Homework 3

Fangda Li
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1 Methodology

1.1 Two-step Method for Projective and Affine Distortion Removal

Given an image that contains both projective and affine distortion, the two-step method first removes projective distortion by mapping the vanishing line in the image back to infinity, using homography matrix H_p. Then, with two pairs of physically orthogonal lines on the projective-distortion-removed image, the two-step method finds the homography matrix that removes affine distortion, H_a^{-1}. As a result, the final rectified image free of both projective and affine distortion is obtained by applying $H_a^{-1}H_p$ on the original image.

1.1.1 Projective Distortion Removal

To remove projective distortion, we need to first obtain two pairs of lines on the distorted image that are parallel in the physical world. Since the two pairs are not parallel on the distorted image because of projective distortion, the two lines in each pair intersect at a vanishing point. Next, on the distorted image, the two vanishing points form the vanishing line, which can be written in HC[1] as in Equation 1.

$$\overrightarrow{l} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \tag{1}$$

Finally, we obtain the transformation matrix H_p that maps $\overrightarrow{l}$ back to $\overrightarrow{l_\infty}$.

$$H_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix} \tag{2}$$

This can be verified by Equation 3.

$$H_p^{-T}\overrightarrow{l} = \begin{bmatrix} 1 & 0 & -l_1/l_3 \\ 0 & 1 & -l_2/l_3 \\ 0 & 0 & 1/l_3 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \overrightarrow{l_\infty} \tag{3}$$

[1] HC refers to Homogeneous Coordinate

1.1.2 Affine Distortion Removal

To remove affine distortion, we need to find the affine transformation matrix H_a, which transforms the world image into the affine distorted image. More specifically, H_a can be written as

$$H_a = \begin{bmatrix} A & \overrightarrow{t} \\ \overrightarrow{0} & 1 \end{bmatrix}. \tag{4}$$

To compute H_a, we now consider two physically orthogonal lines, $\overrightarrow{l} \perp \overrightarrow{m}$. Because of their orthogonality, the cosine of the angle θ between the two lines must be 0,

$$cos(\theta) = \frac{\overrightarrow{l}^T C^*_\infty \overrightarrow{m}}{\sqrt{(\overrightarrow{l}^T C^*_\infty \overrightarrow{l})(\overrightarrow{m}^T C^*_\infty \overrightarrow{m})}} = 0, \tag{5}$$

which implies,

$$\overrightarrow{l}^T C^*_\infty \overrightarrow{m} = 0, \tag{6}$$

where the dual degenerate conic is $C^*_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Let $\overrightarrow{l'} = H_a^{-T}\overrightarrow{l}$ and $\overrightarrow{m'} = H_a^{-T}\overrightarrow{m}$ be the affine transformed lines of $\leftrightarrow l$ and $\overrightarrow{m}$. Then we can substitute $\leftrightarrow l$ and $\overrightarrow{m}$ in Equation 6, which can be rewritten as,

$$\overrightarrow{l}^T C^*_\infty \overrightarrow{m} = \overrightarrow{l'}^T H_a C^*_\infty H_a^T \overrightarrow{m'} = \overrightarrow{l'}^T \begin{bmatrix} S & \overrightarrow{0} \\ \overrightarrow{0} & 0 \end{bmatrix} \overrightarrow{m'} = 0, \tag{7}$$

where the symmetric matrix $S = AA^T = \begin{bmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{bmatrix}$. Equation 7 essentially gives us one linear equation for solving A. As a result, with two orthogonal pairs, we a linear system of two equations for solving S (since the ratios matter, we can let $s_{22} = 1$).

Finally, we apply SVD on S to get $S = UDU^T$, where U and D can be further used in reconstructing A, as shown in Equation 8.

$$A = U\sqrt{D}U^T \tag{8}$$

Now we have obtained H_a.

1.2 One-step Method for Projective and Affine Distortion Removal

The affine distorted dual degenerate conics can represented by

$$C^{*'}_\infty = H_a C^*_\infty H_a^T = \begin{bmatrix} A & \overrightarrow{0} \\ v^T & 1 \end{bmatrix} \begin{bmatrix} I & \overrightarrow{0} \\ \overrightarrow{0}^T & 1 \end{bmatrix} \begin{bmatrix} A^T & v \\ \overrightarrow{0}^T & 1 \end{bmatrix} = \begin{bmatrix} AA^T & Av \\ v^T A^T & v^T v \end{bmatrix}. \tag{9}$$

Subsequently, if we are able to calculate $C^{*'}_\infty$, we can derive H_a by computing A and v in the representation of $C^{*'}_\infty$.

