Cryptology

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Lecture 13

Digital Signatures

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- Signer uses secret key sk to create a signature σ on a message m.
- Verifier uses signer's public key pk to verify the signature σ on a message m.

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    5. Anyone including Alice canverify the $Cert$.

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  - MAC key is private.
  - Even if verifier provides MAC key, there is no way to prove that is the key.

Definition

A signature scheme $\mathfrak{S} = (\mathcal{G}, \mathcal{S}, \mathcal{V})$ is a triple of efficient algorithms, $\mathcal{G}$, $\mathcal{S}$ and $\mathcal{V}$, where $\mathcal{G}$ is called a key generation algorithm, $\mathcal{S}$ is called a signing algorithm, and $\mathcal{V}$ is called a verification algorithm. Algorithm $\mathcal{S}$ is used to generate signatures and algorithm $\mathcal{V}$ is used to verify signatures.

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- $\mathcal{V}$ is a deterministic poly-time algorithm invoked as $\mathcal{V}(pk, m, \sigma)$. It outputs either `accept` or `reject`.

Definition (Cont.)

- We require that a signature generated by $\mathcal{S}$ is always accepted by $\mathcal{V}$. That is, for all (sk, pk) output by $\mathcal{G}$ and all messages m, we have

$$\Pr[V(pk, m, S(sk, m)) = \texttt{accept}] = 1.$$

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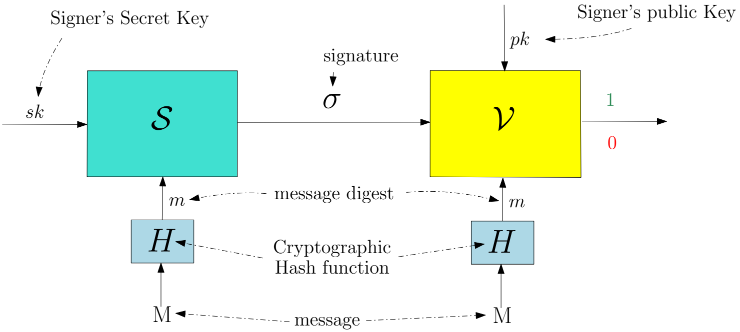
- Messages lie in a finite message space $\mathcal{M}$, and signatures lie in some finite signature space Σ. We say that $\mathfrak{S} = (\mathcal{G}, \mathcal{S}, \mathcal{V})$ is defined over $(\mathcal{M}, \Sigma)$.



Signer's Secret Key
sk
$\mathcal{S}$
signature
σ
Signer's public Key
pk
$\mathcal{V}$
1
0
m
message digest
m
H
Cryptographic
Hash function
H
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Existentially Unforgeable under a Chosen Message Attack

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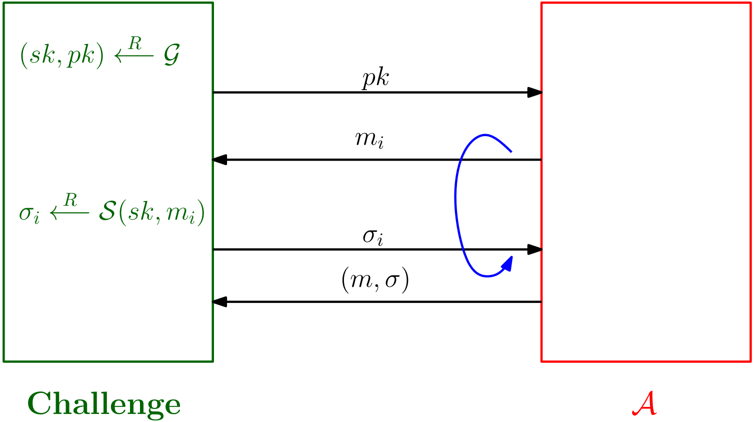
3. Eventually $\mathcal{A}$ outputs a candidate forgery pair $(m, \sigma) \in \mathcal{M} \times \Sigma$.

Security of Digital Signature



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Secure Digital Signature

A digital signature system $\mathfrak{S}$ is secure if for all efficient adversaries $\mathcal{A}$, the value $\mathsf{SIGadv}[\mathcal{A}, \mathfrak{S}]$ is negligible.

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$\mathcal{V}(pk, (m, \sigma) \in \mathbb{Z}_N \times \mathbb{Z}_N)$

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$$\sigma^e = (\sigma_1 \cdot \sigma_2)^e = \left(m_1^d \cdot m_2^d\right)^e = m_1^{ed} \cdot m_2^{ed} = m_1 \cdot m_2 = m \pmod{N}.$$

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- Let $H : \{0,1\}^* \longrightarrow \mathbb{Z}_N$ be a collision resistant hash function.

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Hashed RSA

Security of hashed RSA

It is possible to prove the security-of hashed RSA in an idealized model where H is a truly random function.

- Out-of-the-scope of the course.

- We will study the following three digital signatures whose security relies on the hardness of discrete logarithm problem.
  1. ElGamal Digital Signature,
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Correctness

$$a_1 \equiv g^m \equiv g^{tk+ds} = (g^k)^t (g^d)^s \equiv s^t Q^s.$$

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Correctness

$$u \equiv g^t Q^s \equiv g^t \left(g^d\right)^s \equiv g^{(t+ds)} \equiv g^{d'} \equiv Q'.$$

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\begin{aligned}
s' &= g^{w_1} Q^{w_2} \\
&= g^{w_1} g^{d w_2} \qquad \text{as } Q = g^d
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End