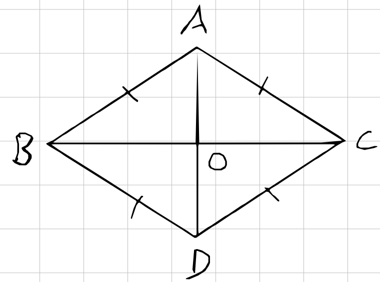
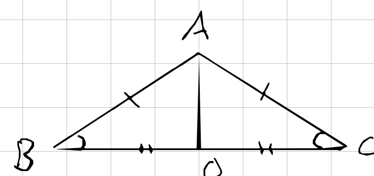
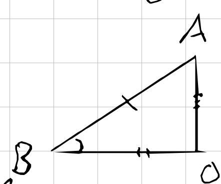
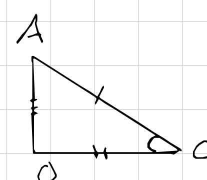
1.4



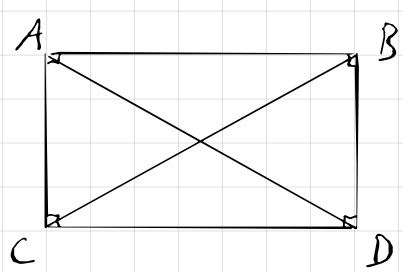
Proof. Given $AB = AC = BD = DC$. Note that BC is a shared line for ΔABC and ΔDBC, so by SSS they're equal. Let line AO cut through line BC. Note Then we have $BO = OC$. Also since AO is a shared line, by SSS we have $\Delta ABO = \Delta ACO$. Hence using the same process $\Delta DBO = \Delta DCO$. Therefore since $\Delta ABC = \Delta DBC$, we have $\Delta ABO = \Delta ACO = \Delta DBO = \Delta DCO$, so by SSS we have $\angle AOB = \angle AOC = DOB = DOC$. ∎

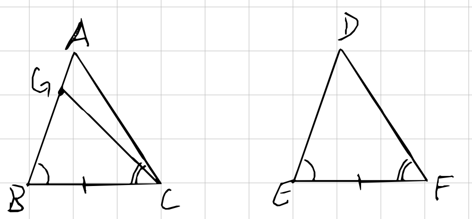






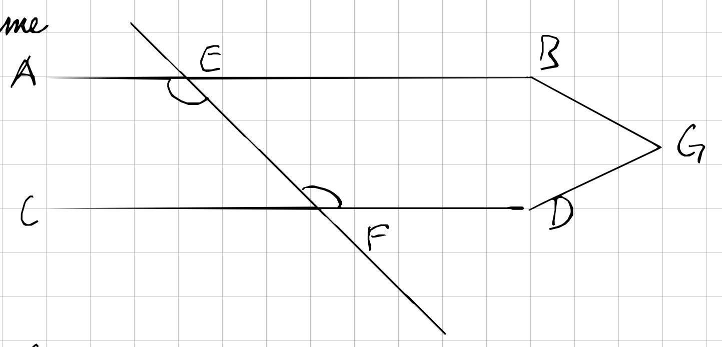
1.5 Proof.



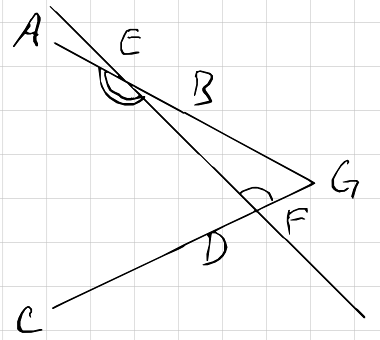


Suppose $AB > DE$. Let $BG = DE$ and $GC = DF$.
Then $\angle GBC = \angle DEF$, so $GC = DF$,
so $\angle BCG = \angle EFD$. Thus $\angle BCG = \angle BCA$,
which is false, so $AB = DE$.

