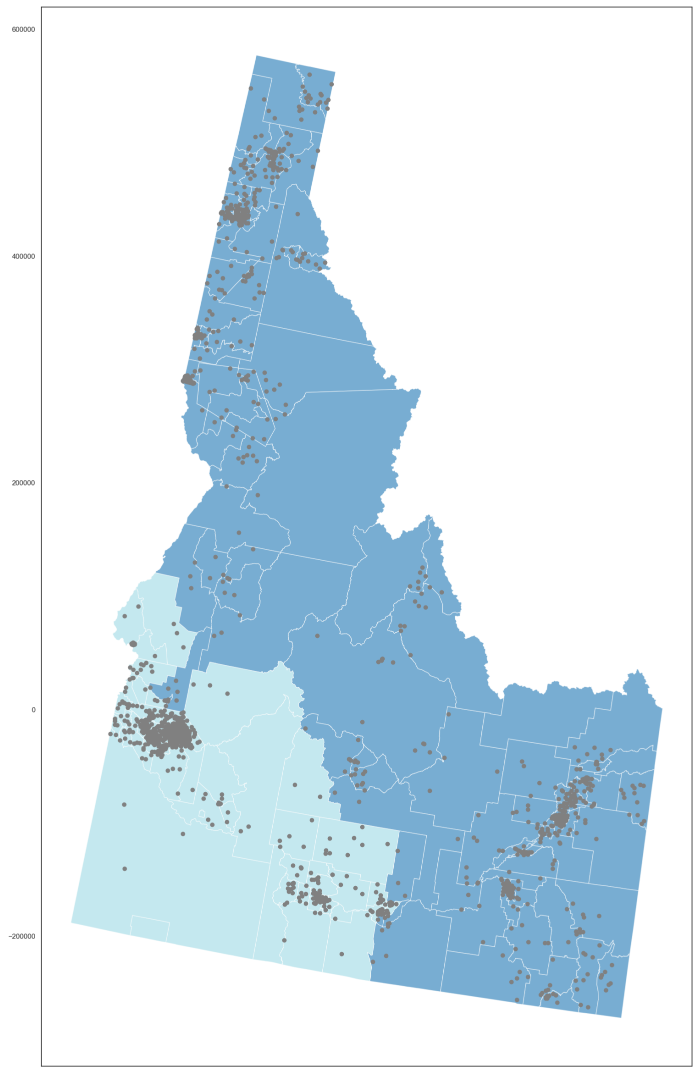

Figure 16: Population density plot of Idaho. Each point represents ~700 people.

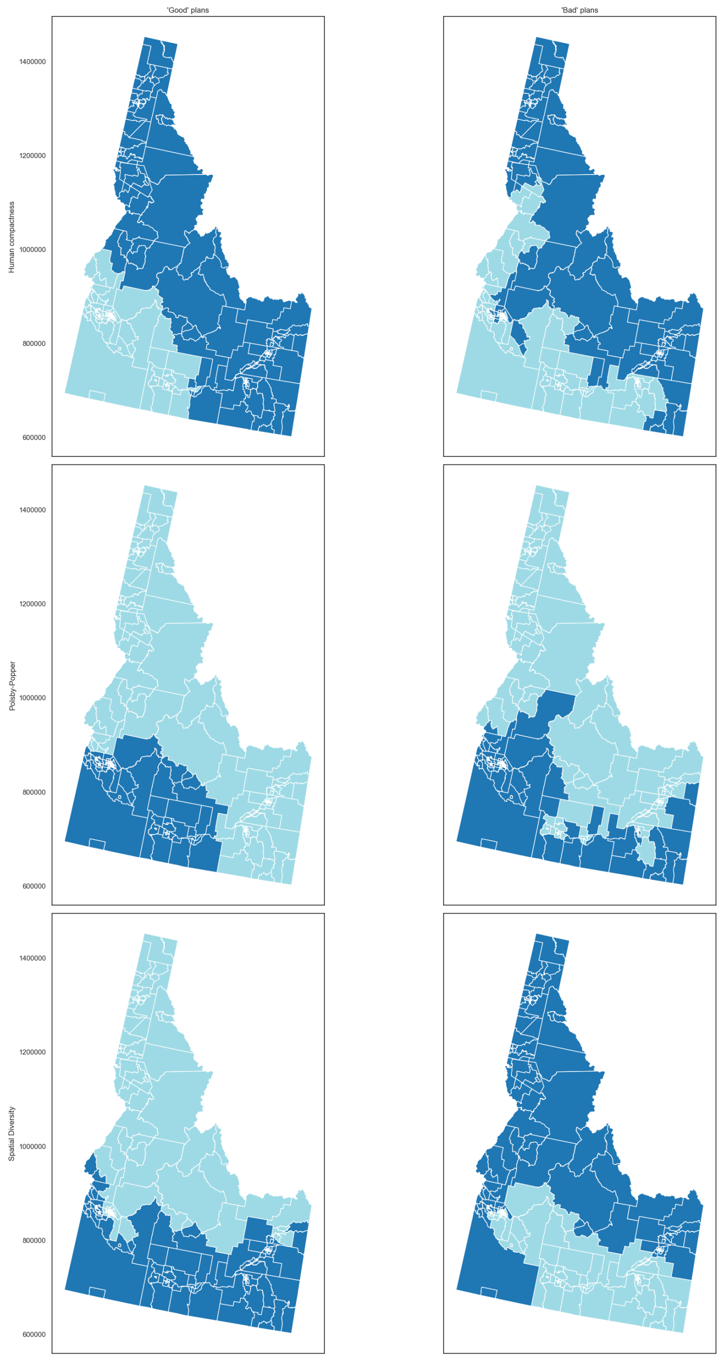

Figure 17: Best and worst districting plans of Idaho under different metrics

Figure 18 is a kernel density estimation (KDE) plot of the distribution of spatial diversity in all districts. As in Stephanopoulos's results, the distribution appears log-normal, with a noticeable tail on the right that contains a number of especially heterogeneous districts.

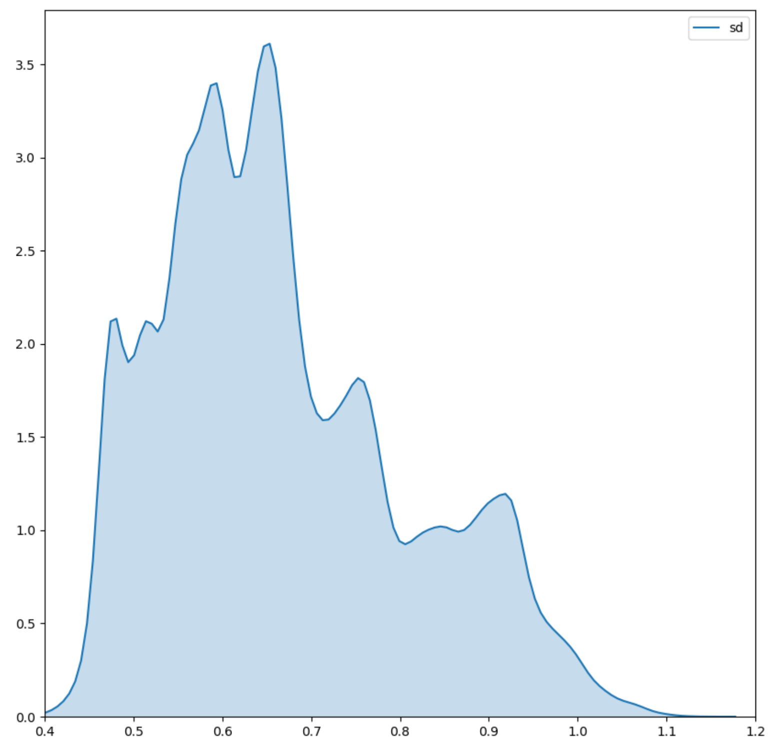

Figure 18: Spatial diversity of all districts

A tempting conclusion to draw from the data is that these districts are equally distributed over the different states. In reality, though, the districts of a state can only take on a small range of values no matter how a districting plan is drawn. Figure 19 demonstrates. The peaks imply a multimodal distribution where individual districts are clustered around certain values and not others. This is most starkly displayed in the states with only two districts. Despite the fact that the redistricting algorithm is continuous, there is a sharp bimodal distribution present in the states of Idaho and Maine, and to a lesser degree Utah and New Hampshire.

