Cryptology

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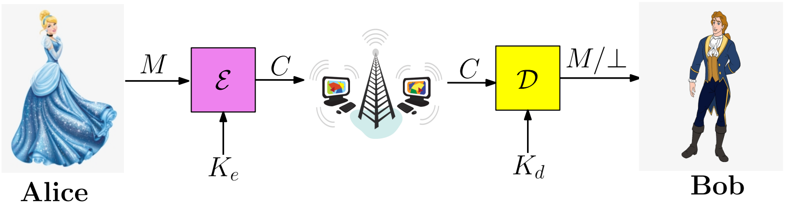
Lecture 02

History and Classical Ciphers

Cryptographic Schemes



- $\mathcal{E}$: Encryption algorithm, and K_e: Encryption key.
- $\mathcal{D}$: Decryption algorithm, and K_d: Decryption key.
- M: Plain text or Message, and C: Cipher text or Encrypted message.

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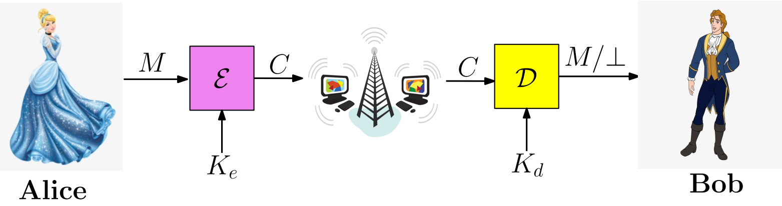

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Settings

- Symmetric Key: $K_e = K_d$.
- Asymmetric Key or Public Key: $K_e \neq K_d$.

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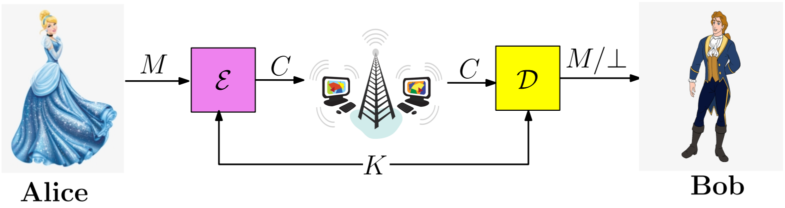

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Definition

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- $\mathcal{C}$: Cipher Space, a finite set of possible ciphertexts.
- $\mathcal{K}$: Key Space, a finite set of possible keys.
- $\mathcal{G}$: The Key-Generation Algorithm, a probabilistic algorithm that outputs a key k from $\mathcal{K}$ chosen according to some distribution.
- $\mathcal{E}$: The Encryption Algorithm,

$$\begin{aligned} \mathcal{E} \quad : \quad & \mathcal{K} \times \mathcal{P} \to \mathcal{C} \\ & \mathcal{E}(k, m) = c, \text{ where } k \in \mathcal{K}, m \in \mathcal{P}, c \in \mathcal{C}. \end{aligned}$$

- $\mathcal{D}$: The Decryption Algorithm,

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Basic Correctness of a Symmetric Encryption Scheme

For all key $k \in \mathcal{K}$ generated by $\mathcal{G}$ and for all plaintext $m \in \mathcal{P}$, the following must hold:

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- Decrypting a ciphertext using the appropriate key yields the original message that was encrypted.

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Therefore, Kerckhoffs' principle demands that everything (algorithm) is public except for the key.

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Importance of Kerckhoffs' Principle through Example

Content Scrambling System (CSS) for DVD content protection, which was broken easily once it was reverse-engineered.

Attack Models

There are four commonly considered attack models:

1. Ciphertext-only attack
2. Known-plaintext attack
3. Chosen-plaintext attack
4. Chosen-ciphertext attack

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  3. Demonstrate that simple approaches to achieving secure encryption are unlikely to succeed.
  4. To identify/learn why that is the case.

Caesar's Cipher (100 B.C. - 44 B.C.)

- It is one of the oldest recorded ciphers.
- Is described in De Vita Caesarum, Divus Iulius (The Lives of the Caesars, The Deified Julius) which was written in approximately 110 C.E.:

> "There are also letters of his to Cicero, as well as to his intimates on private affairs, and in the latter, if he had anything confidential to say, he wrote it in cipher, that is, by so changing the order of the letters of the alphabet, that not a word could be made out. *If anyone wishes to decipher these, and get at their meaning, he must substitute the fourth letter of the alphabet, namely D, for A, and so with the others*."

