QE Tutorial 6

1. Let $\{\varepsilon_t\}$ be *i.i.d.*, with mean 0 and variance σ^2. Consider the following stochastic process $\{x_t\}$ derived from $\{\varepsilon_t\}$:

$$x_t = \alpha + \beta x_{t-1} + \varepsilon_t, \text{ for } t = 1, 2, ... \tag{1}$$

where α, β are constants and x_0 is specified below.

   (a) Show that

$$x_t = \frac{1-\beta^t}{1-\beta}\alpha + \beta^t x_0 + \sum_{j=0}^{t-1} \beta^j \varepsilon_{t-j}.$$

   [Hint: you can do that either by backward substitution, using the fact that $\sum_{i=0}^{n-1} \rho^i = (1-\rho^n)/(1-\rho)$, when $\rho \neq 1$, or by evaluating the above equation at $t-1$ and substituting for x_t in (1).]

   (b) Suppose $|\beta| < 1$ and x_0 is a random variable with mean $\alpha/(1-\beta)$ and variance $\sigma^2/\left(1-\beta^2\right)$, uncorrelated with ε_t, for all $t > 0$. Find the mean, variance and autocovariance function of x_t for any $t \geq 1$? Is the process (weakly) stationary?

   (c) Suppose $x_0 = 0$ and $\beta = 1$.

      i. Express x_t as a moving average of $\varepsilon_1, ..., \varepsilon_t$. [Hint: use result in part a.]

      ii. Hence, derive the mean and variance of x_t. Is x_t stationary?

   (d) Obtain the minimum mean square error j-step ahead forecast of x, $E(x_{t+j}|x_t, x_{t-1}, ...)$, and the associated mean squared error when $\beta \neq 1$.

2. A researcher estimated the following AR(1) model for inflation over the period 1974q1 to 2017q1 (173 observations)

$$\pi_t = \underset{(0.21)}{0.68} + \underset{(0.04)}{0.84}\ \pi_{t-1} + \widehat{v}_t. \tag{2}$$

Numbers in parentheses are standard errors. Inflation is measured as $400 \times \Delta \ln P_t$, where P_t is the price level.

   (a) Perform a 5% level test of the hypothesis that lagged inflation is useful in forecasting future inflation, stating clearly the null and alternative hypotheses of the test.

   (b) Suppose inflation in 2017q1 is 2%. Obtain a one-step ahead minimum mean square forecast error (MSFE) forecast of inflation for 2017q2 based on the results in equation (2). Do you have any concerns about the reliability of your forecast? [Hint: think about possible biases.]

(c) Another researcher estimated the following AR(4) model instead:

$$\Delta\pi_t = \underset{(0.206)}{0.467} - \underset{(0.041)}{0.107}\ \pi_{t-1} - \underset{(0.079)}{0.187}\ \Delta\pi_{t-1} - \underset{(0.076)}{0.254}\ \Delta\pi_{t-2} + \underset{(0.075)}{0.196}\ \Delta\pi_{t-3} + \widehat{v}_t. \tag{3}$$

On the basis of the above estimates, she thinks model (3) should produce a more accurate forecast of inflation than (2). Why?

(d) Test model (3) against the restricted model "$\Delta\pi_t$ is an AR(3)." Carefully state the null and alternative hypotheses.[Hint: you should not use Normal critical values for this test.]

(e) The researcher also had data on unemployment over the same period. She estimated two alternative models for $\Delta\pi$: an AR(3) and an ADL(3,3). Both models included a constant term and their R^2 was 0.200 and 0.344, respectively. Based on this information, test whether unemployment Granger causes inflation. [Hint: note that the F statistic for testing a restricted model with $R^2 = R_r^2$, against an unrestricted model with $R^2 = R_u^2$, is $F = \frac{\left(R_u^2 - R_r^2\right)/(k_u - k_r)}{(1-R_u^2)/(T-k_u)}$, where $k_r < k_u$ are the parameters of the restricted and unrestricted models, respectively.]

3. The following equations present results from regressions of log per capita real tobacco expenditure ($LTOB_t$) on an intercept, log per capita real personal disposable income (LY_t) and the log of the relative price of tobacco (LP_t) estimated by OLS using annual data for the UK over the period 1963 to 1988.

$$LTOB_t = \underset{(0.0438)}{-1.1888} \underset{(0.0330)}{-0.6109}\ LY_t \underset{(0.0544)}{-0.6633}\ LP_t + \widehat{u}_t$$

$$R^2 = 0.9577, RSS_0 = \sum_{t=1}^{T} \widehat{u}_t^2 = 0.0171.$$

Figures in parenthesis are estimated standard-errors. RSS stands for "Residual Sum of Squares." The corresponding regression model is

$$LTOB_t = \beta_0 + \beta_1 LY_t + \beta_2 LP_t + u_t \tag{4}$$

where u_t is assumed to be a normally distributed random variable with mean zero and variance σ^2, and $E\left(u_t | LY_t, LP_t\right) = 0$.

(a) Give a structural interpretation to the coefficients β_1 and β_2. Do the signs and magnitudes of the estimates accord with economic theory?

(b) Consider the following alternative model

$$LTOB_t = \gamma_0 + \lambda LTOB_{t-1} + \gamma_1 LY_t + \gamma_2 LP_t + v_t. \tag{5}$$

What are the long-run and short run income and price elasticities of tobacco expenditure (hint: think about dynamic multipliers)?