Assume

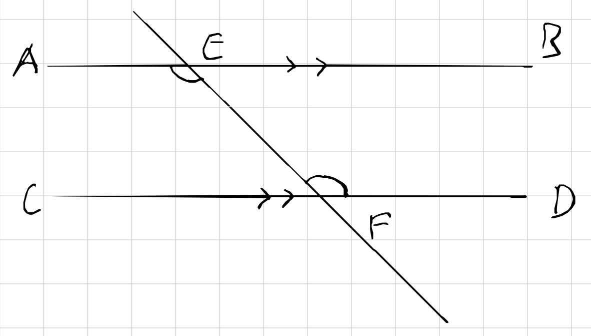


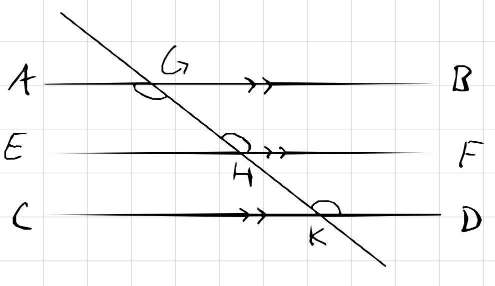
Suppose AB and CD meet at G.

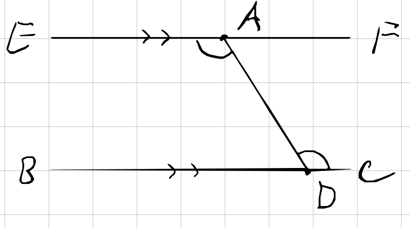


Then at exterior angles of ΔGEF we have
$$\angle AEF = \angle EFG$$
but by proposition 16,
$$\angle AEF \neq \angle GFE,$$
so

so we have

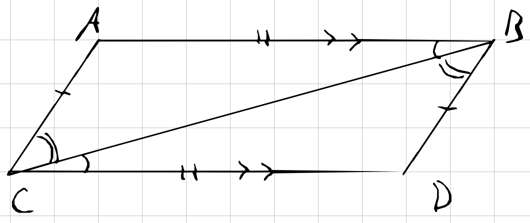




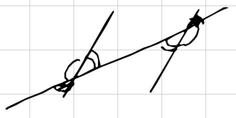


1.13. Given $\varphi = (1+\sqrt{5})/2$. Let $\psi = (1-\sqrt{5})/2$.
We then prove $Fib(n) = (\varphi^n - \psi^n)/\sqrt{5}$.

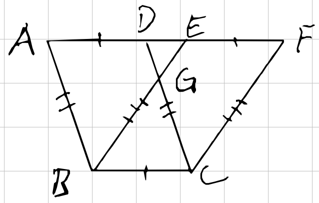
Proof. By induction we prove $n=0$, $n=1$. Then we have
$$Fib(0) = (\varphi^0 - \psi^0)/\sqrt{5} = (1-1)/\sqrt{5} = 0,$$
$$Fib(1) = (\varphi - \psi)/\sqrt{5} = ((2\sqrt{5})/2)/\sqrt{5} = \sqrt{5}/\sqrt{5} = 1.$$
Hence both $n=0,1$ holds. Now suppose that $F(n) = (\varphi^n - \psi^n)/\sqrt{5}$ for all $0<1<n$.
Then we have
$$\begin{aligned} Fib(n+1) &= Fib(n) + Fib(n-1) \\ &= (\varphi^n - \psi^n)/\sqrt{5} + (\varphi^{n-1} - \psi^{n-1})/\sqrt{5} \\ &= ((\varphi^n + \varphi^{n-1}) - (\psi^n + \psi^{n-1}))/\sqrt{5}. \end{aligned}$$
Hence since
$$x^2 = x+1, \text{ we have}$$
$$\varphi^{n-1}\varphi^2 = \varphi\varphi^{n-1} + \varphi^{n-1},$$
$$\varphi^{n+1} = \varphi^n + \varphi^{n-1} \text{ and}$$
$$\psi^{n-1}\psi^2 = \psi\psi^{n-1} + \psi^{n-1},$$
$$\psi^{n+1} = \psi^n + \psi^{n-1}.$$
Thus $Fib(n+1) = (\varphi^{n+1} - \psi^{n+1})/\sqrt{5}$.
Hence since $0 < \psi < 1$, as n increases we have $F(n)$ gets closer to $\varphi^n/\sqrt{5}$. ∎



