Retail Trading and Asset Prices: The Role of Changing Social Dynamics*

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Abstract

Social-media-fueled retail trading poses new risk to institutional investors. This paper examines the origin and pricing of this new risk. Using data on meme stocks, I first establish that aggregate fluctuations in retail sentiment originated from a growing and concentrated social network. I then document that retail sentiment fluctuations induced changes in investor base composition. As sentiment increased throughout 2020 and 2021, retail investors built up long positions, while price-elastic long institutions started to exit the market since early 2020. Short interest stayed high in 2020, then dropped sharply following the price surge in January 2021, and remained low throughout 2021. I develop a model to show that retail sentiment shocks shift investor composition, which in turn determines the price of retail sentiment risk. In particular, following an increase in aggregate retail sentiment, price-elastic long institutions first hit their short-sale constraints. Then short institutions hit the margin constraints, leading to a short squeeze. As a consequence, the market for an individual stock becomes price-inelastic, and a moderate retail sentiment shock can have a large price impact. The model reconciles the price, quantity and retail sentiment dynamics during this period. Finally, I conduct counterfactuals, which show that social network dynamics shape the distribution of sentiment shocks, and have economically large impact on asset prices.

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1 Introduction

Retail trading accounts for an increasing share of U.S. equity trading activity. From mid-2020 to mid-2021, retail investors' activity was responsible for over 20% of all shares traded in the U.S. stock market. And in the first half of 2021, "new brokerage accounts opened by retail investors have roughly matched the total created throughout 2020."[1] This flood of new investors, many of which are young first-time traders, transformed social media platforms (e.g., Reddit, Twitter, and TikTok) into virtual trading clubs, where they can share investment ideas, spur each other, and coordinate actions against institutional investors. And retail sentiment expressed on social media was blamed for adding fuel to market turmoil.

This paper answers four questions in this context: What is the role of social media platforms in shaping retail sentiment? How do retail investors interact with institutional investors? Do they pose new risks to institutional investors? And during the GameStop frenzy in January 2021, why did sophisticated short sellers fail to anticipate the risk of a short squeeze?

I provide empirical evidence on the link between retail sentiment, asset prices, and quantities, using data on meme stocks. I document that aggregate retail sentiment fluctuations originated from a growing and concentrated social network. The influence distribution across users on the network determined the sentiment shock distribution. Retail sentiment shocks then induced changes in investor base composition, which in turn determines the pricing of retail sentiment risk. I present a unified framework that reconciles retail sentiment fluctuations originated from social networks, and the price and quantity dynamics following the sentiment changes.

I first present four facts, in the context of GameStop short squeeze. Fact 1 establishes the relationship between GameStop's price and aggregate retail sentiment from Reddit WallStreetBets (WSB) forum. I document that the price movement of GameStop decoupled from the average retail sentiment movement: Average sentiment had been steadily increasing since early 2020, yet price spiked in January 2021. The price surge coincided with a spike in discussion volume about GameStop on Reddit WallStreetBets (WSB).

A change in retail sentiment effectively shifted the demand curve of retail investors, and its price impact crucially depends on the demand of investors who took the other side of the trade. Hence, I next investigate the change in investor composition of GameStop.

Fact 2 establishes that retail investors gradually built up their positions in GameStop from 2020 Q1 till 2021 Q1, relative to long institutions. And their relative positions remained

[1] Caitlin McCabe (June 18, 2021), It Isn't Just AMC. Retail Traders Increase Pull on the Stock Market, Wall Street Journal; Gunjan Banerji (August 10, 2022), Retail Investors Storm the Market, But Activity Is Well Off Highs, Wall Street Journal.

constant for the rest of 2021. This suggests that retail investors were relatively more optimistic than long institutions. Interestingly, long hedge funds also built up their positions in 2020, but then liquidated almost all their long positions in 2021 Q1. This suggests that long hedge funds were initially "riding the bubble." But their original long strategies were no longer profitable after the price surge in January 2021, as they expected the price to quickly fall back to the pre-January level.

Fact 3 establishes that the short interest of GameStop increased from 60% to 80% from mid- to late 2020. But then it dropped sharply in January 2021, and stayed at below 20% throughout 2021. This is consistent with the narrative that short sellers got squeezed and were forced to cover their short positions.

Long institutions and short sellers are the two group of investors who took the other side of the trade against retail investors. However, they were both constrained in terms of taking (large) short positions. Long institutions typically do not short for institutional reasons, while short sellers face margin constraints. If retail sentiment keeps rising and drives up price, then both group of investors will hit their portfolio constraints. In particular, when short sellers hit their margin constraints, they will be forced to cover their short positions, and price could rise even further.

Next, I explore the role of the WSB social network in driving retail sentiment fluctuations, in particular, how influential users' views shaped aggregate retail sentiment. To do so, I construct daily WSB user networks from their conversations, and measure each user's influence based on their network connections.

Fact 4 establishes that a few influencers dominated the discussions on WSB. And the influence distribution across users was highly skewed, implying that influencers' idiosyncratic sentiment shocks would carry a high weight in the aggregate retail sentiment. As the network grew over time, influencers' sentiment shocks would get amplified more.

Motivated by these observations, I develop a model to reconcile the movement in price, quantity, and aggregate retail sentiment during this period. The model features three groups of investors: a large number of unconstrained retail investors, one long institution facing short-sale constraint, and one short institution facing margin constraint. The three groups of investors trade one risky asset (e.g., GameStop), and they have heterogeneous beliefs (i.e., sentiment) about the asset's payoff.

The aggregate fluctuations in retail sentiment originate from a social network with skewed influence distribution. Specifically, retail investors draw idiosyncratic sentiment shocks, and then communicate on the social network. The network then aggregates individual retail investors' views into an aggregate view. Due to skewed influence distribution on the network, influencers' views carry a disproportionately high weight in the aggregate view. Hence, idiosyncratic sentiment shocks do not average out, and this leads to fluctuations in aggregate retail sentiment.

The price impact of retail sentiment shock depends on the aggregate price elasticity of investors. Consider the case where influencers happen to draw positive sentiment shocks. It first translates into a positive aggregate retail sentiment shock, and then drives up the price of the asset. Under sufficiently large retail sentiment shock, the price increase would make institutions face a binding portfolio constraint, and effectively make them price-inelastic. As a result, the aggregate price elasticity in the market drops, and this amplifies the price impact of the retail sentiment shock. Moreover, wealth redistribution across investors also changes the aggregate price elasticity. As price increases, short sellers lose wealth and carry a smaller weight in the aggregate price elasticity. In the extreme case where short sellers lose all their wealth, they exit the market, and only those investors who remain in the market determine the aggregate price elasticity. If these investors are sufficiently price inelastic, then this also leads to the large price impact of retail sentiment change.

The model thus provides a unified explanation for the retail sentiment fluctuations that originated from social network, and the price and quantity dynamics induced by these sentiment fluctuations. And I demonstrate (through a numerical example) that the model can generate the price and quantity movements observed in the data.

Finally, I analyze two counterfactual scenarios through the lens of the model. First, I consider a scenario where the WSB discussion volume did not spike in January 2021. I show that the resulting retail sentiment shock is lower, and short sellers would not hit their margin constraint. The resulting price impact of retail sentiment shock would be much lower. Then I conduct another counterfactual exercise, where short sellers changed their perceptions of retail sentiment risk after observing the influx of retail investors to the WSB social network. And the change in risk perception could help explain the price and short interest dynamics post January 2021.

My paper contributes to a growing literature that studies the impact of retail trading on asset prices. Barber et al. (2021) study the trading behaviors of Robinhood customers, and document that Robinbood users engage in attention-induced trading. Eaton et al. (2021) use retail brokerage outages to identify the causal impact of retail trading on asset prices. Hu et al. (2021) use social media data from Reddit to study the connections between stock prices, retail trading, shorting selling and social media activities, for a large sample of meme stocks. My work builds on this literature by providing a comprehensive empirical analysis of retail sentiment, price, and quantity. And I present new facts on the changing price impact of retail sentiment, and the changing social dynamics that drives day-to-day fluctuations of retail sentiment.

My model reconciles these facts by bridging three strands of literature: heterogeneous beliefs and wealth effect (Caballero and Simsek (2021), Kyle and Xiong (2001), Martin and Papadimitriou (2022)), limits to arbitrage due to portfolio constraints (Brunnermeier and Pedersen (2009), Gârleanu and Pedersen (2011)), and inelastic demand for aggregate market (Gabaix and Koijen (2022)). In particular, my framework microfounds the time variation of aggregate demand elasticity, and shows that this time variation is important for explaining the price impact of retail sentiment.

I microfound the retail sentiment dynamics using a model of naive learning on social networks, which builds on the DeGroot-type models of learning (DeGroot (1974), DeMarzo et al. (2003), Golub and Jackson (2010), Pedersen (2022)). An important distinction is that my model takes into account interim sentiment shocks and time-varying network. These extensions are important for understanding retail sentiment as a source of risk and the pricing of this risk. This microfoundation is also related to the literature on social learning (Hirshleifer (2020)) and social finance (Kuchler and Stroebel (2021)), which highlights the importance of social dynamics and behavioral factors in shaping investor behavior and asset market outcomes.

2 Data and methodology

2.1 Reddit data

2.1.1 Sample construction

I retrieve historical data on Reddit submissions and comments from the Pushshift API, using the Python Pushshift Multithread API Wrapper (PMAW). I restrict the data download to the subreddit r/WallStreetBets, and to the period from Jan 2020 to Dec 2021.

Occasionally, the Pushshift API does not return any submissions or comments for a given day, due to API outages. The missing data can be retrieved from the Pushshift dump files.[2] For any date that Pushshift API returns zero submissions or comments, I pull data from these dump files.

In the raw data from Pushshift, submissions and comments are labeled with a UTC (Coordinated Universal Time) timestamp, which I convert to the New York time zone – a difference of 5 hours during Eastern Standard Time and 4 hours during Daylight Saving Time.

Next, I construct a sample that includes submissions and comments about CRSP common stocks. To do so, I first obtain the list of tickers for CRSP common stocks, and then tag each

[2]See https://files.pushshift.io/reddit/.

submission with stock tickers through an iterative process of searching for tickers in the title and body text. If a submission is tagged with a ticker, then the associated comments are also tagged with the same ticker. And a submission or a comment can be associated with multiple stock tickers. Appendix A2.2 includes further details on the sample selection and the tagging algorithm.

2.1.2 Network construction

The WSB user network on day t can be represented by a directed graph $\mathcal{G}_t = (\mathcal{V}_t, \mathcal{E}_t)$, where $\mathcal{V}_t = \{1, 2, \cdots, N_t\}$ is the set of users (or nodes, vertices) on the network, and $\mathcal{E}_t \subseteq \mathcal{V}_t \times \mathcal{V}_t \backslash \mathcal{D}_t$ is the set of directed edges between users, with $\mathcal{D}_t = \{(i, i) : i \in \mathcal{V}_t\}$ denoting self loops.

To construct the node set $\mathcal{V}_t$ for day t, I select submissions and comments about CRSP common stocks,[3] made within the time window $[t-30, t-1]$. I define the node set $\mathcal{V}_t$ as the set of unique users who are authors of these selected submissions and comments. Hence, the nodes of the network are the users that have ever participated in the discussion of CRSP common stocks, during the 30-day window prior to day t.