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- $\mathcal{D} : m := (c-3) \pmod{26}$.

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Does not follow Kerckhoffs' Principle.

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Caesar's Cipher is a special case of Shift cipher with $K = 3$

Shift Cipher

Example

- Let $K = 7$

A	B	C	D	E	F	G	H	I	J	K	L	M
H	I	J	K	L	M	N	O	P	Q	R	S	T

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- Is it possible to decrypt a message without knowing the key? Answer: Yes
- **Brute-forced Attack:**
  - Check each key one after another.
  - Stop when the text makes some sense.

K	Decrypted Text	Make Sense?
0	ILNPU AOL HAAHJR UVD	No
1	HKMOT ZNK GZZGIQ TUC	No
2	GJLNS YMJ FYYFHP STB	No
3	FIKMR XLI EXXEGO RSA	No
4	EHJLQ WKH DWWDFN QRZ	No
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- Maximum number of tries required? 26

Shift Cipher

- Should not be vulnerable to such a brute-force attack.
- Else, it can be completely broken, irrespective of how sophisticated the encryption algorithm is.
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- **Present Standard:** $|\mathcal{K}| \geq 2^{80}$.
- **Note:** The above principle gives a *necessary condition* for security, not a sufficient one.
- Our next scheme has a *very large key space* but is still insecure.

Substitute Cipher

- $\mathcal{P} = \mathcal{C} = \{A, B, C, \ldots, Z\}$ (English Alphabet).
- $\mathcal{K} = \Pi$ = All possible permutations of the symbols $\{A, B, C, \ldots, Z\}$.

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- $\mathcal{D} : m := \pi^{-1}(c)$.

Substitute Cipher

Example

π :

A	B	C	D	E	F	G	H	I	J	K	L	M
H	I	J	K	L	M	N	O	P	Q	R	S	T

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
U	V	W	X	Y	Z	A	B	C	D	E	F	G

Substitute Cipher

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Substitute Cipher

Example

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U	V	W	X	Y	Z	A	B	C	D	E	F	G

- **Plaintext:** BEGIN THE ATTACK NOW.
- **Ciphertext:** $\pi(\mathrm{B})\pi(\mathrm{E})\pi(\mathrm{G})\cdots$ = ILNPU AOL HAAHJR UVD.

Substitute Cipher

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  - The Shift Cipher includes only 26 of the 26! possible permutations of 26 elements.
- In both the Shift Cipher and the Substitution Cipher, once a key is chosen, each alphabetic character is mapped to a unique alphabetic character. These type of cryptosystems are called monoalphabetic cryptosystems.

Cryptanalysis of Substitute Cipher

Assumptions

- English-language text is being encrypted.
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Attack Idea

- Use statistical patterns of the English language.
  Note: The same attack works for any language.
- The attack utilizes the following two properties of the cipher:
  1. The mapping of each letter is fixed, that is, $e \mapsto L$ for all occurrences of e in the plaintext.
  2. The probability distribution of individual letters is known.
     - The average frequency counts of the different English letters are quite invariant over different texts.

Statistical Properties of English Language

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Letter	Probability	Letter	Probability
A	0.082	N	0.067
B	0.015	O	0.075
C	0.028	P	0.019
D	0.043	Q	0.001
E	0.127	R	0.060
F	0.022	S	0.063
G	0.020	T	0.091
H	0.061	U	0.028
I	0.070	V	0.010
J	0.002	W	0.023
K	0.008	X	0.001
L	0.040	Y	0.020
M	0.024	Z	0.001

Statistical Properties of English Language

- Statistical analysis gives us the following partition the 26 letters into five groups:
  - E: Having probability of about 0.120.
  - T, A, O, I, N, S, H, R: Probability between 0.06 and 0.09.
  - D, L: Probability around 0.04.
  - C, U, M, W, F, G, Y, P, B: Prob. between 0.015 and 0.028.
  - V, K, J, X, Q, Z: Probability less than 0.01.