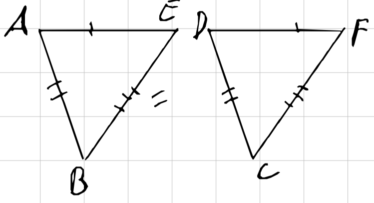
$AC = BD$ due to $\triangle ABC = \triangle DCB$.
Hence $\angle ACB = \angle CBD$.



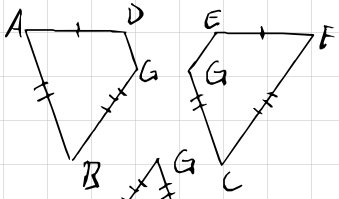
Also since the line BC goes through AB and CD, and has alternate angles, $AC \parallel BD$.

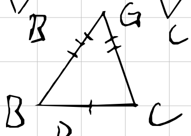


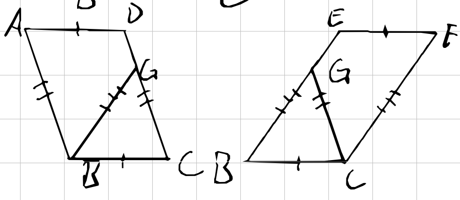
Given $AF \parallel BC$ We show $ABCD = EBCF$.

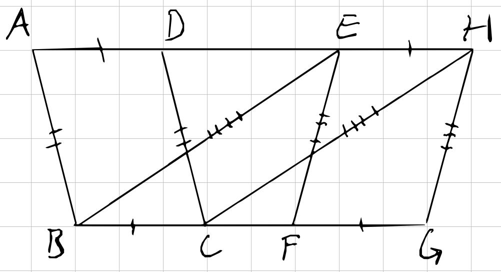


Thus since $ABCD$ is a parallelogram $AD = BC$, and similarly $EF = BC$. Therefore $AD = EF$, so $AD + DE = DE + EF$, so $AB = DF$. We also have $AB = DC$ and $EB = FC$. Hence $\triangle ABE = \triangle DCF$. Then we subtract DGE to get two trapeziums $ABDG = FCGE$. Next we add $\triangle BGC$ to each making two parallelograms $ABCD = EBCF$. Therefore we have two parallelograms that are equal which are on the same base and parallels from each other.

