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Towards Mapping Lift To Deep Neural Networks

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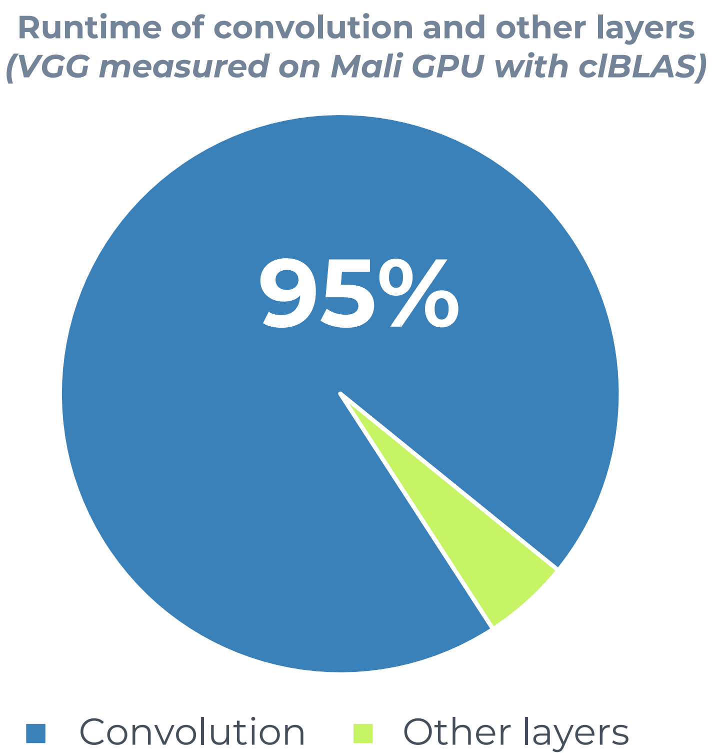
Convolution

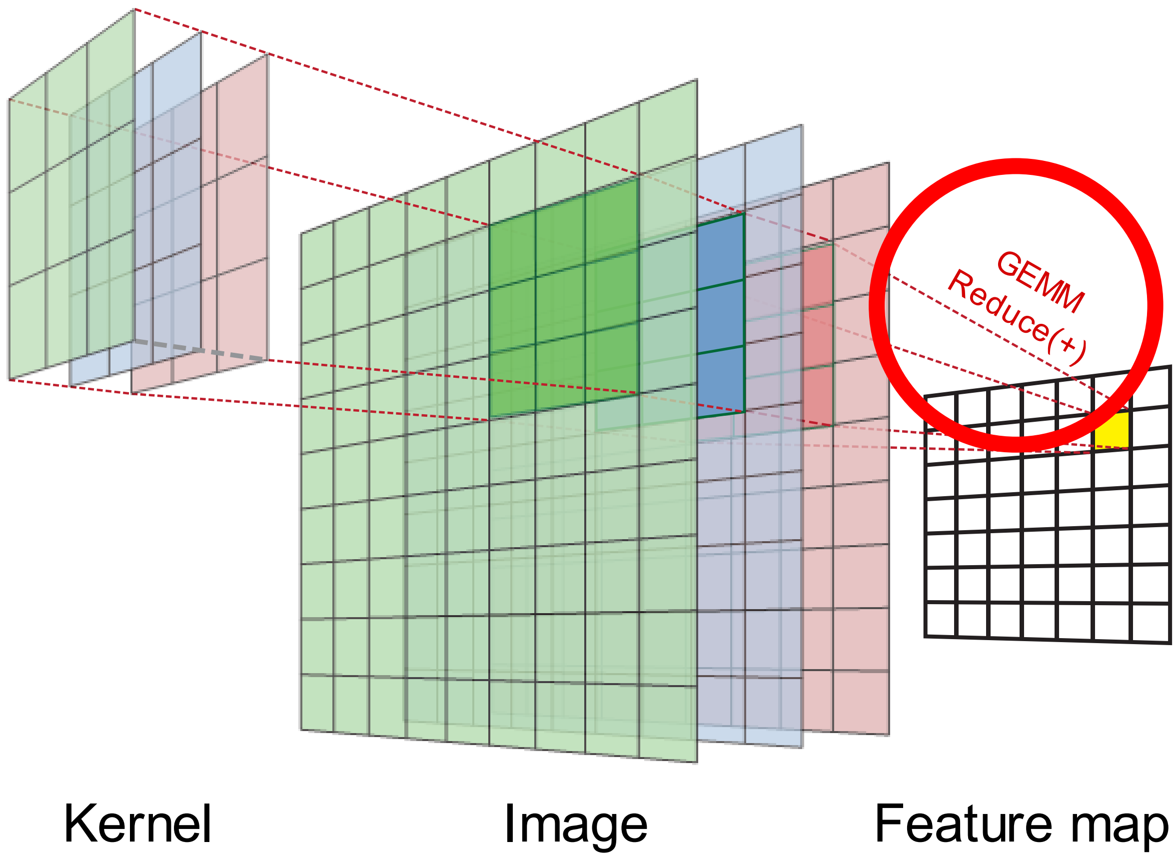
- Many popular NN architectures depend on convolution
  - *AlexNet (2012)*
  - *ZFNet(2013)*
  - *GoogleNet(2014)*
  - *VGGNet(2014)*
  - *ResNet(2015)*



