Introduction

Good job today. To reiterate, basically I think your research question should be something like: "Research shows that districts that consist of more homogeneous groups of voters achieve better representation on several dimensions. Meanwhile, many statutes require that districts be"compact", a term with many interpretations. It is not clear, however, that more compact districting plans (however compactness is measured) result in more homogeneous districts. Are compactness and homogeneity fundamentally conflicting goals? Are some measures of compactness more consistent with homogeneity than others?"

My contribution

- i develop the human compactness metric
- i use MCMC to measure spatial diversity
- first to measure relationship between compactness and normative outcomes?
- at the very least, first to measure relationship between compactness and spatial diversity

Theoretical stuff

Why compactness

- states mandate it
- good check against gerrymandering

Why spatial diversity

> spatial diversity remained a statistically significant predictor of roll-off rate. With these variables held constant at their means, a House district's shift from the tenth to the ninetieth percentile in spatial diversity was associated with an increase in roll-off rate of about six percentage points

The final political gerrymandering issue that I investigate is how spatial diversity relates to common dis- trict plan metrics such as partisan bias and electoral responsiveness. As noted earlier, partisan bias refers to the divergence in the share of seats that each party would win given the same share of the statewide vote.254 For example, if Democrats would win forty-eight percent of the seats with fifty percent of the vote (in which case Republicans would win fifty-two percent of the seats), then a district plan would have a pro-Republican bias of two percent. Electoral responsiveness refers to the rate at which a party gains or loses seats given changes in its statewide vote share. For instance, if Democrats would win ten percent more seats if they received five percent more of the vote,

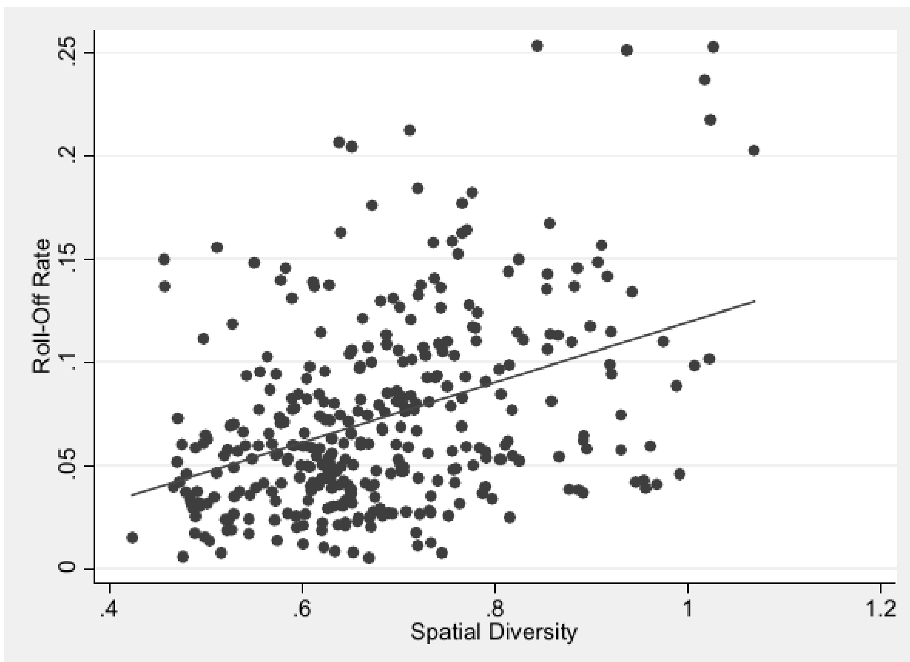


Figure 1: Effect of spatial diversity on electoral rolloff

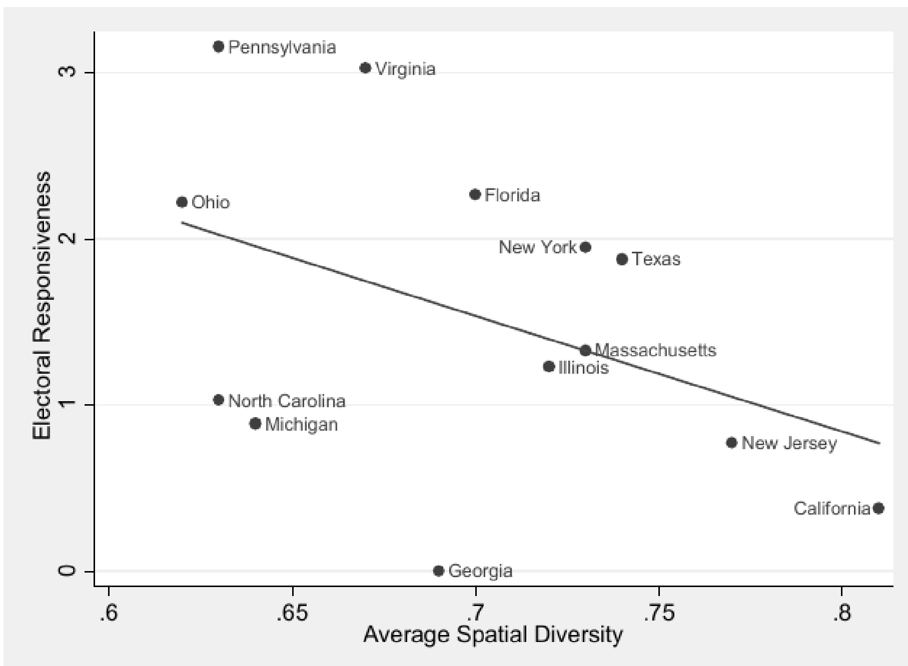


Figure 2: Effect of electoral responsiveness on spatial diversity

then a plan would have a responsiveness of 2.0. 255 As a general matter, the lower a plan's bias, and the higher its responsiveness, the better the plan is thought to be. 256

255 See Gelman & King, supra note 11, at 544–45 (defining bias and responsiveness). 256 Reducing bias all the way to zero is unproblematic. However, very high rates of responsiveness are undesirable because they result in large changes in seat shares despite only small shifts in vote shares. Fortunately, the responsiveness scores reported here are not high enough to raise such concerns.

Figures 12 and 13 show how states' spatial diversity averages were related to partisan bias and electoral responsiveness in the 2006, 2008, and 2010 elections.260 I include only states with at least ten congres- sional districts (because bias and responsiveness are not very meaning- ful for states with small numbers of seats), 261 and I use the absolute value of bias (because I am interested in the metric's magnitude rather than its orientation). As is evident from the first chart, spatial diversi- ty has a curvilinear relationship with bias. At lower levels of spatial diversity, that is, bias tends to decrease as spatial diversity increases; but at higher levels of spatial diversity, bias and spatial diversity tend to move in tandem. The curve as a whole is clearly U-shaped. 262 This result suggests that states seeking to treat the major parties as equitably as possible should not minimize the average spatial diversity of their districts. Consistent with the relevant literature, high levels of geographic variation are associated with high bias; 263 they both imply

The story with responsiveness is more straightforward. As the se- cond chart illustrates, responsiveness simply tends to decrease as aver- age spatial diversity increases. The states whose districts are most homogeneous, on average, are also the states whose elections are most responsive to changes in public opinion. In contrast, the states whose districts are most heterogeneous are also the ones in which even large swings in voter sentiment have little impact on the parties' seat shares. This finding indicates that while high spatial diversity is not a prereq- uisite for a partisan gerrymander (low spatial diversity can also do the trick), it is indeed an effective way to protect incumbents of both par- ties from shifting political tides. Advocates of responsive elections, then, may push without hesitation for spatially homogeneous districts to be drawn, since it is these districts that seem most likely (in the ag- gregate) to reflect the public's evolving preferences. 265

Stephanopoulos [2012]

How compactness might affect spatial diversity

Previous work

- people have done how compactness affects competitiveness (schutzman 2020)

- people have used MCMC approach to look at how districts are more or less competitive (daryl's work)

Overview of research strategy

Two key research questions:

1. Do more compact districts have better, equal, or worse spatial diversity scores?
2. Is there an inherent trade-off between compactness and homogeneity?
3. Does spatial diversity give us a normative basis to select one compactness metric over another?

