Cryptology

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Lecture 10

Cryptographic Hash

Introduction

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- Primary requirement: Avoid collisions.
- Note: Collisions do exists, but finding it should be hard.
- Collision-resistant hash functions have numerous uses, e.g., digital signature schemes, H-MAC etc.

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- But weaker assumption than the existence of PK encryptions.
- They have become ubiquitous in cryptography.
- Are often used in scenarios that require properties much stronger than collision resistance.

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- Compute $y = H(x)$ (message digest), typically short binary strings (e.g. 160 bits).
- Store y in a secure place.
- Suppose x is changed to x'.
- Then, as H is collision resistant, $y \neq y' = H(x')$.
- Therefore ensuring data integrity.

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- Authentication: Bob is assured that the message x originates from Alice.

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Security Notions

- Collision Reistance,
- Second-Preimage Resistance, and
- Preimage Resistance.

Collision Resistance Attack Game

For a keyed hash function H defined over $(\mathcal{K}, \mathcal{M}, \mathcal{T})$, and a given adversary $\mathcal{A}$, the attack game runs as follows

- The challenger picks a random $k \xleftarrow{R} \mathcal{K}$ and sends k to $\mathcal{A}$.
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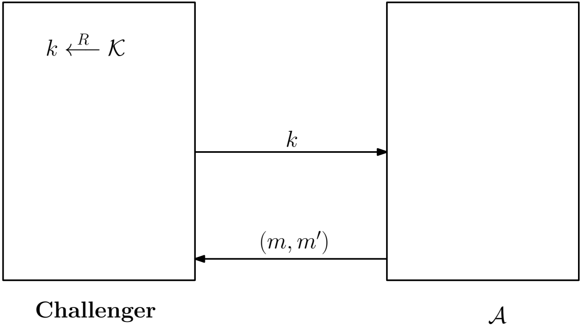

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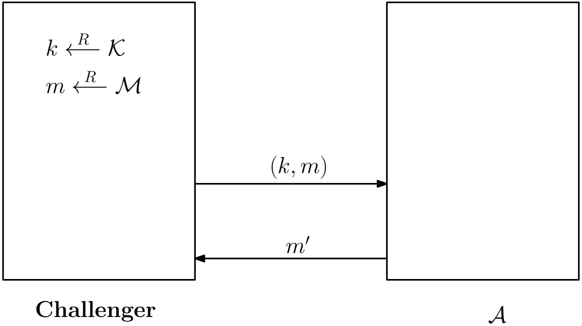

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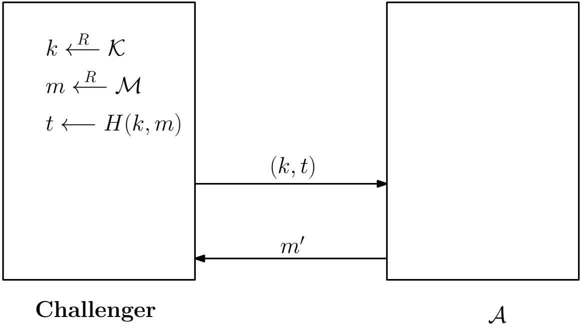
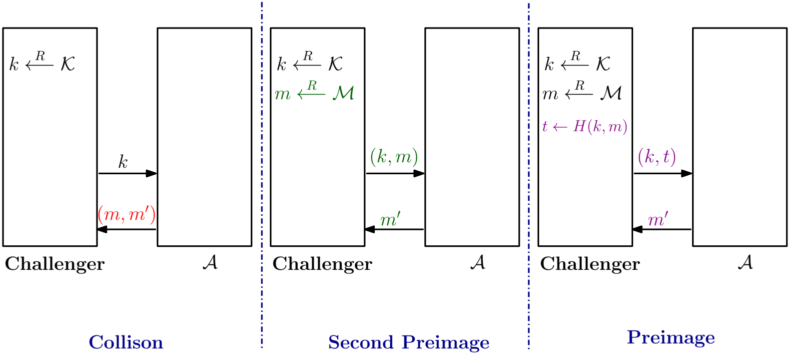