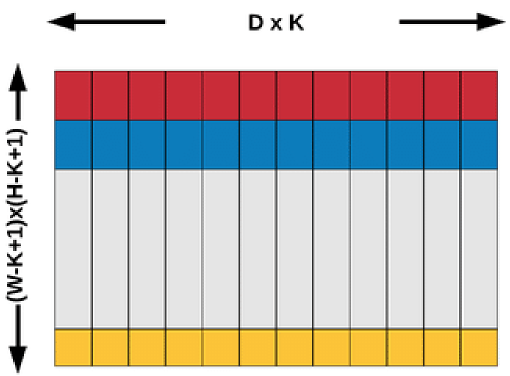
Convolution: algorithms

- Stencils
- *Im2col*
- *Fast Fourier transform*
- *Winograd*

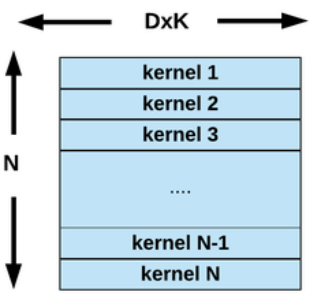
Convolution: stencils



Convolution: im2col



X

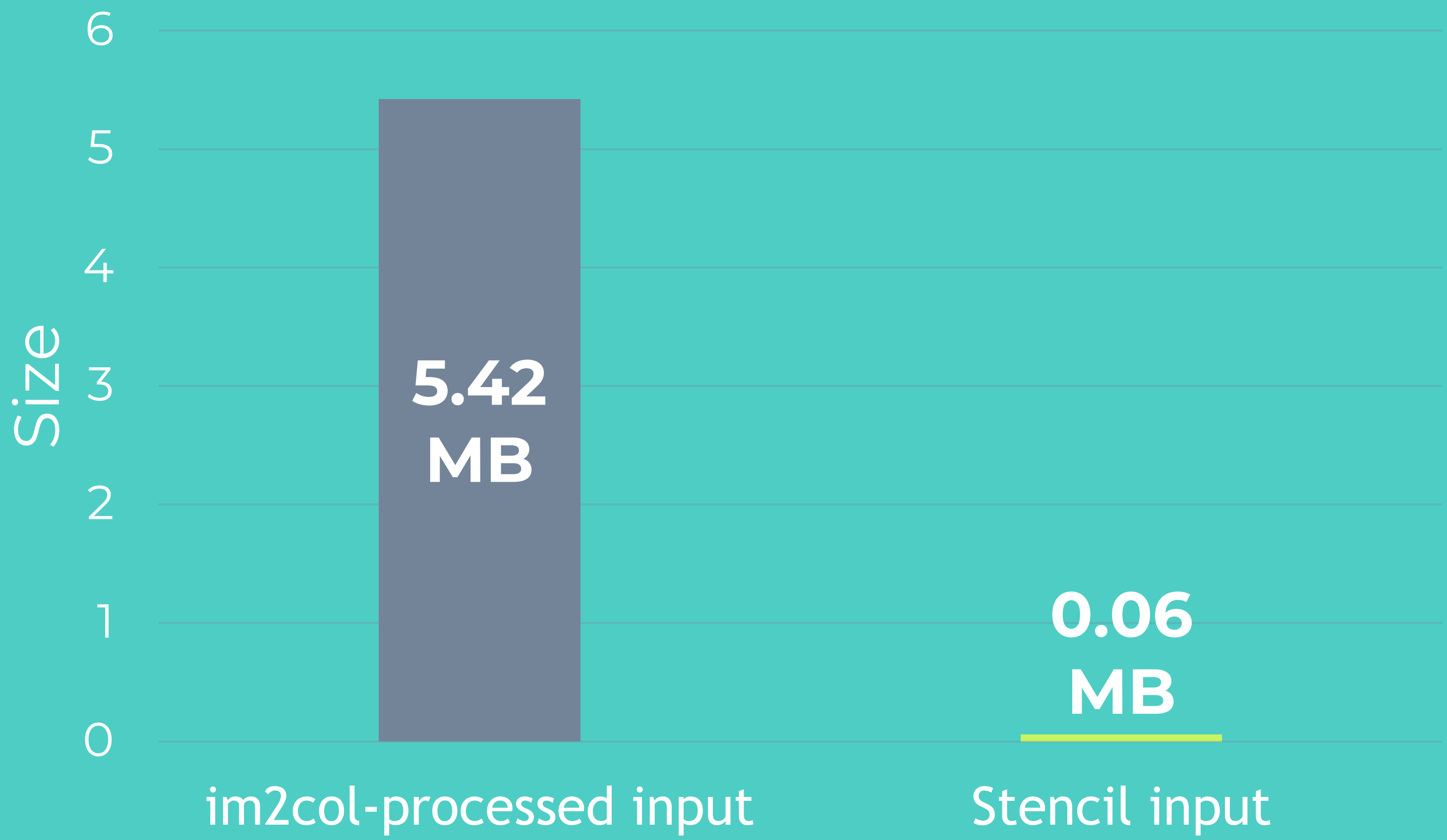


= Convolution result

GEMM
Reduce(+)

Image source: Loukadakis et al (2018)

Input image size in the two methods (1^{st} layer of VGG)



GEMM in neural networks

- Both stencil and im2col methods for convolution rely heavily on GEMM
- GEMM is also the basis of fully connected layers
  - MLPs make for a large portion of server workloads

Hardware accelerators

GPUs

Image source: https://tomshardware.co.uk

FPGAs

Image source: https://eetimes.com

ASICs

Image source: https://blogs.microsoft.com

Examples: TPU, BrainWave, DianNao, Huawei Da Vinci, Movidius Myriad

All accelerators feature:

- Large memory
- High bandwidth
- Multidimensional computational units

Compiling for hardware accelerators

- Multidimensional computational units are exposed in instruction sets:
  - `VVAdd32, VVAdd64`
  - `VVMul32, VVMul64, VVMul128`
  - `MVMul32, MVMul64, MVMul128`

- Finding opportunities to use these primitives is challenging

Compiling for hardware accelerators