This finding is somewhat surprising. It implies that even though the MCMC algorithm explores the entire set of feasible districting plans, any district in any feasible plan will take on a specific form. In other words—no matter how one draws the plan, each district's spatial diversity is largely pinned down by its state's political geography. Some states have very spatially diverse districts, some states have very homogeneous ones, and this is a function of their geography and not the way the districts are drawn.

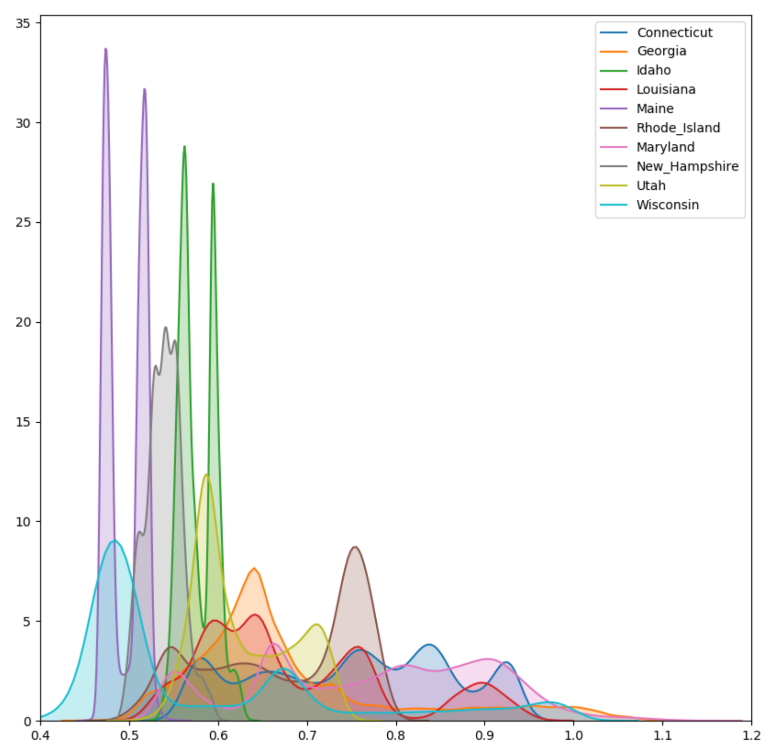

Figure 19: Spatial diversity of districts binned by state

Given that each district's spatial diversity is largely exogenous, we should expect each state's overall spatial diversity not to vary much as well. Indeed, we see in Figure 20 that each state occupies a narrow band in the range of possible spatial diversity scores. While the range of spatial diversity scores ranges from 0.50 to 0.80, the range of a state's spatial diversity score is only 0.05. While this range is small, it is not insignificant. Figure 3 shows that an increase in a state's spatial diversity by 0.05 is correlated with a decrease in electoral responsiveness by 0.3, about 10% of the variance.

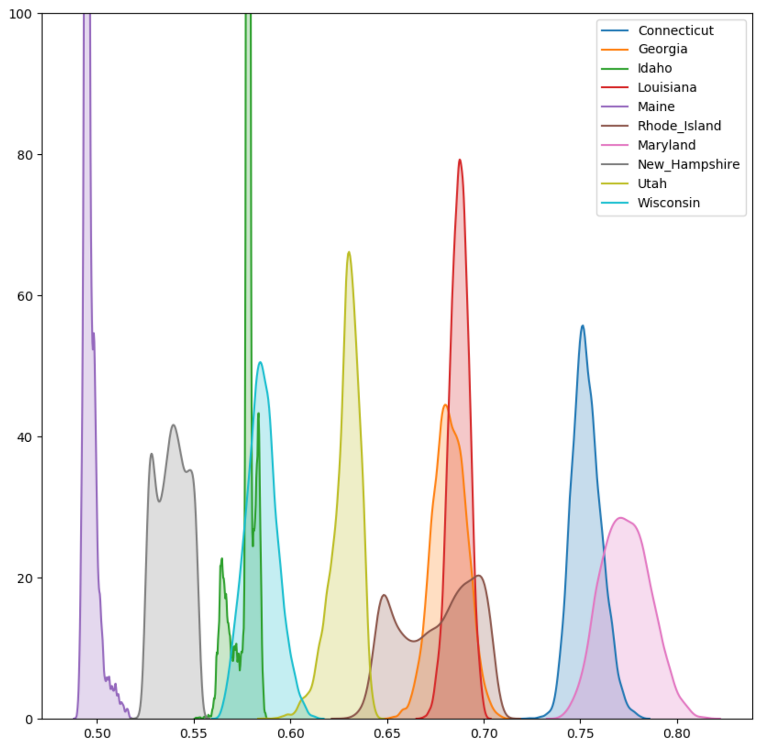

Figure 20: Overall spatial diversity of districting plans by state

5.1.3 Districts that are small and urban are usually more spatially diverse

One finding consistent across all states is that the smaller (by area) the district, the higher the spatial diversity.

Maryland provides the clearest example, although the same pattern repeats in all other states. Figure 21 is a correlation- and KDE plot of different metrics, binned by the area of each district. The most important figure is the KDE plot in the top left-hand corner. We can see that large districts (in blue) occupy the

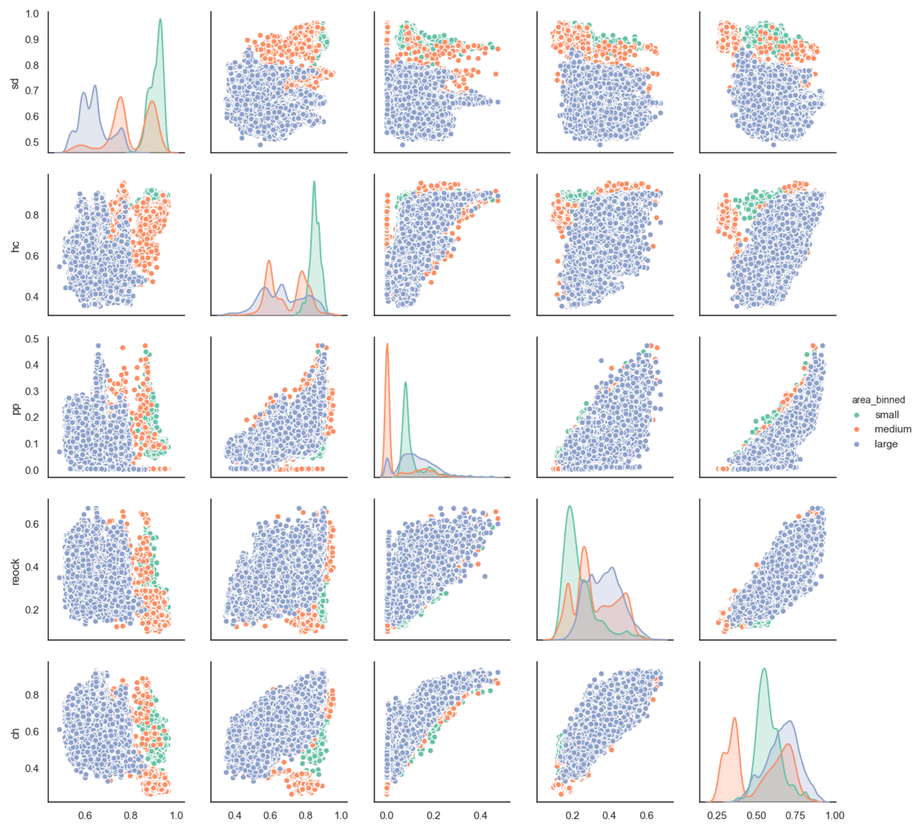

Figure 21: Pairwise plot of Maryland's districts: urban districts have highest SD

low end of the spatial diversity range. Medium-sized districts (orange) have a bimodal distribution, but it is the smallest districts (in green) that have the highest spatial diversity.