Collision Vs. Second Preimage Vs. Preimage



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- Let H is compressing, meaning that the domain of H is bigger than its range.
- Let $|\mathcal{M}| \geqslant s \cdot |\mathcal{T}|$ for some compression factor $s > 1$.
- If s is super-poly

$$\text{collision resistant} \Rightarrow \text{2nd-preimage resistant} \Rightarrow \text{one-way}.$$

- The converse is not true.

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- For example, SHA1 is believed to be 2nd-preimage resistant even though SHA1 is not collision resistant.

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- It is used extensively in practice.
- Examples: MD5 and the SHA family.
- Theoretical point of view: Compressing by a single bit is as easy (or as hard) as compressing by an arbitrary amount.

Assumptions

- Let $\hat{h}$ be a keyed hash function defined over $(\mathcal{K}, \mathcal{X}, \mathcal{T})$ as $\hat{h} : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{T}$.

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- Typically $\mathcal{X}$ is of the form $\{0, 1\}^{n}$ for some n.
- The Merkle-Damgård function derived from h, denoted H_{MD} defined over $\left(\{0, 1\}^{\leqslant L}, \mathcal{X}\right)$.

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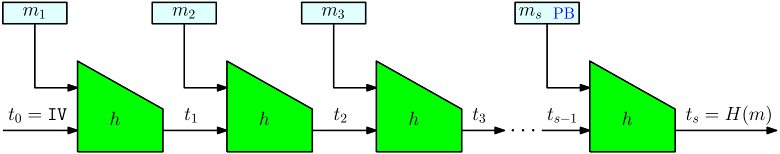

The Merkle-Damgård Transform

The Merkle-Damgård Paradigm

input: $M \in \{0,1\}^{\leqslant L}$
output: a tag in $\mathcal{X}$

1. $\hat{M} \longleftarrow M \| \mathtt{PB}$ //pad with $\mathtt{PB}$ to ensure that the length of M is a multiple of ℓ bits
2. partition $\hat{M}$ into consecutive ℓ-bit blocks so that
3. $\hat{M} = m_1 \| m_2 \| \cdots \| m_s$ where $m_1, m_2, \ldots, m_s \in \{0,1\}^{\ell}$.
4. $t_0 \longleftarrow IV \in \mathcal{X}$
5. for $i = 1$ to s do:
6. $t_i \longleftarrow h(t_{i-1}, m_i)$
7. output t_s

The Merkle-Damgård Paradigm

- The function SHA256 is a Merkle-Damgård function where $\ell = 512$ and $n = 256$.

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- The variables $m_1, m_2, \ldots, m_s$ are called message blocks.
- The variables $t_0, t_1, \ldots, t_s \in \mathcal{X}$ are called chaining variables.

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    - $\mathtt{PB} := 100\ldots00\|\langle s\rangle$.
    - $\langle s\rangle$ encodes the number of ℓ-bit blocks in M.
    - If the message length is such that there is no space for PB in the last block (for example, if the message length happens to be a multiple of ℓ), then an additional block is added just for the padding block.

Theorem

Let L be a poly-bounded length parameter and let h be a collision resistant hash function defined over $(\mathcal{X} \times \mathcal{Y}, \mathcal{X})$. Then the Merkle-Damgård hash function H_{MD} derived from h, defined over $(\{0,1\}^{\leqslant L}, \mathcal{X})$, is collision resistant.