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- It is also useful to consider distributions digrams and trigrams.
- Some of the most common diagrams (in decreasing order) are
  - TH, HE, IN, ER, AN, RE, ED, ON, ES, ST, EN, AT, TO, NT, HA, ND, OU, EA, NG, AS, OR, TI, IS, ET, IT, AR, TE, SE, HI, OF.

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- Some of the most common trigrams (in decreasing order) are
  - THE, ING, AND, HER, ERE, ENT, THE, ING, AND, HER, ERE, ENT, THA, NTH, WAS, ETH, FOR, DTH.

Ciphertext:

YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
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- Tabulate the frequency distribution of characters:
  A appeared 0 times, B appeared 1 times, and so on
- Compare them with the known frequencies of English text.
- Guess parts of the mapping based on the observed frequencies.
  Example: Guess E $\mapsto$ most frequent character, and so on.

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  Example: Guess E $\mapsto$ most frequent character, and so on.
- **Note:** Some of the guesses may be wrong.
- But enough of the guesses will be correct to enable relatively quick decryption.
- For quick decryption use other knowledge of English.
  - u generally follows q.
  - h is likely to appear between t and e.
  - The frequencies of *digrams* and *trigrams*.
  - ...

Empirical distribution

Letter	Frequency	Letter	Frequency
A	0	N	9
B	1	O	0
C	15	P	1
D	13	Q	4
E	7	R	10
F	11	S	3
G	1	T	2
H	4	U	5
I	5	V	5
J	11	W	8
K	1	X	6
L	0	Y	10
M	16	Z	20

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- Most frequencies (≥ 10).

Z - 20;	D - 13
M - 16;	F, J - 11
C - 15;	R, Y - 10

Initial guess: Let $\mathcal{E}(k, \mathrm{E}) = Z$.

- We might expect that

  C, D, F, J, M, R, Y (frequency $\geq$ 10 each)

  are encryptions of (a subset of)

  T, A, O, I, N, S, H, R.

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- But these frequencies really do not vary enough to tell us what the correspondence might be.

Initial guess: Let $\mathcal{E}(k, \mathrm{E}) = Z$.

- So look at diagrams of the form _Z or Z_.
- DZ and ZW: 4 times each.
  NZ and ZU: 3 times each.
  RZ, HZ, XZ, FZ, ZR, ZV, ZC, ZD and ZJ: Twice each.

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- Observations 1:
  - ZW occurs 4 times, WZ occurs 0 times.
  - W occurs less often than many other characters.
  - Subsequent guesses: $\mathcal{E}(k, \mathrm{D}) = W$.

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- Observations 2:
  - DZ occurs 4 times, ZD occurs 2 times.
  - With high probability, $\mathcal{D}(k, D) \in \{R, S, T\}$.
  - But it is not clear which of these 3 possibility is the correct one.

Guess: Let $\mathcal{E}(k, \mathrm{E}) = Z, \mathcal{E}(k, \mathrm{D}) = W$.

- Observations 3:
  - ZRW occurs near the beginning of the ciphertext, and RW occurs again later on.
  - Since R occurs frequently in the ciphertext and ND is a common digram.
  - Subsequent guesses:

$$\mathcal{E}(k, \mathrm{N}) = R.$$

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Partial Decryption:

- - - - - - -END - - - - - - - - - - - E - - - - -NED - - - - E -E - - - -
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
- E - - - - - - - - - - - N - - D - - -EN - - - - E - - - - - E - E - - - N - - - - - - - N - - - - - - - ED - - – E
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
- – - E – - NE -ND - E - E - - ED - - - - - - N - - - - - - – - - - - - - - E - - - - ED - - - - - - - - D - -
VYFZUMRZCRWNZDZJJXZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJ
- E - -N
NZDIR

Guess: Let $\mathcal{E}(k,\mathrm{E}) = Z, \mathcal{E}(k,\mathrm{D}) = W, \mathcal{E}(k,\mathrm{N}) = R.$

- Observations 4:
  - NZ is a common diagram and ZN is not.
  - Subsequent guesses:

$$\mathcal{E}(k,\mathrm{H}) = N.$$

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Further Decryption:

