Cryptology

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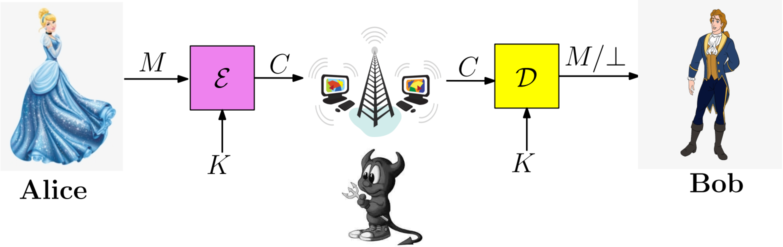
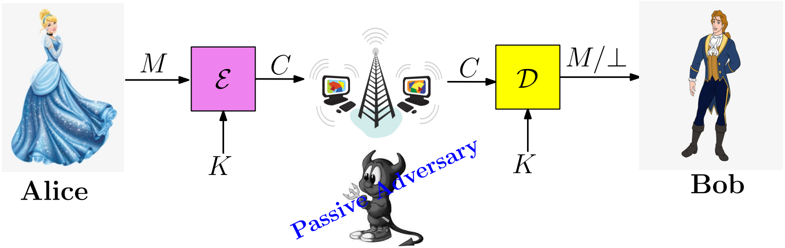
Lecture 09

Message Authentication Code

Secure Communication



Secure Communication



Alice

M

$\mathcal{E}$

K

C

Passive Adversary

C

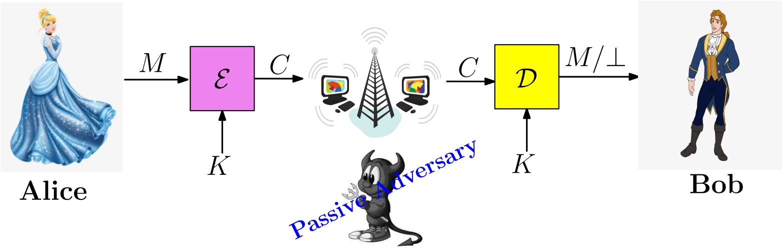
$\mathcal{D}$

K

$M/\perp$

Bob

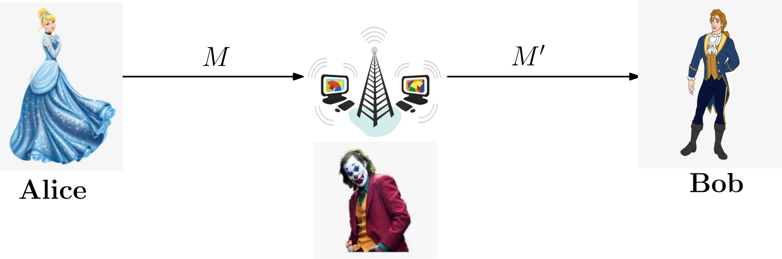
Secure Communication



Attack Models

1. Passive Adversary
2. Adversary must not learn anything about the message.

Message Integrity (or Message Authentication)



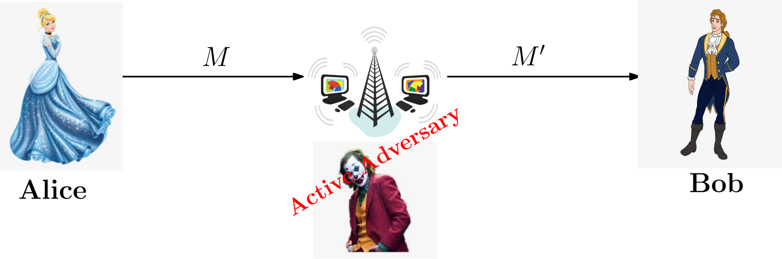
M

M'

Alice

Bob

Message Integrity (or Message Authentication)



M

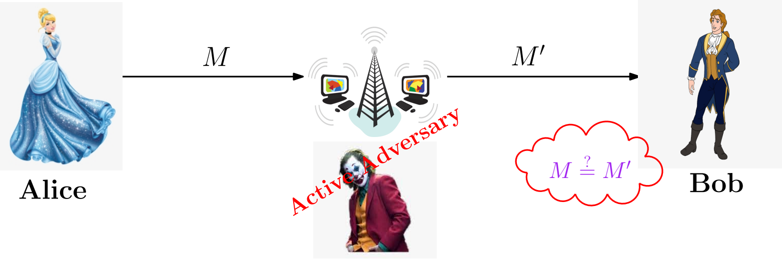
M'

