Quantitative Economics Lecture 1 - Introduction to Quantitative Economics and Probability

Richard Povey

University of Oxford

23rd April 2018

richard.povey@hertford.ox.ac.uk, richard.povey@st-hildas.ox.ac.uk

With thanks to Ian Crawford, Margaret Stevens, Vanessa Berenguer-Rico.

- **Aims**:
  - Testing theories
  - Quantifying / calibrating models
  - Forecasting
  - Policy recommendations
- **Main Basic toolkit**:
  - Probability
  - Statistics

Important:

Not only Economics!

Probability and Statistics are central to Science!

Applications of Probability and Statistics in Economics

- **Econometrics** - "Economic measurement"
- **Microeconometrics** - Analysis of microeconomic data concerning many distinct individual agents
  - **Randomised Controlled Trials : Empirical Programme Evaluation**
    - **Public Economics / Public Policy**
    - **Development Economics**
  - **Competition Policy** - e.g. analysing market structure and outcomes
  - **Fiscal Policy Analysis** - e.g. impact of policies such as taxation on income distribution and social welfare

Applications of Probability and Statistics in Economics

- **Macroeconometrics / Time Series Econometrics** - Analysis of macroeconomic variables over time
    - **Monetary Policy**
    - **Empirical Finance**
    - **Growth Theory**

Introduction to Probability and Statistics

- This is the first of 6 lectures on probability and statistics.

- **Problem**: How to model environments with chance/randomness/stochasticity.
- **Unpredictable outcomes**: tossing a coin, gender of a new born, grade on an exam, first salary after graduation, etc...

- **Probability Theory**: Mathematical formalisation of randomness

---

Prerequisites:

- Differential and integral calculus.
- If you are rusty or not confident with integration, review chapter 10 of the "Maths Workbook".

Introduction to Probability and Statistics

Material based primarily on:

- "Introduction to Probability," by Hoel, Port, Stone
- "Introduction to Econometrics," by Stock and Watson

However, there are other good introductory books on statistics:

- R. L. Thomas, "Using Statistics in Economics", Prerequisites, and Chapters 1, 2
- T. H. Wonnacott and R. J. Wonnacott, "Introductory Statistics for Business and Economics", Chapters 1-5
- Sheldon Ross, "A First Course in Probability", Chapters 1-7
- ... and there are many others out there.

Objectives for Lectures 1 to 6

Primary Objective

- Lay conceptual foundations for remainder of Quantitative Economics course.

Secondary Objectives

- Material hopefully useful for expected utility theory in micro.
- ... and inter-temporal consumption macro topic with future uncertainty.

- We will not be able to cover everything, so you will need to read more broadly to fill in some of the details. We will, however, aim to provide a good overview of how the material fits together.
- 2 problem sets are available on Weblearn containing a variety of applications of this material. Your College tutors will tell you which problems from the problem sets to attempt for which weeks.

- Reality (physical and/or social) might be *fundamentally* stochastic (e.g. quantum mechanics)
- ... or it may just be that our *knowledge* about a deterministic reality is imperfect (e.g. the order of cards in poker is determined once the cards are shuffled, but we do not *know* this order, so we face a stochastic variable for each card until it is revealed).

Causal Explanation

- Statistics show us connections between observable variables. This is necessary but not sufficient to establish *causality*.
- This is where *economic theory* comes in, giving us models of the causal mechanisms underlying or generating the statistical relationships.

Sample Space and Events

Consider an experiment whose outcome is not predictable with certainty:

- **Sample Space:** the set of possible outcomes (Ω). Elements in Ω will be denoted ω.
- **Event**: a subset of the sample space (A)

Examples: Sample Space: set of possible outcomes (discrete)

- Toss a coin: $\Omega = \{H, T\}$
- Flip two coins: $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$
- Throw a die: $\Omega = \{1, 2, 3, 4, 5, 6\}$
- Tossing two dice: $\Omega = \{(i, j) : i, j = 1, 2, 3, 4, 5, 6\}$

Sample Space and Events

Examples: Events: Any subset of the sample space

- Flipping two coins: $A = \{(H, H), (H, T)\}$ i.e., event that a head appears on the first coin
- Tossing two dice: $A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$, i.e., event that the sum of the dice equals 7

Some Special Events:

- Elementary Events: each outcome in the sample space
- Null Event ($\varnothing$): the event consisting of no outcomes
- Certain Event (Ω): the whole sample space

Continuous and Discrete Random Variables

- **Discrete r.v.** takes on a countable number of possible values.
- **Examples:** number of heads when tossing a coin, sum of points when throwing 2 dice, selecting ball from an urn, buying a good or not, etc...
- **Continuous r.v.** takes on a continuum of possible values
- **Examples:** waiting time of a bus, the life time of a transistor, height of a person, the wage of a worker, etc...