The research procedure:

1. Generate a large and representative subset of plausible districting plans

2.

Overview of compactness measures

To empirically evaluate a trade-off between compactness and homogeneity, we must first define the metrics by which we evaluate a proposed districting plan over each of these dimensions. Here, I introduce many different compactness measures. I give a brief overview of the different types of measures, explain the pros and cons of each, present a compactness measure that I develop, and support my decision to use an ensemble of measures to increase robustness.

Geometric compactness metrics

Shape-based versus dispersion-based

The most

Polsby-Popper

The Polsby-Popper measure, introduced by Polsby and Popper in 1991, is a ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district.

$$4\pi \times \frac{A}{P^2}$$

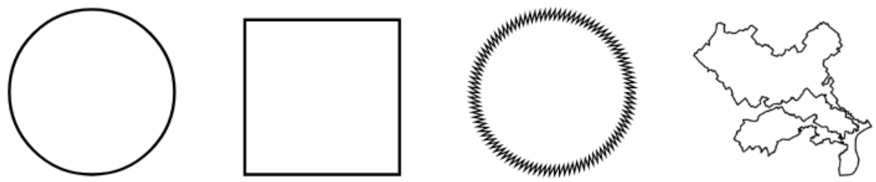


Figure 3: Polsby-Popper scores of four example regions: a perfect circle, a square, a circle with a ragged boundary, an an example district from a Pennsylvania plan. Taken from Schutzman [2020].

Reock

The Reock score is a measure of the ratio of the district to the area of the minimum bounding circle that encloses the district's geometry.

$$\frac{Area}{AreaOfMinimumBoundingCircle}$$

Convex Hull

The Convex Hull metric is a ratio of the area of the district to the area of the minimum convex polygon that can enclose the district's geometry.

$$\frac{Area}{AreaOfMinimumConvexPolygon}$$

Choosing a compactness metric

Which compactness measure should we choose? All three compactness measures are well-cited in the literature and enjoy widespread use. They have been cited in U.S. Supreme Court cases, *amici* briefs, and redistricting commissions [Moncrief, 2011]. Despite their widespread acceptance, however, the problems with compactness measures are many, and well-covered in the literature. No compactness measure is perfect: as an example, the most popular compactness measure in the literature—Polsby-Popper—is sensitive to small perturbations in data resolution (the coastline problem). [1] It is therefore important to use an

[1] The Polsby-Popper metric measures the ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district. But depending on the resolution of the map, the perimeter can be effectively infinite. Barnes and Solomon find that the choice of resolution has "a substantial impact on compactness scores, with the Polsby-Popper score especially affected."

ensemble of compactness measures to make sure that one's data and conclusions are robust.

But even this is not enough. Because most compactness measures are purely geometric, they are all vulnerable to a specific family of geographic perturbations. Indeed, Barnes and Solomon [Forthcoming] show that minimally tweaking the geometric features of states is enough for the four most popular compactness measures (Polsby-Popper, Convex Hull, Reock, Schwartzberg) to give very different conclusions on nominally identical data.

Thus, it is important to include a non-geometric compactness measure in the ensemble to guard against the possibility that the results are driven by a specific quirk in geography. Many such measures have been proposed. For instance, Dube and Clark [2016] bring in a discipline of mathematics—graph theory—to formulate a new metric of compactness.

However, one particular class of metrics I term *point-wise distance metrics* stands out for its ease of understanding (critical if it is to be persuasive to Supreme Court judges), theoretical attractiveness, and academic consensus. This class of compactness metrics tries to measure the distance between two voters in a district, and assigns higher scores the lower that distance is.

This class of metrics enjoy strong theoretical grounding. Paramount to the idea of single-member districts is that there is some value in voters who live in the same area being put into the same district. Eubank and Rodden [2019]:

> "Voters in the same area are likely to share political interests; voters in the same area are better able to communicate and coordinate with one another; politicians can better maintain connections with voters in the same area; voters in the same area are especially likely to belong to the same social communities — all suggest the importance of voters being located in districts with their geographic peers."

In contrast, districts that carve voters out of their natural communities and pool them with unrelated, distant voters are bad ones. Therefore, we should be sensitive not just to geometric shape, but rather whether or not voters live close to one another. Point-wise distance metrics are more readily understandable to laymen and possess a normative bent that more abstract mathematical compactness measures lack. It has therefore been an active area of development in the literature. Chambers and Miller [2010] present a measure of "bizarreness", which is the "expected relative difficulty in traveling between two points within the district". And Fryer Jr and Holden [2011] measures "the distance between voters within the same district relative to the minimum distance achievable".

I build upon the literature by developing a new metric which I call human compactness. The metric measures the ratio of driving durations between one's nearest neighbours and one's fellow districtors. The higher this ratio is, the more compact the district. Intuitively, it encourages drawing districts that put one's next-door neighbours together in the same district. This metric makes two key improvements over existing measures that have been proposed.

TODO explain my metric a bit more before talking about the improvements Maybe change tack — criticise the existing literature approach/ talk about shortcomings, then introduce my metric.

First, I use driving durations rather than Euclidean (as-the-crow-flies) distances between voters. This keeps the metric robust to quirks in political geography like mountains and lakes, and better represents the notion of natural communities. In fact, while Fryer Jr and Holden [2011] used Euclidean distance in his metric, he points out its shortcomings:

> Suppose there is a city on a hill. On the West side is [a] mild, long incline toward the rest of the city, which is in a plane. On the East side is a steep cliff, either impassable or with just a narrow, winding road that very few people use. While the next residential center to the East is much closer to the hilltop on a horizontal plane, it is much further on all sorts of distances that we think might matter: transportation time, intensity of social interactions, sets of shared local public goods and common interests, etc. Thus, for all practical purposes, one probably wants to include the hilltop in a Western district rather than an Eastern one. More general notions of distance can handle this.

In this case, driving durations would better reflect this quirk in political geography. The "impassable" region on the East would have a short Euclidean distance, and any districting plan that put the hilltop with the Eastern district would be unfairly penalised by these point-wise distance metrics. On the other hand, the impassable region would have a long driving duration, accurately reflecting the political geography. After acknowledging the shortcomings of Euclidean distance, Fryer Jr and Holden specifically suggest using driving durations to improve their metric: "one can extend much of [our analysis] by using driving distance or what legal scholars refer to as 'communities of interest' ".

There are thus strong theoretical grounds for using driving durations in point-wise distance metrics. Driving durations also enjoys empirical support—the use of driving durations seems strictly superior in many cases involving human-scale distances. Working with Nicholas Eubank and Jonathan Rodden, I update their gerrymandering-detection metric to use driving durations instead [Eubank and Rodden, 2019]. We find a consistently different picture of the social context of American suburban voters, raising the possibility of false positives under the Euclidean distance measure [Eubank, Lieu, and Rodden, Forthcoming].

The second improvement I make is algorithmic and computational. My metric improves upon the algorithmic complexity of Fryer Jr and Holden's algorithm from an NP-hard problem to one with a $O(n^2)$ polynomial runtime. This is an exponential decrease in algorithmic complexity, which means the disparity between my metric and Fryer Jr and Holden's increases as the input size grows. Similarly, Chambers and Miller's measure cannot feasibly be improved with driving durations due to the algorithmic complexity of finding point-to-point travel distances without passing through another district [2]. Given that there are strong theoretical and empirical reasons to adopt driving durations, this is a large improvement. I also use programming techniques like precomputation and memoisation to decrease the time taken to compute the metric greatly. My implementation is competitive with geometry-based compactness measures like Reock: on my machine, both metrics took roughly the same amount of time (~0.20s per step). Further details on the metric can be found in Appendix A.