This finding is quite intuitive. Cities tend to be the most heterogeneous parts of a state, with people of different races, ages, and socioeconomic classes. This finding suggests that more urban states will simply have larger spatial diversity scores—another factor pinning down the spatial diversity of a state's districting plans.

5.1.4 Conclusions of initial data exploration

We have seen that the overall distribution of districts and plans lie within a tight bound, largely determined by each state's political geography. This suggests that while districting can exert an effect on political outcomes, we should not expect optimising for compactness to change spatial diversity very much.

5.2 Comparing different compactness measures

5.2.1 Compactness measures largely agree with one another, but human compactness less so

The next key finding is that compactness measures largely agree with one another, meaning that a proposed plan that scores highly on one compactness metric will likely score highly on another. The correlations are strongest between the three geometric compactness measures, and lower (but still significantly positive) between the geometric and human compactness measures.

This finding is somewhat intuitive—we would expect the different geometric compactness measures to track each other very closely as they are measuring very similar concepts. It is much less obvious, however, that a purely geometric measure would agree with a metric that measures driving durations between points. This result is encouraging because it shows that these metrics are able to get at the same concept despite having completely different theoretical backgrounds.

One way to find the relationship between compactness measures would be to aggregate all the observations from each state into a pooled data set, and calculate the pairwise correlations between all such observations. However, looking at these aggregate results in this way can be highly misleading, as a single outlier state can bias the results. I therefore look at the correlations for each individual state instead.

One way to visualise these correlations is through the use of a heatmap. Figure 22 plots the correlation coefficients between each pair of metrics. Firstly, we can see the correlation coefficients between spatial diversity (sd) and the compactness metrics. Here, it seems like human compactness has a significant negative correlation with spatial diversity, with the other compactness metrics having little correlation. We can also see that the correlation between human compactness and geometric compactness measures are somewhat lower (~0.46) than the correlations between different geometric measures.

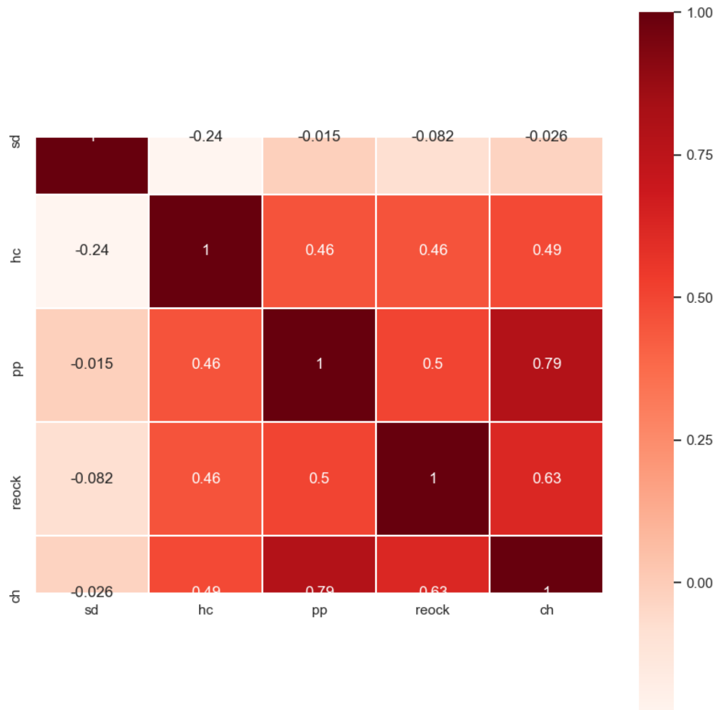

Figure 22: Correlation heatmap of Connecticut

The correlation heatmap of Utah shows a case where human compactness and the other geometric measures disagree. Here, the correlation between geometric compactness measures is very high (0.89—almost 1), but there is in fact a negative correlation between human compactness and the geometric measures.

These results vindicate my choice to use an ensemble of compactness metrics rather than relying on a single measure. While the correlation between metrics is high, it is not perfect, and indeed we observe cases like Utah where the compactness measures disagree.

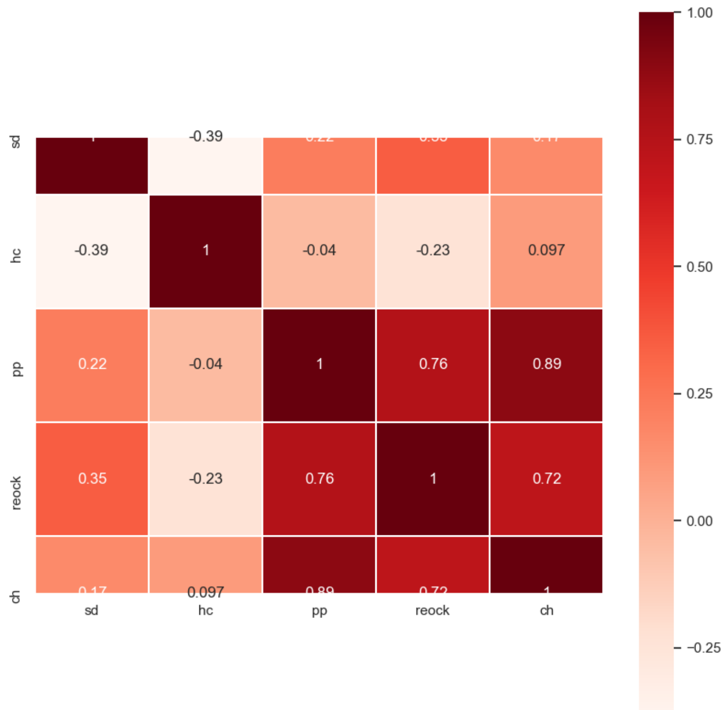

Figure 23: Correlation heatmap of Utah

Another way to visualise the findings is through pairwise scatterplots. Figure 24 is a correlation plot between spatial diversity and the various compactness metrics for the state of Georgia. These plots have the advantage of being able to visualise the scatterplots, which can surface non-linear relationships that a simple correlation coefficient cannot. In all of the states, however, the relationship between compactness metrics is always linear.