Examples: Sample Space: set of possible outcomes (continuous)

- Time that it takes for a particle to decay: $\Omega = [0, \infty)$
- Choose a point at random from a subset of $\Omega = [a, b]$

Probability Space

- Probability Space: $(\Omega, \mathcal{A}, P)$
  - Ω : Sample Space
  - $\mathcal{A}$: Collection of events to which we assign probabilities.
  - P: is a probability measure on $\mathcal{A}$

- How to choose $\mathcal{A}$?
- $\mathcal{A}$: should be a non-empty collection of subsets of Ω so that:
  - If A is in $\mathcal{A}$ so is A^c
  - If A and B are in $\mathcal{A}$ so are $A \cup B$ and $A \cap B$

Definition

- A non-empty collection of subsets $\mathcal{A}$ of Ω is called a σ-field of subsets of Ω provided that:
  - If A is in $\mathcal{A}$, then A^c is also in $\mathcal{A}$
  - If A_n is in $\mathcal{A}$ for $n = 1, 2, ...$ then $\cup_{n=1}^{\infty} A_n$ and $\cap_{n=1}^{\infty} A_n$ are both in $\mathcal{A}$

Probability Space

- Assignment of Probabilities: P

Definition

- A **probability measure** P on a σ-sigma field of subsets $\mathcal{A}$ of a set Ω is a real-valued function having domain $\mathcal{A}$ satisfying:
  - $P(\Omega) = 1$
  - $P(A) \geq 0$ for all A in $\mathcal{A}$
  - If A_n for $n = 1, 2, 3...$ are mutually disjoint sets in $\mathcal{A}$ then

$$P(\cup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} [P(A_n)]$$

Definition

- A **probability space**, denoted $(\Omega, \mathcal{A}, P)$, is a set Ω, a σ-field of subsets $\mathcal{A}$, and a probability measure P defined on $\mathcal{A}$

Discrete Random Variables

Example: Experiment - Tossing a coin

- Toss a coin where the probability of a head is $1/2$
- If head we win £1, if tail nothing. Quantity of interest: X =total winnings.
- X can take two values: $X = 1$, $X = 0$

A	$X(A)$	$P(A)$
H	1	$1/2$
T	0	$1/2$

- Event $\{\omega : X(\omega) = 1\}$ corresponds to: $A_1 = \{H\}$
- $P(\{\omega : X(\omega) = 1\}) = P(A_1) = P(X = 1) = 1/2$

Discrete Random Variables

Example: Experiment -Throw two dice

- Throw two dice where the probability of any outcome on an individual die is 1/6. X =sum of the outcomes from each die

A	$X(A)$	$P(A)$
$A_1 = \{(1,1)\}$	2	1/36
$A_2 = \{(1,2),(2,1)\}$	3	2/36
$A_3 = \{(1,3),(2,2),(3,1)\}$	4	3/36
$A_4 = \{(1,4),(2,3),(3,2),(1,4)\}$	5	4/36
$A_5 = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$	6	5/36
$A_6 = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$	7	6/36
$A_7 = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$	8	5/36
$A_8 = \{(3,6),(4,5),(5,4),(6,3)\}$	9	4/36
$A_9 = \{(4,6),(5,5),(6,4)\}$	10	3/36
$A_{10} = \{(5,6),(6,5)\}$	11	2/36
$A_{11} = \{(6,6)\}$	12	1/36

- Event $\{\omega : X(\omega) = 7\}$
- Corresponds to: $A_6 = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$
- $P(\{\omega : X(\omega) = 7\}) = P(A_6) = P(X = 7) = 6/36$

Discrete Random Variables

- We would like to formally define random variables on a probability space $(\Omega, \mathcal{A}, P)$, i.e. talk about the probability that $P(X = x_i)$ for each i. To do so we need to know that for each i, $\{\omega \in \Omega : X(\omega) = x_i\}$ is an event (i.e. is a member of $\mathcal{A}$) because then we can use the probability measure P. (Note: If $\mathcal{A}$ is a σ-field of all subsets of Ω then this is indeed the case.)
- **Solution**: Define a random variable X as a function on Ω for which this property holds.

Definition

A discrete real-valued random variable X on a probability space $(\Omega, \mathcal{A}, P)$ is a function with domain Ω and range of a finite or countably infinite subset $\{x_1, x_2, ...\}$ of the real numbers $\mathbb{R}$ such that $\{\omega \in \Omega : X(\omega) = x_i\}$ is an event for all i.

Discrete Random Variables

Example: Experiment - Flip two coins

- $\Omega = \{HH, HT, TH, TT\}$
- $\mathcal{A} = \{\Omega, \varnothing, \{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\}, \{HH, TH\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\}, \{TH, TT\}, \{HH, HT, TH\}, \{HH, HT, TT\}, \{HH, TH, TT\}, \{HT, TH, TT\}\}$: power set: $2^4 = 16$ elements
- $P\{\Omega\} = 1$, $P\{\varnothing\} = 0$, $P\{HH\} = 1/4, ...,$ $P\{HH, HT\} = 1/2$, $P\{HT, TH, TT\} = 3/4...$
- X : number of heads we obtain when flipping two coins.
- Then $P(X = 2) = 1/4$, $P(X = 1) = 1/2$, and $P(X = 0) = 1/4$

Discrete Random Variables

Example: Experiment - Throw two dice

- $\Omega = \{(1,1), (1,2), ..., (6,6)\}$ (36 elementary events)
- $\mathcal{A}$: power set $2^{36} = 68719476736$
- $P\{\Omega\} = 1$, $P\{\varnothing\} = 0$, $P\{(1,1)\} = 1/36, ...,$
  $P\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} = 6/36, ...$
- X : sum of the outcomes from each die
- $P(X = 1) = 1/36$, $P(X = 7) = 6/36, ...$

Next Lecture Tomorrow : Probability Distributions