- - - - - - -END - - - - - - - - - - - E - - - - -NEDH - - -E - - - - - - - - - - - - - H - - - - - - - E - - - -
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
- E - - - - - - - - - - - NH- -D - - EN - - - - E - H- - - EHE - - - N - - - - - - N - - - - - - ED - - - -E
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
- – - E – - NE - NDHE - E - - ED - - - - - - NH - - - -H - - - - - - - E - - - -ED - - - - - - - D - -
VYFZUMRZCRWNZDZJJXZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJ
HE - -N
NZDIR

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- Observations 5:
  - AND is a very common trigram.
  - Subsequent guesses: (from the segment NE - NDHE suggests)

$$\mathcal{E}(k, \mathrm{A}) = C.$$

Guess: Let $\mathcal{E}(k, \mathrm{E}) = Z, \mathcal{E}(k, \mathrm{D}) = W, \mathcal{E}(k, \mathrm{N}) = R, \mathcal{E}(k, \mathrm{H}) = N.$

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Further Decryption:

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YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
- E - - - - - - - - - - - NH- -D - - EN - - - - E - H- - - EHE - - - N - - - - - - - N - - - - - - - ED - - - -E
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
- – - E – - NE - NDHE - E - - ED - - - - - - - NH - - - -H - - - - - - - - E - - - -ED - - - - - - - - D - -
VYFZUMRZCRWNZDZJJXZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJ
HE - -N
NZDIR

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Further Decryption:

- - - - - - -END - - - - - -A - - - E -A - - NEDH - - -E - - - - - - A - - - - - - H - - - - - - -EA - - -
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
- E - A - - - A - - -NHAD -A-EN - - A -E - H- - - EHE -A - N - - - - - - N - - - - - - ED - - - E
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
- – - E – - NEANDHE - E - - ED - -A - - - NH - - - -HA - - - A - E - - - -ED - - - - -A -D - -
VYFZUMRZCRWNZDZJJXZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJ
HE - -N

NZDIR

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- Observations 6: Consider M
  - Second most common ciphertext character.
  - Partial decryption of $\mathcal{D}(k, \text{RNM}) = \text{NH}-$, suggests that H- is the beginning of a word.
  - So M probably encrypts a vowel.

Guess: Let $\mathcal{E}(k, \mathrm{E}) = Z, \mathcal{E}(k, \mathrm{D}) = W, \mathcal{E}(k, \mathrm{N}) = R, \mathcal{E}(k, (\mathrm{H}) = N, \mathcal{E}(k, \mathrm{A}) = C$

- Observations 6: Consider M
    - Second most common ciphertext character.
    - Partial decryption of $\mathcal{D}(k, \mathrm{RNM}) = \mathrm{NH}-$, suggests that H- is the beginning of a word.
    - So M probably encrypts a vowel.
    - A and E are already accounted, then

$$\mathcal{D}(k, \mathrm{M}) = \mathrm{I} \text{ or } \mathrm{O}.$$

    - “AI” is a much more likely digram than “AO”
    - Subsequent guesses: (from the digram CM)

$$\mathcal{E}(k, \mathrm{I}) = \mathrm{M}.$$

Guess: Let
$\mathcal{E}(k,\mathrm{E}) = Z, \mathcal{E}(k,\mathrm{D}) = W, \mathcal{E}(k,\mathrm{N}) = R, \mathcal{E}(k,\mathrm{H}) = N, \mathcal{E}(k,\mathrm{A}) = C, \mathcal{E}(k,\mathrm{I}) = \mathrm{M}.$

Further Decryption:

- - - - - -IEND - - - - - - - A - I - E -A - INE D H I - E - - - - - - - A - - - - I -H - - - - - I - EA - -I
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
- E - A - - - A -I- -NHAD -A-EN - - A -E - H I - EHE -A - N - - - - - I N - I - - - - ED - - – E
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
- – - E – I NEANDHE - E - - ED - -A - - I NH I - - -HA I - - A - E -I- - -ED - - - - - -A -D - -
VYFZUMRZCRWNZDZJJXZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJ
HE - -N
NZDIR

Guess: Let
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- O is a common plaintext character.
- Guess $\mathcal{E}(k,O) \in \{D,F,J,Y\}$.
- Y seems to be the most likely possibility.
- Otherwise,

$$\begin{aligned} \mathcal{D}(k,\mathrm{CFM}) &= \mathrm{AOI}; \quad \text{if } \mathcal{E}(k,\mathrm{O}) = \mathrm{F} \text{ or} \\ \mathcal{D}(k,\mathrm{CJM}) &= \mathrm{AOI}; \quad \text{if } \mathcal{E}(k,\mathrm{O}) = \mathrm{J}, \end{aligned}$$

  giving long strings of vowels in both cases.
- Hence, assume $\mathcal{E}(k,\mathrm{O}) = \mathrm{Y}$.