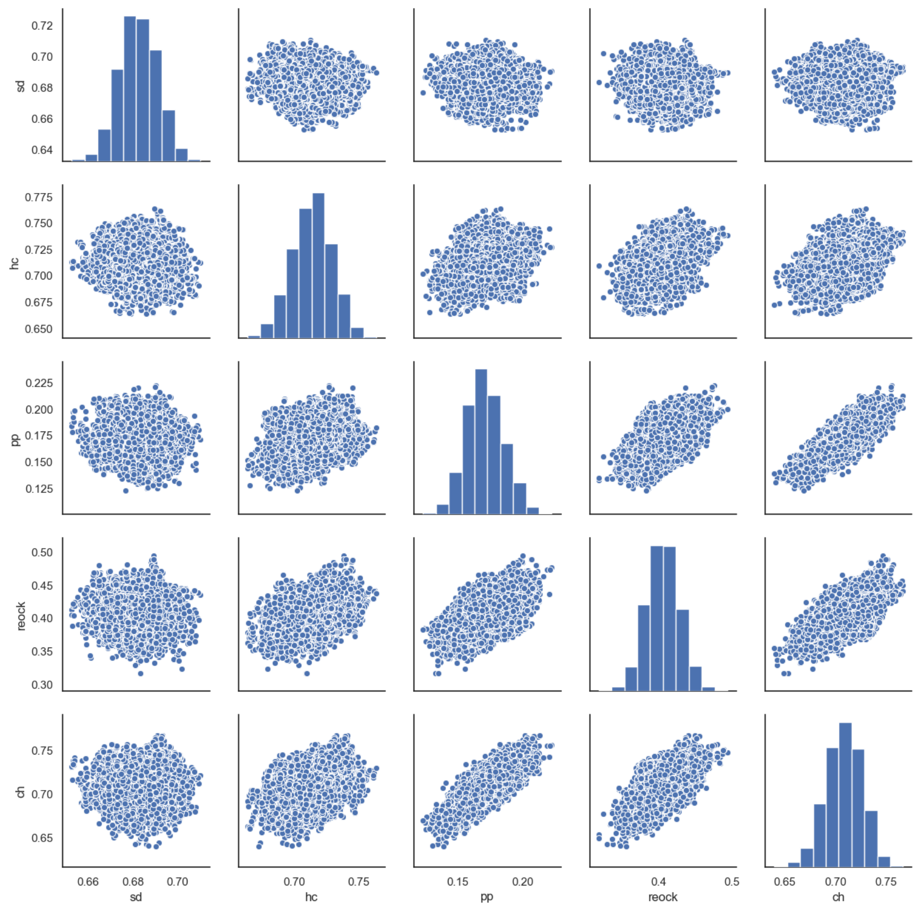

Figure 24: Correlation plot of Georgia

I have included correlation matrices and pairwise scatterplots for all ten states in the Supplementary Information. The overall correlation is positive for most states and most metrics, with human compactness being less correlated with the other metrics.

5.2.2 Only human compactness positively correlates with district homogeneity

Next, I run multivariate OLS regressions with country dummies and difference-in-means tests, and find no significant effects of geometric compactness on spatial diversity. I find that human compactness has a significant negative effect on spatial diversity: increasing human compactness from 0 to 1 decreases spatial diversity by 0.04 points.

We cannot simply run a regression aggregating every single district as each state has a unique distribution of spatial diversity and compactness. Consider the following. Within each state, increasing compactness decreases spatial diversity. But on the aggregate, states with high spatial diversity also have low compactness. In this case, regressing spatial diversity on the aggregate level would give an inflated estimate of the actual effect, falling afoul of the *ecological fallacy*. I illustrate this in figures 25 and 26. In Figure 25, I plot a graph of human compactness on the x-axis and spatial diversity on the y-axis. The overall trend seems to be slightly negative: in most of the groups, there is a slight negative correlation between human compactness and spatial diversity. However, we would obtain erroneous results if we aggregated the different states and ran a singular regression. This is depicted in Figure 26: due to the *between-group* correlation of compactness and spatial diversity, the estimate of the effect is biased. We must therefore control for state when running the regression. Thus, I run a multivariate regression with the functional form

$$SpatialDiversity = \beta_0 + \beta_1 Compactness + \beta_2 State$$

where *State* is a dummy variable, taking care to avoid the dummy variable trap.

Table 1 shows the results for human compactness. I run the same regression for each compactness metric and obtain the following:

```
HC: -0.0404, t-value -40.632
PP: +0.0251, t-value 29.841
Reock: +0.0209, t-value 27.645
CHull: -0.0016, t-value -1.801
```

I find that only human compactness has a statistically significant negative coefficient on spatial diversity, while Polsby-Popper and Reock have a significant positive effect on spatial diversity. This initial result suggests two things: firstly, and rather disappointingly, that optimising over the two most popular compactness measures may have adverse effects on electoral competitiveness and responsiveness. More encouragingly, though, these effects can be mitigated by the judicious choice of compactness measure. The results show that optimising over Convex Hull does not come at the cost of diversity, and that increasing human compactness actually decreases spatial diversity.

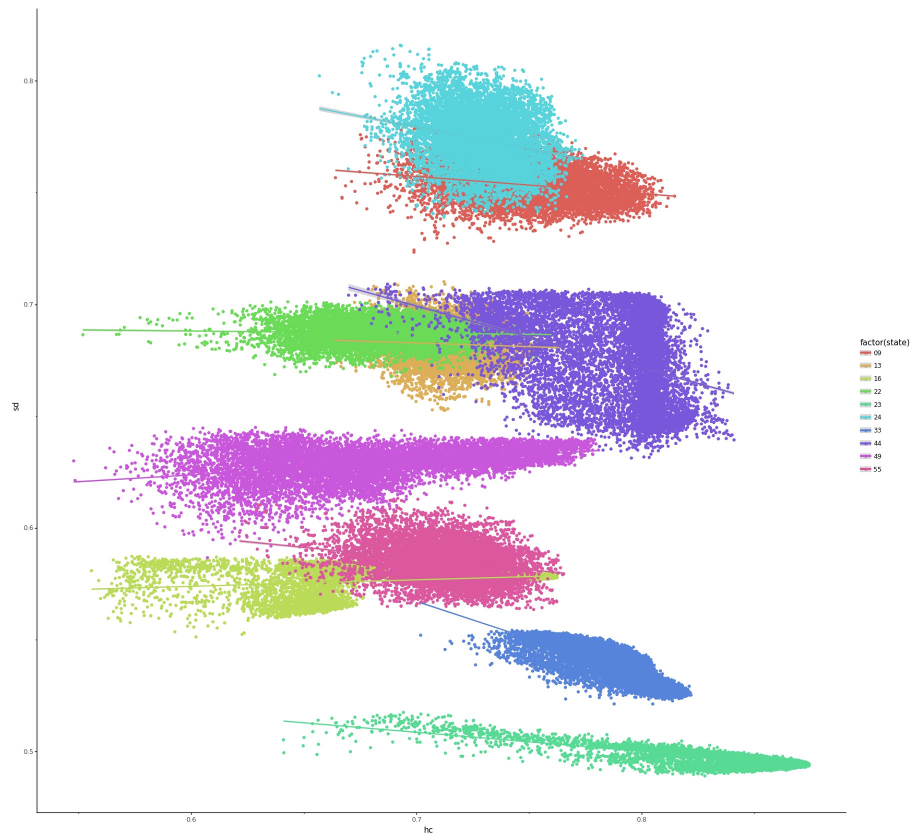

Figure 25: The individual-level regressions show a weak negative correlation between human compactness and spatial diversity

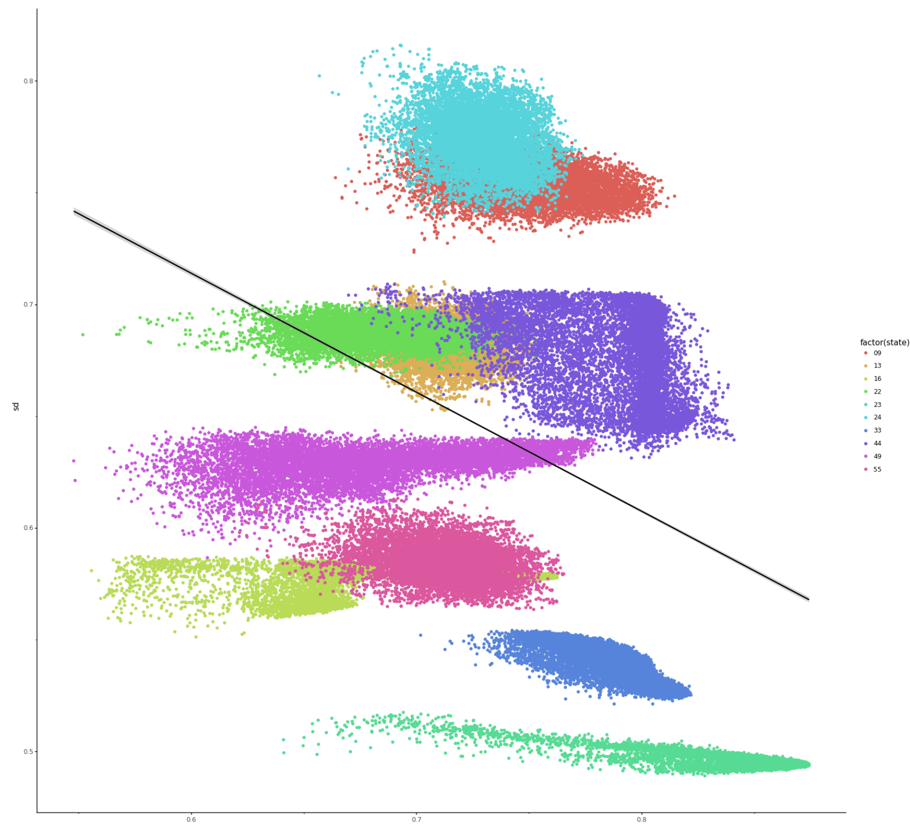

Figure 26: Aggregating the individual states gives an inflated estimate of the effect of compactness and commits the ecological fallacy