Given these considerations, I settle on an ensemble of four different compactness measures: Polsby-Popper, Reock, Convex Hull, and Human Compactness. I exclude the Schwartzberg metric as the Schwartzberg and Polsby-Popper measure are mathematically equivalent. Finally, I include my point-wise distance metric. This ensures the robustness and validity of my results.

Overview of automated districting algorithms

In order to find out whether compactness measures track spatial diversity, we have to generate many counterfactual plausible plans that span the entirety of possible districting plans and measure the correlation between compactness and spatial diversity. This requires using a computer to draw a large number of plans according to some minimal criteria.

The idea of drawing a large number of districting plans with a computer has a long and storied history, starting in the 60s and 70s. The approach has almost always been used to identify gerrymandering; for instance Cirincione et al. [2000] build an algorithm to "quantitatively [assess] whether the [1990 South Carolina] plan is a racial gerrymander". More recently, Chen et al. [2013] "generat[e] a large number of hypothetical alternative districting plans that are blind as to party and race, relying only on criteria of geographic contiguity and compactness." They do this using a Markov Chain simulation algorithm, a procedure that makes iterative changes for a large number of steps until a unique districting plan emerges. At each step of Cirincione et al.'s algorithm, they randomly select a Census Block Group to serve as a "seed" of the district, then randomly add its neighbouring block groups to it until a district with the desired population is formed. Similarly, Chen et al. begin by initialising all precincts as an individual, separate district, then randomly agglomerating neighbouring precincts until the desired number of districts is reached.

While this "standard simulation algorithm" enjoys a certain degree of success, it has one crippling weakness. The way in which this class of algorithms operates necessarily explores only a tiny subset of all possible districting plans.

[2]The metric uses the probability that the shortest path between any two points in the district is exactly the shortest path between any two points in the state. This requires finding the shortest path between all points in the district subgraph, which has O(n^3) complexity in the best case, but in practice has a larger complexity because we are querying a routing engine for travel times.

Subsequent work pointed out this flaw: Magleby and Mosesson wrote that automated processes "may take a biased sample of all possible legislative maps... and fail to efficiently produce a meaningful distribution of all alternative maps". And Fifield et al. contend that "[standard Monte Carlo algorithms] are unlikely to yield a representative sample of redistricting plans for a target population." [3] This poses a huge issue for the validity of any statistical analysis, because any correlation that we discover on a biased subset of plans may be spurious when measured over the actual distribution of plans. [4]

Thankfully, scholars have developed an improvement over the standard algorithm with stronger theoretical guarantees. This second class of algorithms reframe the districting problem as a *graph partition* problem (borrowing insights from graph theory and computer science), and use a *Markov Chain Monte Carlo* (MCMC) approach to sample possible districting plans. This approach is best laid out in Fifield et al. [Working Paper]. Broadly speaking, the approach initialises a specific graph partition as a step in the Markov Chain, then *flips* a random node of the graph to get another valid partition. This process is repeated until the Markov Chain approaches its steady state distribution: when this happens, the Markov chain is called "well-mixed".

This class of algorithms inherit desirable well-known theoretical guarantees of the Markov Chain.[5] They are therefore much less likely (both theoretically and empirically) to generate a biased subset of plans. Conducting a small-scale validation study on a 25-precinct set, Fifield et al. compare the distribution of plans generated by their algorithm to those generated by the standard redistricting algorithm. They prove that their algorithm produces plans that hew much more closely to the *actual* distribution of all possible districting plans.

Due to the many advantages of the MCMC approach, I use it in all my analyses. I use an superior proposal distribution called Recombination (Recom) by DeFord et al., which uses a spanning tree method to bipartition pairs of adjacent districts at each step [DeFord et al., 2019a]. This proposal distribution improves upon the `Flip` proposal in Fifield et al. in two significant ways: it generates plans in much fewer steps [6], and it generates plans that are much more realistic. The `Flip` proposal tends to generate very uncompact, snakelike districts, as can be

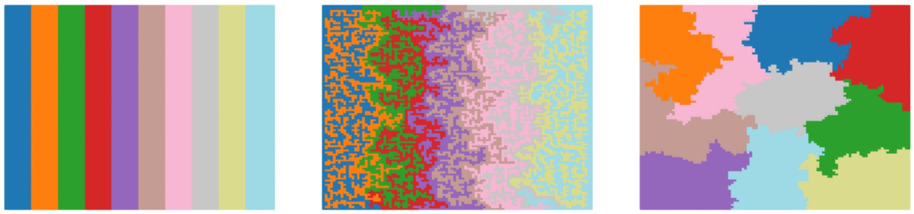
[3] See Fifield et al. [Working Paper], pg. 16, for a technical explanation of why these algorithms don't produce uniform redistricting plans: "For example . . . , the creation of earlier districts may make it impossible to yield contiguous districts. These algorithms rely on rejection sampling to incorporate constraints, which is an inefficient strategy. More importantly, the algorithms come with no theoretical result and are not even designed to uniformly sample redistricting plans."

[4] Generating a biased sample is not necessarily a problem if all you want to do is *optimise*, e.g. draw the most compact plan possible. Recent work builds upon this standard algorithm, using Voronoi diagrams or iterative flood fill procedures rather than random chance, to assign the precincts to be agglomerated. See Levin and Friedler [2019] for a technical overview.

[5] See DeFord et al. [2019b] for a technical overview.

[6] the "mixing time" of the Markov Chain—that is, the number of steps it takes for the Markov Chain to be "close enough" to the stationary distribution—is order of magnitudes smaller in `Recom` compared to `Flip`.

seen in the figure.



(a) Initial Partition (b) 1,000,000 Flip steps (c) 100 ReCom steps

Figure 4: The `Recom` proposal generates more realistic plans in much fewer steps. Taken from DeFord et al. [2019b].

Data generation

1. Generate 100,000 districting plans

I download Census Tract level. These can be downloaded from the United States Census Bureau website.

I use the open-source software library GerryChain to generate the ensembles. Replication code and data are included in the Supplementary Information. I obtain the ReCom Markov chain procedure from one of the co-authors of the DeFord et al. [2019b] paper, and generate 10,000 districting plans for 10 states (Connecticut, Georgia, Idaho, Louisiana, Maine, Maryland, New Hampshire, Rhode Island, Utah, and Wisconsin) for a total of 100,000 plans.

2. Calculate spatial diversity and compactness scores for each of the 100,000 districting plans

I obtain data on spatial diversity from Professor Nicholas Stephanopoulos. The dataset gives *factor scores* for each Census Tract in the country. A district's spatial diversity score is calculated by the sum of the standard deviation of each factor score, normalised by the proportion of the variance each factor score explains. As an example, consider a district made up of three Census Tracts (A, B, C) and let each Tract have three factor scores (1, 2, 3). Let the proportion of the variance explained by each factor score be 50%, 30% and 20% respectively. Then the total spatial diversity score would be

$$\sigma(A_1, B_1, C_1) \times 0.5 + \sigma(A_2, B_2, C_2) \times 0.3 + \sigma(A_3, B_3, C_3) \times 0.2$$

Because

I obtain voter data from Eubank and Rodden,

In order to do this, I have to

3. Analyse

As a robustness check, I rerun all the analyses

by taking the sum of square roots rather than the arithmetic mean.

This penalises districting plans that have a large difference between districts e.g. one very good district and one very bad one.