Alice

Active Adversary

Bob

Message Integrity (or Message Authentication)



M

M'

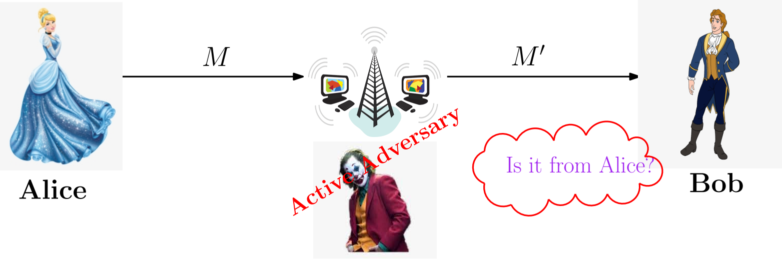
Active Adversary

$M \overset{?}{=} M'$

Alice

Bob

Message Integrity (or Message Authentication)



Alice

M

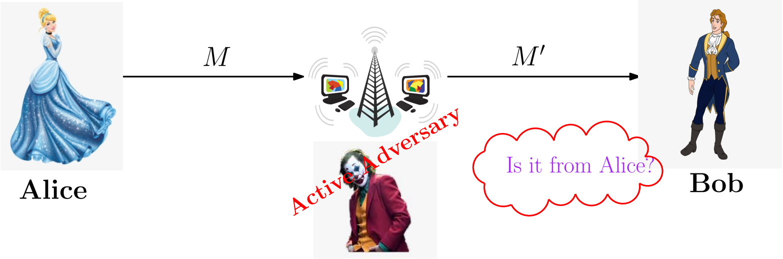
Active Adversary

M'

Is it from Alice?

Bob

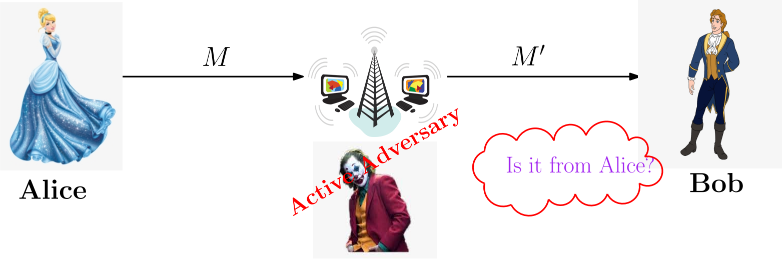
Message Integrity (or Message Authentication)



Attack Models

1. Active Adversary
2. Message may be public.

Message Integrity (or Message Authentication)



Attack Models

1. Active Adversary
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Question

How to provide Message Integrity?

Encryption Does Not Provide Integrity?

A note on Encryption

- Encryption does not (in general) provide any integrity.

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- Encryption should never be used with the intent of achieving message authentication unless it is specifically designed with that purpose in mind (Authenticated encryption).

Encryption Does Not Provide Integrity?

A note on Encryption

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A Incorrect reasoning

- Ciphertext hides the message.
- An adversary cannot modify an encrypted message in any meaningful way.
- All the encryption schemes that we have seen thus far do not provide message integrity.

Encryption Does Not Provide Integrity?

Stream Cipher

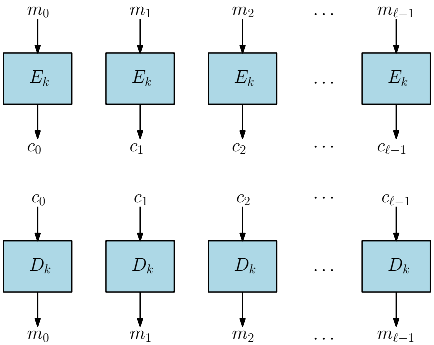
- We have already seen that we can change a part of a message in a meaningful way:

Encryption Does Not Provide Integrity?

Stream Cipher

- We have already seen that we can change a part of a message in a meaningful way:
  - If we know the text, and
  - Its position.