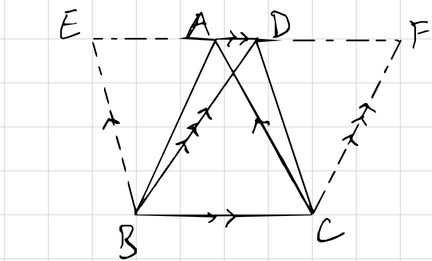


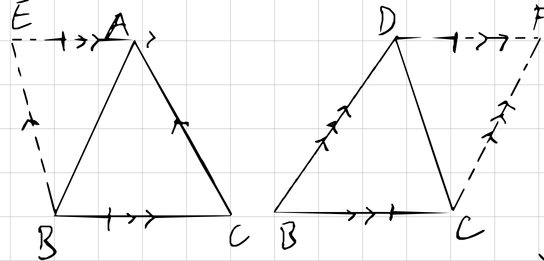




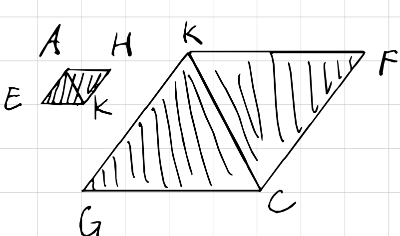


Suppose $ABCD = EFGH$.
Next join BE and CH.
Then since $BC = EH$, we have $BE = CH$. Thus since $BC \parallel EH$, we also have $BE \parallel CH$, so $BCEH$ is a parallelogram.
Thus this meant $BCEH = ABCD = EFGH$ since $BC \parallel AD$, $EH \parallel BC$.
Therefore parallelograms are equal if their bases and parallels are the same.

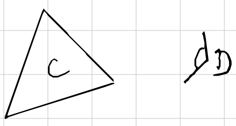


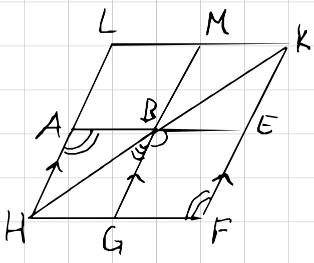


We extend line AD to be within EF. Next join BE and CF. Next ensure $BE \parallel AC$ and $BD \parallel CF$. Then we have two equal parallelogram $ACBE = DBCF$ since they're parallel and of the same base BC. Note that $\Delta ABC = \Delta DBC$ since the half of their parallelogram are their respective triangle. Therefore triangles of the same base and parallels equal each other.

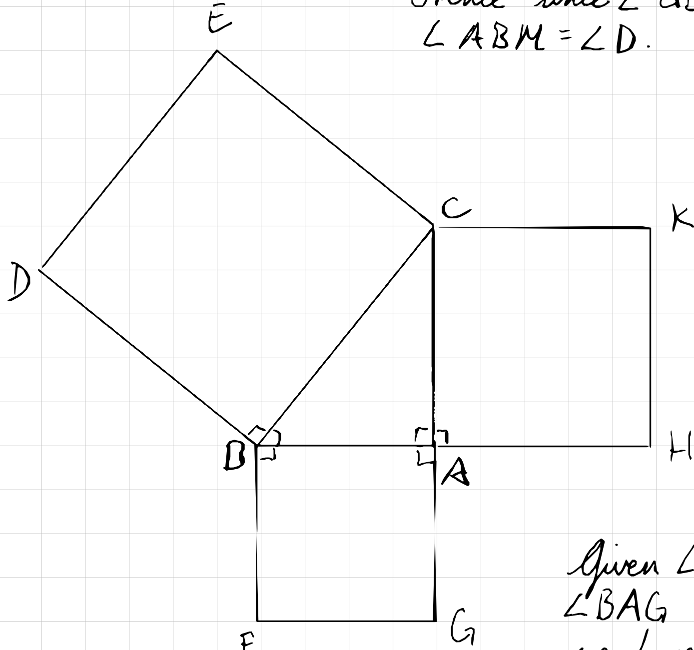


Given ABCD is a parallelogram, and AC is its diameter. Let EH and FG be parallelograms extended from some point K in line AC, and let BK and KD be complements. We show that $BK = KD$. Then since ABCD is a parallelogram, we have $\Delta ABC = \Delta ACD$. In a similar manner for EH and FG, we have $\Delta AEK = \Delta AHK$, and $KGC = KFC$. Therefore since $\Delta ABC = \Delta ADC$, we have $BK = KD$ remaining.