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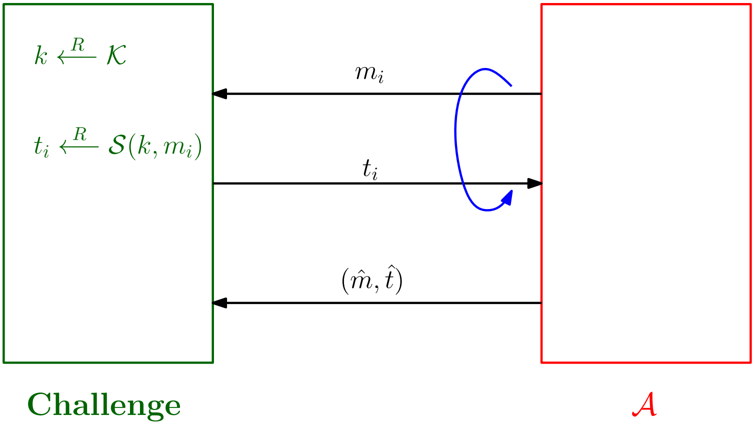
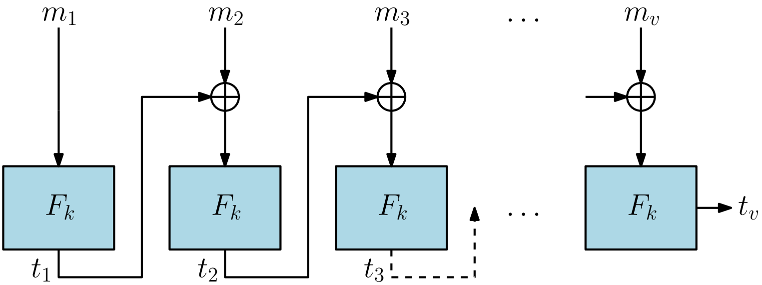
In particular, for every collision finder $\mathcal{A}$ attacking H_{MD} there exists a collision finder $\mathcal{A}$ attacking h, where $\mathcal{B}$ is an elementary wrapper around $\mathcal{A}$, such that

$$\mathsf{CRadv}[\mathcal{A}, H_{\text{MD}}] = \mathsf{CRadv}[\mathcal{B}, h].$$



MAC Attcak Game

Recap: CBC-MAC



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Attack on CBC-MAC

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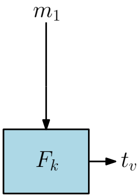
- Challenger returns (m_1, t_v).
- Adversary returns $(\hat{m}, \hat{t})$, where

$$\begin{aligned} \hat{m} &:= m_1 \| (m_1 \oplus t_v) \\ \hat{t} &:= t_v. \end{aligned}$$

CBC-MAC: Problems



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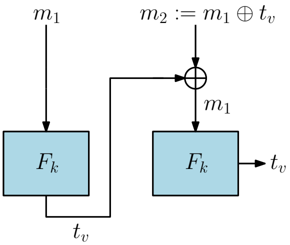

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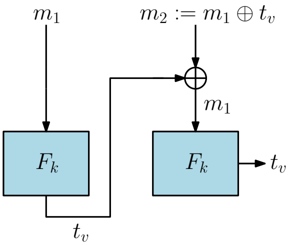

Correctness of the Attack

$$S(k,(m_1 \| m_2)) = F(k, m_2 \oplus F(k,m_1)) = F(k,(m_1 \oplus t_v) \oplus t_v) = F(k,m_1) = t_v.$$

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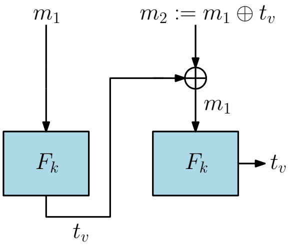

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Refusal

- Many practitioners refused to use CBC-MAC.
    - They claim it is also too slow.