Results

My key results are as follows:

1. Political geography largely pins down the spatial diversity of each individual district.
    - Small urban districts have high SD, large rural ones have low SD.
2. Different compactness measures are correlated with one another.
3. OLS regressions and difference-in-means tests suggest that only the human compactness measure is negatively correlated with spatial diversity: geometric/dispersion based measures have either no or a positive (bad) effect on spatial diversity.
4. A difference-in-means test suggests that the most compact districting plans are indeed less spatially diverse than average.
5. A difference-in-means test suggests that the most compact plans under human compactness are less spatially diverse than the most compact plans under geometric/dispersion based measures.

Overall, the evidence suggests that optimising over compactness will give you less spatially diverse districts, and human compactness will do the best job of it.

Descriptive analysis

After having obtained all the plans and their corresponding scores, I plot the plans with the best and worst spatial diversity and compactness scores to get an understanding for the types of plans that each metric encourages. This will give us valuable intuition for understanding the subsequent results.

For ease of exposition we consider states with only two districts, but the analysis extends to states with any number of districts. I also use Polsby-Popper to represent the other two dispersion-based compactness metrics as my explanations are similarly applicable to those metrics.

Figure 5 plots the best and worst plans according to several metrics. Let us begin with the middle row (Polsby-Popper), as its interpretation is the most straightforward. The Polsby-Popper (and other dispersion-based) metric penalises districts that are very “snakelike” and prefers districts that have regular shapes like squares or circles. This is clearly reflected in the plot. The best

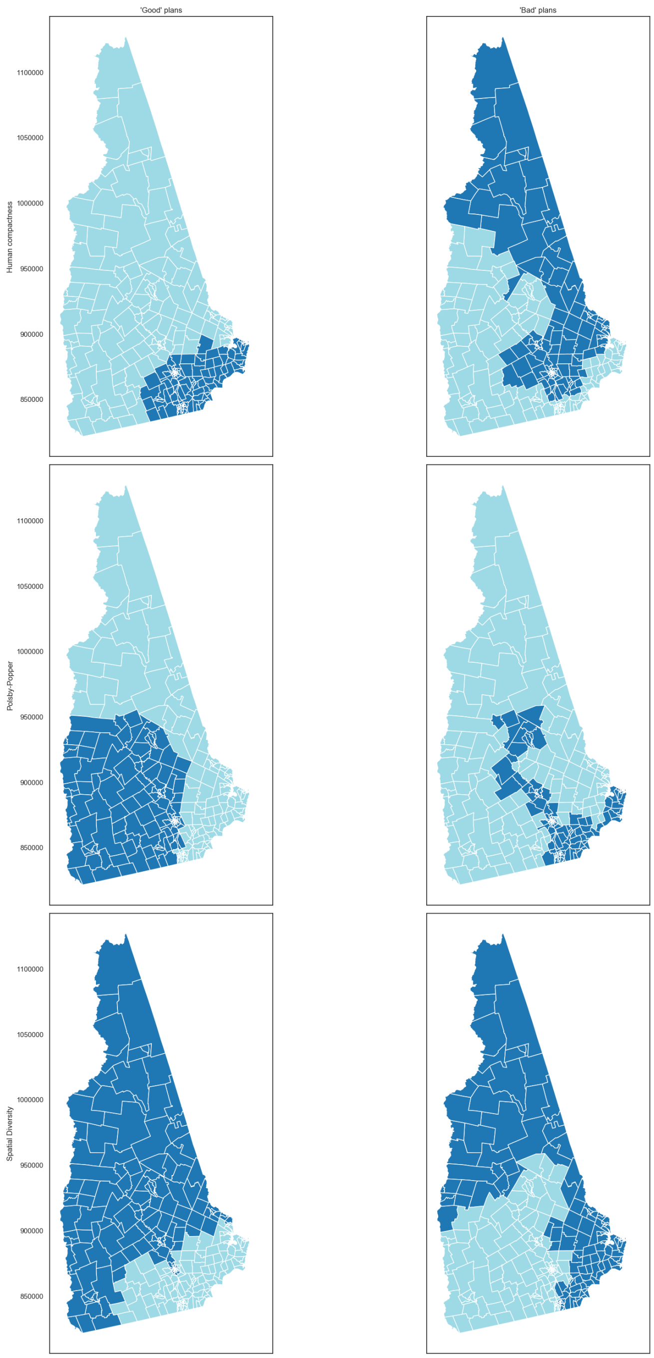


Figure 5: Best and worst districting plans of New Hampshire under different metrics

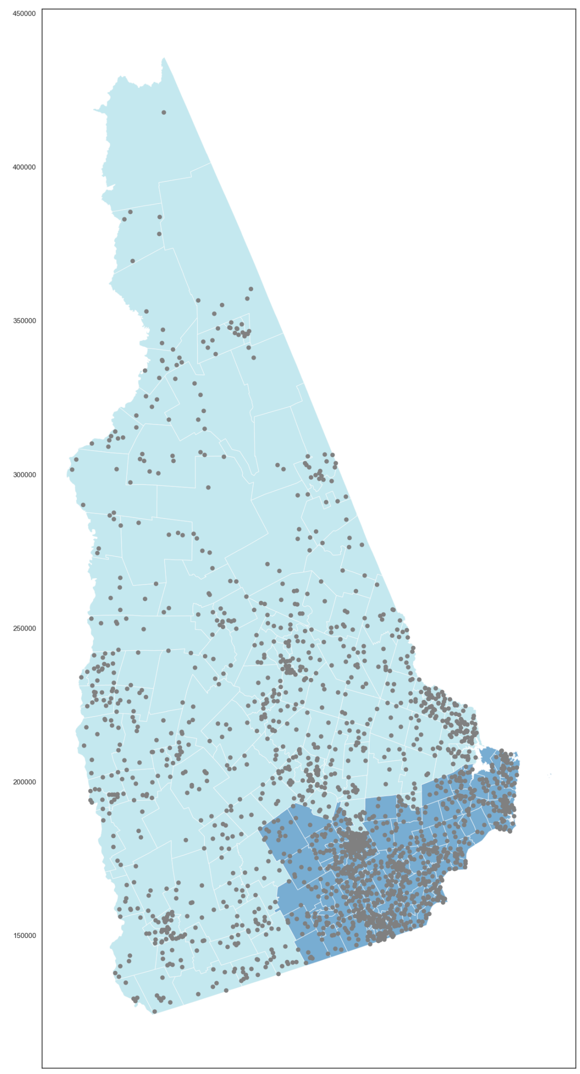


Figure 6: Population density plot of New Hampshire. Each dot represents roughly 600 people.

plan has a district with a very regular shape, and the worst plan has a snakelike district that contorts through half the state.

On the top row is human compactness. A good plan under human compactness minimises the total travel times between every member of the district. This encourages small, compact districts that avoid splitting urban centers.

We can see that the top plan under human compactness corresponds well to the actual population density of New Hampshire as seen in Figure 6. The top plan puts the two most populous and urban counties in New Hampshire—Rockingham and Hillsborough—together in the same district. The worst plan under human compactness splits the counties in such a way that one's co-districtors are far away, and one's nearest neighbours are in a separate district.

As expected, the top plan under spatial diversity (bottom row) closely resembles the top plan under human compactness. In relatively homogeneous New Hampshire, the main source of spatial diversity is the urban-rural divide. A plan that keeps urbanites together in one district is favoured under spatial diversity.

And while the worst plan under spatial diversity looks different from that under human compactness at first glance, they are actually quite similar. Both plans split up the two populous urban counties, having a "fish-hook" shaped district that starts from the rural north of the state and swoops down to the south to carve out a large part of the counties.

This case study shows that dispersion-based measures may not always reflect existing communities of interest. This seems to fuel criticism of dispersion-based measures on exactly that basis ("it makes no sense to combine areas that have nothing in common except that they fit neatly into a square" [Wolf, 2015]). In this example, human compactness and spatial diversity agree neatly on what the best districting plans should look like.

While human compactness generally tracks spatial diversity better than other compactness metrics (I provide evidence for this later), it does not always do so.