- Using built-in primitives is made harder by:
  - **Other optimisations**
    - Parallelisation
    - Tiling *(for limited shared memory)*
    - Memory coalescing
    - Prefetching
    - *(etc)*
  - **Individual device characteristics**

Compiling for hardware accelerators

GEMM

```
for (int i = 0; i < M; i++) {
  for (int j = 0; j < N; j++) {
    for (int k = 0; k < K; k++) {
      temp[k + K*N*i + K*j] =
        A[k + K*i] * B[k + K*j];
    }
    for (int k = 0; k < K; k++) {
      C[j + N*i] +=
        temp[k + K*N*i + K*j];
    }}}
```

Compiling for hardware accelerators

GEMM

```
for (int i = 0; i < M; i++) {
  for (int j = 0; j < N; j++) {
    for (int k = 0; k < K; k++) {
      temp[k + K*N*i + K*j] =
        A[k + K*i] * B[k + K*j];
    }
    for (int k = 0; k < K; k++) {
      C[j + N*i] +=
        temp[k + K*N*i + K*j];
    }}}
```

GEMMtiled

```
for (int i = 0; i < M; i++) {
  for (int j = 0; j < N; j++) {
    for (int k = 0; k < K/4; k++) {
      for (int l = 0; l < 4; l++) {
        temp[l + k*4 + K*N*i + K*j] =
          A[l + k*4 + K*i] * B[l + k*4 + K*j];
      }
    }
    for (int k = 0; k < K; k++) {
      C[j + N*i] +=
        temp[k + K*N*i + K*j];
    }}}
```

Questions:

Data contiguity

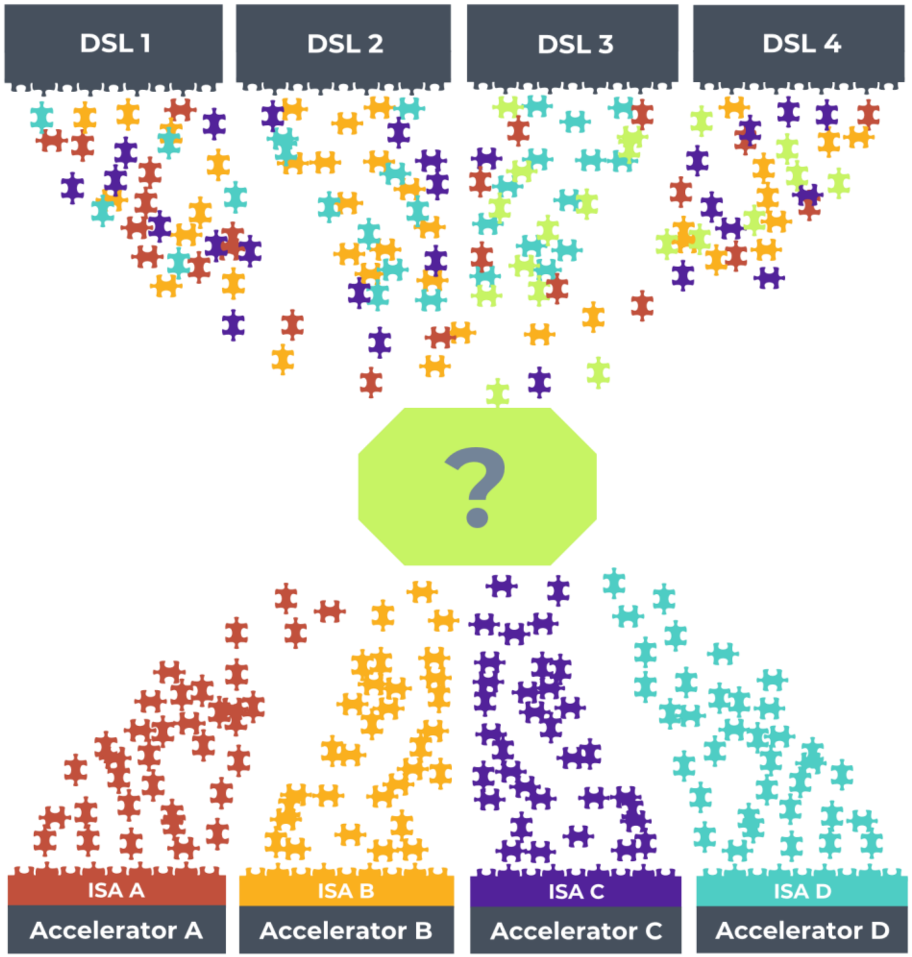
Data size

Efficient memory accesses

Data type

The problem

- Combine device-specific operators optimally
- Design a performance portable approach
- Automate and abstract the process from the user



Lift: the approach

1. Separate algorithm (WHAT) from implementation (HOW)

GEMM imperative

```
for (int i = 0; i < M; i++) {
  for (int j = 0; j < N; j++) {
    for (int k = 0; k < K; k++) {
      temp[k + K*N*i + K*j] =
        A[k + K*i] * B[k + K*j];
    }
    for (int k = 0; k < K; k++) {
      C[j + N*i] +=
        temp[k + K*N*i + K*j];
    }}}
```

→

GEMM functional

```
A >> Map(Arow =>
  B >> Map(Bcol =>
    Zip(Arow, Brow) >>
    Map(ScalarMul) >>
    Reduce(0, Add)
  )
)
```

Lift: the approach

2. Detect and rewrite patterns

GEMM$^{\text{functional}}$

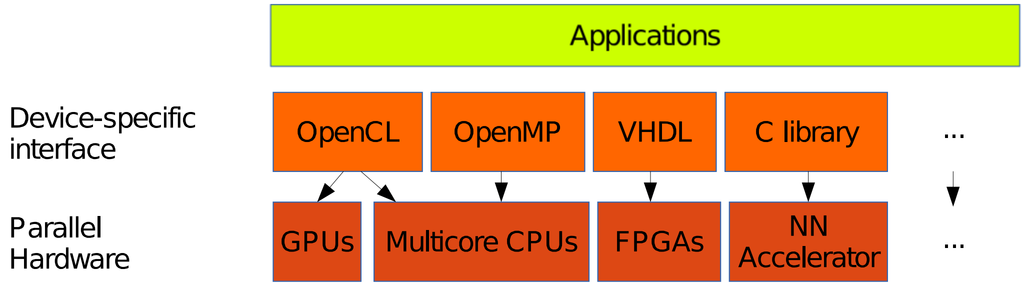
```
A >> Map(Arow =>
  B >> Map(Bcol =>
    Zip(Arow, Brow) >>
    Map(ScalarMul) >>
    Reduce(0, Add)
  )
)
```