To construct the edge set $\mathcal{E}_t$, I start by representing conversation threads as comment trees. A conversation thread consists of a particular submission and the associated comments. Figure 2 shows an example of a conversation thread. This thread consists of a submission made by user Deep*******Value, and comments on this submission made by five other users. In particular, two of the users, YoloFDs4Tendies and FroazZ directly commented on the submission made by Deep*******Value. And the other three users, smols1, GrowerNotAShower11, and DingLeiGorFei commented on FroazZ's comment. This thread is represented as a comment tree on the left side of Figure 3 panel (a). The comments made by YoloFDs4Tendies and FroazZ are called level-1 comments, since they were directly replying to the submission. The comments made by smols1, GrowerNotAShower11, and DingLeiGorFei are called level-2 comments, since they replied to a level-1 comment. The right panel of Figure 3 panel (a) shows another tree, with quantkim being the author of the submission, and FroazZ being the common user across the two trees.

I simplify each comment tree following Gianstefani et al. (2022). Specifically, I assume any level-k comment is a direct reply to the submission, even if the comment was originally replying to some other comments. Figure 3 panel (b) shows the simplified trees that correspond to the original two trees in (a).

I construct one simplified tree for each selected submission within the $[t-30, t-1]$ time

[3] The network constructed in this section is common to all stocks. Alternatively, one could also construct stock-specific networks, by selecting submissions and comments about a specific stock ticker, and perform the rest of the construction in a similar way.

window. The nodes in each tree are the users who authored the submission or the associated comments. And the set of directed edges are from users who commented on the submission to the user who authored the initial submission.

Finally, I define the edge set $\mathcal{E}_t$ as the union of the directed edges of all conversation trees. For example, in the two trees of Figure 3 panel (b), there is a common user FroazZ. When I take the union of the two trees, there are two edges that come out of FroazZ, one pointing to Deep*******Value (who is the author of the submission in the first conversation), and the other pointing to quantkim (who is the author of the submission in the second conversation). Figure 3 panel (c) shows the resulting network. Note that there are also cases where two distinct users i and j belong to multiple trees, and in each tree there is a directed edge from user i to user j. Then I only keep one edge from i to j in the edge set $\mathcal{E}_t$.[4] Furthermore, I drop self-loops, i.e., any edge from a user to himself.

To summarize, the user network on day t consists of node set $\mathcal{V}_t$ and edge set $\mathcal{E}_t$. The node set $\mathcal{V}_t$ is the set of unique users who are authors of the selected submissions and comments. The edge set $\mathcal{E}_t$ captures the connections between users. For any two distinct users $i, j \in \mathcal{V}_t$, if user j made a submission within the $[t-30, t-1]$ time window, and user i commented on that submission, then there is a directed edge from i to j, i.e., $(i, j) \in \mathcal{E}_t$.

2.1.3 Influence measures

Based on the day-t network, I can measure the "influence" of each user on the network. And in Section 3.4.1, I will explore the time variation and cross-sectional distribution of user influence.

First define the adjacency matrix $\boldsymbol{A}_t = (a_{ij,t})$, which is an $N_t \times N_t$ square matrix with

$$a_{ij,t} \equiv \begin{cases} 1, & (i,j) \in \mathcal{E}_t \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

In other words, in the day t network, there is a directed edge from user i to user j if and only if $a_{ij,t} = 1$. Hence, the adjacency matrix encodes the same information about user connections as the edge set $\mathcal{E}_t$. And $a_{ij,t} = 1$ indicates that user i "listens to" or "attends to" user j, in the sense that i has commented on j's submission during the past 30 days.

Then I normalize the rows of the adjacency matrix to be 1 to get the weighting matrix

[4] Alternatively, one could assign a positive weight to the edge from i to j, where the weight corresponds to the number of trees that have an edge from i to j, which is also the number of times user i commented on user j's submission within the specific time window.

$\boldsymbol{W} = (\omega_{ij,t})$, where

$$\omega_{ij,t} \equiv \frac{a_{ij,t}}{\sum_{j=1}^{N_t} a_{ij,t}}. \tag{2}$$

And I define the in-degree of user j on day t as

$$d_{j,t}^{in} \equiv \sum_{i=1}^{N_t} \omega_{ij,t}. \tag{3}$$

I call $d_{j,t}^{in}$ the "influence" of user j on day t. Intuitively, $\omega_{ij,t}$ captures the weight that user i assigns to user j, among all users that i listens to. Then $d_{j,t}^{in}$ sums up the weights that user j gets from all other users. A higher value of $d_{j,t}^{in}$ indicates that more users listen to or attend to j, and thus j is more influential.

2.1.4 Retail sentiment measures

For each submission (or comment), I conduct textual analysis on its augmented body text[5], using the Python sentiment analysis tool Valence Aware Dictionary and sEntiment Reasoner (VADER). VADER is a lexicon and rule-based sentiment analysis tool that is specifically attuned to sentiments expressed in social media (Hutto and Gilbert (2014)). And the lexicon includes emojis and emoticons. I further augment the VARDER dictionary with the WSB-specific jargons listed in Table 4, following Mancini et al. (2022).

For a submission (or comment) l about stock n made by user i on day t, VADER returns a weighted composite sentiment score $Sent_l$ normalized to the range $[-1, 1]$.[6] A score in $[-1, -0.05]$ indicates that the submission has a negative tone, while a score in $[0.05, 1]$ indicates positive tone. And a score in $(-0.05, 0.05)$ indicates neutral tone.

Then I aggregate sentiment to stock-day level. I first compute an equal-weighted sentiment measure for stock n on day t, defined as

$$Sent_t^{EW}(n) \equiv \frac{1}{|\mathcal{L}_t(n)|} \sum_{l \in \mathcal{L}_t(n)} Sent_l. \tag{4}$$

where $\mathcal{L}_t(n)$ is the set of submissions and comments about stock n that came out within the window (4pm on day $t-1$, 4pm on day t], and $|\mathcal{L}_t(n)|$ is the number of submissions and

[5] A submission has its title and body text. I obtain the augmented body text by appending the body text to the title, separated by a white space. A comment only has body text (without title).

[6] I use the compound score returned from VARDER. The compound score is computed by summing the valence scores of each word in the lexicon, adjusted according to the rules, and then normalized to be between -1 (most extreme negative) and $+1$ (most extreme positive).

comments in this set. For Monday sentiment, in addition to including the 4pm-midnight articles from Sunday, I also include articles from 4pm to midnight on the prior Friday.

I also construct an influence-weighted sentiment measure, $Sent_t^{IW}(n)$, for stock n on day t. It is the average sentiment across users weighted by their influence, i.e.,

$$Sent_t^{IW}(n) \equiv \frac{1}{|\mathcal{J}_t(n)|} \sum_{j \in \mathcal{J}_t(n)} d_{j,t}^{in} \cdot Sent_{j,t}(n), \tag{5}$$

where $Sent_{j,t}(n) \equiv \frac{1}{|\mathcal{K}_{j,t}(n)|} \sum_{l \in \mathcal{K}_{j,t}(n)} Sent_l$ is the average sentiment of all submissions and comments about stock n made by user j on day t, $\mathcal{K}_{j,t}(n)$ is the set of submissions and comments about stock n made by user j on day t, $\mathcal{J}_t(n)$ is the set of users who made submissions or comments about stock n on day t, and $d_{j,t}^{in}$ is the influence of user j on day t defined in equation (3).

I use $Sent_t^{EW}(n)$ and $Sent_t^{IW}(n)$ as measures of retail investors' sentiment about a stock n on day t. By construction, both measures are within the range $[-1, 1]$.

2.2 Financial data

I obtain data on stock price and shares outstanding from CRSP, short interest from IHS Markit and Compustat, holdings of 13F institutions from FactSet, and retail order flows from TAQ.

2.2.1 Institutional and household holdings

I retrieve quarterly portfolio holdings of 13F institutions from FactSet. Following Gabaix and Koijen (2022) and Koijen et al. (2022), I classify institutions into five groups: Hedge Funds, Brokers, Private Banking, Investment Advisors, and Long-Term Investors. And I compute the total number of shares held by institutions in each group. Appendix A3 includes further details on the data construction.

I back out household holdings from the market clearing condition. I assume that households do not short, and short sellers is a separate group of investors that are distinct from households and the long institutions in the 13F data. Then the market clearing condition can be written as

$$Q_t^{\text{Households}}(n) + \sum_{g \in G} Q_t^g(n) = \bar{S}_t(n) + SS_t(n). \tag{6}$$

For stock n at the end of quarter t: $Q_t^{\text{Households}}(n)$ is the number of shares held by Households; $Q_t^g(n)$ is the total number of shares held by institutional group $g \in G$, where

$G = \{$Hedge Funds, Brokers, Private Banking, Investment Advisors, Long-Term Investors$\}$; $\bar{S}_t(n)$ is the total number of shares outstanding, and $SS_t(n)$ is the number of shares sold short (from Compustat).

Equation (6) is an accounting identity. It says that the total number of shares held by long investors is equal to the number of shares outstanding, plus the additional supply of shares from short selling. In the data, I observe holdings of long institutions $\{Q_t^g(n)\}_{g \in G}$, shares outstanding $\bar{S}_t(n)$, and number of shares sold short $SS_t(n)$. Hence, I can back out the number of shares held by households from equation (6), i.e.,

$$Q_t^{\text{Households}}(n) = \bar{S}_t(n) + SS_t(n) - \sum_{g \in G} Q_t^g(n).$$

Then for each investor group $k \in G \bigcup \{\text{Households}\}$, I compute two measures of its percentage holdings:

- Shares held by investor group k as a percentage of total number of shares outstanding:

$$q_t^k(n) \equiv \frac{Q_t^k(n)}{\bar{S}_t(n)}. \tag{7}$$

- Shares held by investor group k as a percentage of total number of shares outstanding plus short interest:

$$\hat{q}_t^k(n) \equiv \frac{Q_t^k(n)}{\bar{S}_t(n) + SS_t(n)}. \tag{8}$$

  Note that $\sum_k \hat{q}_t^k(n) = 1$.

For the rest of the paper, I treat households and retail investors as the same group of investors, and use household holdings as a measure of retail investors' positions.

Figure A1 and A2 in the Appendix plot the total institutional holdings versus shares outstanding plus short interest, for GameStop and AMC. After correcting for the additional supply from shorting, the total institutional holdings do not exceed total supply.

2.2.2 Retail order flows

Section 2.2.1 constructs an indirect measure of retail investors' positions. While in this Section, I present a direct yet noisy measure based on retail order flows. It serves as a cross check to the indirect measure.

BJZZ proposed an algorithm to identify off-exchange trades made by retail investors, based on sub-penny price improvement. Importantly, they assumed that the bid-ask spread is equal to one cent, and thus the price improvement has to be a fraction of one cent. If a trade was executed at less (more) than 0.4 (0.6) of a cent, then they labeled it as a retail sell (buy) trade.

However, Schwarz et al. (2022) conducted an experiment to show that if the bid-ask spread is much larger than one cent, the BJZZ algorithm might incorrectly classify retail trades. And they proposed a modified algorithm to address this misclassification problem. I use the modified BJZZ algorithm to identify retail buy trades and sell trades. Appendix A4 includes further details.

For stock n on day t, I compute total volume of retail buy orders $Mrbvol_t(n)$, and the total volume of retail sell orders $Mrsvol_t(n)$. Then I define cumulative net retail buy volume on day t as a fraction of shares outstanding plus short interest.

$$Cum\ Net\ Retail\ Buy_t(n) = \frac{\sum_{s=0}^{t} Mrbvol_s(n) - Mrsvol_s(n)}{\bar{S}_t(n) + SS_t(n)} \tag{9}$$

3 Facts

3.1 Price and aggregate retail sentiment

On January 28, 2021, GameStop hit an intra-day high price of \$483, compared to a price of less than \$20 throughout 2020. This price surge was believed to be driven by retail investors who communicated on WSB. So I begin by analyzing the relationship between GameStop's price and aggregate retail sentiment from WSB.