Political geography largely pins down the spatial diversity of each individual district

While spatial diversity varies enormously between districts, this is to a large extent dependent on the state's political geography.

However, plotting individual states reveals great homogeneity.

Compactness measures largely agree with one another, although an ensemble is still required due to possibility of outliers (actually occurs)

sd	hc	pp	reock	ch

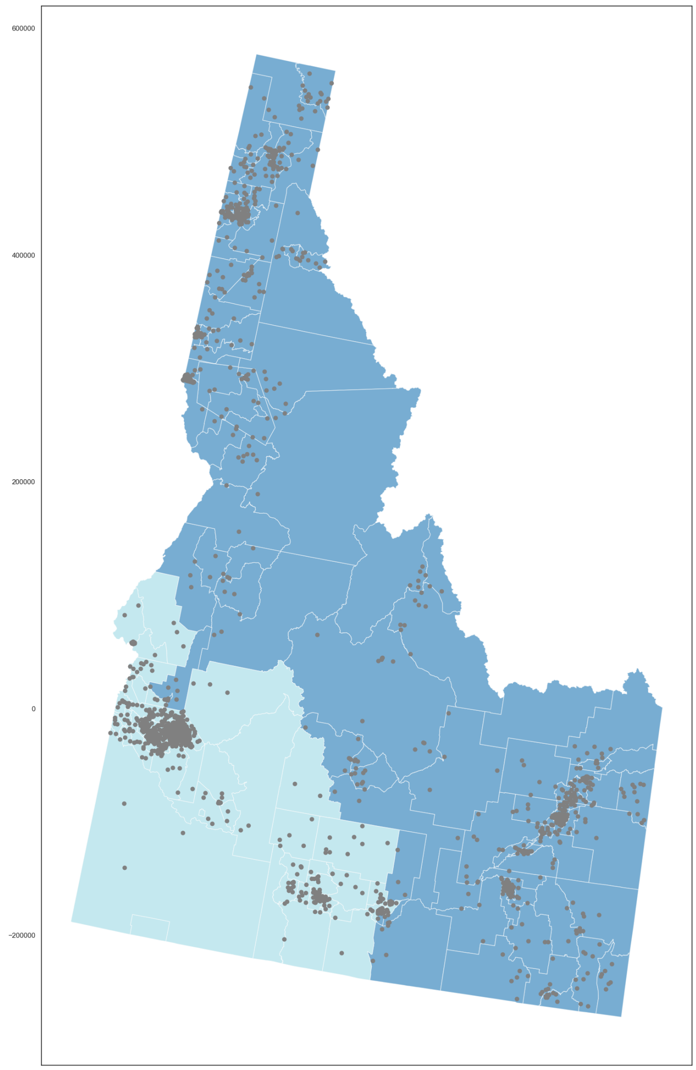


Figure 7: Population density plot of Idaho. Each point represents ~700 people.