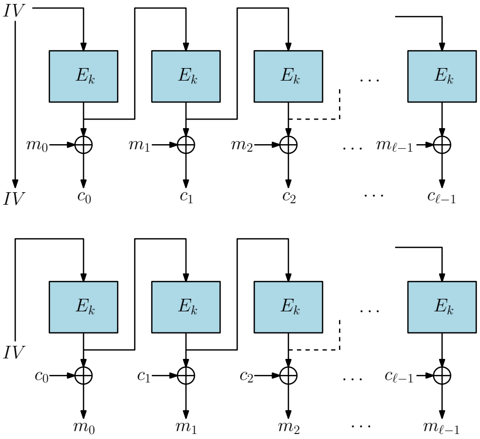
Encryption Does Not Provide Integrity?



Block Cipher (ECB mode)

- Changing one bit in cipher text means changing only one data block.

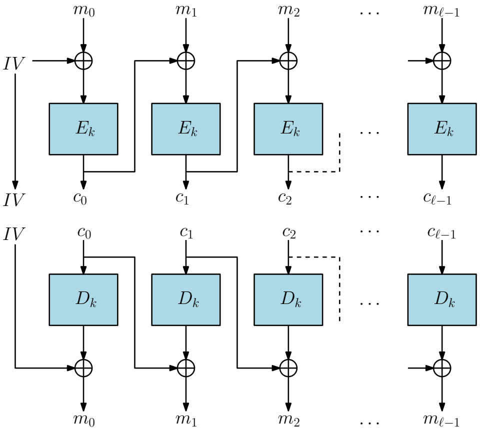
Encryption Does Not Provide Integrity?



Block Cipher (OFB mode)

- Similar to stream cipher.

Encryption Does Not Provide Integrity?



Block Cipher (CBC mode)

- Change in the IV of cipher text only changes the first data block.

How to provide Message Integrity?

Requirement

Providing message integrity between two communicating parties requires that the sending party has a secret key unknown to the adversary. Because adversary

- knows the algorithm,
- knows the message, and
- need nothing extra to compute a tag.

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Keyless Integrity Mechanism

- Often used in communication designed not for security
- CRC32 in Ethernet,
- 16-bit checksum in TCP.
- These keyless integrity mechanisms are designed to detect random transmission errors, not malicious errors.

Message Authentication Code

Message Authentication Code

A MAC system $\mathcal{I} = (\mathcal{S}, \mathcal{V})$ is a pair of efficient algorithms, $\mathcal{S}$ and $\mathcal{V}$, where $\mathcal{S}$ is called a signing algorithm and $\mathcal{V}$ is called a verification algorithm. Algorithm $\mathcal{S}$ is used to generate tags and algorithm $\mathcal{V}$ is used to verify tags.

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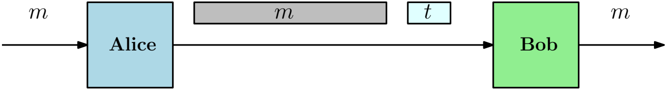

Generate tag t

$t \xleftarrow{R} \mathcal{S}(k, m)$

Verify Message-tag pair (m, t)

$\mathcal{V}(k, m, t) \overset{?}{=} \texttt{accept}$

Message Authentication Code

We say that a MAC system $\mathcal{I} = (\mathcal{S}, \mathcal{V})$ is defined over $(\mathcal{K}, \mathcal{M}, \mathcal{T})$.

- $\mathcal{K}$ is a finite key space
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- $\mathcal{V}$ is a deterministic algorithm that is invoked as $r \longleftarrow \mathcal{V}(k, m, t)$, where k is a key, m is a message, t is a tag, and the output r is either `accept` or `reject`.

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- $\mathcal{V}$ is a deterministic algorithm that is invoked as $r \longleftarrow \mathcal{V}(k, m, t)$, where k is a key, m is a message, t is a tag, and the output r is either `accept` or `reject`.
- Correctness: We require that tags generated by $\mathcal{S}$ are always accepted by $\mathcal{V}$; that is, the MAC must satisfy the following correctness property: for all keys k and all messages m,

$$\Pr[\mathcal{V}(k, m, \mathcal{S}(k, m)) = \texttt{accept}] = 1.$$

Message Authentication Code

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- Deterministic MAC
  - For a given key k, and a given message m, there is a unique valid tag for m under k.