Figure 4 plots the daily close price of GameStop (solid blue line), together with the equal-weighted retail sentiment from WSB (dotted red line).[7] The equal-weighted sentiment started at close to 0 in 2020 Q2, steadily increased to 0.2 till 2021 Q1, and remained stable for the rest of 2021. Recall from Section 2.1.4 that a sentiment score in $[0.05, 1]$ indicates optimistic views. Then the sentiment level of 0.2 in 2021 suggests that retail investors were indeed optimistic, but far from being extremely optimistic.

More importantly, at different points in time, the same change in average retail sentiment had had dramatically different price impact. For example, the equal-weighted sentiment increased by 15% from mid to late December 2020, and also from early to late January 2021. Yet the price of GameStop increased by 1700% in the latter period, compared to 36% in the former. Moreover, there was no significant movement in average retail sentiment in the latter half of 2021, but despite that, GameStop price still exhibited substantial volatility.

[7] In Figure 4, I plot 30-day moving averages of the daily sentiment series.

The price impact of average retail sentiment change not only had significant time variation, but also differed across stocks. Figure 5 panel (a) compares the equal-weighted sentiment of GameStop with two tech stocks – Amazon and Microsoft.[8] From late 2020 to early 2021, the retail sentiment of Amazon and Microsoft had similar increase as GameStop. However, Figure A3 and A4 show that the prices of these two stocks did not surge as GameStop did in January 2021.

However, the aggregate retail sentiment is a combination of average sentiment across users and the number of users who participated in the discussion. Figure 11 shows that, despite the moderate increase in average sentiment, the discussion volume about GameStop spiked in January 2021. Hence, the aggregate retail sentiment would have a larger increase than the average sentiment.

The change in aggregate retail sentiment effectively shifted the demand curve of retail investors, and its price impact crucially depends on the demand of investors who took the other side of the trade. In the extreme case where other investors (who traded GameStop) are perfectly price elastic, they would willingly take the other side and push down the price. And thus retail sentiment change would have zero price impact. On the hand, a lack of price-elastic investors in this market could help explain the price surge of GameStop in late January 2021. In Section 3.2 and 3.3, I present facts on who took the other side of the trade and how their positions changed over time.

As a robustness check, I plot the price and sentiment of AMC in Figure A5. The price of AMC had a similar spike in late January 2021 as GameStop. And its equal-weighted sentiment had a similar steady increasing trend.

I summarize the findings of this section into the following fact.

Fact 1: In the time series, the average retail sentiment of GameStop has been steadily increasing since the beginning of 2020, while the discussion volume on WSB about GameStop spiked in January 2021. The spike in discussion volume coincided with the price surge of GameStop. In the cross section, there are tech stocks with similar change in average sentiment but did not have a price surge as GameStop did.

3.2 Positions of long investors

Figure 6 plots the quarterly holdings of households and long institutions of GameStop, as a fraction of number of shares outstanding plus number of shares sold short (see equation (8)). Households (blue shaded area) gradually built up their positions in GameStop from 2020 Q1 till 2021 Q1, relative to long institutions. And their relative positions remained constant

[8]In Figure 5, I plot 30-day moving averages of the daily sentiment series.

for the rest of 2021. This suggests that households (or retail investors) were relatively more optimistic than long institutions. And the dynamics of household holdings is consistent with the dynamics of retail sentiment documented in Section 3.1.

Interestingly, long hedge funds (red shaded area) also built up their positions in 2020, but then liquidated almost all their long positions in 2021 Q1. One story is that long hedge funds were "riding the bubble" (Brunnermeier and Nagel (2004)). But their original long strategies were no longer profitable after the price surge in January 2021, as they expected the price to quickly fall back to the pre-January level.

Figure 7 panel (d), (e), (f) plot the holdings of households, investment advisors and hedge funds, as a fraction of the number of shares outstanding (see equation (7)). Hence, these figures show the "absolute holdings" of each group. The absolute holdings had similar patterns as the relative holdings in Figure 6 and Figure 7 panel (a), (b), (c). For AMC, Figure A6 and A7 show similar patterns in the holdings of households versus long institutions.

In Figure 8 (and Figure A8 for AMC), I compare the quarterly household holdings measure in equation (8) with the daily cumulative net retail buy measure in equation (9). Both measures show an increasing trend, though the latter has a temporary drop in late January of 2021, and the change in the latter from early 2020 to late 2021 is only half of the change in the former.

I summarize the key results in the following fact.

Fact 2: Households built up their positions in GameStop from 2020-2021, while long institutions reduced their positions. In particular, long hedge funds initially built up their positions throughout 2020, then liquidated almost all their positions after 2021 Q1.

3.3 Positions of short sellers

Section 3.2 documents that long institutions reduced their positions in GameStop, possibly because they thought the price was "too high" in January 2021, and it would quickly drop to the pre-January level. If short sellers (e.g., short hedge funds) held the same belief, they would short more of GameStop in January, hoping to profit from the subsequent price drop.

However, data suggests the opposite. Figure 9 plots the daily short interest of GameStop (dotted red line) together with the price (solid blue line). Short interest started out high at 80% of the outstanding shares till the end of 2020. But surprisingly, it dropped sharply in January 2021, and stayed at below 20% throughout 2021.[9] Given the high price of GameStop in 2021, it would be profitable for short sellers to take even larger short positions.

[9] A short interest of 20% of outstanding shares is still considered high relative to an average stock. So the puzzle here is not the absolute level of the short interest in January 2021, but the time series patterns of the short interest of GameStop.

But instead, they seem to have dropped out the market since January 2021.

Anecdotally, some short sellers were squeezed and lost capital. For example, Melvin Capital were forced to cover its short positions in GameStop, and lost 53% on its investments in January 2021.[10] If they account for a large fraction of the short positions opened prior to January, then the sharp drop in short interest is consistent with the fact that they lost capital and exited the market.

Moreover, the short squeeze might have been triggered by the 15% retail sentiment increase from early to late January 2021 (see Section 3.1). Consider a short seller who already had a large short position in GameStop prior to January, and who faced a margin constraint. A further 15% increase in retail sentiment could make the margin constraint bind, and force the short seller to liquidate part of the short position.

Then the remaining question is how sophisticated short sellers failed to anticipate the increase in retail sentiment, and still maintained a large short position till January 2021. In Section 3.4, I explore the changing social dynamics on WSB, which likely lead to the "unexpected" retail sentiment increase from short sellers' perspective.

I sum up the findings of this section into the following fact.

Fact 3: Short interest of GameStop started out high at 80% of the outstanding shares till the end of 2020. But then it dropped sharply in January 2021, and stayed at below 20% throughout 2021.

Long institutions and short sellers are the two group of investors who took the other side of the trade against retail investors. However, they were both constrained in terms of taking (large) short positions. Long institutions like Fidelity typically do not short for institutional reasons, while short sellers like Melvin Capital face margin constraints. If retail sentiment keeps rising and drives up price, then both group of investors will hit their constraints at some point. At the time short sellers hit their margin constraints, they will be forced to cover their short positions, and price could rise even further. In Section 4, I present a model to formalize this idea.

3.4 Changing social dynamics on WallStreetBets

In this section, I document the changing dynamics of WSB community leading up to January 2021. If short sellers failed to anticipate these changes, then they would likely make "mistakes" in opening or covering their short positions, or even get squeezed.

I first examine the aggregate dynamics of WSB community. Figure 10 presents some

[10]Thyagaraju Adinarayan (January 27, 2021), Explainer: How retail traders squeezed Wall Street for bets against GameStop., Reuters. Juliet Chung (January 31, 2021), "Melvin Capital Lost 53% in January, Hurt by GameStop and Other Bets.", Wall Street Journal.

descriptive statistics of daily submissions, comments, and user activities on WSB. Panel (a) shows that the number of subscribers to WSB (solid blue line) grew exponentially in late January of 2021, and then the growth rate reverted back to its pre-January level. Consistent with the growth of subscribers, there was a concurrent surge in the daily number of new submissions (panel (b) solid blue line), daily number of new comments (panel (b) dotted red line), and the daily number of users who participated[11] in the discussion of CRSP stocks (panel (c)), in late January of 2021. Moving to the subjects of the discussions, panel (d) shows that the number of stock tickers mentioned (on a given day) also spiked in late January – over 700 tickers were mentioned on a given day, compared to less than 200 tickers before January.

These facts suggest that WSB users became more engaged in the discussions in January 2021, and the engagement coincided with the price surge of GameStop. But how exactly did individual users' engagement lead to "collective actions" that could squeeze out short sellers? And how is it related to the 15% sentiment increase from early to late January of 2021?

To answer these questions, I inspect the day-to-day activities of WSB users, and in particular, how influential users spurred others. Figure 12 shows the user communications on January 14, 2021.[12] Panel (a) plots user activities from 6-9am, right before market open. Each node represents a unique user who made a new submission or comment within this 3-hour window. For any two users i and j in this figure, if i commented on j's submission (within the 3-hour window), then I draw a directed edge from i to j. For example, the largest red dot represents the AutoModerator, and the dots clustered around it represent the users who commented on AutoModerator's submission.

The AutoModerator created "Daily Discussion Thread for January 14, 2021" at 06:00:18 on January 14, 2021. This thread quickly became the center of WSB discussions, as it received 46,228 comments, which is 94.26% of the comments received by new threads that came out between 6-9am. A similar discussion "hub" emerged right after market close: At 16:00:16 on the same day, the AutoModerator started another thread titled "What Are Your Moves Tomorrow, January 15, 2021". Just like the morning discussion thread, this afternoon thread was the dominant thread on WSB between 4-7pm (Figure 12 panel (b)), which received 80.28% of the comments. These two types of threads are routine discussions on WSB. On each weekday, the AutoModerator will publish a new "Daily Discussion Thread" before market open, and a new "What Are Your Moves Tomorrow" after market close. Users

[11] I define "participation" as follows: A user participated in the discussion about CRSP stocks on a given day, if and only if he made a new submission or a new comment about CRSP stock(s) on that day.

[12] This figure is inspired by Mancini et al. (2022).

typically discuss the market conditions and their trading strategies under these threads (Boylston et al. (2021), Mancini et al. (2022)).

"Daily Discussion Thread" and "What Are Your Moves Tomorrow" are two prominent examples of "megathreads" on WSB, which are user-initiated discussions designated for a specific topic or issue. There were other megathreads for discussing individual stocks, e.g., GME megathreads. Figure 13 plots the discussions between 6-8am on January 21, 2021. At 07:49:03, user grebfar created "GME Megathread - Lemon Party 2: Electric Boogaloo" and it received 67.84% of the comments, which is twice more than the comments received by the daily discussion thread.

Figure 15 shows further evidence on the relative influence of GME megathreads versus the daily discussion threads, and how the relative influence evolves over time. The y-axis is the fraction of comments (on each day) received by a particular type of thread. The solid black line represents "GME Megathread", the dotted red line represents "Daily Discussion Thread" at market open, and the dash-dotted blue line represents "What Are Your Moves Tomorrow" at market close.[13] On January 20, 2021, the first GME megathread appeared, and garnered as many comments as the daily discussion threads. It continued to be as influential as the daily discussion threads until mid-April, after which no new GME megathreads were created.

Megathreads could facilitate "collective actions" in the following sense: They make users' views visible to each other at a designated place. A particular user is able to gain influence within a short period of time. And his view can suddenly dominate the community, which then leads to the kind of "collective actions" short sellers failed to anticipate. In Section 3.4.1 and 3.4.2, I explore the dynamics of the influence distribution among users and the dynamics of influencers' views.