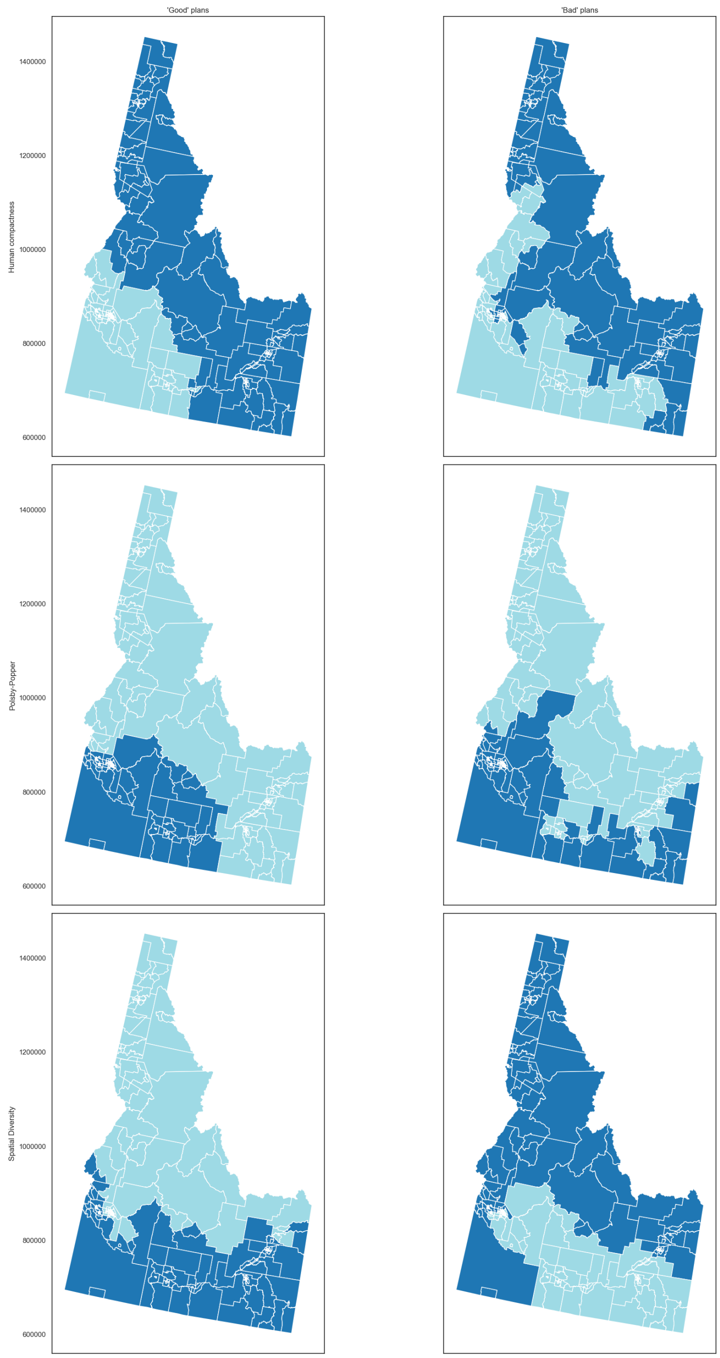


Figure 8: Best and worst districting plans of Idaho under different metrics

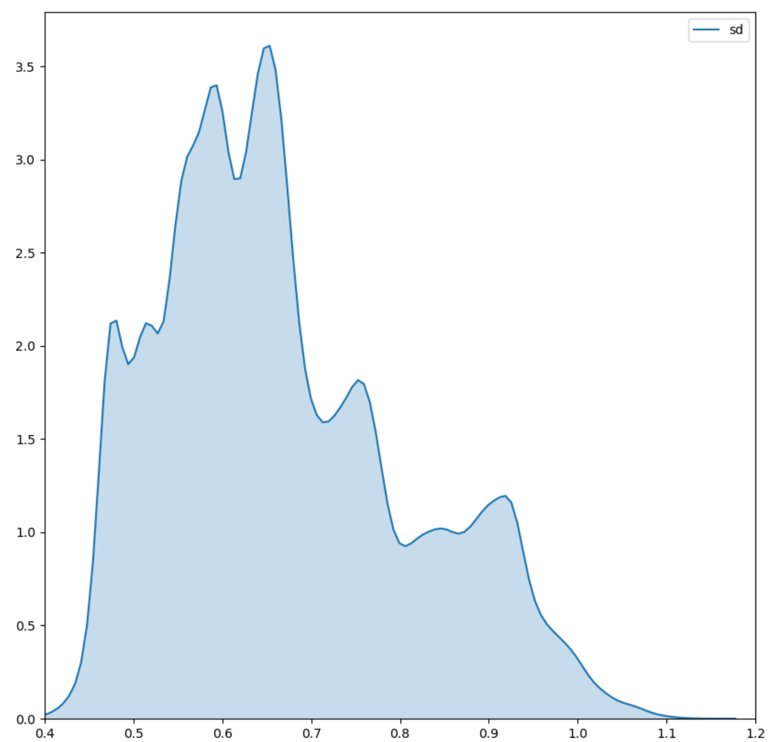


Figure 9: Spatial diversity of all districts

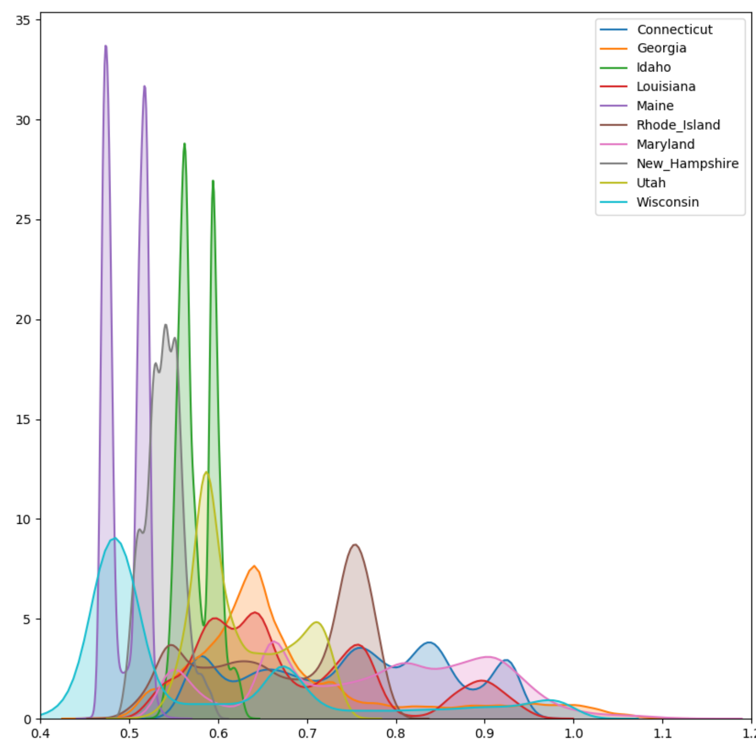


Figure 10: Spatial diversity of districts binned by state

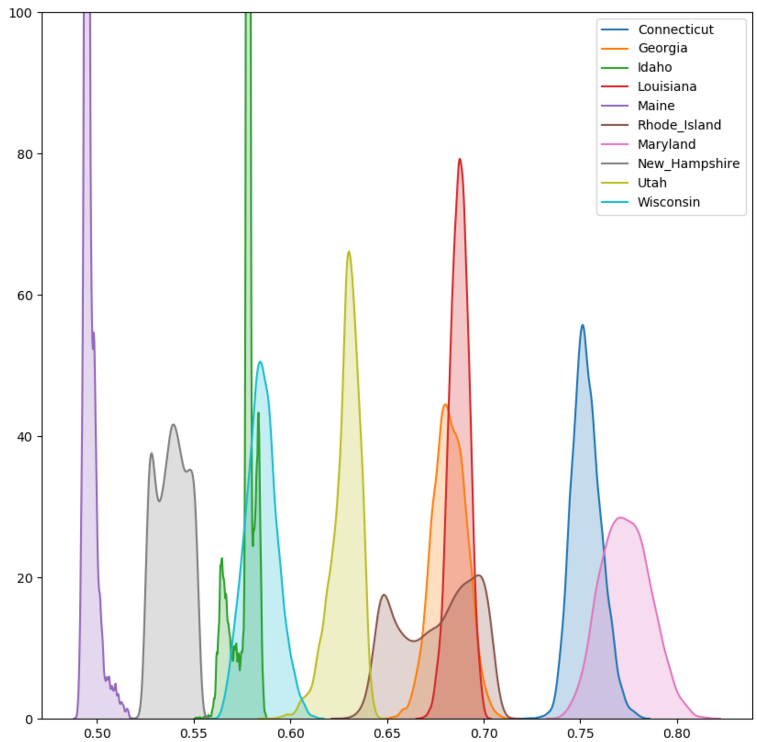
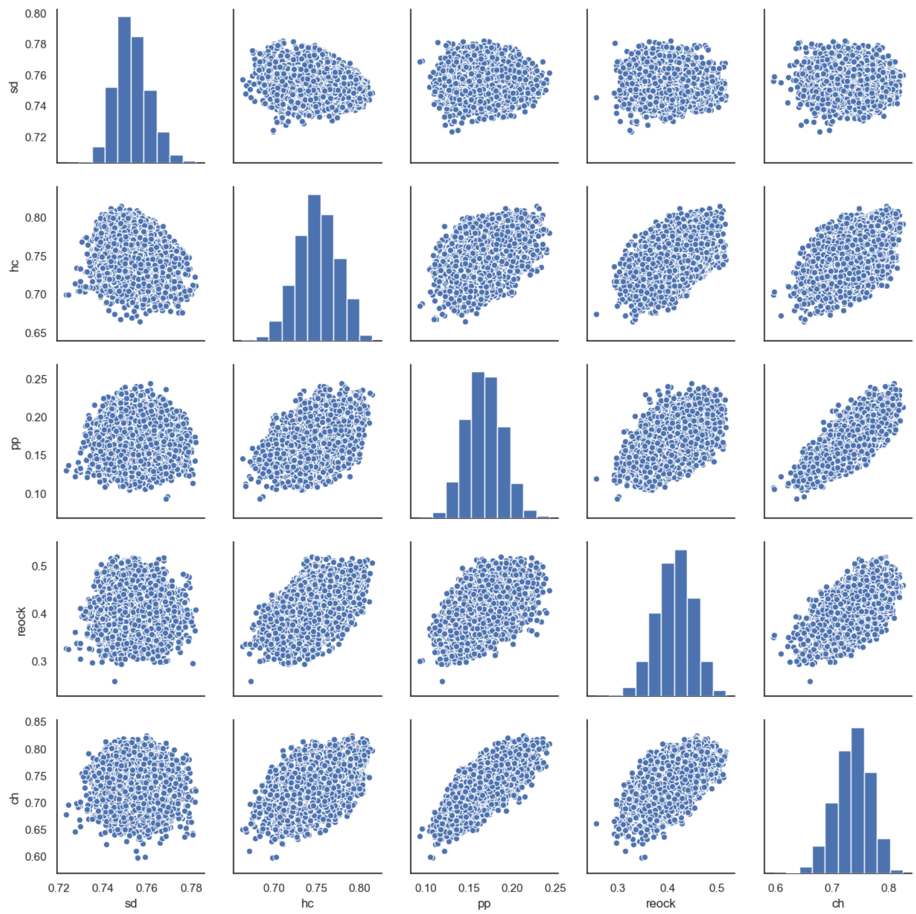


Figure 11: Overall spatial diversity of districting plans by state

sd	1.000000	-0.357926	-0.094629	-0.007729	-0.455142
hc	-0.357926	1.000000	-0.409995	-0.111860	0.110439
pp	-0.094629	-0.409995	1.000000	0.480940	0.606623
reock	-0.007729	-0.111860	0.480940	1.000000	0.629797
ch	-0.455142	0.110439	0.606623	0.629797	1.000000



The overall effect of compactness on spatial diversity is equivocal. Only human compactness has a significant negative effect on spatial diversity

Multivariate regression with country dummies

We cannot simply run a regression aggregating every single district as each state has a unique distribution of spatial diversity and compactness. Consider the following. Within each state, increasing compactness decreases spatial diversity. But on the aggregate, states with high spatial diversity also have low compactness.

In this case, regressing spatial diversity on the aggregate level would give an inflated estimate of the actual effect, falling afoul of the *ecological fallacy*. I illustrate this in figures 12 and 13. In Figure 12, I plot a graph of human compactness on the x-axis and spatial diversity on the y-axis. The overall trend seems to be slightly negative: in most of the groups, there is a slight negative correlation between human compactness and spatial diversity. However, we would obtain erroneous results if we aggregated the different states and ran a singular regression. This is depicted in Figure 13: due to the *between-group* correlation of compactness and spatial diversity, the estimate of the effect is biased. We must therefore control for state when running the regression. Thus, I run a multivariate regression with the functional form

$$SpatialDiversity = \beta_0 + \beta_1 Compactness + \beta_2 State$$

where *State* is a dummy variable, taking care to avoid the dummy variable trap.