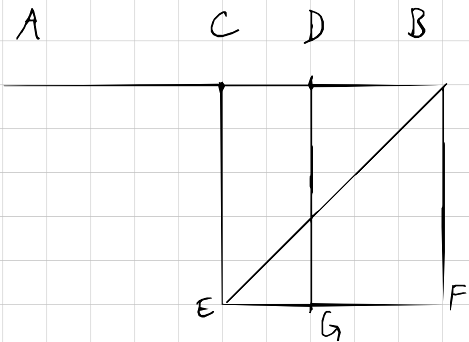




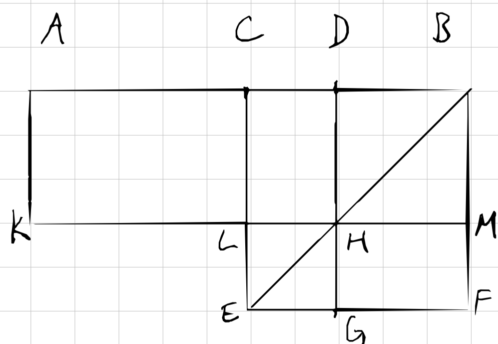
Suppose that $\Delta C = BEFG$ and $\angle EBG = \angle D$. Next extend line EB to A. Next do the same with FG to have H, which is $AH \parallel BG \parallel EF$ then join HB. Thus since $AH \parallel EF$, then $\angle AHF + \angle HFE$ is the angle of a straight line. Therefore $\angle BHG + \angle GFE < \angle AHF + \angle GFE$, so $\angle BHG < \angle AHF$, so line HB and EF will intersect at K. Next extend K to be parallel to AE while extend HA to meet the line from K and label the intersection point L. Next extend line GB to meet a point on line LK, label that point M. Then we have parallelogram $HFKL$ with diameter HK with AG and ME to be parallelograms. Thus $LB = BF$ since they're complements. Thus $LB = \Delta C$. Hence since $\angle GBE = \angle ABM$, we have $\angle ABM = \angle D$.



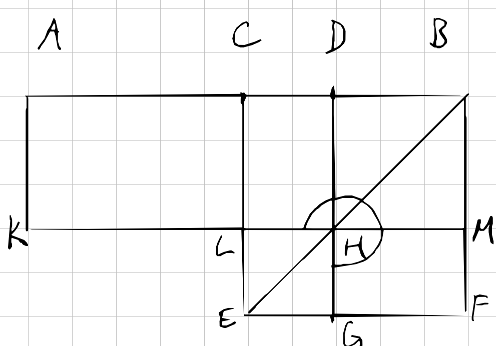
Given $\angle BAC$ is $\perp$ and $\angle BAG$ is also $\perp$. Then we have CG to be a straight line. Thus BH is too in a similar manner. Thus since $\angle DBC = \angle FBA$ and are $\perp$, we have $\angle DBA = \angle FBC$.



Let AB be a line, $AC = CB$, and $AD \neq DB$. Now construct the square $CEFB$ on CB. Next join points BE. Next create a line DG where $DG \parallel BF \parallel CE$, and $DG = BF = CE$.

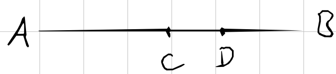


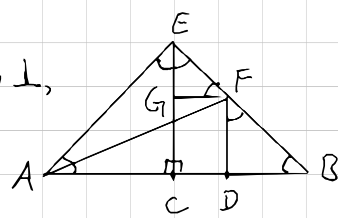
Next draw the line KM such that $KM = AB$ and $KM \parallel AB$. Then join AK where $AK = BM$ and $AK \parallel BM$. Note we have an intersect H from DG and KM. In a similar manner we have L. Note $CH = HF$ via their complements. Thus $CM = DF$. Hence since lines $AC = CB$, rectangles $AL = CM$.



Hence since AH = line AD * line DB, since $DH = DB$, thus gnomon LBG also equal AH next we have square of $LG = CD^2$. Therefore $LBG + LG = AD \cdot DB + CD$

We show
$AD^2 + DB^2 = 2\,(AC^2 + CD^2)$.





