Homework 1

Fangda Li
ECE 661 - Computer Vision

August 29, 2016

Question 1.

Points in representational space $\Re^3$ that represent the origin in physical space $\Re^2$ are

$$\begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}, k \in \Re, k \neq 0$$

Question 2.

No, points can approach infinity form in different directions and they form a straight line in $\Re^2$.

Question 3.

A degenerated conics can be represented by summation of two outer products:

$$C = lm^T + ml^T$$

Since rows of each outer product matrix are linearly dependent, its rank must be 1. Therefore, the matrix of a degenerated conics, which is an summation of two rank 1 matrices, cannot exceed rank 2.

Question 4.

Each line can be represented by the cross product of its two points in HC. Then, the intersection of the two lines can be represented by the cross product of two lines, as shown below:

$$\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right) \times \left(\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} \frac{3}{4} \\ \frac{9}{8} \\ 1 \end{pmatrix}$$

The same procedure above applies to lines passing through any two given points.

Question 5.

The intersection p between the two lines can be found as following:

$$p = \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \right) \times \left(\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} -18 \\ -18 \\ 0 \end{pmatrix}$$

Since p is an idea point, the two lines intersect at infinity, meaning that they are *parallel*.

Question 6.

The implicit form for a conic is:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Our conic circle can be represented by $(x + 6)^2 + (y + 6)^2 = 1$, which can be expanded as shown below:

$$x^2 + y^2 + 12x + 12y + 71 = 0$$

where $a = 1$, $b = 0$, $c = 1$, $d = 12$, $e = 12$, $f = 71$. Then, the HC representation of our conic is:

$$C = \begin{pmatrix} a & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{pmatrix} = \begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 6 \\ 6 & 6 & 71 \end{pmatrix}$$

Next, the polar line l can be calculated by

$$l = Cx = \begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 6 \\ 6 & 6 & 71 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 71 \end{pmatrix}$$

Finally, the intersection between l and *x-axis* is:

$$\begin{pmatrix} 6 \\ 6 \\ 71 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{71}{6} \\ 0 \\ 1 \end{pmatrix}$$

Similarly, the intersection between l and *y-axis* is:

$$\begin{pmatrix} 6 \\ 6 \\ 71 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{71}{6} \\ 1 \end{pmatrix}$$