Table 1 shows the results for human compactness. I run the same regression for each indivi

find that only human compactness has a statistically significant negative coefficient on spatial diversity, as shown here:

```
HC: -0.0404, t-value -40.632
PP: +0.0251, t-value 29.841
Reock: +0.0209, t-value 27.645
CHull: -0.0016, t-value -1.801
```

The most compact plans have better spatial diversity than average

I therefore consider the top 500 plans

I run a differences-in-means test using Welch's t-test, as Student's t-test relies on a homogeneity in variances assumption. When the assumption of equal variances is not met, Student's t-test yields unreliable results, while Welch's t-test controls Type 1 error rates as expected. Delacre et al. [2017]

I find a very strong result that

This result is even more robust than one would expect, because

```
Mean SD of plans with highest Human Compactness scores: 0.635558
Mean SD of plans with highest Polsby-Popper scores: 0.640954
Mean SD of plans with highest Reock scores: 0.639897
Mean SD of plans with highest Convex Hull scores: 0.639985
Mean SD of all plans: 0.66879
```

```
Ttest_indResult(statistic=array([-26.95882013]), pvalue=array([4.92355005e-150]))
Ttest_indResult(statistic=array([-23.01140913]), pvalue=array([1.00564593e-111]))
Ttest_indResult(statistic=array([-23.65074847]), pvalue=array([1.33953995e-117]))
Ttest_indResult(statistic=array([-23.36762755]), pvalue=array([5.64959214e-115]))
```

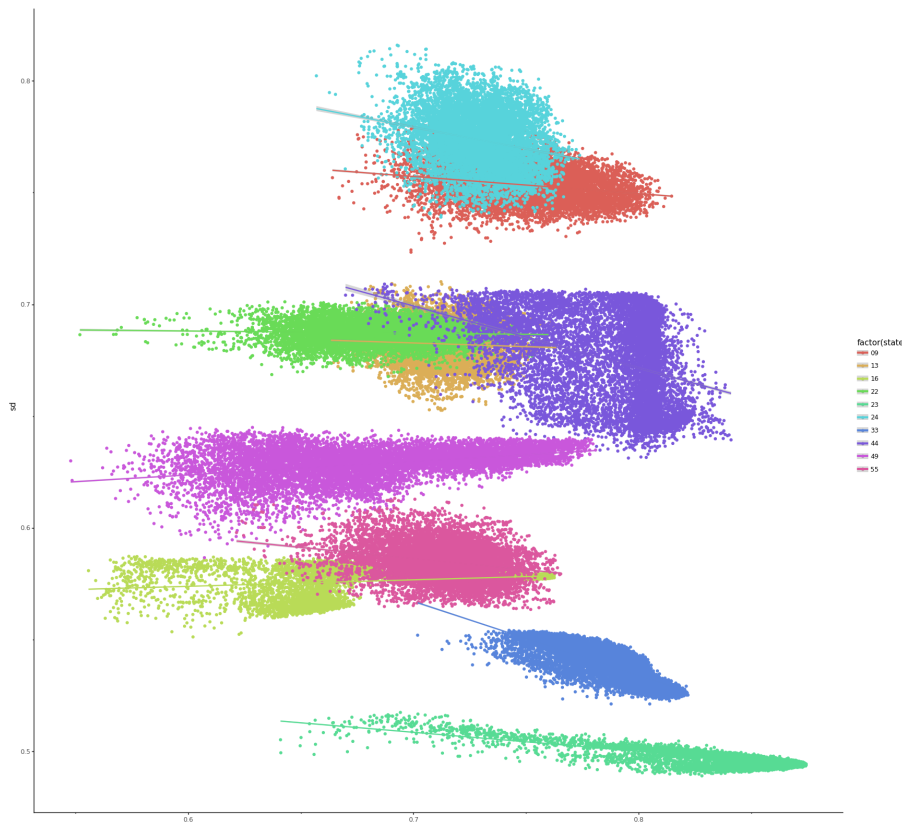


Figure 12: The individual-level regressions show a weak downward trend between human compactness and spatial diversity

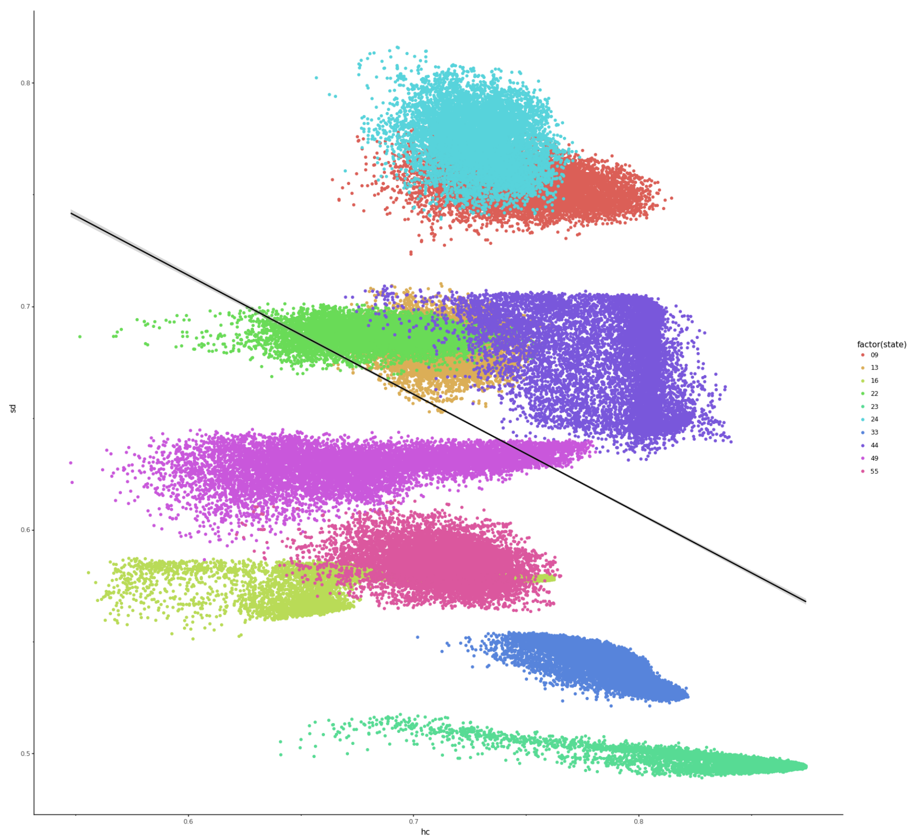


Figure 13: Aggregating the individual states gives an inflated estimate of the effect of compactness and commits the ecological fallacy

Dep. Variable:	sd	**R-squared:**	0.988
Model:	OLS	**Adj. R-squared:**	0.988
Method:	Least Squares	**F-statistic:**	8.188e+05
Date:	Tue, 10 Mar 2020	**Prob (F-statistic):**	0.00
Time:	16:04:33	**Log-Likelihood:**	3.2365e+05
No. Observations:	100000	**AIC:**	-6.473e+05
Df Residuals:	99989	**BIC:**	-6.472e+05
Df Model:	10		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
Intercept	0.7837	0.001	1042.069	0.000	0.782	0.785
C(district)[T.13]	-0.0725	0.000	-520.351	0.000	-0.073	-0.072
C(district)[T.16]	-0.1783	0.000	-1255.474	0.000	-0.179	-0.178
C(district)[T.22]	-0.0687	0.000	-460.458	0.000	-0.069	-0.068
C(district)[T.23]	-0.2533	0.000	-1540.668	0.000	-0.254	-0.253
C(district)[T.24]	0.0194	0.000	142.303	0.000	0.019	0.020
C(district)[T.33]	-0.2132	0.000	-1534.138	0.000	-0.213	-0.213
C(district)[T.44]	-0.0763	0.000	-549.486	0.000	-0.077	-0.076
C(district)[T.49]	-0.1276	0.000	-843.636	0.000	-0.128	-0.127
C(district)[T.55]	-0.1698	0.000	-1216.197	0.000	-0.170	-0.170
hc	-0.0404	0.001	-40.632	0.000	-0.042	-0.038