Table 1: OLS Regression of Spatial Diversity on Human Compactness with Country Dummies

Dep. Variable:	sd	**R-squared:**	0.988
Model:	OLS	**Adj. R-squared:**	0.988
Method:	Least Squares	**F-statistic:**	8.188e+05
Date:	Wed, 11 Mar 2020	**Prob (F-statistic):**	0.00
Time:	20:23:45	**Log-Likelihood:**	3.2365e+05
No. Observations:	100000	**AIC:**	-6.473e+05
Df Residuals:	99989	**BIC:**	-6.472e+05
Df Model:	10		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
C(state)[09]	0.7837	0.001	1042.069	0.000	0.782	0.785
C(state)[13]	0.7111	0.001	993.725	0.000	0.710	0.713
C(state)[16]	0.6054	0.001	856.490	0.000	0.604	0.607
C(state)[22]	0.7149	0.001	1039.373	0.000	0.714	0.716
C(state)[23]	0.5303	0.001	626.929	0.000	0.529	0.532
C(state)[24]	0.8030	0.001	1097.735	0.000	0.802	0.804
C(state)[33]	0.5705	0.001	725.232	0.000	0.569	0.572
C(state)[44]	0.7073	0.001	899.177	0.000	0.706	0.709
C(state)[49]	0.6561	0.001	959.927	0.000	0.655	0.657
C(state)[55]	0.6138	0.001	858.803	0.000	0.612	0.615
hc	-0.0404	0.001	-40.632	0.000	-0.042	-0.038

Omnibus:	3979.140	**Durbin-Watson:**	1.171
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	9332.569
Skew:	-0.236	**Prob(JB):**	0.00
Kurtosis:	4.420	**Cond. No.**	67.9

5.2.3 Only under human compactness are top plans more homogeneous than average

While the results of the overall regression are discouraging, it may not be the last word. The neutral ensemble approach means that the generated plans run the whole gamut of compactness scores, including both highly compact plans and highly noncompact ones in the sample of 100,000. In reality, though, legislators will try to optimise for compactness to some degree. A plan proposed in real life—while not being optimally compact—would be reasonably so. Rather than regressing over the entire sample, then, we should specifically check the spatial diversity of plans which exceed the threshold of "reasonable compactness".

But what is the threshold of "reasonable compactness"? The choice of the threshold cannot be determined *a priori*. One would have to know the distribution of compactness in a sample of plans generated in real life. Of course, as real-life expert districtors do not produce a distribution of plans, this is also a tall order. I therefore run the same OLS regression for different thresholds of "reasonable compactness", ranging from the top 90% (excluding the bottom 10%) of plans to the top 10% of plans.[11] The results are as follows:

10th percentile:

	coef	std err	t	P>|t|
hc	-0.0256	0.001	-19.660	0.000
pp	0.0339	0.001	34.757	0.000
reock	0.0287	0.001	34.723	0.000
ch	-0.0001	0.001	-0.143	0.886

25th percentile:

	coef	std err	t	P>|t|
hc	-0.0274	0.002	-16.018	0.000
pp	0.0357	0.001	30.861	0.000
reock	0.0307	0.001	33.635	0.000
ch	-0.0024	0.001	-2.153	0.031

50th percentile:

	coef	std err	t	P>|t|
hc	-0.0514	0.003	-15.722	0.000
pp	0.0154	0.001	10.269	0.000
reock	0.0267	0.001	24.400	0.000
ch	-0.0119	0.001	-8.627	0.000

75th percentile:

	coef	std err	t	P>|t|
hc	-0.1154	0.007	-17.023	0.000
pp	-0.0338	0.002	-14.814	0.000
reock	0.0185	0.002	12.136	0.000
ch	-0.0589	0.002	-30.961	0.000

90th percentile:

	coef	std err	t	P>|t|
hc	-0.0396	0.013	-3.098	0.002
pp	0.0412	0.005	7.756	0.000
reock	0.0074	0.003	2.285	0.022
ch	-0.0274	0.004	-6.862	0.000

The results vary somewhat depending on our choice of threshold, but are on the whole remarkably consistent. The Reock measure performs poorly in all thresholds. The Polsby-Popper metric is not much better either. Only when the threshold is set to the top 25% of plans does the coefficient go below 0,

[11] The results are similar when we take the top 5% or 2% of plans, but the small sample sizes of those thresholds mean that it is difficult to get statistical significance.

and the effect reverses when we look at the top 10% of plans. I am inclined to believe that is an outlier. The Convex Hull metric is the best of the dispersion-based metrics. It consistently has a negative coefficient, although the negative coefficients are very small—particularly when the threshold is low. Finally, the human compactness metric performs well on all subsamples. The coefficient on human compactness is larger than all the other metrics on all the thresholds—a strong indication that it is the metric that best minimises spatial diversity.

5.2.4 The most humanly-compact plans are significantly more homogeneous than the most geometrically-compact plans

The OLS regressions we run give the relationship between compactness and spatial diversity. But perhaps one is not concerned about the marginal effect of compactness on diversity. One might ask a more basic question: if we mandate that plans are "reasonably compact"—whatever that means—and force legislators to propose only plans that cross a threshold of reasonable compactness, will that adversely affect spatial diversity?

If there is indeed a fundamental trade-off between compactness and spatial diversity, then we should observe the average spatial diversity of highly compact plans to be higher than the spatial diversity across all plans. I therefore compare the mean spatial diversity of top 500 plans under each compactness metric to the mean spatial diversity of all plans. As a robustness check, I look at different proportions (top 10%/5%/2%) and obtain almost-identical results. The results are as follows:

```
Mean SD of plans with highest Human Compactness scores: 0.635558
Mean SD of plans with highest Polsby-Popper scores: 0.640954
Mean SD of plans with highest Reock scores: 0.639897
Mean SD of plans with highest Convex Hull scores: 0.639985
Mean SD of all plans: 0.639758
```

Encouragingly, there seems to be no trade-off between compactness and spatial diversity: the mean spatial diversity in top compactness plans is not higher than the overall mean spatial diversity. But only human compactness has a mean spatial diversity *significantly lower* than the mean spatial diversity of all plans. In order to check the significance of this result, I run a differences-in-means test using Welch's t-test. I use Welch's t-test as Student's t-test relies on a homogeneity in variances assumption. When the assumption of equal variances is not met, Student's t-test yields unreliable results, while Welch's t-test controls Type 1 error rates as expected [Delacre et al., 2017]. In this case, since the top plans come from different distributions, it is unlikely that the variances are homogeneous. The results are as follows:

```
Welch's t-tests for the top 5% of plans

HC vs All: statistic=[-3.36526759]), pvalue=[0.00076992]
```

```
Reock vs All: statistic=[0.97597048]), pvalue=[0.32912173]
PP vs All: statistic=[0.11228718]), pvalue=[0.91059979]
CHull vs All: statistic=[0.18211076]), pvalue=[0.85550249]
```

Only human compactness had a statistically significant difference in mean spatial diversity. For completeness, I also ran pairwise differences-in-means tests between all four metrics, for a total of 6 tests. The results are as follows:

```
Welch's t-tests for the top 5% of plans (significant results only)

HC vs PP: statistic=[-3.16361084]), pvalue=[0.00156292]
HC vs Reock: statistic=([-2.53127357]), pvalue=[0.01138011]
HC vs CHull: statistic=[-2.57101923]), pvalue=array([0.0101543]))
```

As expected, there were no significant differences in means between any of the geometric compactness metrics, but there was a significant difference in the means between human compactness and the other compactness metrics. Similar results obtain when I rerun the tests for the top 10% and top 2% of plans under each compactness metric. The results show that the top plans under human compactness have significantly lower spatial diversity than the top plans under other compactness metrics.