GEMM$^{\text{optimised}}$

```
MMul(A, B)
```

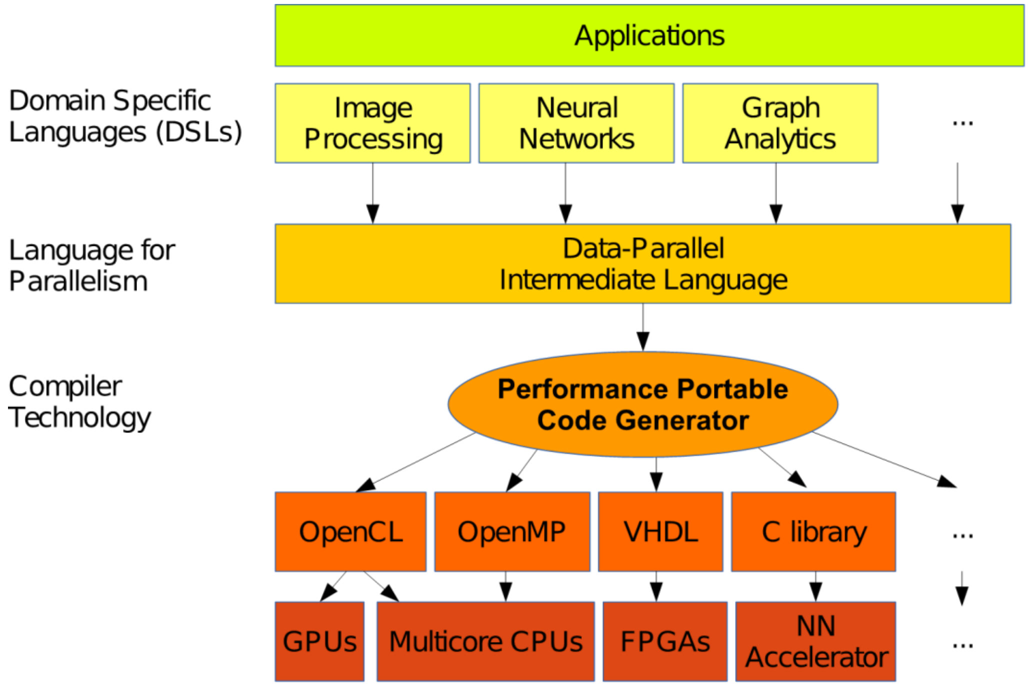
Lift in context

Current landscape



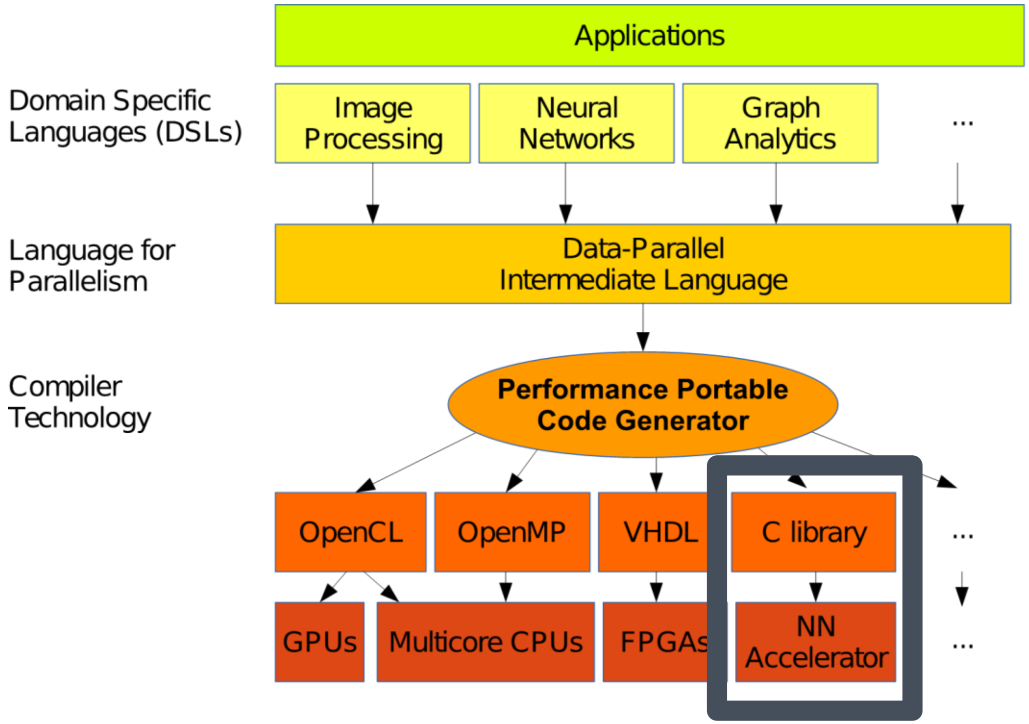
Lift in context

The goal



Lift in context

The focus of this talk



Lift: IR

- Data types
  - `Int, Float8 / Float16 / Float32, Arrays`
- Algorithmic patterns
  - `Map, Slide, Reduce, Zip, Join, Split`
- Address space operators
  - `toChip, toDram, toOutput`
- Arithmetic operators
  - `ScalarAdd, VVAdd, MVAdd, MMAdd`
  - `ScalarMul, VVMul, MVMul, MMMul`
  - `VVRelu, VVTanh`

Lift: example rewrite rules

Split-join rule

```
Map(f)
   ↧   ↧   ↧
Split(n) >>
Map(Map(f)) >>
Join
```

Map fusion rule

```
Map(f) >> Map(g)
   ↧   ↧   ↧
Map(f >> g)
```

GEMV rule

```
matrix >> Map(row =>
  VVMul(row, vector) >>
  Reduce(ScalarAdd, 0))
      ↧   ↧   ↧
MVMul(matrix, vector)
```

Rewrite rule system

- Generic and customisable
- 3 levels: DSL, algorithmic, hardware
- Extensible

Lift: rewriting

A fully connected layer

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = B[i];
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
```

?

```
temp = MVMul(W, X);
OUT = VVAdd(temp, B);
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, neuronB)) >>
VVRelu() >> toOutput()
```

↑

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = B[i];
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, neuronB)) >>
VVRelu() >> toOutput()
```

```
// GEMV rewrite
matrix >> Map(row =>
  VVMul(row, vector) >>
  Reduce(ScalarAdd, 0))
       ↧    ↧    ↧
MVMul(matrix, vector)
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = B[i];
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, neuronB)) >>
VVRelu() >> toOutput()
```

`<Extract Initializer From Reduce>`

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = B[i];
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, neuronB)) >>
VVRelu() >> toOutput()
```

→

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = B[i];