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HMAC

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- Problem 1: H construction from h is a keyless hash function.
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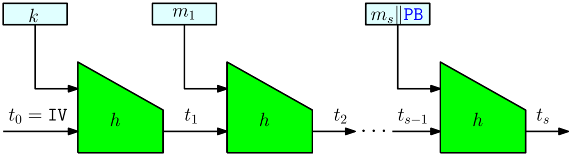
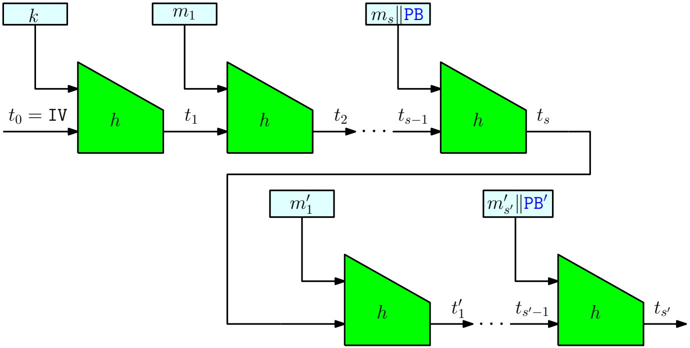
- A Common Solution: $S(k,m) := H(k\|m)$.
  - We can create a new tag on the message $(m\|PB\|m')$ where m' can be chosen randomly.

HMAC



HMAC


k
m_1
$m_s \| \mathsf{PB}$
$t_0 = \mathtt{IV}$
h
t_1
h
t_2
$\cdots$
t_{s-1}
h
t_s
m'_1
$m'_{s'} \| \mathsf{PB}'$
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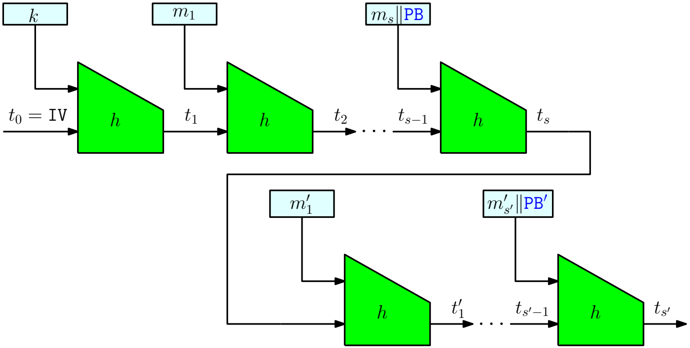
$\cdots$
$t_{s'-1}$
h
$t_{s'}$

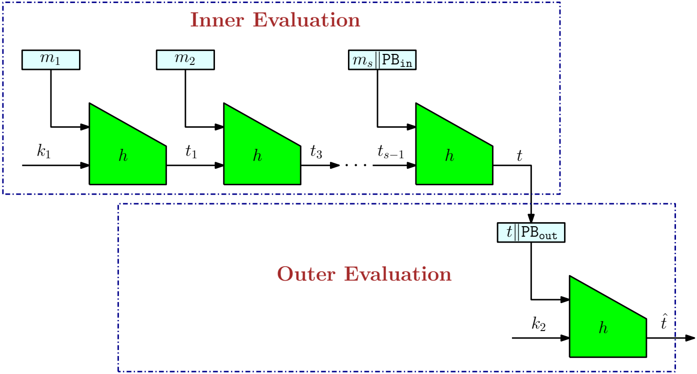
HMAC



HMAC

- One standard solution to the problem: A two keyed approach.
- One such standard construction is HMAC.
- Before HMAC, we will discuss something called Nested MAC or NAMC.