Omnibus:	3979.140	**Durbin-Watson:**	1.171
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	9332.569
Skew:	-0.236	**Prob(JB):**	0.00
Kurtosis:	4.420	**Cond. No.**	53.4

Table 1: OLS Regression of Spatial Diversity on Human Compactness with Country Dummies

The average spatial diversity of top plans under human compactness is significantly lower than the average spatial diversity of top plans under other compactness metrics

Do a t-test

Means:

```
Mean SD of top plans under Human Compactness: 0.635558
Mean SD of top plans under Polsby-Popper: 0.640954
Mean SD of top plans under Reock: 0.639897
Mean SD of top plans under Convex Hull: 0.639985
```

Top 10%:

Top 5%:

```
Ttest_indResult(statistic=array([-3.16361084]), pvalue=array([0.00156292]))
```

```
Ttest_indResult(statistic=array([-2.53127357]), pvalue=array([0.01138011]))
Ttest_indResult(statistic=array([-2.57101923]), pvalue=array([0.0101543]))
```

Top 2%:

Appendix A

Appendix B: Results of difference-in-means tests for individual states

Here I compare the average spatial diversity of all 10,000 plans per state to the average spatial diversity of the 500 most compact plans per state.

I present the results for each state and each metric in the ensemble, using Welch's t-test.

```
0 Connecticut
hc
Ttest_indResult(statistic=array([-6.49740127]), pvalue=array([8.53280274e-11]))
pp
Ttest_indResult(statistic=array([0.86781049]), pvalue=array([0.3855318]))
reock
Ttest_indResult(statistic=array([0.83762575]), pvalue=array([0.40227492]))
ch
Ttest_indResult(statistic=array([-1.68248732]), pvalue=array([0.09251047]))
1 Georgia
hc
Ttest_indResult(statistic=array([-1.88909516]), pvalue=array([0.05904629]))
pp
Ttest_indResult(statistic=array([-3.3741558]), pvalue=array([0.00075958]))
reock
Ttest_indResult(statistic=array([0.39887338]), pvalue=array([0.69005064]))
ch
Ttest_indResult(statistic=array([-3.13625521]), pvalue=array([0.00174518]))
2 Idaho
hc
Ttest_indResult(statistic=array([10.42666125]), pvalue=array([2.16882037e-25]))
pp
Ttest_indResult(statistic=array([5.86780509]), pvalue=array([4.9782383e-09]))
reock
Ttest_indResult(statistic=array([5.33166925]), pvalue=array([1.24215121e-07]))
ch
Ttest_indResult(statistic=array([0.03089443]), pvalue=array([0.9753569]))
3 Louisiana
```

```
hc
Ttest_indResult(statistic=array([-3.55779128]), pvalue=array([0.00037512]))
pp
Ttest_indResult(statistic=array([-3.21856506]), pvalue=array([0.00129316]))
reock
Ttest_indResult(statistic=array([-1.77611238]), pvalue=array([0.07574051]))
ch
Ttest_indResult(statistic=array([-2.95273326]), pvalue=array([0.00315286]))
4 Maine
hc
Ttest_indResult(statistic=array([-13.2240253]), pvalue=array([9.28561364e-40]))
pp
Ttest_indResult(statistic=array([24.8919901]), pvalue=array([3.52063173e-110]))
reock
Ttest_indResult(statistic=array([11.72570569]), pvalue=array([1.36024936e-31]))
ch
Ttest_indResult(statistic=array([-13.6832433]), pvalue=array([1.9646117e-42]))
5 Rhode_Island
hc
Ttest_indResult(statistic=array([-26.07171472]), pvalue=array([3.68436833e-134]))
pp
Ttest_indResult(statistic=array([13.82562159]), pvalue=array([8.51520228e-42]))
reock
Ttest_indResult(statistic=array([5.01734031]), pvalue=array([5.92797527e-07]))
ch
Ttest_indResult(statistic=array([5.64708581]), pvalue=array([1.93856794e-08]))
6 Maryland
hc
Ttest_indResult(statistic=array([-7.72406455]), pvalue=array([1.8720555e-14]))
pp
Ttest_indResult(statistic=array([-10.69599062]), pvalue=array([7.44090023e-26]))
reock
Ttest_indResult(statistic=array([-11.43688714]), pvalue=array([2.07114299e-29]))
ch
Ttest_indResult(statistic=array([-2.44035376]), pvalue=array([0.01480235]))
7 New_Hampshire
hc
Ttest_indResult(statistic=array([-77.79139703]), pvalue=array([0.]))
pp
Ttest_indResult(statistic=array([5.66328049]), pvalue=array([2.13182847e-08]))
reock
Ttest_indResult(statistic=array([-5.62644692]), pvalue=array([2.32800657e-08]))
ch
Ttest_indResult(statistic=array([10.76371698]), pvalue=array([4.81248181e-25]))
8 Utah
hc
```

```
Ttest_indResult(statistic=array([18.41406631]), pvalue=array([8.86499654e-75]))
pp
Ttest_indResult(statistic=array([8.17458673]), pvalue=array([3.54250317e-16]))
reock
Ttest_indResult(statistic=array([6.8129706]), pvalue=array([1.05271654e-11]))
ch
Ttest_indResult(statistic=array([7.40371108]), pvalue=array([1.49308888e-13]))
9 Wisconsin
hc
Ttest_indResult(statistic=array([-6.45251298]), pvalue=array([1.16333085e-10]))
pp
Ttest_indResult(statistic=array([-2.37112098]), pvalue=array([0.01776865]))
reock
Ttest_indResult(statistic=array([-0.73082387]), pvalue=array([0.46492516]))
ch
Ttest_indResult(statistic=array([-2.87931061]), pvalue=array([0.00400501]))
```

Very similar paper

We posit that this is due to thepolitical geographies of the two states, and examining this effect isan important thread for understanding what kinds of reforms mightor might not be effective in various jurisdictions. Future work coulduse more sophisticated mathematical and statistical techniques todescribe a relationship between political geography and the trade-offs we consider here. Our analysis suggests that a one-size-fits-allapproach to drawing 'fair' districts is inappropriate and that indi-vidual states and localities should carefully consider the relevanttrade-offs when redistricting or implementing redistricting reformini-tiatives. One factor ignored in this analysis, which is critical to theprocess of drawing districts, isrespecting communities-of-interest.Even defining and locat-ing geographically such communities is avery difficult problem, let alone the determination of whether ornot to preserve that group in a single district. We therefore pro-pose our analysis as a framework for discussion about trade-offs inredistricting rather than as a policy recommendation.In this work, we have demonstrated with a simple model thatdemanding districts be drawn to be as compact as possible anddemanding that they satisfy a notion of partisan symmetry areincompatible, but to different degrees depending on the particu-larfeatures of the geographic region in question. Since existing propos-als and methodologies for automated and algorithmic redistrictinginvolve finding an approximate solution to an optimization problem,it is important to understand how changing the objective functionof these procedures can affect the outcome. As more jurisdictionsconsider redistricting reforms, they should be cautious about abdi-cating the line drawing process to algorithms which encode valuesdifferent from those of the voters who use the districts to elect theirrepresentatives.

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