While this analysis is suggestive, there are two rejoinders to this. Firstly, one could argue that the difference in means is quite small: only 1.5% of the total variance in spatial diversity. Secondly, one might think that looking only at the aggregated results could be misleading. A difference in means in the aggregate could be due to one or a few outlier states driving the results.

To address these two criticisms, I run Welch's t-tests for each metric for all ten states (giving a total of 40 t-tests). The full list of t-tests is available in Appendix B. Once again, human compactness performs the best. The top plans under the Reock metric have statistically significant negative differences in spatial diversity means in 3 out of 10 states. Polsby-Popper and Convex Hull do a little better with 4 out of 10 states. Human Compactness has a whopping seven states. If we look at *meaningful* differences—not just statistically significant ones (instances where the mean is lower by more than 5% of the total variance)—then human compactness outperforms by a wide margin. Human compactness has a statistically significant and meaningfully lower spatial diversity in six of the states. Reock does in two states, and Convex Hull and Polsby-Popper only in one. Finally, in two cases (both under the human compactness metric), the difference is so meaningful that it makes up 25% and 35% of the total variance. Concretely, the spatial diversity of all 10,000 New Hampshire plans lie within a range of 0.03. The top 1,000 plans under human compactness have a spatial diversity that is 0.01 lower than the mean — a very meaningful effect that spans one-third of the total range. Far from being a small effect, it seems that the choice of compactness metric to optimise over can have very meaningful impacts.

What do the difference in means actually imply in terms of proposed plans? Table 2 shows what percentile the top 10 percent of plans under each metric would occupy in the distribution of 10,000 plans (lower is better). If there is no relationship between a compactness metric and spatial diversity, then we should expect the mean percentile to lie around 50 percent. If, however, the top plans under a metric are significantly less spatially diverse, then we should see a low percentile for many of the states. In the table, I have **bolded** the best-performing metric in each row, subject to it being less than the median (<50th percentile). As before, I run robustness checks and get qualitatively similar results for various threshold cut-offs.

Table 2: What percentile the top 10 percent of plans under each metric occupy (lower is better)

	hc	pp	reock	ch
0 Connecticut	**34.31**	54.02	55.61	48.25
1 Georgia	48.29	**44.24**	48.34	47.62
2 Idaho	59.92	48.62	**20.90**	26.88
3 Louisiana	**39.03**	39.12	42.45	41.24
4 Maine	26.22	92.48	78.12	**23.56**
5 Rhode Island	**23.32**	56.46	53.71	52.70
6 Maryland	36.99	**33.00**	**33.00**	48.68
7 New Hampshire	**8.25**	58.08	40.30	65.73
8 Utah	77.05	61.72	58.57	59.92
9 Wisconsin	**34.09**	42.14	47.26	43.07
Mean percentile	**38.75**	52.99	47.83	45.77

The table shows that the human compactness metric consistently outperforms the other metrics in many of the states, forestalling the criticism that the results may be driven by one or two outliers. While human compactness does particularly well in New Hampshire and Rhode Island, it still performs best overall even if we remove those two states from consideration.

6 Discussion

6.1 I found no consistent trade-off between compactness and communities of interest

Is there a fundamental trade-off between compactness and communities of interest, as proxied by district homogeneity? The answer seems to be: it depends on how you measure compactness. For geometric compactness measures, the results are equivocal: OLS regressions indicate that there is some trade-off between compactness and homogeneity, while difference-in-means tests indicate no such trade-off. Point-based distance metrics seem to fare better. In fact the results show that rather than a trade-off, there is a synergy between human compactness and district homogeneity.

It was certainly the right call to use many different compactness metrics, due to the frequency at which even very similar compactness measures disagree. The Maine entry in Table 2 is a good example. The top Polsby-Popper plans lie in the 92nd percentile of all plans—shockingly high—but looking at the Reock and Convex Hull measures paint a much less one-sided picture. In fact, it is surprising that the Reock and Convex Hull percentiles differ so radically, seeing as the measures differ only in the bounding shape (convex polygon versus a circle) of the district.

If we had used only the Polsby-Popper metric in our analyses, we would have (erroneously) concluded that Maine's political geography was fundamentally incompatible with compactness. This casts doubt upon work that uses only a singular compactness metric to score districting plans. Without wishing to single out any work in particular (many other papers do the same thing), Schutzman [2020] uses only the Polsby-Popper measure to analyse only two states. My data suggest that this analysis is insufficient—severely curtailing the generalisability of the work.

6.2 Human compactness seems to best encompass communities of interest

[TODO – reword to talk about communities of interest/homogeneity]

Does spatial diversity give us a good reason to choose one compactness metric over another? Yes. The data show that human compactness better tracks spatial diversity, which in turn correlates with democratic outcomes like participation, responsiveness and competitiveness. This finding consistently repeats itself throughout different analyses, different thresholds, and different aggregation functions. The implication is clear: if we believe Stephanopoulos's work on the benefits of lower spatial diversity, then adopting human compactness will give us better plans.

To be fair, there are many other considerations that go into choosing a compactness metric, and I have alluded to several in the previous sections. First is objectivity. Geometric compactness measures were invented in the first place—almost six decades ago—to measure and prosecute gerrymandering objectively: "[compactness] remains subjective in that no method of measurement has gained general acceptance" [Reock, 1961, p. 74].

But second—and possibly far more important—is explainability. Compactness metrics feature prominently in spheres outside academic political science, from general political discourse to amicus briefs for the Supreme Court. The seminal work by Reock almost sixty years ago says "the best use for the method of measuring compactness outlined here is *as a tool for the courts and as a weapon for public opinion*". It is thus incredibly important that a compactness metric be intuitive and explainable to laymen. This almost entirely rules out overly mathematical measures like Dube and Clark [2016] that use graph theory and minimise cut edges, or uninterpretable measures like Kaufman et al. [Forthcoming] that build a "black box" machine learning model.

While geometric compactness metrics are simple enough to explain, they lack a normative appeal. It is almost too easy to criticise geometric compactness metrics on the basis of irrelevance. If we ask: *why* should districts follow some regular shape? the answer is not immediately forthcoming, and in fact many have pointed out correctly that there is little reason to do so *eo ipso*.

Human compactness seems to meet both these criteria. It encapsulates the notion of "communities of interest", while sidestepping the problem of having to define, delineate and make subjective judgement calls on these communities. And while it's not obvious that districts should conform to some regular polygon, the idea of putting people who live together in the same voting district has a strong normative force with great intuitive appeal. Finally, the lower (but still substantially positive) correlation between human compactness and the other compactness measures suggests that human compactness qualitatively differs from geometric compactness.

6.3 Directions for future work

Future work should look at expanding the scope of the analysis in three ways: the number of states, the number of compactness measures, and the number of outcomes of interest.

My work analyses 10 out of the 50 states. Restricting analysis to a subset of states is common in other redistricting work, due to the onerous computational burdens of the procedure. DeFord et al. [2019a] measure the effect of competitiveness on partisanship for five states, and Schutzman [2020] looks at the trade-off between compactness and partisan symmetry for only two states. We know, however, that this has implications on external validity. While my analysis covers more states than much of the literature, further work should nonetheless extend the analysis to cover more states—especially large states like Texas, Florida and California. Future work should also analyse more compactness measures. Of particular interest would be other point-wise distance metrics like bizarreness, and Kaufman et al.'s (Forthcoming) metric that attempts to imitate human perception.