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  - Are not necessary to achieve security.

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  - Are not necessary to achieve security.
  - Yield better efficiency/security trade-offs.

Security of Randomized MAC

Existentially Unforgeable under a Chosen Message Attack

A given MAC system $\mathcal{I} = (\mathcal{S}, \mathcal{V})$, defined over $(\mathcal{K}, \mathcal{M}, \mathcal{T})$, and a given adversary $\mathcal{A}$, the attack game runs as follows:

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1. The challenger picks a random $k \overset{R}{\longleftarrow} \mathcal{K}$.
2. $\mathcal{A}$ queries the challenger several times. For $i = 1, 2, \ldots$, the i-th signing query is a message $m_i \in \mathcal{M}$. Given m_i, the challenger computes a tag $t_i \overset{R}{\longleftarrow} \mathcal{S}(k, m_i)$, and then gives t_i to $\mathcal{A}$.

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3. Eventually $\mathcal{A}$ outputs a candidate forgery pair $(m, t) \in \mathcal{M} \times \mathcal{T}$ that is not among the signed pairs, i.e.,

$$(m, t) \notin \{(m_1, t_1), (m_2, t_2), \ldots\}.$$

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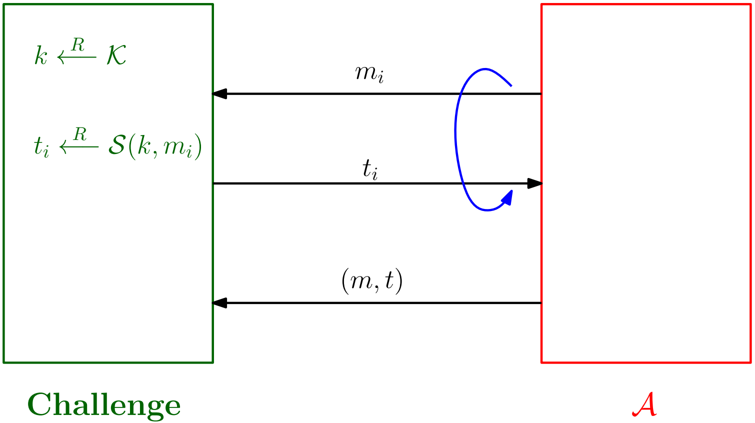
$k \xleftarrow{R} \mathcal{K}$
$t_i \xleftarrow{R} \mathcal{S}(k, m_i)$
m_i
t_i
(m, t)
Challenge
$\mathcal{A}$

MAC Attcak Game

Security of Randomized MAC

Advantage of $\mathcal{A}$

- We say that $\mathcal{A}$ wins the above game if (m, t) is a valid pair under k (i.e., $\mathcal{V}(k, m, t) = \texttt{accept}$).

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Secure MAC

A MAC system $\mathcal{I}$ is secure if for all efficient adversaries $\mathcal{A}$, the value $\mathsf{MACadv}[\mathcal{A},\mathcal{I}]$ is negligible.

Security of Deterministic MAC

Attack Game

- $\mathcal{A}$ wins the Attack Game if it produce a valid message-tag pair (m,t) for some new message
$$m \notin \{m_1, m_2, \ldots\}.$$

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$$m \notin \{m_1, m_2, \ldots\}.$$
- Security in this case just means that S is unpredictable.

MAC Security with Verification Queries

Existentially Unforgeable under a Chosen Message Attack

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3. Verification query: For $j = 1, 2, \ldots$, the j-th verification query consists of a message-tag pair $(\hat{m}_j, \hat{t}_j) \in \mathcal{M} \times \mathcal{T}$ that is not among the previously signed pairs, i.e.,

   $$(\hat{m}_j, \hat{t}_j) \notin \{(m_1, t_1), (m_2, t_2), \ldots\}.$$

   The challenger responds to $\mathcal{A}$ with $\mathcal{V}(k, \hat{m}, \hat{t})$.

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Theorem

If $\mathcal{I}$ is a secure MAC system, then it is also secure in the presence of verification queries.