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

<Map fission>

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W,
  B) >>
Map((neuronW, neuronB) =>
  VVMul(neuronW, toChip(X)) >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X))),
  B) >>
Map((neuronWX, neuronB) =>
  neuronWX >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }

  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X))),
  B) >>
Map((neuronWX, neuronB) =>
  neuronWX >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X))),
  B) >>
Map((neuronWX, neuronB) =>
  neuronWX >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

<A couple rewrites later...>

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X))),
  B) >>
Map((neuronWX, neuronB) =>
  neuronWX >>
  Reduce(ScalarAdd, 0) >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

→

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X)) >>
    Reduce(ScalarAdd, 0)),
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
  OUT[i] += B[i];
}
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X)) >>
    Reduce(ScalarAdd, 0)),
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X)) >>
    Reduce(ScalarAdd, 0)),
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
// GEMV rewrite
matrix >> Map(row =>
  VVMul(row, vector) >>
  Reduce(ScalarAdd, 0))
      ↧   ↧   ↧
MVMul(matrix, vector)
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  W >> Map(neuronW =>
    VVMul(neuronW, toChip(X)) >>
    Reduce(ScalarAdd, 0)),
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
Zip(
  MVMul(W, X >> toChip)
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
for (int i = 0; i < n_neurons; i++) {
  for (int j = 0; j < x_length; j++) {
    temp[j + x_length*i] =
      X[j] * W[j + x_length*i];
  }
  OUT[i] = 0;
  for (int j = 0; j < x_length; j++) {
    OUT[j] += temp[j + x_length*i];
  }
}
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

