CWVS: Critical Window Variable Selection

Statistical Model

$$Y_i|\boldsymbol{\beta}, \boldsymbol{\alpha} \stackrel{\text{ind}}{\sim} \text{Bernoulli} \{p_i(\boldsymbol{\beta}, \boldsymbol{\alpha})\}, i = 1, ..., n;$$

$$\log \left\{ \frac{p_i\left(\boldsymbol{\beta}, \boldsymbol{\alpha}\right)}{1 - p_i\left(\boldsymbol{\beta}, \boldsymbol{\alpha}\right)} \right\} = \mathbf{x}_i^{\mathrm{T}} \boldsymbol{\beta} + \sum_{j=1}^{m_i} \mathbf{z}_{ij} \alpha\left(j\right);$$

$$\alpha(j) = \theta(j) \gamma(j), j = 1, ..., m;$$

$$\gamma\left(j\right)|\pi\left(j\right)\stackrel{\mathrm{ind}}{\sim}\mathrm{Bernoulli}\left\{ \pi\left(j\right)\right\} ,\ \Phi^{-1}\left\{ \pi\left(j\right)\right\} =\eta\left(j\right),\ j=1,...,m;$$

$$\left[\begin{array}{c} \theta\left(j\right) \\ \eta\left(j\right) \end{array}\right] = A \left[\begin{array}{c} \delta_{1}\left(j\right) \\ \delta_{2}\left(j\right) \end{array}\right], \ A = \left[\begin{array}{cc} A_{11} & 0 \\ A_{21} & A_{22} \end{array}\right];$$

$$\boldsymbol{\delta}_{k} = \left\{ \delta_{k} \left(1 \right), ..., \delta_{k} \left(m \right) \right\}^{\mathrm{T}} | \phi_{k} \stackrel{\mathrm{ind}}{\sim} \mathrm{MVN} \left\{ \mathbf{0}_{m}, \Sigma \left(\phi_{k} \right) \right\}, \ k = 1, 2.$$

- $m = \max\{m_i : i = 1, ..., n\};$
- $\mathbf{0}_m$: Length m vector with each entry equal to zero.

Prior Information

$$\beta_{j} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \sigma_{\beta}^{2}\right), \ j=1,...,p.$$

- p: Length of \mathbf{x}_{ij} vector (same for all i, j);
- Default setting: $\sigma_{\beta}^2 = 10,000$.

 $\ln (A_{11}), \ln (A_{22}), A_{21} \stackrel{\text{iid}}{\sim} \text{N} (0, \sigma_A^2).$

• Default setting: $\sigma_A^2 = 1.00$.

 $\phi_k \overset{\text{iid}}{\sim} \text{Gamma}\left(\alpha_{\phi_k}, \beta_{\phi_k}\right), \ k = 1, 2.$

• Default setting: $\alpha_{\phi_k} = 1.00, \ \beta_{\phi_k} = 1.00, \ k = 1, 2.$

Default Initial Values

- $\beta_j = 0$ for all j;
- $\gamma(j) = 1$ for all j;
- $\delta_k(j) = 0$ for all j, k;
- $\phi_k = -\ln(0.05)/(m-1)$ for all k;
- $A_{kk} = 1$ for all k;
- $A_{21} = 0$.

Alternate Likelihood: Gaussian

 $Y_{i}|\boldsymbol{\beta}, \boldsymbol{\alpha}, \sigma_{\epsilon}^{2} \stackrel{\text{ind}}{\sim} \text{Normal}\left(\mathbf{x}_{i}^{\text{T}}\boldsymbol{\beta} + \sum_{j=1}^{m_{i}} \mathbf{z}_{ij}\alpha\left(j\right), \sigma_{\epsilon}^{2}\right), i = 1, ..., n.$

- $\sigma_{\epsilon}^2 \sim \text{Inverse Gamma}\left(a_{\sigma_{\epsilon}^2}, b_{\sigma_{\epsilon}^2}\right)$;
- Default setting: $a_{\sigma_{\epsilon}^2} = 0.01, b_{\sigma_{\epsilon}^2} = 0.01;$
- Default initial value: $\sigma_{\epsilon}^2 = 1.00$.

Alternate Likelihood: Negative Binomial

 $Y_{i}|\boldsymbol{\beta},\boldsymbol{\alpha},r\overset{\mathrm{ind}}{\sim}\mathrm{Negative\ Binomial}\left\{ r,\lambda_{i}\left(\boldsymbol{\beta},\boldsymbol{\theta}\right)\right\} ,\ i=1,...,n;$

$$\ln \left\{ \frac{\lambda_{i}(\boldsymbol{\beta}, \boldsymbol{\alpha})}{1 - \lambda_{i}(\boldsymbol{\beta}, \boldsymbol{\alpha})} \right\} = \mathrm{O}_{i} + \mathbf{x}_{i}^{\mathrm{T}} \boldsymbol{\beta} + \sum_{j=1}^{m_{i}} \mathrm{z}_{ij} \alpha\left(j\right).$$

- $r \sim \text{Discrete Uniform}[a_r, b_r];$
- Default setting: $a_r = 1, b_r = 100;$
- Default initial value: $r = b_r$.

Likelihood Indicator

- likelihood_indicator = 0: Bernoulli;
- likelihood_indicator = 1: Gaussian;
- likelihood indicator = 2: Negative binomial.