MAC Security with Verification Queries

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If $\mathcal{I}$ is a secure MAC system, then it is also secure in the presence of verification queries.
In particular, for every MAC adversary $\mathcal{A}$ that attacks $\mathcal{I}$ as in MAC Attack Game with verification query, and which makes at most Q_v verification queries and at most Q_s signing queries, there exists a Q_s-query MAC adversary $\mathcal{B}$ that attacks $\mathcal{I}$ as in MAC Attack Game without verification query, where $\mathcal{B}$ uses $\mathcal{A}$ as a subroutine, such that

$$\mathsf{MAC}^{\mathsf{vq}}\mathsf{adv}[\mathcal{A},\mathcal{I}] \leqslant \mathsf{MACadv}[\mathcal{B},\mathcal{I}] \cdot Q_v.$$

MAC Security with Verification Queries

Proof

- Adversary $\mathcal{B}$ plays the role of challenger to $\mathcal{A}$ in Attack Game with verification query.
- Adversary $\mathcal{B}$ plays the role of adversary in Attack Game without verification query.

MAC Security with Verification Queries

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Initialization:

$$\omega \xleftarrow{R} \{1, \ldots, Q_v\}.$$

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Construction of $\mathcal{B}$

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Initialization:
    $\omega \overset{R}{\longleftarrow} \{1, \ldots, Q_v\}$.
Upon receiving a signing query $m_i \in \mathcal{M}$ from $\mathcal{A}$ do:
    forward $m_i$ to the MAC challenger, obtaining the tag $t_i$
    send $t_i$ to $\mathcal{A}$
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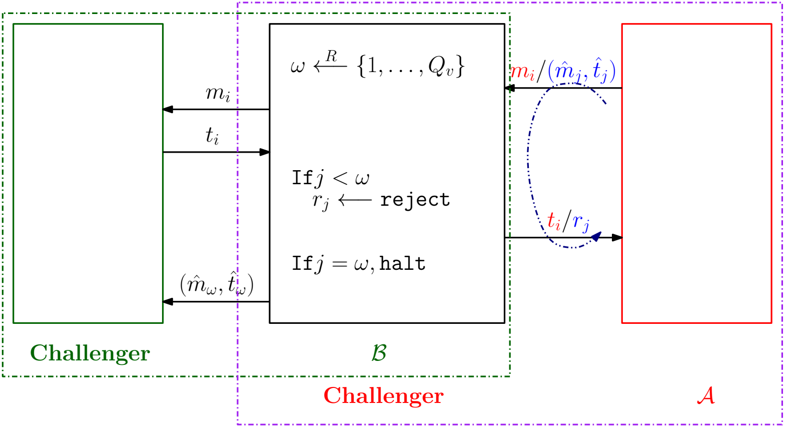
Upon receiving a verification query $(\hat{m}_j, \hat{t}_j) \in \mathcal{M} \times \mathcal{T}$ from $\mathcal{A}$ do:
    if $j = \omega$
        then output $(\hat{m}_j, \hat{t}_j)$ as a candidate forgery pair and halt
        else send reject to $\mathcal{A}$.
```

MAC Security with Verification Queries


$\omega \xleftarrow{R} \{1,\ldots,Q_v\}$
m_i
t_i
If $j < \omega$
$r_j \longleftarrow$ reject
If $j = \omega$, halt
$(\hat{m}_\omega, \hat{t}_\omega)$
$m_i/(\hat{m}_j, \hat{t}_j)$
t_i/r_j
Challenger
$\mathcal{B}$
Challenger
$\mathcal{A}$

MAC Security with Verification Queries

Game 0: MAC attack game with verification between challenger and $\mathcal{A}$

Initialization:

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```

$\mathcal{A}$ wins

Let W_0 be the event that in Game 0, $r_j = \texttt{accept}$ for some j.

$$\Pr[W_0] = \mathsf{MAC}^{\mathsf{vq}}\mathsf{adv}[\mathcal{A}, \mathcal{I}].$$

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- Let W_1 be the event that in Game 1, $r_j =$ `accept` for some j.
- Even though the challenger does not notify $\mathcal{A}$ that W_1 occurs, both Games 0 and 1 proceed identically until this event happens.
- Therefore, events W_0 and W_1 are really the same.

$$\Pr[W_0] = \Pr[W_1].$$

MAC Security with Verification Queries

Game 2

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- By construction,

$$\mathsf{MACadv}[\mathcal{B}, \mathcal{I}] = \Pr[W_2] \geqslant \Pr[W_1]/Q_v = \Pr[W_0]/Q_v = \mathsf{MAC}^{\mathsf{vq}}\mathsf{adv}[\mathcal{A}, \mathcal{I}]/Q_v.$$

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- Any PRF with a large (i.e., super-poly) output space is unpredictable.

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- Any PRF with a large (i.e., super-poly) output space is unpredictable.
- Therefore, the above construction yields a secure MAC.

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Theorem