```
temp = MVMul(W, X);
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  MVMul(W, X >> toChip)
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
temp = MVMul(W, X);
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  MVMul(W, X >> toChip)
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
< Vectorise Map-Zip >
```

```
temp = MVMul(W, X);
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

Lift: rewriting

```
Zip(
  MVMul(W, X >> toChip)
  B) >>
Map((neuronWXreduced, neuronB) =>
  neuronWXreduced >>
  ScalarAdd(neuronB)) >>
VVRelu() >> toOutput()
```

```
VVAdd(
  MVMul(W, X >> toChip)
  B) >>
VVRelu() >> toOutput()
```

```
temp = MVMul(W, X);
for (int i = 0; i < n_neurons; i++) {
  OUT[i] += B[i];
}
```

```
temp = MVMul(W, X);
OUT = VVAdd(temp, B);
```

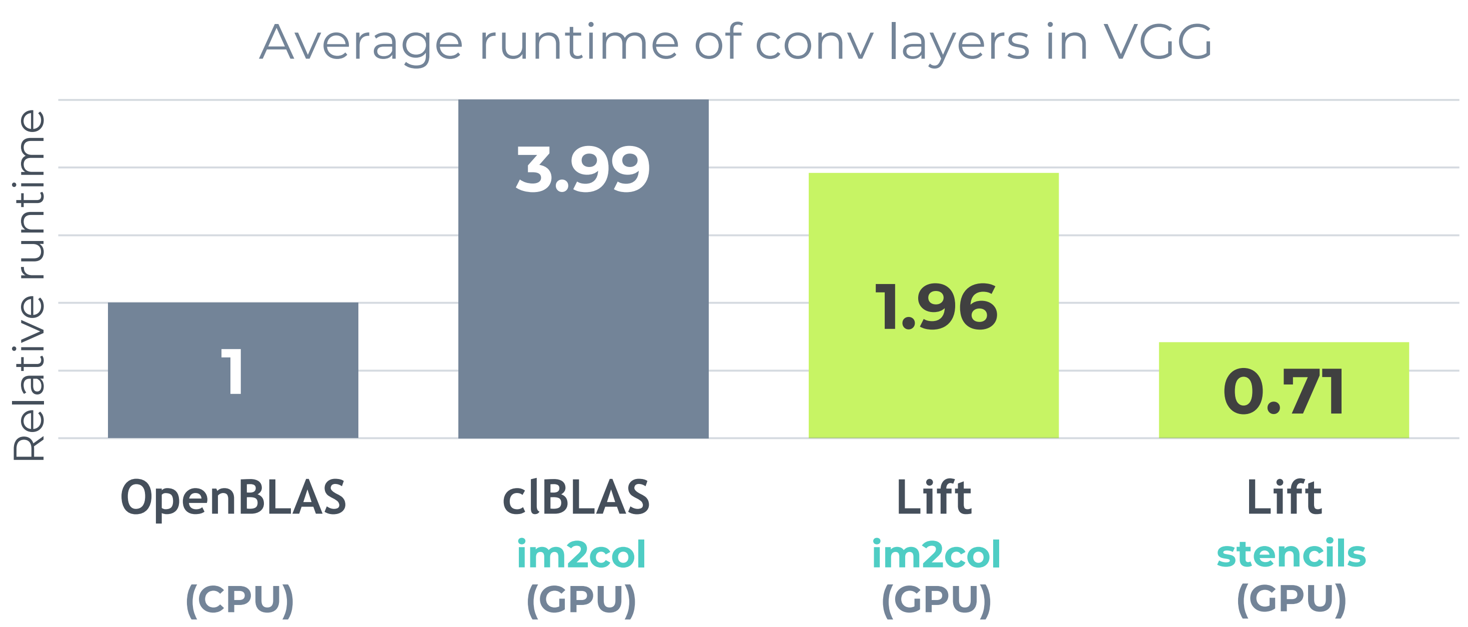
Lift: rewriting

```
VVAdd(
  MVMul(W, X >> toChip)
  B) >>
VVRelu() >> toOutput()
```

```
temp = MVMul(W, X);
OUT = VVAdd(temp, B);
```

Preliminary results

- Functional correctness on the BrainWave accelerator
- Performance measurements on Mali GPU



Future work

- Generation of both OpenCL kernel and host runtime
- Performance evaluation on popular DNN architectures
- Support for more hardware accelerators