# CWVS: Critical Window Variable Selection

## Statistical Model

$$Y_i|\boldsymbol{\beta}, \boldsymbol{\alpha} \overset{\text{ind}}{\sim} \text{Bernoulli}\left\{p_i\left(\boldsymbol{\beta}, \boldsymbol{\alpha}\right)\right\},\ i = 1, ..., n;$$

$$\log\left\{\frac{p_i\left(\boldsymbol{\beta}, \boldsymbol{\alpha}\right)}{1 - p_i\left(\boldsymbol{\beta}, \boldsymbol{\alpha}\right)}\right\} = \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\alpha\left(j\right);$$

$$\alpha\left(j\right) = \theta\left(j\right)\gamma\left(j\right),\ j = 1, ..., m;$$

$$\gamma\left(j\right)|\pi\left(j\right) \overset{\text{ind}}{\sim} \text{Bernoulli}\left\{\pi\left(j\right)\right\},\ \Phi^{-1}\left\{\pi\left(j\right)\right\} = \eta\left(j\right),\ j = 1, ..., m;$$

$$\begin{bmatrix} \theta\left(j\right) \\ \eta\left(j\right) \end{bmatrix} = A\begin{bmatrix} \delta_1\left(j\right) \\ \delta_2\left(j\right) \end{bmatrix},\ A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix};$$

$$\boldsymbol{\delta}_k = \left\{\delta_k\left(1\right), ..., \delta_k\left(m\right)\right\}^{\text{T}}|\phi_k \overset{\text{ind}}{\sim} \text{MVN}\left\{\mathbf{0}_m, \Sigma\left(\phi_k\right)\right\},\ k = 1, 2;$$

- $m = \max\left\{m_i : i = 1, ..., n\right\};$
- $\mathbf{0}_m$: Length $m$ vector with each entry equal to zero.

## Prior Information

$\beta_j \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_\beta^2\right),\ j = 1, ..., p;$

- $p$: Length of $\mathbf{x}_{ij}$ vector (same for all $i$, $j$);
- Default setting: $\sigma_\beta^2 = 10,000$.

$\ln\left(A_{11}\right), \ln\left(A_{22}\right), A_{21} \overset{\text{iid}}{\sim} \text{N}\left(0, \sigma_A^2\right);$

- Default setting: $\sigma_A^2 = 1.00$.

$\phi_k \overset{\text{iid}}{\sim} \text{Gamma}\left(\alpha_{\phi_k}, \beta_{\phi_k}\right),\ k = 1, 2;$

- Default setting: $\alpha_{\phi_k} = 1.00$, $\beta_{\phi_k} = 1.00,\ k = 1, 2$.

## Default Initial Values

- $\beta_j = 0$ for all $j$;
- $\gamma\left(j\right) = 1$ for all $j$;
- $\delta_k\left(j\right) = 0$ for all $j$, $k$;
- $\phi_k = -\ln\left(0.05\right)/\left(m - 1\right)$ for all $k$;
- $A_{kk} = 1$ for all $k$;
- $A_{21} = 0$.

## Alternate Likelihood: Gaussian

$Y_i|\boldsymbol{\beta}, \boldsymbol{\alpha}, \sigma^2_\epsilon \overset{\text{ind}}{\sim} \text{Normal}\left(\mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\alpha\left(j\right), \sigma^2_\epsilon\right),\ i = 1, ..., n.$

- $\sigma^2_\epsilon \sim \text{Inverse Gamma}\left(a_{\sigma^2_\epsilon}, b_{\sigma^2_\epsilon}\right);$
- Default setting: $a_{\sigma^2_\epsilon} = 0.01$, $b_{\sigma^2_\epsilon} = 0.01$;
- Default initial value: $\sigma^2_\epsilon = 1.00$.

## Alternate Likelihood: Negative Binomial

$Y_i|\boldsymbol{\beta}, \boldsymbol{\alpha}, r \overset{\text{ind}}{\sim} \text{Negative Binomial}\left\{r, \lambda_i\left(\boldsymbol{\beta}, \boldsymbol{\theta}\right)\right\},\ i = 1, ..., n;$

$\ln\left\{\frac{\lambda_i(\boldsymbol{\beta}, \boldsymbol{\alpha})}{1-\lambda_i(\boldsymbol{\beta}, \boldsymbol{\alpha})}\right\} = \text{O}_i + \mathbf{x}_i^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_i} \text{z}_{ij}\alpha\left(j\right).$

- $r \sim \text{Discrete Uniform}\left[a_r, b_r\right];$
- Default setting: $a_r = 1$, $b_r = 100$;
- Default initial value: $r = b_r$.

## Likelihood Indicator

- likelihood_indicator = 0: Bernoulli;
- likelihood_indicator = 1: Gaussian;
- likelihood_indicator = 2: Negative binomial.