Guess: Let $\mathcal{E}(k,\mathrm{E}) = Z, \mathcal{E}(k,\mathrm{D}) = W, \mathcal{E}(k,\mathrm{N}) = R, \mathcal{E}(k,\mathrm{H}) = N, \mathcal{E}(k,\mathrm{A}) = C, \mathcal{E}(k,\mathrm{I}) = \mathrm{M}, \mathcal{E}(k,\mathrm{O}) = \mathrm{Y}$.

- **Three most frequent ciphertext letters remaining:** D, F, J.
- **Conjecture:** $\{\mathrm{D},\mathrm{F},\mathrm{J}\} = \{\mathrm{r},\mathrm{s},\mathrm{t}\}$.
- Two occurrences of the trigram NMD suggest that

$$\mathcal{D}(k,\mathrm{D}) = \mathrm{S} \Rightarrow \mathcal{D}(k,\mathrm{NMD}) = \mathrm{HIS}.$$

- $\mathcal{D}(k,\mathrm{HNCMF}) = -\mathrm{HAI}-$ could be a decryption of CHAIR.
- Then let $\mathcal{D}(k,\mathrm{F}) = \mathrm{R}$ and $\mathcal{D}(k,\mathrm{H}) = \mathrm{C}$, which implies

$$\mathcal{D}(k,\mathrm{J}) = T \quad \text{[By process of elimination].}$$

Guess: Let $\mathcal{E}(k,\mathrm{E}) = Z, \mathcal{E}(k,\mathrm{D}) = W, \mathcal{E}(k,\mathrm{N}) = R, \mathcal{E}(k,\mathrm{H}) = N, \mathcal{E}(k,\mathrm{A}) = C, \mathcal{E}(k,\mathrm{I}) = \mathrm{M}, \mathcal{E}(k,\mathrm{O}) = \mathrm{Y}, \mathcal{E}(k,\mathrm{R}) = \mathrm{F}, \mathcal{E}(k,\mathrm{C}) = \mathrm{H}, \mathcal{E}(k,\mathrm{T}) = J.$

Further Decryption:

O-R -R IEND – RO - -ARI SE - A - I NE D H I S E - -T - - - ASS - -ITHS -R-R I SE AS I
YIFQFMZRWQFYVECFMDZPCVMRZWNMDZVEJBTXCDDUMJNDIFEFMDZCDM
- E - A -ORATIONHA DTA-EN - - ACE - H I - EHE - ASNT- OO - I N - I - O- REDSO - E
QZKCEYFCJMYRNCWJCSZREXCHZUNMXZNZUCDRJXYYSMRTMEYIFZWDYVZ
- ORE – I NEANDHESETT- ED - -AC - I NH I SCHA I - - ACET I- -TED- -TO -ARDST
VYFZUMRZCRWNZDZJJXZWGCHSMRNMDHNCMFQCHZJMXJZWIEJYUCFWDJ
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Partially Decrypted Text

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Plaintext

Our friend from Paris examined his empty glass with surprise, as if evaporation had taken place while he wasn't looking. I poured some more wine and he settled back in his chair, face tilted up towards the sun.

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Conclusion

Although the mono-alphabetic substitution cipher has a large key space, it is still *insecure*.

Vigener Cipher (16'th Century, Rome)

- This cipher is named after Blaise de Vigenere.
- Poly-Alphabetic cipher.

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- $\mathcal{P} = \mathcal{C} = \{A, B, C, \ldots, Z\}$ (English Alphabet).