In order to compute $C_\infty^{*'}$, we first rewrite Equation 6 as following,

$$\overrightarrow{l}^T C_\infty^* \overrightarrow{m} = \overrightarrow{l}^T (H^{-1} C_\infty^* H^{-T}) \overrightarrow{m} = (H^{-T} \overrightarrow{l}^T) C_\infty^{*'} (H^{-T} \overrightarrow{m}) = \overrightarrow{l'}^T C_\infty^{*'} \overrightarrow{m'} = 0. \tag{10}$$

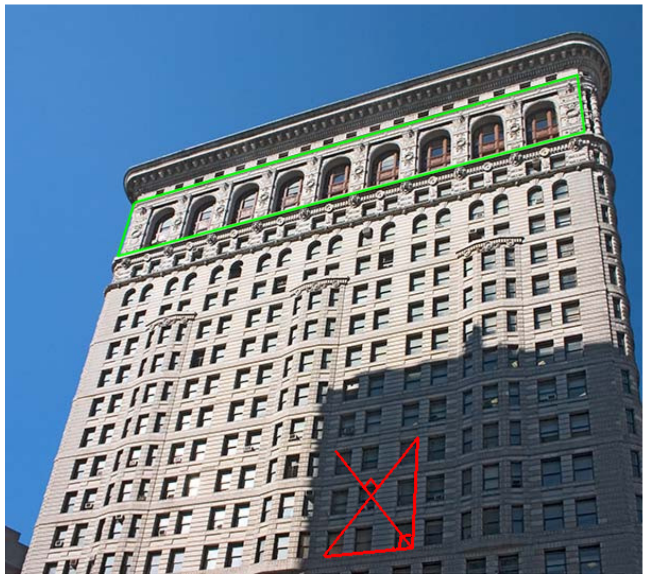
Since the HC representation of a conic can be written as $C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$, we can use 5 pairs of physically orthogonal line pairs of $(\overrightarrow{l}, \overrightarrow{m})$ in Equation 9 to compute the five unknowns in $C_\infty^{*'}$ (only ratio matters).

After we have obtained $C_\infty^{*'}$, we can go back to Equation 9 to compute A and v and finally H_a.

2 Results

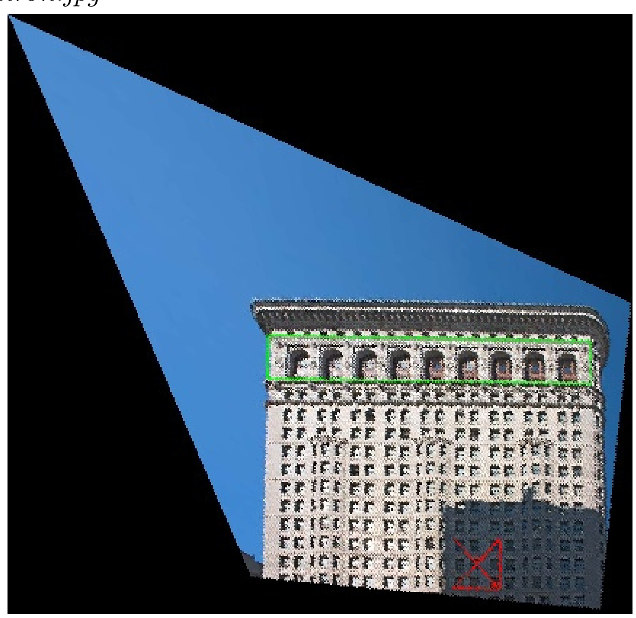
2.1 Two-step Method

Note that the rectangle used for vanishing line is marked in green. Orthogonal line pairs are marked in red.



(a) *flatiron.jpg*

(b) Perpective distortion rectified



(c) Affine distortion rectified

Figure 1: Two-step method for flatiron.jpg

2.2 One-step Method

Unfortunately my time ran out on this one.

2.3 Discussion

First, the one-step method requires 5 pairs of orthogonal lines to solve for H, while the two-step method requires only 3 pairs of lines, including two pairs of lines that form a rectangle and one orthogonal pair. As a result, from the user's perspective, the two-step method is easier to use. Second, it seems that the one-step method are more sensitive to the quality of input, since more line pairs picked by user means the process to be more error-prone. Also one can observe that faulty choices of orthogonal lines can significantly degrade the final result.