Since $\overline{AC} = \overline{CE}$ and ACE is $\perp$,
$\angle EAC = \angle AEC$ and
$\angle EAC + \angle AEC = \perp$.
Hence since $\Delta ACE =$
ΔBCE, $\angle CEA = \angle CEB$.
Hence since FGE is $\perp$,
$\angle GFE = \angle GEF$ and
$\angle GFE + \angle GEF = \perp$.
Hence $\angle DFB = DBF$, so $\overline{DF} = \overline{DB}$,
Thus since $AC = CE$, $AC^2 = CE^2$, so

$$2(AC)^2 = (AC)^2 + (CE)^2,$$

also $(EA)^2 = (AC)^2 + (CE)^2$,
so $(EA)^2 = 2(AC)^2$;
similarly $(EF)^2 = 2(GF)^2 = 2\,(CD)^2$.
Thus since $\angle AEF$, we have

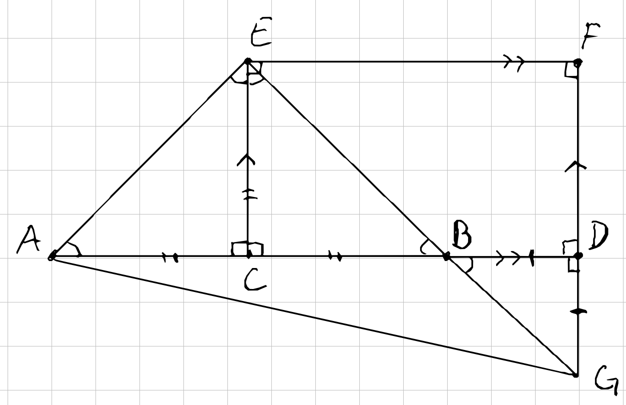
$$(AF)^2 = (AE)^2 + (EF)^2,$$

so

$$(AF)^2 = 2(AC)^2 + 2\,(CD)^2$$
$$= 2(\,(AC)^2 + (CD)^2),$$
$$(AE)^2 + (EF)^2 = 2\,(\,(AC)^2 + (CD)^2)$$
$$(AD)^2 + (DF)^2 = 2\,(\,(AC)^2 + (CD)^2).$$

Given

We prove $(AD)^2 + (BD)^2 = 2((AC)^2 + (CD)^2)$.



Therefore $\angle AEB$ is $\perp$. Hence $\angle DBG$ is half a $\perp$ since proposition 15 from book 1, so $\angle DBG = \angle DGB$. Thus $\overline{BD} = \overline{DG}$. Hence since $\angle EGF = \angle FEG$, we have $\overline{EF} = \overline{FG}$. Thus $(AC)^2 + (CB)^2 = 2(AC)^2$, so $(EA)^2 = 2(AC)^2$. Hence since $\overline{GF} = \overline{EF}$, we have $(EG)^2 = (EF)^2 + (FG)^2 = 2(EF)^2$. Thus $EF = CD$, so $(EG)^2 = 2(CD)^2$. Thus