3.4.1 Dynamics of the influence distribution

Figure 14 plots the user network for GameStop discussion on January 14, 2021.[14] The red dots represent the top 5 most influential users. And for each of these influencers, the percentage in the parenthesis is the fraction of users (on this network) that have commented on his posts within the past 30 days. Deep*******Value turns out to be the most influential user for GME discussion, and he attracted 20% of the users to comment on his posts.

[13] To identify GME megathreads, I search for the keyword "GME Megathread" (in a case-insensitive way) in the title of the threads. And I identify "Daily Discussion Thread" and "What Are Your Moves Tomorrow" in a similar way. On a given day, there could be multiple threads of the same type, for example, multiple threads with "GME Megathread" in their titles. In that case, I take the total number of comments received by each type of thread, and then compute the fraction of comments each type received, which is what I plot on the y-axis of Figure 15.

[14] Here I only use submissions and comments about GameStop to construct the network, and the rest of the construction follows Section 2.1.2.

Figure 14 also reveals that the influence distribution is highly skewed, with a few influencers receiving a lot of attention. This is a common feature of many empirical social networks, and the heavy right tail of the influence distribution can be approximated by a power-law distribution (Newman (2005), Rantala (2019)). If user influence d_j^{in} (defined in equation (3)) is drawn from a power-law distrituion, then it has PDF

$$f_{d_j^{in}}(x) \quad = \quad \frac{\xi - 1}{d_{\min}} \left(\frac{x}{d_{\min}} \right)^{-\xi}, \xi > 1 \tag{10}$$

with support $[d_{\min}, +\infty)$. The exponent ξ captures the skewness of the influence distribution. Lower values of ξ correspond to heavier tails and more right-skewed influence distribution. And there must be some lowest value $d_{\min}$ at which the power law is obeyed (Newman (2005)).

The power-law relationship implies that the log of influence d_j^{in} and the log of the corresponding empirical frequencies have a linear relationship. Figure 16 plots this relationship for January 14, 2021. The x-axis is the log of user influence, and the y-axis is the log empirical frequency. The relationship is approximately linear, which is consistent with the power-law distribution.

I then fit the power-law distribution to the vector of user influence on each day. Following Rantala (2019), I estimate the exponent $\hat{\xi}_t$ and the cutoff value $\hat{d}_{\min,t}$ for each day t using the maximum likelihood method, and compute confidence bands using bootstrap methods. Appendix A5 includes the computational details.

Figure 17 plots the time series of the $\hat{\xi}_t$ estimates with the bootstrapped confidence intervals. $\hat{\xi}_t$ is below 3 throughout the sample, which means the influence distribution is highly skewed. From the beginning to the end of January 2021, $\hat{\xi}_t$ dropped by 10%, from 2.1 to 1.9. This suggests that the influence distribution became increasingly skewed, which would allow influencers to spur more people.

Figure 18 plots the times of the cutoff value $\hat{d}_{\min,t}$, which remains relatively stable within the range $[5, 15]$. Furthermore, Figure A10 plots the p-value of the Kolmogorov-Smirnov test. Small p-values (less than 0.05) indicate that the test rejected the hypothesis that the original data could have been drawn from the fitted power-law distribution. For all the dates from Dec 2020-Jan 2021, the test cannot reject the hypothesis that the original data was drawn from a power-law distribution.

Taken together, the influence distribution on WSB was highly skewed. This implies that influencers' views would quickly become dominant. If they happened to be optimistic, then the WSB community would quickly become optimistic as well. This could help explain the 15% average sentiment increase from early to late January. Next, I document that influencers were indeed optimistic about GameStop.

3.4.2 Dynamics of influencers' views

In Section 3.4.1, I document that Deep*******Value was the most influential user in mid-January 2021. Figure 19 plots some examples of his posts. The title of the posts always started with "GME YOLO". "YOLO" means "You Only Live Once", which is a jargon on WSB and is considered as a positive word. Hence, the influencer Deep*******Value was indeed optimistic about GameStop, and his influence would allow him to spur a large group of users in the community.

Figure 4 shows the time variation of influencers' views. The dash-dotted green line is the influence-weighted sentiment for GameStop defined in equation (5), while the dotted red line is the equal-weighted sentiment in equation (4). From July to Nov 2020, the influence-weighted sentiment lead the equal-weighted sentiment, which is consistent with the hypothesis that influencers happened to be optimistic, and they spurred other users on the network.

I collect the results from this section in the following fact.

Fact 4: The distribution of user influence on WSB follows a power-law distribution with heavy right-tails. Moreover, influencers on WSB happened to be optimistic leading up to January 2021.

3.5 Proposed mechanism

Section 3.1-3.4 present a complete picture of the price, quantity and retail sentiment movement pre- and post- the GameStop frenzy. In this section, I propose a mechanism that reconciles these facts. And in Section 4, I formalize this idea within a model.

At the beginning of 2020, short sellers like Melvin Capital were pessimistic about GameStop's future prospects, and believed that GameStop's was "over-valued". Hence, they maintained large short positions, hoping to profit from a future price drop.

In mid-2020, influencers on WSB like Deep*******Value started to express their optimistic views about GameStop. Other users on WSB adopted this optimistic view and started to take long positions in GameStop. This resulted in a moderate price increase, which "drove out" price-elastic long institutions and attracted more short sellers to further increase their short positions, because they all thought the price was too high.

In January 2021, WSB went through a structural change – more users joined the network and the influence distribution remained highly skewed, which allowed influencers like Deep*******Value to be more influential and spur more people. Aggregate retail sentiment further increased, which drove up prices, and pushed short sellers towards their margin constraints. Short sellers did not expect this further sentiment increase, i.e., they were "surprised".

In late January 2021, short sellers had to cover their short positions and suffered losses. And due to the short covering, price increased even further, and short sellers suffered from more significant loss. This ultimately lead to the price surge on January 28, 2021. Some short sellers lost a large fraction of their capital, and exited the market.

For the rest of 2021, retail investors and price-inelastic long institutions like index funds remained in the market. And retail investors continued to be optimistic throughout 2021. Price-elastic long institutions and short sellers both dropped out of the market, and no longer took the other side against optimistic retail investors. Then a small retail sentiment shock would have large price impact, due to a lack of price-elastic investors in this market.

Short sellers also changed their perceptions of retail sentiment risk, after observing a large influx of retail investors to the WSB forum in January 2021. They traded less aggressively in the latter half of 2021, being aware that the social network structure could change dramatically within a short period of time and this is a new risk they need to cope with.

4 The pricing of retail sentiment risk

In this section, I present a model to reconcile the price, quantity and retail sentiment dynamics documented in Section 3. In particular, I show that a moderate increase in aggregate retail sentiment can have a large price impact, if it drives out price-elastic long institutions and gets short sellers squeezed. The price of retail sentiment risk then crucially depends on this investor composition change.

4.1 Setup

Time is discrete and is indexed by $t \in \{-1, 0, 1, 2\}$. There are $\bar{N} + 2$ investors who are divided into three groups: $\bar{N}$ retail investors indexed by j, a long institution (IL), and a short institution (IS). Investors trade a risky asset and a risk-free asset. And they differ in their beliefs about the risky asset's payoff, their risk aversion, and the portfolio constraints they face.

Assets Assets are traded at time $t \in \{0, 1\}$. The risk-free asset is in zero net supply, and has raw return $R_{f,t} = 1$ which is exogenously given.

The risky asset has a constant supply of $\bar{S}$ shares, and pays a one-time dividend $\tilde{D}$ at time 2. Let $\tilde{d} \equiv \log \tilde{D}$ denote its log payoff. The dividend payment is unobserved at time $t \in \{-1, 0, 1\}$. And the time-t conditional distribution of $\tilde{d}$ is truncated normal on the interval $\left[\underline{d}, \bar{d}\right]$, with post-truncation mean μ_d, variance σ_d^2. Let P_t and $p_t \equiv \log P_t$ denote the price and log price of the risky asset at time t, and let $\log X_t$ denote its log payoff at time t. Then

$$\log X_0 = p_0, \log X_1 = p_1, \log X_2 = p_2 = \tilde{d}.$$

Further define $\mathbb{E}_t \left[\log X_{t+1}\right]$ and $\sigma_t^2 \equiv \mathrm{Var}_t \left(\log X_{t+1}\right)$ as the time-t conditional mean and variance of next period's log payoff, respectively. And note that $\sigma_1^2 = \sigma_d^2$.

Then the risky asset has one-period raw return $R_{t+1} \equiv \frac{X_{t+1}}{P_t}$ from time t to $t+1$. Define $r_{t+1} \equiv \log R_{t+1}$ as the one-period log return of the risky asset, $r_{f,t} \equiv \log R_{f,t} = 0$ as the one-period log return of the risk-free asset.

Investors' subjective beliefs Investors have subjective beliefs about the risky asset's payoff. Specifically, at time $t \in \{0, 1\}$, investor i believes that the log payoff of the risky asset at time $t+1$ has mean $\mathbb{E}_t^i \left[\log X_{t+1}\right]$ and variance $\mathrm{Var}_t^i \left(\log X_{t+1}\right)$. I assume that investors know the true variance of the log payoff, i.e.,

$$\mathrm{Var}_t^i \left(\log X_{t+1}\right) = \sigma_t^2, \forall i. \tag{11}$$

Investors disagree about the mean of the log payoff. First consider the institutional investors. At time 0, the two institutions IL and IS have subjective beliefs (about the mean)

$$\mathbb{E}_0^{IL} \left[\log X_1\right] = \mathbb{E}_0 \left[p_1\right] + \delta_0^{IL}, \tag{12}$$

$$\mathbb{E}_0^{IS} \left[\log X_1\right] = \mathbb{E}_0 \left[p_1\right] + \delta_0^{IS}, \tag{13}$$

where $\mathbb{E}_0 \left[p_1\right]$ is the conditional mean of time-1 log price, which is an equilibrium outcome. δ_0^{IL} and δ_0^{IS} capture the wedges between the subjective beliefs and the objective beliefs, and are given exogenously. At time 1, the two institutions have subjective beliefs that are consistent with the objective mean, i.e.,

$$\mathbb{E}_1^{IL} \left[\log X_2\right] = \mathbb{E}_1^{IS} \left[\log X_2\right] = \mathbb{E}_1 \left[p_2\right] = \mu_d. \tag{14}$$

Hence, at time 0, institutions disagree about the mean, while at time 1 they know the "true" mean.

There are two types of retail investors: At time t, the first N_t retail investors (labeled as "type 1") have subjective beliefs that deviate from the objective beliefs, while the rest $\bar{N} - N_t$ retail investors (labeled as "type 2") have subjective beliefs that conform with the objective ones. In particular, at time $t \in \{0, 1\}$, the subjective belief of type-1 retail investor $j \in \{1, 2, \cdots, N_t\}$ is

$$\mathbb{E}_t^j \left[\log X_{t+1}\right] = \mathbb{E}_t \left[p_{t+1}\right] + y_t^j, \tag{15}$$

where y_t^j is the deviation of j's belief from the objective expectation. I call y_t^j the "sentiment" of retail investor j.

Type-1 retail investors communicate on a social network, and form their subjective beliefs (and thus sentiment) by "listening to" other people on the network. In Section 5, I microfound their sentiment dynamics using a model of naive learning on networks. The model yields the conditional distribution of retail investor sentiment $\left\{y_t^j\right\}_{j=1}^{N_t}$.