NMAC



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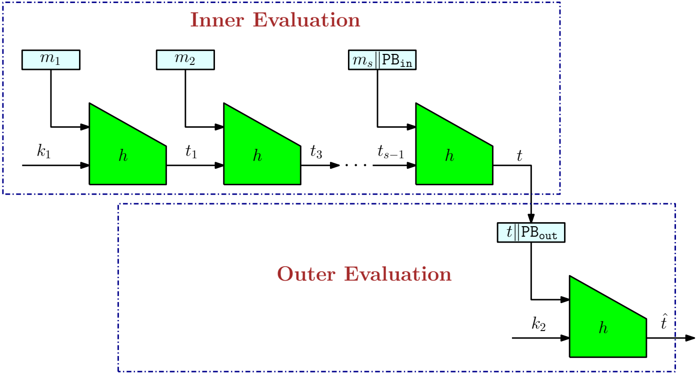
NMAC



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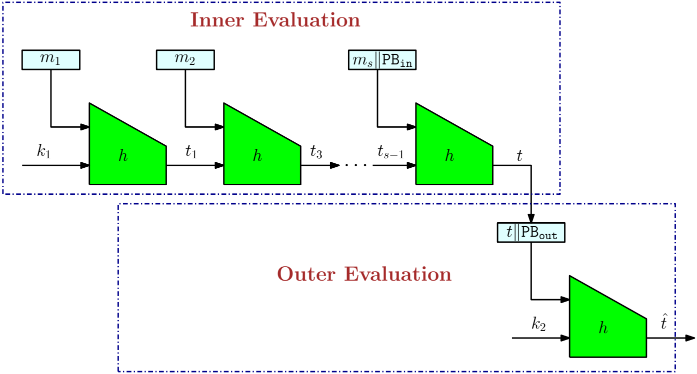

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$$\mathcal{S}((k_1, k_2), m) := H_{k_2}(H_{k_1}(m)),$$

where H_{k_i} is $H(\cdot)$ with $\mathtt{IV} = k_i$.

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- Verification is done in natural way.

Theorem

Let H denote the Merkle-Damgård transform applied to h and let $\mathcal{I}_\circ$ denote the fixed length MAC constructed from h. If h is collision resistant and $\mathcal{I}_\circ$ is a secure MAC, then NMAC is existentially unforgeable under an adaptive chosen-message attack (for arbitrary-length messages).

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- An efficient $\mathcal{A}$ adversary makes queries and based on the received queries, constructs the following lists:

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  4. Contradicts the fact $\mathcal{I}_\circ$ is a secure MAC.

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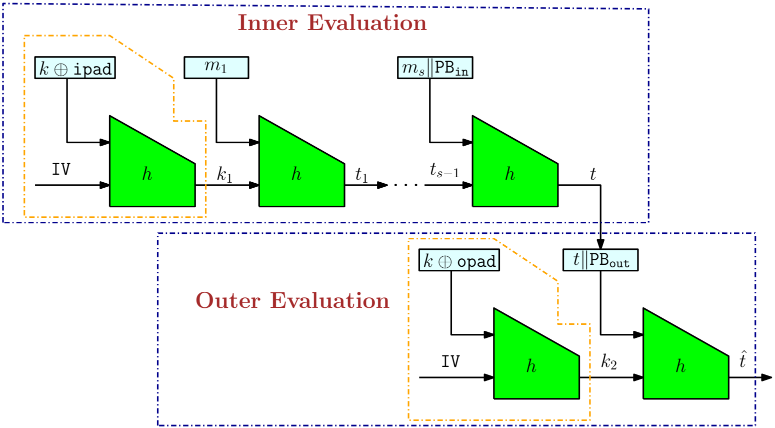

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- $\texttt{ipad}$ = the byte 0x36 repeated $\frac{\ell}{8}$ times.
- $\texttt{opad}$ = the byte 0x5C repeated $\frac{\ell}{8}$ times.

Theorem

Let us define $G(k)$ as

$$G(k) := h(\mathtt{IV} \| k \oplus \mathtt{ipad}) \| h(\mathtt{IV} \| k \oplus \mathtt{opad}) = k_1 \| k_2.$$

If h is collision resistant, and $\mathcal{I}_\circ$ is a secure MAC and if G is a pseudorandom generator, then HMAC is existentially unforgeable under an adaptive chosen-message attack (for arbitrary-length messages).

End