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- $\mathcal{E}$: Let the key be a n character long keyword $k = k_0 k_1 \ldots k_{n-1}$. If the plaintext is $m = m_0 m_1 m_2 \ldots$, then we encrypt is as

$$c_i := \left(m_i + k_{i \pmod n}\right) \pmod{26},$$

and $c = c_0 c_1 c_2 \ldots$.

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  and $c = c_0 c_1 c_2 \dots$.
- $\mathcal{D}$: Let the key be a n character long keyword $k = k_0 k_1 \dots k_{n-1}$. If the ciphertext is $c = c_0 c_1 c_2 \dots$, then we decrypt is as

$$m_i := \left(c_i - k_{i \pmod n}\right) \pmod{26},$$

  and $m = m_0 m_1 m_2 \dots$.

Vigener Cipher (16'th Century, Rome)

Example

- Let $n = 6$ and Keyword is $k = \text{CIPHER}$.

Vigener Cipher (16'th Century, Rome)

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- Let $n = 6$ and Keyword is k = CIPHER.
- **Plaintext:** THISCRYPTOSYSTEMISNOTSECURE.

Vigener Cipher (16'th Century, Rome)

Example

- Let $n = 6$ and Keyword is $k =$ CIPHER.
- **Plaintext:** THISCRYPTOSYSTEMISNOTSECURE.

T	H	I	S	C	R	Y	P	T	O	S	Y	S	T	E	M	I	S	N	O	T	S	E	C	U	R	E
C	I	P	H	E	R	C	I	P	H	E	R	C	I	P	H	E	R	C	I	P	H	E	R	C	I	P
V	P	X	Z	G	I	A	X	I	V	W	P	U	B	T	T	M	J	P	W	I	Z	I	T	W	Z	T

- **Ciphertext:** VPXZGIAXIVWPUBTTMJPWIZITWZT.

Cryptanalysis of Vigener Cipher

Let n is known

- Make the matrix from the ciphertext $c = c_0 c_1 c_2 \ldots$ as given below:

c_0	c_1	c_2	$\cdots$	c_{n-1}
c_n	c_{n+1}	c_{n+2}	$\cdots$	c_{2n-1}
c_{2n}	c_{2n+1}	c_{2n+2}	$\cdots$	c_{3n-1}
c_{3n}	c_{3n+1}	c_{3n+2}	$\cdots$	c_{3n-1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

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c_{3n}	c_{3n+1}	c_{3n+2}	$\cdots$	c_{3n-1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- All the ciphertext symbols of each column of the above table are encrypted by a single character.
- Find the most frequent character of each column and it most likely the encryption of E.
- For each column, the difference between most frequent character and E is the part of the key.

Let n is unknown

- Repeat the above procedure for $n = 1, 2, 3, \ldots$, until the decryption makes sense.

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- More improved version: Use Kasiski Test. (proposed by Friedrich Kasiski in 1863).
  - Charles Babbage discovered it in 1854.

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- More improved version: Use Kasiski Test. (proposed by Friedrich Kasiski in 1863).
  - Charles Babbage discovered it in 1854.
- Further improvement: Use of Index of Coincidence by William Friedman in 1920.

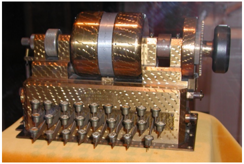

Figure: The Hebern machine (Single Rotor)

Rotor Machines (1870-1943)

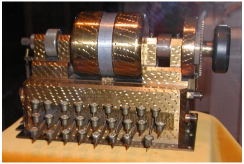

Figure: The Hebern machine (Single Rotor)

- Broken just using letter frequencies, diagrams and trigrams if sufficient ciphertexts are given.
- Key can be recovered.

Rotor Machines (1870-1943)

Figure: Most Famous Enigma (Three Rotor)

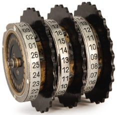

Figure: Example of Rotor

- Employed extensively by Nazi Germany during World War II.
- Was broken.
- **Alan Turing** was part of the team of those who broke Enigma.

Data Encryption Standard (1974)

- DES: 56 bit keys, 64-bit block size
- AES (2001)
- Salsa20 (2008) and so on.

Study At Home

- Affine cipher and its Cryptanalysis.
- Hill cipher and its Cryptanalysis.
- Permutation cipher and its Cryptanalysis.

End