Note that the number of type-1 retail investors, N_t, is time-varying. Assume that $0 \leq N_t \leq \bar{N}$, and define the fraction of type-1 retail investors at time t as

$$\theta\left(N_t\right) \equiv \frac{N_t}{\bar{N}} \in [0, 1]\,. \tag{16}$$

Investors' preferences, budget constraint, and wealth share dynamics Investor i solves the following myopic portfolio choice problem

$$\max_{w_t^i} w_t^i \left(\mathbb{E}_t^i \left[r_{t+1}\right] - r_{f,t}\right) + \frac{1}{2} w_t^i \left(1 - w_t^i\right) \operatorname{Var}_t^i\left(r_{t+1}\right) + \frac{1}{2}\left(1 - \gamma^i\right)\left(w_t^i\right)^2 \operatorname{Var}_t^i\left(r_{t+1}\right), \tag{17}$$

where γ^i is his constant relative risk aversion, and w_t^i is the fraction of end-of-period wealth invested in the risky asset, i.e., the portfolio weight on the risky asset. Define risk tolerance $\tau^i \equiv \frac{1}{\gamma^i}$. I assume that institutional investors (IL and IS) have the same relative risk tolerance $\tau^I = \frac{1}{\gamma^I}$. The $\bar{N}$ retail investors have the same risk tolerance $\tau^R = \frac{1}{\gamma^R}$.

The budget constraint for investor i is

$$A_{t+1}^i = A_t^i \left(w_t^i \exp\left(r_{t+1}\right) + \left(1 - w_t^i\right) \exp\left(r_{f,t}\right)\right), \tag{18}$$

where A_t^i is the investor's wealth entering period t.

Since the risk-free asset is in zero net supply, the aggregate wealth is equal to the market value of the risky asset. Hence, the time-1 wealth share of investor i is

$$\alpha_t^i \equiv \frac{A_t^i}{P_t \bar{S}}. \tag{19}$$

Appendix A1.1 shows that the budget constraint (18) implies the following wealth share dynamics

$$\alpha_{t+1}^i = \alpha_t^i \left(\left(1 - w_t^i\right) \exp\left(p_t - p_{t+1}\right) + w_t^i \right). \tag{20}$$

Non-negative wealth constraint All investors are subject to the non-negative wealth constraint

$$A_t^i \geq 0, \forall t.$$

If an investor loses all his wealth, then he cannot invest and has to exit the market.

Portfolio constraints Institutional investors face portfolio constraints. The long institution IL faces short-sale constraint of the following form

$$w_t^{IL} \geq 0. \tag{21}$$

The short institution IS faces margin constraint on short selling. Following Gârleanu and Pedersen (2011), I assume the margin constraint limits the leverage short sellers can take, i.e.,

$$w_t^{IS} \geq -\frac{1}{m}, \tag{22}$$

where $m \in (0,1)$.

Market clearing Following Caballero and Simsek (2021), I show in Appendix A1.2 that the market clearing conditions for the risky asset and the risk-free asset are equivalent to the set of conditions

$$\sum_i A_t^i = \sum_i w_t^i A_t^i = P_t \bar{S}. \tag{23}$$

Equation (23) says that aggregate wealth is equal to the market value of the risky asset, both before and after investors make portfolio decisions. The conditions in equation (23) are also equivalent to

$$\sum_i \alpha_t^i w_t^i = 1, \tag{24}$$

where the wealth share α_t^i is defined in equation (19). This condition says that the wealth-share-weighted sum of portfolio weights is equal to 1.

Endowment and implicit price at time -1 At time -1, investor i is endowed with wealth share α_{-1}^i and portfolio weight w_{-1}^i. I assume that at time -1, investors do not anticipate future sentiment shocks. They all believe that the prices at time 0 and 1 will reflect the present value of the final dividend payment. In Appendix A1.10, I derive the implicit price p_{-1} that is consistent with this belief. Under this price, investors do not want to trade at time -1 and they enter time 0 with their initial endowment.

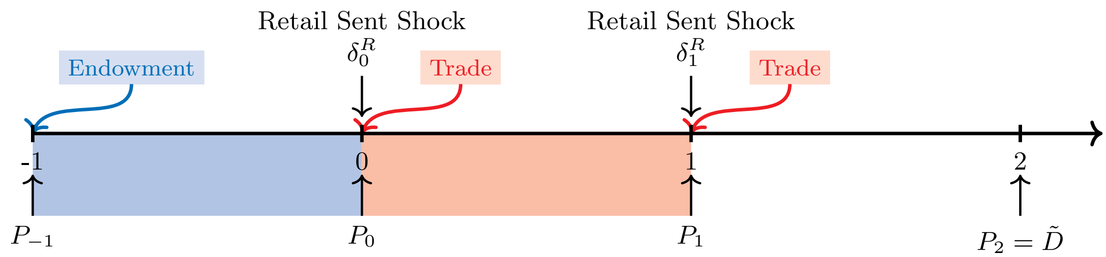

Figure 1. Timeline of the model.

Timeline Figure 1 shows the timeline of the model. At time -1, investors receive their endowment. At time 0 and 1, investors form subjective beliefs about next period's asset payoff, and then trade according to their beliefs. At time 2, the risky asset pays dividend.

In addition, I impose the following assumption.

Assumption 1. *At time $t \in \{0, 1\}$ before trading, retail investors first split their time $t-1$ end-of-period wealth equally among themselves. In particular, they split their aggregate stock position as well as aggregate bond position equally. Then they make portfolio choices based on their wealth after the splitting.*

Assumption 1 says that retail investors split their wealth equally before trading. This assumption together with linear demand implies that there exists an aggregate retail investor whose sentiment matters for asset prices. Lemma 1 in Section 4.2 formalizes this argument.

4.2 Investor demand

In this section, I first derive the asset demand of individual investors. Then I show that there exists an aggregate retail investor whose sentiment matters for asset prices.

Retail investors Type-1 retail investor j solves the portfolio problem in (17). His subjective expectation deviates from the objective expectation by y_t^j. Appendix A1.3.1 shows that his optimal portfolio weights on the risky asset are

$$w_0^j = \tau^R \left(\frac{\mathbb{E}_0 [p_1] + y_0^j - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{25}$$

$$w_1^j = \tau^R \left(\frac{\mu_d + y_1^j - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{26}$$

Type-2 retail investors' subjective beliefs conform with the objective beliefs. Hence, a type-2 retail investor j' chooses portfolio weights

$$w_0^{j'} = \tau^R \left(\frac{\mathbb{E}_0 [p_1] - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{27}$$

$$w_1^{j'} = \tau^R \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{28}$$

Long institution The long institution solves the portfolio problem in (17), subject to the short-sale constraint in (21). Appendix A1.3.2 shows that his optimal portfolio weights on the risky asset are

$$w_0^{IL} = \max \left\{ 0, \tau^I \left(\frac{\mathbb{E}_0 [p_1] + \delta_0^{IL} - p_0}{\sigma_0^2} + \frac{1}{2} \right) \right\}, \tag{29}$$

$$w_1^{IL} = \max \left\{ 0, \tau^I \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right) \right\}. \tag{30}$$

Short institution The short institution solves the portfolio problem in (17), subject to the margin constraint in (22). Appendix A1.3.3 shows that his optimal portfolio weights on the risky asset are

$$w_0^{IS} = \max \left\{ -\frac{1}{m}, \tau^I \left(\frac{\mathbb{E}_0 [p_1] + \delta_0^{IS} - p_0}{\sigma_0^2} + \frac{1}{2} \right) \right\}, \tag{31}$$

$$w_1^{IS} = \max \left\{ -\frac{1}{m}, \tau^I \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right) \right\}. \tag{32}$$

For the rest of the paper, I focus on scenarios where in equilibrium, the portfolio constraints for institutions do not bind at time 0, while they may bind at time 1 depending on the retail sentiment realization $\left\{ y_1^j \right\}_{j=1}^N$.

Before characterizing the equilibrium, I first show that there exists an aggregate retail investor, whose sentiment drives asset prices.

Lemma 1 (Existence of an aggregate retail investor). *Under Assumption 1, the aggregate demand of the $\bar{N}$ retail investors is equal to the demand of an aggregate retail investor (R).*

- *The aggregate retail investor has subjective beliefs*

$$\mathbb{E}_0^R[p_1] = \mathbb{E}_0[p_1] + \delta_0^R, \mathrm{Var}_0^R(p_1) = \sigma_0^2,$$
$$\mathbb{E}_1^R\left[\tilde{d}\right] = \mu_d + \delta_1^R, \mathrm{Var}_1^R\left(\tilde{d}\right) = \sigma_d^2.$$

  His time-t sentiment δ_t^R ($t \in \{0,1\}$) aggregates individual retail investors' sentiment in the following way

$$\delta_t^R = \theta(N_t)\, y_t^R, \tag{33}$$
$$y_t^R \equiv \frac{1}{N_t}\sum_{j=1}^{N_t} y_t^j, \tag{34}$$

  where N_t is the number of type-1 retail investors at time t, and $\theta(N_t)$ is the fraction of type-1 retail investors defined in equation (16).

- *The aggregate retail investor's demands for the risky asset (in terms of portfolio weights) take the form*

$$w_0^R = \tau^R\left(\frac{\mathbb{E}_0[p_1] + \delta_0^R - p_0}{\sigma_0^2} + \frac{1}{2}\right), \tag{35}$$
$$w_1^R = \tau^R\left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right). \tag{36}$$

- *The aggregate retail investor's time-t wealth aggregates individual retail investors' wealth*

$$A_t^R = \sum_{j=1}^{\bar{N}} A_t^j, \alpha_t^R = \sum_{j=1}^{\bar{N}} \alpha_t^j,$$

  where A_t^R and α_t^R are his dollar wealth and wealth share, respectively. And his wealth share evolves according to

$$\alpha_{t+1}^R = \alpha_t^R\left(\left(1 - w_t^R\right)\exp\left(p_t - p_{t+1}\right) + w_t^R\right). \tag{37}$$

- *The time-t equilibrium price of the risky asset is determined by the market clearing* *condition*

$$\alpha_t^R w_t^R + \alpha_t^{IL} w_t^{IL} + \alpha_t^{IS} w_t^{IS} = 1. \tag{38}$$

Proof. See Appendix A1.4. □

This existence result comes from Assumption 1 and the linearity of investors' demand. From equations (25) and (26), an individual investor's demand is linear in his own sentiment. After retail investors split their wealth equally, their aggregate demand will be linear in the aggregate retail sentiment δ_t^R.

Lemma 1 allows me to study the pricing of aggregate retail sentiment risk in Section 4.3 and Section 4.4, for a given distribution of sentiment risk.

The aggregate retail sentiment δ_t^R depends on the fraction of type-1 investors in the retail investor population ($\theta(N_t)$), and also the average sentiment among the type-1 investors (y_t^R). In Section 5, I will show that the average sentiment y_t^R depends on the network geometry, in particular, the skewness of influence distribution on the network.

4.3 Equilibrium at time 1

At time 1, the aggregate retail sentiment δ_1^R drives the price of the risky asset. And the time-1 equilibrium price $p_1\left(\delta_1^R\right)$ is a function of retail sentiment. I assume that the time-0 conditional distribution of δ_1^R is truncated normal on the interval $\left[\underline{\delta}_1, \bar{\delta}_1\right]$, with CDF $\Psi(\cdot)$.

Under certain realizations of the retail sentiment, the portfolio constraints will be binding for institutions, and there will be multiple equilibria. I focus on the class of monotone equilibria defined below.

Definition 1 (Monotone equilibrium at time 1**).** *A monotone equilibrium at time* 1 *is an equilibrium where the price of the risky asset is strictly increasing in the retail sentiment realization, i.e.,* $p_1\left(\delta_1^R\right)$ *is strictly increasing in* δ_1^R.