Let F be a secure PRF defined over $(\mathcal{K},\mathcal{X},\mathcal{Y})$, where $|\mathcal{Y}|$ is super-poly. Then the deterministic MAC system $\mathcal{I}$ derived from F is a secure MAC.

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Let F be a secure PRF defined over $(\mathcal{K}, \mathcal{X}, \mathcal{Y})$, where $|\mathcal{Y}|$ is super-poly. Then the deterministic MAC system $\mathcal{I}$ derived from F is a secure MAC.
In particular, for every Q-query MAC adversary $\mathcal{A}$ that attacks $\mathcal{I}$ as in MAC Attack Game (without verification query), there exists a $(Q+1)$-query PRF adversary $\mathcal{B}$ that attacks F as in PRF Indistinguishability, where $\mathcal{B}$ uses $\mathcal{A}$ as a subroutine, such that

$$\mathsf{MACadv}[\mathcal{A}, \mathcal{I}] \leqslant \mathsf{PRFadv}[\mathcal{B}, F] + \frac{1}{|\mathcal{Y}|}.$$

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Assumptions

- $\mathcal{I} = (\mathcal{S}, \mathcal{V})$: A secure fixed-length MAC for messages of length n.
- We parse the message m to be authenticated as a sequence of blocks $m := (m_1, \ldots, m_d)$, where $|m_i| = n$ for all i.

Domain Extension to Variable-Length Messages

Construction 1

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- Attack:
  - Block reordering attack possible.
  - If $(m,t) := ((m_1, m_2), (t_1, t_2))$ is a valid message-tag pair then so is $(m', t') := ((m_2, m_1), (t_2, t_1))$.

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    - Constructs message-tag pair $(\hat{m}, \hat{t}) := ((m_1, m'_2, m_3, m'_4, \ldots), (t_1, t'_2, t_3, t'_4, \ldots))$.

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Construction 4: Signing

- $S'(k, m)$:
  - Inputs: a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^*$ of (nonzero) length $\ell < 2^{n/4}$.

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    - For $i = 1, \ldots, d$, Compute $t_i \leftarrow \mathcal{S}(k, r||\ell||i||m_i)$.
    - The tag $t := (r, t_1, \ldots, t_d)$, where i, ℓ are encoded as strings of length $n/4$ (as $i, \ell < 2^{n/4}$).

Domain Extension to Variable-Length Messages

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- $\mathcal{V}'(k, m, t)$:
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  - Parse m into d blocks $m_1, \ldots, m_d$, each of length $n/4$.
  - Pad the final block with 0's to make it a multiple of $n/4$.
  - Output 1 if and only if
    1. $d' = d$, and
    2. $\mathcal{V}(k, r||\ell||i||m_i, t_i) = 1$, $\forall\ 1 \leq i \leq d$.

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Construction 4: Theorem

If $\mathcal{I}$ is a secure fixed-length MAC for messages of length n, then Construction 4 is a secure MAC for arbitrary-length messages.

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- Asymptotically, $2^{n/4}$ is an exponential bound.

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Construction 4: Comments

- Asymptotically, $2^{n/4}$ is an exponential bound.
- Honest parties will not authenticate messages that long.
- Any polynomial-time adversary cannot submit messages that long to its MAC oracle.
- In practice, when n is fixed, one must ensure that $2^{n/4}$ is acceptable.

Domain Extension to Variable-Length Messages

Construction 4: Comments (cont.)

- A secure MAC for arbitrary-length messages from a pseudorandom function taking inputs of fixed length n.

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- A secure MAC for arbitrary-length messages from a pseudorandom function taking inputs of fixed length n.