$$(AE)^2 + (EG)^2 = 2(AC)^2 + 2(CD)^2 = 2((AC)^2 + (CD)^2).$$

Hence since $(AG)^2 = (AE)^2 + (EG)^2$, we also have

$$(AG)^2 = (AD)^2 = (DG)^2,$$

hence since $DG = BD$, we have

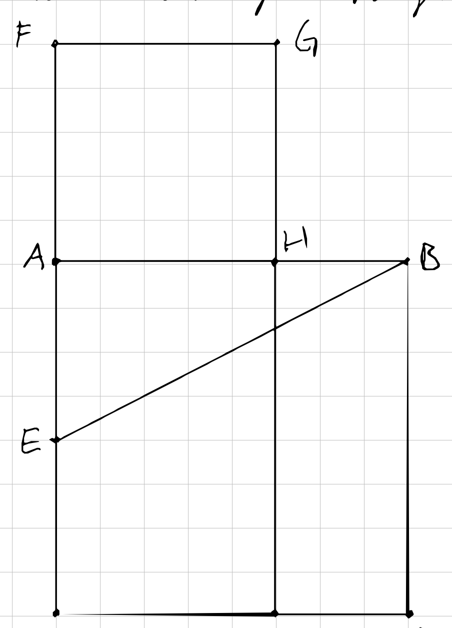
$$(AG)^2 = (AD)^2 + (BD)^2.$$

Therefore

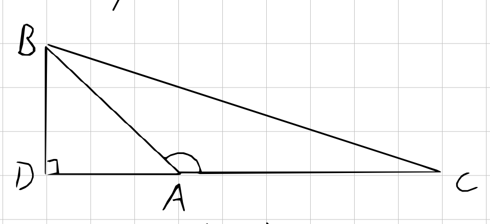
$$(AD)^2 + (BD)^2 = 2((AC)^2 + (CD)^2). \blacksquare$$

Given our line AB.

we then cut at point H giving us



Proposition 12.
Let ABC be a triangle such that $\angle BAC$ is obtuse, and let BD be a $\perp$ line from B while CA has been extended as follow:



We show that $(BC)^2 > (BA)^2$, by $+ 2(AC)(AD)$.
Since CD is a straight line and A is some point of that line, we have $(DC)^2 = (CA)^2 + (AD)^2 + 2(AC)(AD)$, so

$$(DC)^2 + (DB)^2 = (CA)^2 + (AD)^2 + 2(AC)(AD) + (DB)^2,$$

since $(BC)^2 = (DC)^2 + (DB)^2$, we have

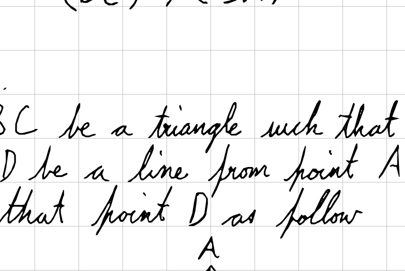
$$(BC)^2 = (CA)^2 + (AD)^2 + 2(AC)(AD) + (DB)^2,$$

since $(AD)^2 + (DB)^2 = (BA)^2$, we also have

$$(BC)^2 = (BA)^2 + 2(AC)(AD),$$

so $(BC)^2 > (BA)^2$. $\blacksquare$

Proposition 13.
Let ABC be a triangle such that $\angle ABC$ is acute, and let AD be a line from point A to be $\perp BC$, we call that point D as follow



We show that $(AC)^2 < (CB)^2 + (AB)^2$
Note that

$$(CB)^2 + (BD)^2 = 2(CB)(BD) + (DC)^2,$$

so

$$(CB)^2 + (BD)^2 + (DA)^2 = 2(CB)(BD) + (DC)^2 + (DA)^2,$$

hence since

$$(DC)^2 + (DA)^2 = (AC)^2$$

and

$$(BD)^2 + (DA)^2 = (AB)^2,$$

we have

$$(CB)^2 + (AB)^2 = 2(CB)(BD) + (AC)^2.$$

Hence

$$(CB)^2 + (AB)^2 > (AC)^2. \blacksquare$$

$$T_{pq}(a,b) = (bq + aq + ap,\ bp + aq)$$

$$T_{pq}(T_{pq}(a,b)) = T_{pq}(bq + aq + ap,\ bp + aq)$$
$$= ((bp + aq)q + (bq + aq + ap)q + (bq + aq + ap)p,\ (bp + aq)p + (bq + aq + ap)q)$$

$$= (\cancel{bpq} + \cancel{aq^2} + bq^2 + \cancel{aq^2} + \cancel{apq} + bpq + \cancel{apq} + ap^2,\ bp^2 + \cancel{apq} + bq^2 + aq^2 + \cancel{apq})$$

$$= (2bpq + 2aq^2 + 2apq + bq^2 + aq^2,\ bp^2 + 2apq + bq^2 + aq^2)$$