To characterize the time-1 equilibrium, I first derive two cutoff prices p_1^m and p_1^h such that: if $p_1 < p_1^m$, then none of the investors are constrained; if $p_1 \in \left[p_1^m, p_1^h\right)$, then the long institution is constrained by the short sale constraint, while the short institution is unconstrained: if $p_1 \geq p_1^h$, then both the long institution and the short institution are constrained. Since p_1^m is the cutoff price at which the short-sale constraint exactly binds for the long institution, we can calculate p_1^m by setting IL's unconstrained demand to 0, which yields

$$p_1^m \equiv \mu_d + \frac{1}{2}\sigma_d^2. \tag{39}$$

Similarly, p_1^h is the cutoff price at which the margin constraint exactly binds for the short institution, which yields

$$p_1^h \equiv \mu_d + \left(\frac{1}{2} + \frac{1}{m\tau^I}\right)\sigma_d^2. \tag{40}$$

Importantly, $p_1^m < p_1^h$, which means in the type of monotone equilibrium of Definition 1, these corresponds to two cutoff sentiment shocks $\delta_1^m = (p_1)^{-1}(p_1^m)$ and $\delta_1^h = (p_1)^{-1}\left(p_1^h\right)$[15], with $\delta_1^m < \delta_1^h$. Impose market clearing condition (24) to derive these cutoffs

$$\delta_1^m \equiv \frac{\sigma_d^2}{\alpha_1^R(p_1^m)\tau^R}, \tag{41}$$

$$\delta_1^h \equiv \frac{\frac{1}{m\tau^I}\hat{\tau}_1\left(p_1^h\right)+1}{\alpha_1^R\left(p_1^h\right)\tau^R}\sigma_d^2, \tag{42}$$

where $\hat{\tau}_1\left(p_1^h\right) \equiv \alpha_1^R\left(p_1^h\right)\tau^R + \alpha_1^{IS}\left(p_1^h\right)\tau^I$.

For low retail sentiment shock realization, $\delta_1^R < \delta_1^m$, none of the investors are constrained. For intermediate shock realization $\delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$, the long institution is constrained while the short institution is unconstrained. And for $\delta_1^R > \delta_1^h$, both the long institution and the short institution are constrained. If $\underline{\delta}_1 < \delta_1^m$ and $\delta_1^h < \bar{\delta}_1$, then as sentiment increases from $\underline{\delta}_1$ to $\bar{\delta}_1$, the long institution first hits the short-sale constraint, and then the short institution hits the margin constraint. Table 1 below summarizes the features of each sentiment region.

Table 1
Sentiment Regions and Binding Constraints

Sentiment region	Shock realization	Constrained: Rep. Retail	Long Inst.	Short Inst.
Low	$\delta_1^R \in [\underline{\delta}_1, \delta_1^m)$	No	No	No
Medium	$\delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$	No	Yes	No
High	$\delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$	No	Yes	Yes

For the rest of the paper, I focus on equilibria where the three sentiment regions are non-empty, i.e., $\underline{\delta}_1 < \delta_1^m < \delta_1^h < \bar{\delta}_1$.

Proposition 1 (Time-1 price). *Suppose a monotone equilibrium of Definition 1 exists at time 1, and $\underline{\delta}_1 < \delta_1^m < \delta_1^h < \bar{\delta}_1$. Take time-0 portfolios $\{w_0^i\}$ and wealth shares $\{\alpha_0^i\}$ as given, the time-1 equilibrium price function $p_1\left(\delta_1^R\right)$ is determined as follows.*

[15] $(p_1)^{-1}(\cdot)$ denotes the inverse function of $p_1(\cdot)$.

- *For $\delta_1^R \in [\underline{\delta}_1, \delta_1^m)$, the equilibrium features a price $p_1 < p_1^m$ that solves*

$$J\left(p_1, \delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R\left(p_1\right)\tau^R\delta_1^R - \sigma_d^2}{\tau_1\left(p_1\right)}\right) - p_1 = 0, \tag{43}$$

where $\tau_1(p_1)$ is the aggregate risk tolerance of unconstrained investors, which is defined as

$$\tau_1\left(p_1\right) \equiv \alpha_1^R\left(p_1\right)\tau^R + \left(1 - \alpha_1^R\left(p_1\right)\right)\tau^I. \tag{44}$$

- *For $\delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$, the equilibrium features a price $p_1 \in \left[p_1^m, p_1^h\right)$ that solves*

$$H\left(p_1, \delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R\left(p_1\right)\tau^R\delta_1^R - \sigma_d^2}{\hat{\tau}_1\left(p_1\right)}\right) - p_1 = 0, \tag{45}$$

where $\hat{\tau}_1(p_1)$ is the aggregate risk tolerance of unconstrained investors, which is defined as

$$\hat{\tau}_1\left(p_1\right) \equiv \alpha_1^R\left(p_1\right)\tau^R + \alpha_1^{IS}\left(p_1\right)\tau^I. \tag{46}$$

- *For $\delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the equilibrium features a price $p_1 > p_1^h$ that solves*

$$G\left(p_1, \delta_1^R\right) \equiv \mu_d + \delta_1^R + \left(\frac{1}{2} - \frac{1 + \alpha_1^{IS}\left(p_1\right)\frac{1}{m}}{\alpha_1^R\left(p_1\right)\tau^R}\right)\sigma_d^2 - p_1 = 0. \tag{47}$$

The cutoff prices p_1^m and p_1^h are defined in equations (39) and (40), and the cutoff sentiment shocks δ_1^m and δ_1^h are defined in equations (41) and (42).

Proof. See Appendix A1.5. □

Proposition 1 shows that in each of the sentiment region, the equilibrium price solves an implicit function. This is because the equilibrium price not only enters investors' demand, but also determines their wealth shares. These implicit functions may have multiple solutions, which means there could be multiple equilibria. As retail sentiment realization δ_1^R increases, certain class of equilibria may disappear, this gives rise to endogenous discontinuity in equilibrium price. Proposition 2 below presents the formal argument.

Proposition 2 (Endogenous discontinuity in time-1 price). *Consider an equilibrium with the following properties:*

- *Investors' time-0 optimal portfolios satisfy: $w_0^R > 1$, $w_0^{IS} < 0 < w_0^{IL} < w_0^R$.*

- *For any sentiment shock realization $\delta_1^R \in \left(\underline{\delta}_1, \bar{\delta}_1\right)$, the equilibrium price $p_1\left(\delta_1^R\right)$ is such that all investors have strictly positive wealth at time 1.*
- *The time-1 equilibrium is a monotone equilibrium of Definition 1.*

If $p_1\left(\delta_1^R\right)$ is continuous on $\left[\underline{\delta}_1, \delta_1^h\right)$ and $\left.\frac{\partial G\left(p_1, \delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} > 0$, then $p_1\left(\delta_1^R\right)$ jumps discontinuously at $\delta_1^R = \delta_1^h$, i.e.,

$$\lim_{\delta_1^R \to \left(\delta_1^h\right)^-} p_1\left(\delta_1^R\right) < \lim_{\delta_1^R \to \left(\delta_1^h\right)^+} p_1\left(\delta_1^R\right).$$

Proof. See Appendix A1.7. □

To understand the endogenous jump, I provide a numerical example and Section **??** explains the parameter choices. Figure 20 plots the time-1 equilibrium price $p_1\left(\delta_1^R\right)$ as a function of the sentiment shock δ_1^R. There is an endogenous jump at the cutoff δ_1^h, at which the margin constraint exactly binds for the short institution. Figure 24 plots all the time-1 equilibria in this numerical example. Generically, for a given sentiment shock realization δ_1^R, there are one or three equilibria. And in the knife edge cases, there are two equilibria. In particular, there are two equilibrium prices at $\delta_1^R = \delta_1^h$, with p_1^h being the lower price. As sentiment increases further above δ_1^h, the low-price equilibrium disappears and the high-price equilibrium becomes the unique equilibrium, and this gives rise to the endogenous jump. Moreover, under this set of parameter values, we cannot find a price path $p_1\left(\delta_1^R\right)$ that is continuous in the sentiment shock δ_1^R. Hence, we can pick any other class of equilibrium (i.e., not necessarily the low-price equilibrium), and there will still be a price jump at certain sentiment shock realization.

Hence, the endogenous jump in price is a result of multiple equilibria. Next, I show that margin constraint and wealth effect generate multiple equilibria. I first analyze demand and supply around the cutoff sentiment δ_1^h, from the short institution's perspective. The demand curve of the short institution can be written as

$$\frac{Q_1}{\bar{S}} = \begin{cases} \alpha_1^{IS}\left(p_1\right)\tau^I\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right), & p_1 \in \left[p_1^m, p_1^h\right] \\ -\frac{1}{m}\alpha_1^{IS}\left(p_1\right), & p_1 > p_1^h \end{cases}.$$

Around the cutoff δ_1^h, long institution demands zero shares due to the binding short-sale constraint (recall from Table 1). Hence, the "supply curve" faced by short institution is 1 minus the demand of the retail investor, i.e.,

$$\frac{Q_1}{\bar{S}} = 1 - \alpha_1^R\left(p_1\right)\tau^R\left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right).$$

Figure 25 plots the inverse demand curve (solid black line), and the inverse supply curves (blue lines) under different sentiment shock realizations. The demand curve is downward sloping for $p_1 \leq p_1^h$, but is upward sloping for $p_1 < p_1^h$. Under a price higher than p_1^h, margin constraint binds for the short institution and he can only allocate a constant fraction $-\frac{1}{m}$ of his wealth to the risky asset. As price increases, he loses wealth on the short position. This wealth effect together with the margin constraint limits the number of shares he can short, and make the demand curve upward sloping. The supply curves are upward sloping for $p_1 > p_1^h$, but is downward sloping for $p_1 < p_1^h$ due to the wealth effect. In this numerical example, the retail investor has a levered position in the risky asset. As price decreases below p_1^h, he loses wealth and demands less shares. This effectively "increases" the number of shares supplied to the short institution.

The yellow dots represent the three equilibria under a sentiment shock that is slightly below δ_1^h. As sentiment increases to δ_1^h, the lower and middle equilibria collapse into one, so there are two equilibria represented by the two green dots. As sentiment increases further above δ_1^h, the low-price equilibrium disappears, and price jumps discontinuously to the red dot (high-price equilibrium).

Intuitively, when sentiment increases further above δ_1^h, an unconstrained short seller would increase his short position and there will still be a low-price equilibrium. With the margin constraint, short seller would short less than in the unconstrained case, and the low-price equilibrium no longer clears the market and price has to rise further. As price rises further, the short seller loses wealth and has to short even less, this again drives up price. This feedback loop implies that market only clears at a very high price, which is the high-price equilibrium.

This phenomenon has a tight connection to Gennotte and Leland (1990), who analyze an endogenous price drop due to multiple equilibria. To see this, I define the short institution's "excess demand" as his demand minus "supply" from the retail investor, i.e.,

$$
\begin{aligned}
&\frac{Q_1^{IS}}{\bar{S}} + \frac{Q_1^R}{\bar{S}} \\
= &\begin{cases} \left(\alpha_1^{IS}(p_1)\tau^I + \alpha_1^R(p_1)\tau^R\right)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) + \alpha_1^R(p_1)\tau^R\frac{\delta_1^R}{\sigma_d^2}, & p_1 \in \left[p_1^m, p_1^h\right] \\ -\frac{1}{m}\alpha_1^{IS}(p_1) + \alpha_1^R(p_1)\tau^R\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) + \alpha_1^R(p_1)\tau^R\frac{\delta_1^R}{\sigma_d^2}, & p_1 > p_1^h \end{cases}
\end{aligned}
$$

Then market clearing implies that the "excess supply" is equal to 1. Figure 26 plots the "excess demand" and "excess supply", which is a mirror image of the scenario in Gennotte and Leland.

Proposition 2 shows that price can jump discontinuously at certain sentiment cutoff.

And the jump is one reason why moderate sentiment shock can have large price impact. Proposition 3 then characterizes the price impact within each sentiment region.