Finally, future work should analyse a variety of other outcomes of interest apart from spatial diversity. As the primary draw of point-based distance measures is that it should keep communities of people together in the same district, I would particularly like to see future work whether human compactness does a better job of keeping communities of interest together. We should also examine the effect of compactness on a wider range of normative outcomes—not just procedural ones. Districting affects many other things: political knowledge, turnout, and federal spending [Snyder Jr and Strömberg, 2010], but work so far has been focused almost entirely on electoral competitiveness.

6.4 Conclusion

[TODO]

6.5 Acknowledgements

- Big thanks to Bassel;
- Big thanks to Daryl Deford, for explaining MCMC, Gerrychain, and generating the districting plans;
- and Filip, for walking through with me all my ideas from May 2019 until now;
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- Zun Yuan, for letting me bounce optimisation ideas off him;
- Sergi, for being the tutor that got me interested in Politics;
- Am I allowed to thank Andy or name him as my supervisor? Ask Tak Huen about this

Images of Reock and PP metric taken from fisherzachary.github.io

7 Technical Appendix A: evaluating different methods of plan generation

The idea of drawing a large number of districting plans with a computer has a long and storied history, starting in the 60s and 70s. The approach has almost always been used to identify gerrymandering; for instance Cirincione et al. [2000] build an algorithm to "quantitatively [assess] whether the [1990 South Carolina] plan is a racial gerrymander". More recently, Chen et al. [2013] "generat[e] a large number of hypothetical alternative districting plans that are blind as to party and race, relying only on criteria of geographic contiguity and compactness." They do this using a Markov Chain simulation algorithm, a procedure that makes iterative changes for a large number of steps until a unique districting plan emerges. At each step of Cirincione et al.'s algorithm, they randomly select a Census Block Group to serve as a "seed" of the district, then randomly add its neighbouring block groups to it until a district with the desired population is formed. Similarly, Chen et al. begin by initialising all voting precincts as an individual, separate district, then randomly agglomerating neighbouring precincts until the desired number of districts is reached.

While this standard iterative algorithm enjoys a certain degree of success, it has one crippling weakness. The way in which this class of algorithms operates necessarily explores only a tiny subset of all possible districting plans. Subsequent work pointed out this flaw: Magleby and Mosesson wrote that automated processes "may take a biased sample of all possible legislative maps… and fail to efficiently produce a meaningful distribution of all alternative maps". And Fifield et al. contend that "[standard Monte Carlo algorithms] are unlikely to yield a representative sample of redistricting plans for a target population." [12] This poses a huge issue for the validity of any statistical analysis, because any correlation that we discover on a biased subset of plans may be spurious when measured over the actual distribution of plans. [13]

7.0.0.1 Markov Chain algorithms

Thankfully, scholars have developed an improvement over the standard algorithm with stronger theoretical guarantees. This second class of algorithms reframe the districting problem as a *graph partition* problem (borrowing insights from

[12] See Fifield et al. [Working Paper], pg. 16, for a technical explanation of why these algorithms don't produce uniform redistricting plans: "For example . . . , the creation of earlier districts may make it impossible to yield contiguous districts. These algorithms rely on rejection sampling to incorporate constraints, which is an inefficient strategy. More importantly, the algorithms come with no theoretical result and are not even designed to uniformly sample redistricting plans."

[13] Generating a biased sample is not necessarily a problem if all you want to do is *optimise*, e.g. draw the most compact plan possible. Recent work builds upon this standard algorithm, using Voronoi diagrams or iterative flood fill procedures rather than random chance, to assign the precincts to be agglomerated. See Levin and Friedler [2019] for a technical overview.

graph theory and computer science), and use a *Markov Chain Monte Carlo* (MCMC) approach to sample possible districting plans. This approach is best laid out in Fifield et al. [Working Paper]. The approach first initialises a specific graph partition. A graph partition is an assignment of Census Tracts/Blocks to districts — basically a districting plan. This is the first step of the Markov Chain. Then it *flips* a random node of the graph to get another valid partition. This process is repeated until the Markov Chain approaches its steady state distribution: when this happens, the Markov chain is called “well-mixed”.

This class of algorithms inherit desirable well-known theoretical guarantees of the Markov Chain.[14] They are therefore much less likely (both theoretically and empirically) to generate a biased subset of plans. Conducting a small-scale validation study on a 25-precinct set, Fifield et al. compare the distribution of plans generated by their algorithm to those generated by the standard redistricting algorithm. They prove that their algorithm produces plans that hew much more closely to the *actual* distribution of all possible districting plans.

Due to the many advantages of the MCMC approach, I use it in all my analyses. However, there are many ways to conduct an MCMC analysis. The key question is how one should sample from the near-infinite pool of possible plans. State-of-the-art literature in this space use one of three main approaches, all of which have their pros and cons.

The first is to get a sense for the properties of extremely compact plans under each compactness measure by using a local optimization technique, starting at a whole bunch of different initial seeds using the single node `Flip` proposal. This approach gives us the most compact plans, and is often used to find the “maximal” or “best” districting plans. However, it will—by design—only explore a very tiny subset of all plausible districting plans. Also, because the `Flip` proposal is very state-dependent, the initial state can affect the results greatly.

The second is a middle-of-the-road approach, using a global proposal distribution and a Metropolis-Hasting acceptance function to sample from a distribution over plans that is proportional to $e^{(-\beta \times Compactness)}$. This will give us a distribution of plans that is biased towards compact ones, but also contains some noncompact plans.

One can get different distributions of plans depending on the specific acceptance (score) function. For instance, DeFord and Duchin [2019] prioritises plans that have fewer locality splits and/or sustain a Black majority-minority district. Herschlag et al. [2018] use a complicated score function that takes into account county splitting, population deviation, compactness and minority representation. If I were to use this approach, I would define four different score functions corresponding to the different compactness measures, and compare the resulting distributions that result from each measure.

Finally, one can sample from a distribution that doesn’t incorporate any com-

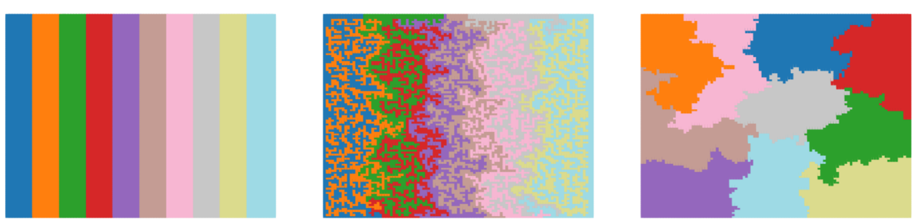
[14]See DeFord et al. [2019b] for a technical overview.

pactness score at all and extract the plans that achieve a good score under each metric. This approach is used in DeFord et al. [2019a], where they generate a large neutral ensemble of districting plans and then subsequently filter the plans according to increasingly strict vote-band constraints. The advantage of this approach is that it casts the widest net: all plausible districts (subject to the equal population bound) are explored. The disadvantage is that the odds of sampling an 'optimal' district are incredibly low, which makes it suboptimal for algorithms that aim to build the "best" plan.

7.0.0.2 Choosing the best MCMC approach

To recap, there are three plausible MCMC approaches to generate a large subset of redistricting plans: local optimisation, score function, or neutral ensemble. I examine them each in turn and decide on the neutral ensemble approach because it generates the largest and most representative subset of redistricting plans, which best represents the plans that legislators are likely to draw in real life.

The first proposal is local optimisation. Local optimisation approaches like the `Flip` proposal have one key problem. The "mixing time" of the Markov Chain under the `Flip` proposal—that is, the number of steps it takes for the Markov Chain to be "close enough" to the stationary distribution—is very large. What that means is that the `Flip` proposal tends to generate very uncompact, snakelike districts in the beginning, as can be seen in Figure 27. It will take millions of steps for plans under the `Flip` proposal to reach a satisfactory districting plan. As such, I prefer the Recombination (Recom) distribution by DeFord et al., which uses a spanning tree method to bipartition pairs of adjacent districts at each step [DeFord et al., 2019a]. This proposal distribution improves upon the `Flip` proposal by decreasing the mixing time needed to reach a satisfactory districting plan.