- Thus secure MACs can be constructed from block ciphers.
- But, Construction 4 is extremely inefficient:
  - $4d$ evaluations of block cipher are required to compute a tag on a message of length dn.
  - The tag is more than $4dn$ bits long.

Domain Extension to Variable-Length Messages

Construction 4: Comments (cont.)

- A secure MAC for arbitrary-length messages from a pseudorandom function taking inputs of fixed length n.
- Thus secure MACs can be constructed from block ciphers.
- But, Construction 4 is extremely inefficient:
  - $4d$ evaluations of block cipher are required to compute a tag on a message of length dn.
  - The tag is more than $4dn$ bits long.
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Given a secure PRF on short inputs construct a secure PRF on long inputs.

CBC-MAC

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- We present here a basic version of CBC-MAC.
- **Caution:** This basic scheme is not secure in the general case when messages of different lengths may be authenticated.

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Theorem

Let ℓ be a polynomial. If F is a pseudorandom function, then CBC-MAC construction $\mathcal{I}_{\mathsf{CBC}}$ is a secure MAC for messages of length $\ell(n) \cdot n$.

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CBC-MAC $\mathcal{I}_{\mathsf{CBC}}$ is much more efficient than Construction 4

- Requires d block-cipher evaluations for a message of length dn.
- Tag length is n.

CBC-MAC: Arbitrary-length messages

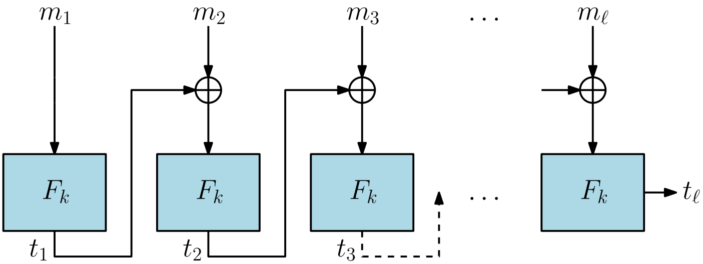

Figure: MAC-CBC

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- Output t_v as the tag.

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- Inputs: Key $k \in \{0,1\}^n$, a message m of length $v \cdot n$ where $1 \leqslant v \leqslant \ell(n)$, and tag t.

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- Output $t \overset{?}{=} t'_{v'}$.

CBC-MAC

Prefix-free Secure PRF

Let F be a PRF defined over $(\mathcal{K}, \mathcal{X}^{\leqslant \ell}, \mathcal{Y})$. We say that a PRF adversary $\mathcal{A}$ playing PRF Indistinguishability Attack Game with respect to F is a prefix-free adversary if all of its queries are non-empty strings over $\mathcal{X}$ of length at most ℓ, no one of which is a proper prefix of another. We denote $\mathcal{A}$'s advantage in winning the game by $\mathsf{PRF^{pf}adv}[\mathcal{A}, F]$. Further, let us say that F is a prefix-free secure PRF if $\mathsf{PRF^{pf}adv}[\mathcal{A}, F]$ is negligible for all efficient, prefix-free adversaries $\mathcal{A}$.

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Theorem

Let F be a secure PRF defined over $(\mathcal{K}, \mathcal{X}, \mathcal{X})$ where $\mathcal{X} = \{0,1\}^n$ and $|\mathcal{X}| = 2^n$ is super-poly. Then for any poly-bounded value ℓ, we have that $\mathcal{I}_{\mathsf{CBC}} - 2$ is a prefix-free secure PRF defined over $(\mathcal{K}, \mathcal{X}^{\leqslant \ell}, \mathcal{X})$.
In particular, for every prefix-free PRF adversary $\mathcal{A}$ that attacks $\mathcal{I}_{\mathsf{CBC}} - 2$ as in PRF Indistinguishability Attack Game, and issues at most Q queries, there exists a PRF adversary $\mathcal{B}$ that attacks F as in PRF Indistinguishability Attack Game, where $\mathcal{B}$ calls $\mathcal{A}$ as subroutine, such that

$$\mathsf{PRF}^{\mathsf{pf}}\mathsf{adv}[\mathcal{A}, \mathcal{I}_{\mathsf{CBC}} - 2] \leqslant \mathsf{PRFadv}[\mathcal{B}, F] + \frac{(Q\ell)^2}{2|\mathcal{X}|}.$$

End