Proposition 3 (Price impact of time-1 aggregate retail sentiment shock). *Consider an equilibrium where $p_1\left(\delta_1^R\right)$ is continuous and differentiable in the interior of the three sentiment regions. The price impact of aggregate retail sentiment shock, $\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R}$, can be decomposed into two components – the direct effect and the redistribution effect.*

- *Low sentiment region $\delta_1 \in [\underline{\delta}_1, \delta_1^m)$:*

$$\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = \underbrace{\frac{\alpha_1^R\left(p_1\right)\tau^R}{\tau_1\left(p_1\right)}}_{\text{direct effect}} \cdot \underbrace{\frac{1}{1-\frac{1}{\tau_1(p_1)}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R+\frac{d\tau_1(p_1)}{dp_1}\left(\mu_d+\frac{1}{2}\sigma_d^2-p_1\right)\right)}}_{\text{redistribution effect}}.$$

- *Medium sentiment region $\delta_1 \in \left(\delta_1^h, \delta_1^h\right)$:*

$$\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = \underbrace{\frac{\alpha_1^R\left(p_1\right)\tau^R}{\hat{\tau}_1\left(p_1\right)}}_{\text{direct effect}} \cdot \underbrace{\frac{1}{1-\frac{1}{\hat{\tau}_1(p_1)}\left(\frac{d\left(\alpha_1^R(p_1)\right)}{dp_1}\tau^R\delta_1^R+\frac{d\hat{\tau}_1(p_1)}{dp_1}\left(\mu_d+\frac{1}{2}\sigma_d^2-p_1\right)\right)}}_{\text{redistribution effect}}.$$

- *High sentiment region $\delta_1 \in \left(\delta_1^h, \bar{\delta}_1\right]$:*

$$\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = \underbrace{1}_{\text{direct effect}} \cdot \underbrace{\frac{1}{1-\frac{1}{\alpha_1^R(p_1)\tau^R}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R+\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\left(\mu_d+\frac{1}{2}\sigma_d^2-p_1\right)-\frac{d\alpha_1^{IS}(p_1)}{dp_1}\frac{1}{m}\sigma_d^2\right)}}_{\text{redistribution effect}}.$$

Proof. See Appendix A1.8. □

Within each sentiment region, the price impact can be decomposed into the direct effect and the redistribution effect. The direct effect says that the effect of retail sentiment shock depends on the contribution of retail investor's risk tolerance to aggregate risk tolerance. In the high sentiment region, both institutions are constrained, and thus retail investor's risk aversion is the aggregate risk aversion, and thus the direct effect is large. The redistribution effect captures the wealth redistribution triggered by the retail sentiment shock.

4.4 Equilibrium at time 0

Proposition 4 characterizes the time-0 equilibrium.

Proposition 4 (Equilibrium at time 0). *Consider an equilibrium where the short-sale constraint for the long institution and the margin constraint for the short institution are not binding at time 0 (under the equilibrium price p_0), and the time-1 equilibrium is a monotone equilibrium of Definition 1. Then the time-0 price is determined as follows.*

1. *Investors' time-0 beliefs about time-1 price distribution is consistent with the time-1 pricing function $p_1\left(\delta_1^R\right)$ and the shock distribution $\Psi\left(\delta_1^R\right)$, i.e.,*

$$\mathbb{E}_0^i\left[p_1\left(\delta_1^R\right)\right] = \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] + \delta_0^i = \int_{\underline{\delta}_1}^{\bar{\delta}_1} p_1\left(\delta_1^R\right) d\Psi\left(\delta_1^R\right) + \delta_0^i,$$
$$\mathrm{Var}_0^i\left(p_1\left(\delta_1^R\right)\right) = \sigma_0^2 = \int_{\underline{\delta}_1}^{\bar{\delta}_1} \left(p_1\left(\delta_1\right) - \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right]\right)^2 d\Psi\left(\delta_1^R\right).$$

2. *Given the time-1 pricing function $p_1\left(\delta_1^R\right)$, time-0 equilibrium price p_0 clears the market at $t=0$:*

$$p_0 \quad = \quad \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] + \left(\frac{1}{2}\sigma_0^2 + \frac{\sum_i \alpha_0^i\left(p_0\right)\tau^i\delta_0^i - \sigma_0^2}{\tau_0\left(p_0\right)}\right)$$

where $\tau_0\left(p_0\right)$ is the aggregate risk tolerance at time 0, defined as

$$\tau_0\left(p_0\right) \quad \equiv \quad \alpha_0^R\left(p_0\right)\tau^R + \left(1-\alpha_0^R\left(p_0\right)\right)\tau^I$$

Hence, the equilibrium is a fixed problem. The time-0 price depends on the shape of the time-1 pricing function $p_1\left(\delta_1^R\right)$ through investors' beliefs, while the time-1 pricing function depends on p_0 through the wealth shares.

5 The network origins of aggregate retail sentiment fluctuations

Section 4 shows how investor composition matters for the pricing of retail sentiment risk. In this section, I microfound the sentiment risk distribution. I assume that type-1 retail investors communicate on a social network and update their beliefs by "listening to" other investors on the network. The influence distribution on the network is highly skewed, which means influencers' views will carry a disproportionately high weight in the aggregate view of retail investors. Then idiosyncratic sentiment shocks to retail investors would not cancel out, and would instead translate into an aggregate retail sentiment shock.

This microfoundation allows me to study two counterfactual scenarios in Section 6. These two counterfactuals shed light on why short sellers got squeeze in January 2021, and why they exited the market thereafter.

5.1 Naive learning on a growing random network

At time $t = 1$, type-1 retail investor j draws a noisy signal

$$x_t^j = \rho y_{t-1}^R + \varepsilon_t^j,$$

where y_{t-1}^R is the average retail sentiment at time $t-1$, ε_t^j is an error term that is i.i.d. across investors and time. I assume that ε_t^j follows a truncated normal distribution on $[-\bar{\varepsilon}, \bar{\varepsilon}]$, with post-truncation mean 0 and variance σ_ε^2. $\rho \in (0, 1]$ is a parameter that captures the persistence of sentiment.

Type-1 retail investors communicate on a social network, and reveal their signals to others. Then each investor on the network updates his belief by "listening to" other people on the network. I use the adjacency matrix $\boldsymbol{A}_t = (a_{jk,t})$ to capture the relationship between pairs of investors. If investor j "listens to" (or "attends to") investor k at time t, then $a_{jk,t} = 1$, otherwise $a_{jk,t} = 0$. Investor j assigns weight $\omega_{jk,t}$ to investor k's signal, and $\omega_{jk,t}$ is defined as

$$\omega_{jk,t} \equiv \frac{a_{jk,t}}{\sum_{k=1}^{N_t} a_{jk,t}}.$$

Hence, each investor on the network assigns equal weights to people he listens to. Also note that $\sum_{k=1}^{N_t} \omega_{jk,t} = 1$.

After the updating, investor j's view becomes

$$y_t^j = \sum_{k=1}^{N_t} \omega_{jk,t} x_t^k = \sum_{k=1}^{N_t} \omega_{jk,t} \left(\rho y_{t-1}^R + \varepsilon_t^k\right) = \rho y_{t-1}^R + \sum_{j=1}^{N_t} \omega_{jk,t} \varepsilon_t^k.$$

y_t^j is the sentiment of investor j in equation (15).

Dynamics of aggregate retail sentiment Using the definition in equation (33), time-t aggregate retail sentiment is

$$\delta_t^R \equiv \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} y_t^j = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R + \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} d_{j,t}^{in} \varepsilon_t^j, \tag{48}$$

where $d_{j,t}^{in}$ is the time-t "influence" (or in-degree) of retail investor j, defined as

$$d_{j,t}^{in} \equiv \sum_{i=1}^{N_t} \omega_{ij,t}.$$

This is the same definition of influence as in equation (3). δ_t^R has support $\left[\underline{\delta}_t, \bar{\delta}_t\right]$, where

$$\begin{aligned} \underline{\delta}_t &= \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R - \theta\left(N_t\right) \bar{\varepsilon}, \\ \bar{\delta}_t &= \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R + \theta\left(N_t\right) \bar{\varepsilon}. \end{aligned}$$

Motivated by the findings in Section 3.4, I assume that $d_{j,t}^{in}$ is drawn from a power-law distribution, and is i.i.d. in the cross section of the N_t retail investors on the social network. The PDF of $d_{j,t}^{in}$ is

$$f_{d_{j,t}^{in}}(x) = \frac{\xi - 1}{d_{\min}} \left(\frac{x}{d_{\min}}\right)^{-\xi}, \xi > 1, \tag{49}$$

with support $\left[d_{\min}, d_{\max}\left(N_t\right)\right]$. The exponent ξ captures the skewness of the influence distribution. Lower values of ξ correspond to heavier tails and more right-skewed influence distribution. The upper bound $d_{\max}\left(N_t\right) = d_{\min} \cdot N_t^{\frac{1}{\xi-1}}$.[16] Lemma 2 computes the moments of the influence distribution.

Lemma 2 (Moments of the influence distribution). *In the cross section of N_t retail*

[16]Following Newman (2005) and Acemoglu et al. (2012), the upper bound can be computed in a heuristic way.

$$\Pr\left(d_j^{in} > x\right) = \int_x^{+\infty} \frac{\xi - 1}{d_{\min}} \left(\frac{y}{d_{\min}}\right)^{-\xi} dy = -\int_x^{+\infty} d\left(\frac{y}{d_{\min}}\right)^{1-\xi} = \left(\frac{x}{d_{\min}}\right)^{1-\xi}$$

$d_{\max}(N)$ is computed from

$$\Pr\left(d_j^{in} > d_{\max}(N)\right) = \frac{1}{N} \implies d_{\max}(N) = d_{\min} \cdot N^{\frac{1}{\xi-1}}$$

investors (type-1), the m-th moment of influence $d_{j,t}^{in}$ is

$$\mathbb{E}^{CS}\left[\left(d_{j,t}^{in}\right)^m\right] = \frac{\xi-1}{\xi-m-1}\frac{1}{d_{\min}^{1-\xi}}\left(d_{\min}^{m+1-\xi}-\left(d_{\max}\left(N_t\right)\right)^{m+1-\xi}\right).$$

The cross-sectional variance of $d_{j,t}^{in}$ is

$$\begin{aligned}\mathrm{Var}^{CS}\left(d_{j,t}^{in}\right) = &\frac{\xi-1}{3-\xi}\frac{1}{d_{\min}^{1-\xi}}\left(\left(d_{\max}\left(N_t\right)\right)^{3-\xi}-d_{\min}^{3-\xi}\right)\\ &-\left(\frac{\xi-1}{\xi-2}\right)^2\frac{1}{d_{\min}^{2-2\xi}}\left(d_{\min}^{2-\xi}-\left(d_{\max}\left(N_t\right)\right)^{2-\xi}\right)^2.\end{aligned} \tag{50}$$

And $\mathrm{Var}^{CS}\left(d_{j,t}^{in}\right) = O\left(N_t^{\frac{3-\xi}{\xi-1}}\right)$ *for* $\xi > 1$,

Proof. See Appendix A1.11. □

5.2 Aggregate fluctuations in retail sentiment

Proposition 5 below relates the volatility of the aggregate sentiment shock to the volatility of idiosyncratic shock σ_ε and the network parameters. This is a direct application of Acemoglu et al. (2012) Theorem 2 and Corollary 1.