(c) In the light of model (5), explain the potential pitfalls in the interpretation of the coefficient in model (4). Moreover, discuss when the estimation results presented above for model (4) would be spurious. Hence, what additional information would you like to see reported in order to assess the reliability of the above estimates?

(d) The following results are for OLS estimates of model (4) over subsamples

$$\underset{}{LTOB_t} = \underset{(0.1462)}{-1.2035} \underset{(0.1447)}{-0.5958}\, LY_t \underset{(0.1914)}{-0.6625}\, LP_t + \widehat{u}_t$$
$$R^2 = 0.5416, RSS_1 = 0.0117, t = 1963, ..., 1980.$$

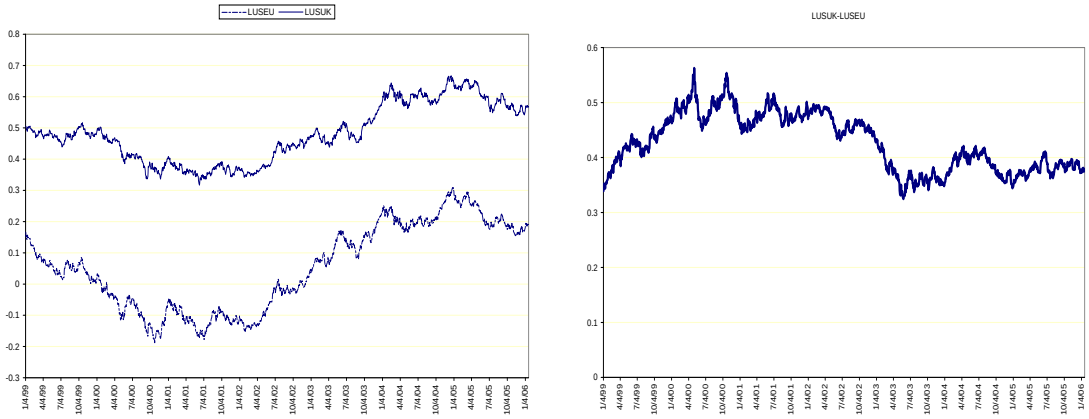
$$\underset{}{LTOB_t} = \underset{(0.1844)}{-1.9077} \underset{(0.1247)}{-0.1246}\, LY_t \underset{(0.1159)}{-0.8284}\, LP_t + \widehat{u}_t$$
$$R^2 = 0.9796, RSS_2 = 0.0007082, t = 1981, ..., 1988.$$

How would you assess the effect of anti-smoking propaganda during the 1980's? Do you find any evidence of that?
You may use the fact that

$$F_{chow} = \frac{\left(RSS_0 - \left(RSS_1 + RSS_2\right)\right)/K}{\left(RSS_1 + RSS_2\right)/\left(T - 2K\right)} \sim F_{K,T-2K}$$

where T is the sample size, K is the number of estimated parameters in the model, and $F_{q,p}$ denotes the F distribution with degrees of freedom q and p.

4. The time series of the log of the daily US-Euro and US-Pound exchange rates, LUSEU and LUSUK, and their difference is plotted in the following graph.



The following table reports ADF tests using 22 lagged differences for each series:

Series	Constant only	Constant + Trend
LUSUK	-0.827	-1.939
LUSEU	-0.868	-2.427
ΔLUSUK	-9.342	-9.383
ΔLUSEU	-7.854	-7.944
LUSUK-LUSEU	-1.671	-3.113
ΔLUSUK-ΔLUSEU	-9.675	-9.759

CVs: Constant -2.86 (5%), -3.44 (1%), Const+Trend: -3.41 (5%), -3.97 (1%)

(a) Determine the order of integration of LUSUK and LUSEU.

(b) Determine the order of integration of the Euro-Pound exchange rate. What does this result tell you about the independence of UK monetary policy relative to the Euro area?

(c) A financial analyst runs the following regressions (standard errors in parentheses):

$$\widehat{\text{LUSEU}}_t = \underset{(.004)}{-0.63} + \underset{(.009)}{-1.42}\,\text{LUSUK}_{t-1}, \quad R^2 = 0.93$$

$$\widehat{\text{LUSEU}}_t = \underset{(.005)}{-0.63} + \underset{(.010)}{-1.41}\,\text{LUSUK}_{t-5}, \quad R^2 = 0.92$$

$$\widehat{\text{LUSEU}}_t = \underset{(.006)}{-0.61} + \underset{(.012)}{-1.38}\,\text{LUSUK}_{t-22}, \quad R^2 = 0.87$$

He claims that these results show that the dollar-euro exchange rate can be successfully predicted by the dollar-pound exchange rate at various horizons (one day, one week or even one month in advance). He bases this conclusion on the very high t-statistics on the predictor and the very high R^2 of each regression. Do you agree with this conclusion?

5. Suppose you work for the Treasury and you are asked to evaluate the impact of a possible fiscal contraction of 1% of GDP on the economy. You have quarterly UK data on real GDP and real government deficit as a fraction of real GDP for the past 60 years. Write down a model that you might use to estimate the dynamic fiscal multipliers, and critically discuss possible threats to its validity.