(a) Initial Partition (b) 1,000,000 Flip steps (c) 100 ReCom steps

Figure 27: The `Recom` proposal generates more realistic plans in much fewer steps. Taken from DeFord et al. [2019b].

Mixing time aside, the extreme compactness of local optimisation is in fact something that I want to avoid. I aim to find out if mandating compactness in state constitutions can inadvertently adversely affect democratic representation. But restricting one's analysis to extremely compact plans means that we cannot say much about the relationship between compactness and spatial diversity. In addition, extremely compact plans are *not* a representative subset of the plans unbiased redistrictors might draw. This is because redistrictors care about a lot of other considerations apart from compactness, and therefore most definitely do not optimise solely over compactness. State constitutions demand that plans be "reasonably compact", not "maximally compact": it's vanishingly unlikely that those extremely compact plans would resemble the types of plans that would be drawn in real life. As such, *even* if I found that optimally compact plans had greater spatial diversity, this would have very little bearing on redistricting policy. It's far more instructive to see whether the relationship holds in the plans that legislators could actually be expected to draw.

How can we get a representative subset of plans that legislators could actually be expected to draw? Given that legislators care a lot about many different considerations, might it be better to try and include these considerations into the score function? This is what the second approach does. While this approach holds strong theoretical merit, I find that this approach introduces too many degrees of freedom. The choice of what factors to include in the score function is contentious: Herschlag et al. use population deviation, Polsby-Popper score, county boundaries and minority deviation. But they could just as easily have included factors such as proportionality or number of cut edges (proposed in Dube and Clark [2016]) for instance. Even if there is a strong justification for including exactly those factors, there is still significant researcher freedom to operationalise the scores. For instance, Herschlag et al. and DeFord and Duchin both include a population deviation score, but operationalise the metric differently.

Furthermore, any score function has to be assigned specific weights— but this assignment is somewhat arbitrary and open to argument. For instance, Herschlag et al. "chose a VRA score function which awards lower scores to districting plans which had one district close to 44.48% African-Americans and a second district close to 36.20% African-Americans", on the basis that the 2016 districting plan which was accepted by the Court had districts with those proportions. But this is incredibly arbitrary. Obviously, just because a particular district was accepted by the Court with those proportions of African-Americans doesn't imply that those exact proportions of African-Americans are optimal.

To be clear, these problems are not insurmountable. If there is a strong theoretical basis for one particular operationalisation over another, then the criticism of researcher fiat largely loses its bite. Furthermore, the results obtained are robust to a variety of perturbations. Herschlag et al. [2018] change the weights and threshold values as a robustness check and find qualitatively similar results. Nonetheless, different results can occur. And if two different operationalisations or factor weights yield qualitatively different results, how would we adjudicate between them? For these reasons, I choose not to use the second approach.

Finally, the neutral ensemble approach is the most permissive, and thus gives us the best chance of getting a representative sample of legislators' plans. It generates a neutral ensemble and does not favour one plan over another (except for some minimal compactness and population deviation requirements). This approach gives us the largest space of plausible plans, which has a key advantage: it allows the results to be applicable even for districting algorithms that do not use an MCMC approach. This includes not only the regular low-tech way of drawing districts, but also other automated districting algorithms like Magleby and Mosesson [2018] and Levin and Friedler [2019].

Therefore, I elect to use the last, "neutral walk" approach. I use a global `Recom` proposal to generate the states, but accept every proposal subject to minimal population deviation requirements. This gives me a neutral ensemble of 10,000 plans for every state.

8 Technical Appendix B: Optimisations used in computing the human compactness metric

Maybe not necessary, but talk about the precomputation steps and saving the pointwise distances

9 Appendix B: Results of difference-in-means tests for individual states

Here I compare the average spatial diversity of all 10,000 plans per state to the average spatial diversity of the 500 most compact plans per state.

I present the results for each state and each metric in the ensemble, using Welch's t-test.

	state	metric	mean_diff	variance	pct_variance	t-stat	p-value
0	0	hc	-0.003460	0.058642	-5.900607	-17.425785	1.366961e-61
1	0	pp	0.000069	0.058642	0.118009	0.288166	7.732681e-01
2	0	reock	0.000381	0.058642	0.650317	1.624462	1.045297e-01
3	0	ch	-0.001042	0.058642	-1.776135	-5.014481	6.033771e-07
4	1	hc	-0.000513	0.057499	-0.892208	-1.868482	6.193359e-02
5	1	pp	-0.001423	0.057499	-2.475505	-4.986335	7.054193e-07
6	1	reock	-0.000498	0.057499	-0.865298	-1.692770	9.076060e-02
7	1	ch	-0.000678	0.057499	-1.178930	-2.231754	2.581874e-02
8	2	hc	0.001489	0.036047	4.131827	26.809567	2.038788e-153
9	2	pp	0.001104	0.036047	3.062205	10.321991	2.820313e-24
10	2	reock	-0.000188	0.036047	-0.520417	-0.859779	3.900941e-01
11	2	ch	0.000383	0.036047	1.063637	2.841225	4.560090e-03
12	3	hc	-0.001257	0.033457	-3.756204	-9.240446	9.523388e-20
13	3	pp	-0.001245	0.033457	-3.720159	-7.632057	4.670461e-14
14	3	reock	-0.000776	0.033457	-2.318159	-5.132091	3.320205e-07
15	3	ch	-0.000927	0.033457	-2.770633	-7.108140	1.896994e-12
16	4	hc	-0.001902	0.028376	-6.704063	-49.155427	0.000000e+00
17	4	pp	0.005131	0.028376	18.081320	38.153281	1.090249e-206
18	4	reock	0.001304	0.028376	4.596054	20.334160	7.714653e-84
19	4	ch	-0.002035	0.028376	-7.171113	-50.341694	0.000000e+00
20	5	hc	-0.019707	0.077819	-25.324736	-43.785027	7.817121e-271
21	5	pp	0.007385	0.077819	9.490310	14.029691	8.033314e-42
22	5	reock	0.005601	0.077819	7.197869	10.059549	5.666063e-23
23	5	ch	0.004848	0.077819	6.229592	8.615116	2.011837e-17
24	6	hc	-0.004913	0.076917	-6.386934	-12.541515	4.676097e-34
25	6	pp	-0.006333	0.076917	-8.233653	-16.445177	3.655560e-55
26	6	reock	-0.006334	0.076917	-8.235342	-17.317992	1.527167e-60
27	6	ch	-0.000795	0.076917	-1.033852	-1.809545	7.061978e-02
28	7	hc	-0.011556	0.032940	-35.083239	-120.004988	0.000000e+00
29	7	pp	0.002150	0.032940	6.527335	9.218455	1.208411e-19
30	7	reock	-0.002165	0.032940	-6.573630	-11.615082	6.541658e-30
31	7	ch	0.004050	0.032940	12.294876	17.193270	1.023553e-59
32	8	hc	0.005538	0.058276	9.503582	42.778404	4.401068e-291
33	8	pp	0.002962	0.058276	5.082034	18.477814	2.578165e-69
34	8	reock	0.002492	0.058276	4.275665	14.864941	8.132217e-47
35	8	ch	0.002689	0.058276	4.613737	16.787183	1.984654e-58
36	9	hc	-0.003290	0.049699	-6.619743	-13.092609	8.687410e-37
37	9	pp	-0.001645	0.049699	-3.309711	-6.053633	1.889349e-09
38	9	reock	-0.000677	0.049699	-1.361577	-2.476624	1.340008e-02
39	9	ch	-0.001482	0.049699	-2.982079	-5.561983	3.278783e-08

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