Proposition 5 (Moments of aggregate retail sentiment). *Suppose the network size N_t evolves deterministically over time. Then at time $t-1$, the conditional mean and variance of aggregate retail sentiment δ_t^R are*

$$\mathbb{E}_{t-1}\left[\delta_t^R\right] = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)}\rho\delta_{t-1}^R, \tag{51}$$

$$\mathrm{Var}_{t-1}\left(\delta_t^R\right) = \left(\theta\left(N_t\right)\right)^2\frac{2d_{\min}^{\xi-1}}{N_t}\frac{1}{3-\xi}\left(\left(d_{\max}\left(N_t\right)\right)^{3-\xi}-d_{\min}^{3-\xi}\right)\sigma_\varepsilon^2. \tag{52}$$

And the conditional volatility satisfies

$$\sqrt{\mathrm{Var}_{t-1}\left(\delta_t^R\right)} = O\left(N_t^{\frac{2-\xi}{\xi-1}}\right).$$

Proof. See Appendix A1.12. □

Proposition 5 shows that the volatility of aggregate retail sentiment shock decreases with ξ. Intuitively, a smaller ξ corresponds to a more skewed influence distribution. Then idiosyncratic shocks to influencers will carry a higher weight in the aggregate sentiment, which leads to more aggregate fluctuations.

$\xi = 3$ corresponds to the standard Central Limit Theorem, which says that the aggregate volatility decreases at a rate of $\sqrt{N_t}$. Section 3.4.1 shows that for the Reddit WSB social network, $\xi < 3$. Hence, volatility decreases at a much lower rate. Even with a large number of users on the network, idiosyncratic sentiment shocks may still lead to large aggregate sentiment fluctuations. The 15% increase in average sentiment in January 2021 is thus a result of influencers' idiosyncratic sentiment shocks and a small $\hat{\xi}_t$.

5.3 Numerical example

Table 2
Model Parameters

Description	Parameter	Value
Risky Asset		
Mean of log dividend	μ_d	4
Volatility of log dividend	σ_d^2	0.1
Lower bound of log dividend	$\underline{d}$	-2.5
Upper bound of log dividend	$\bar{d}$	10.5
Supply of shares	$\bar{S}$	100
Endowment		
Retail investors	α_{-1}^R	0.3
	w_{-1}^R	1.194
Long institution	α_{-1}^{IL}	0.14
	w_{-1}^{IL}	4.800
Short institution	α_{-1}^{IS}	0.56
	w_{-1}^{IS}	-0.054
Risk Aversion		
Retail investors	γ^R	2
Institutions	γ^I	1

Description	Parameter	Value
Constraints		
Margin constraint	m	0.5
Sentiment Shocks		
Retail investors	δ_0^R	1.028
	$\bar{\varepsilon}$	2.872
	σ_ε^2	0.081
Long institution	δ_0^{IL}	0.256
Short institution	δ_0^{IS}	-0.505
Network		
Population of type-1 retail investors	N_L	80000
	N_H	140000
Population of retail investors	$\bar{N}$	200000
Exponent of power-law distribution	ξ	2.1
Cutoff value of power-law distribution	$d_{\min}$	10
Persistence of agg. retail sent shock	ρ	1

I present a numerical example that matches the price and quantity patterns observed in the data. Table 2 shows the parameters.

I assume that the network size remains constant over time, with $N_0 = N_1 = N_L$. When investors form their subjective expectations, they also perceive the network size as constant. When drawing time-1 sentiment shocks, I assume that the aggregate sentiment shock δ_1^R follows a truncated normal distribution with post-truncation mean and variance given by equations (51) and (52), and support $\left[\underline{\delta}_1, \bar{\delta}_1\right]$. Appendix A1.13 shows that the true distribution of δ_1^R (by aggregating the y_1^j's) can be approximated by this truncated normal distribution, if the influence distribution is skewed.

Figure 20 plots the time-1 price as a function of the aggregate sentiment shock realization. And Figure 21 plots the pricing function together with the PDF of the aggregate retail sentiment shock. As shown in Section 4.3, the price impact within each sentiment region is determined by the direct effect and wealth redistribution. At the cutoff sentiment δ_1^h, there is an endogenous jump in price, due to margin constraint and wealth effect.

In this example, investors' time-0 portfolio weights are $w_0^R = 1.90$, $w_0^{IL} = 1.76$, and $w_0^{IS} = -0.25$. Both the aggregate retail investor and the long institution take a levered position in the risky asset. Hence, as retail sentiment drives up price, wealth redistributes from the short institution to retail investors and the long institution (Figure 23 panel (c)).

Figure 22 shows the time series predictions from the model. The time-1 values correspond to an aggregate sentiment shock $\delta_1^R = 2.18$. The model can match the price and quantity patterns documented in Section 3.1-3.3. In particular, panel (a) shows that short sellers increase their short positions following the first retail sentiment shock δ_0^R, while significantly reduce their short positions after the second sentiment shock.

6 Counterfactuals

I conduct two counterfactuals, which shed light on the reasons why short sellers got squeezed in January 2021, and why they stayed out of the market thereafter.

6.1 Why did short sellers get squeezed in January 2021?

In Section 3.1, I document that average retail sentiment on GME had been steadily increasing from mid-2020 to Jan 2021, while the discussion volume on GME spiked in January 2021. Both forces would contribute to a large positive realization of aggregate retail sentiment, as is shown in equation (33). This realized retail sentiment shock not only drove out price-sensitive long investors, but also got short sellers squeezed.

I formalize this idea through the lens of the model, using the parameters for the numerical example in Section 5.3. In particular, an increase in discussion volume in the data corresponds to an unexpected increase in network size in the model, i.e., an "MIT shock" to network size. Given the skewness of the influence distribution and how optimistic influencers are, the growth of the network translates into a large sentiment realization, which exceeds the short squeeze cutoff δ_1^h in equation (40), i.e., the long institution liquidates his position and the short institution gets squeezed under this sentiment shock realization. Next, I consider a counterfactual scenario where the discussion volume does not spike in January, i.e., the network size does not change in the model. In this case, the average sentiment still remains positive, but the aggregate sentiment is lower than the realized sentiment and short sellers would not get squeezed.

I begin by analyzing the factors that contribute to the large positive realization of aggregate retail sentiment: network size, network geometry (or influence distribution), and optimism of individual retail investors on the network. I assume that the network size grows from time 0 to time 1, with $N_0 = N_L < N_H = N_1$, and the values of N_L and N_H are given in Table 2. Substitute into equation (48) to get the realized aggregate retail sentiment

$$\delta_1^R = \underbrace{\frac{\theta(N_H)}{\theta(N_L)}\rho\delta_0^R}_{\text{persistence}} + \underbrace{\theta(N_H)\frac{1}{N_H}\sum_{j=1}^{N_H} d_{j,1}^{in}\varepsilon_1^j}_{\text{aggregation of idio. shocks}}. \tag{53}$$

The first component captures the persistence of aggregate retail sentiment ($\rho\delta_0^R$) and the amplification effect through a growing network ($\frac{\theta(N_H)}{\theta(N_L)}$). $\frac{\theta(N_H)}{\theta(N_L)} = \frac{N_H}{N_L} > 1$ captures the growth of the social network from time 0 to time 1. $\rho > 0$ captures the persistence of aggregate retail sentiment. Suppose $\delta_0^R > 0$, i.e., at time 0, retail investors are optimistic in aggregate. Then retail investors who newly join the network will adopt the optimistic views from existing investors, and the average optimism of existing investors will get amplified and be reflected in the aggregate retail sentiment.

The second component ($\frac{1}{N_H}\sum_{j=1}^{N_H} d_{j,1}^{in}\varepsilon_1^j$) captures the aggregation of idiosyncratic sentiment shocks to investors on the network. Since the influence distribution is skewed, idiosyncratic sentiment shocks do not average out across investors, and influencers' sentiment shocks will carry a higher weight in the aggregate sentiment, leading to fluctuations in aggregate sentiment. If influencers happen to draw a positive sentiment shock, then aggregate sentiment will also be positive. And importantly, on the intensive margin, the aggregate optimism will depend on the skewness of the influence distribution and the network size. To see this, if we have a large number of retail investors on the network, i.e., $N_H \to +\infty$, then first apply the Law of Large Numbers in the cross section of retail investors,

$$\frac{1}{N_H}\sum_{j=1}^{N_H} d_{j,1}^{in}\varepsilon_1^j \xrightarrow{p} \mathbb{E}\left[d_{j,1}^{in}\varepsilon_1^j\right] = \text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)\sqrt{\text{Var}\left(d_{j,1}^{in}; N_H\right)}\sigma_\varepsilon. \tag{54}$$

$\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)$ is the cross-sectional correlation between users' influence and the idiosyncratic shocks they draw. If $\text{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) > 0$, then it means that influencers are optimistic. And how much influencers' views carry in the aggregate view depends on the cross-sectional dispersion in user influence, which is captured by $\sqrt{\text{Var}\left(d_{j,1}^{in}; N_H\right)}$. In Section 3.4.1, I estimated that the power-law exponent $\hat{\xi}_t \in (1, 3)$. Then it immediately follows from Lemma 2 that, as the network grows, the influence distribution is more dispersed in the cross section of retail investors, and influencers' views will get amplified more and carry a higher weight.

Next, I consider a counterfactual scenario where the network size remains constant from time 0 to time 1, i.e., $N_0 = N_1 = N_L$. Using (54) to approximate the aggregation of idiosyncratic sentiment shocks, the realized aggregate sentiment in (53) can be approximated by

$$\delta_1^R \approx \frac{\theta(N_H)}{\theta(N_L)} \rho \delta_0^R + \theta(N_H) \operatorname{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) \sqrt{\operatorname{Var}\left(d_{j,1}^{in}; N_H, \xi\right)} \sigma_\varepsilon. \tag{55}$$

The counterfactual aggregate retail sentiment is

$$\hat{\delta}_1^R \approx \rho \delta_0^R + \theta(N_L) \operatorname{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) \sqrt{\operatorname{Var}\left(d_{j,1}^{in}; N_L, \xi\right)} \sigma_\varepsilon \tag{56}$$

In this counterfactual scenario, influencers remain as optimistic as they are in the realized scenario, i.e., $\operatorname{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)$ remains the same. But due to a smaller network size, the counterfactual aggregate retail sentiment shock is smaller, i.e., $\hat{\delta}_1^R < \delta_1^R$.

The model in Section 4 allows me to quantify the pricing impact of the counterfactual sentiment shock. From the pricing function $P_1\left(\delta_1^R\right)$ in Figure 20 and the price of GameStop observed from the data ($P_1 = 349.73$ in January 2021), I can back out the realized aggregate retail sentiment $\delta_1^R = 2.18$. Given the network parameters $\left(N_L, N_H, \bar{N}, d_{\min}, \xi\right)$ in Table 2, and using equation (55), I then back out how optimistic influencers are, which is $\operatorname{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right) = 0.00135$. Now fix the optimism of influencers, I can calculate the counterfactual retail sentiment from equation (56), which yields $\hat{\delta}_1^R = 1.20$. Finally, using the pricing function $P_1\left(\delta_1^R\right)$ in Figure 20, the counterfactual price is thus $P_1\left(\hat{\delta}_1^R\right) = 65.63$.

Figure 27 plots the equilibrium price under the realized sentiment shock $\delta_1^R = 2.18$ versus that under the counterfactual sentiment shock $\hat{\delta}_1^R = 1.20$. In the latter case, short sellers do not get squeezed, since the counterfactual sentiment shock is smaller than the short squeeze cutoff δ_1^h.

As is discussed above, we can decompose the gap between the realized sentiment and counterfactual sentiment into two parts: one captures the persistence of aggregate retail sentiment and the amplification through a growing network, while the other captures the aggregation of idiosyncratic shocks on a network with skewed influence distribution. Formally, compare equation (55) with equation (56), and compute the difference

$$\begin{aligned}\delta_1^R - \hat{\delta}_1^R = & \underbrace{\left(\frac{\theta\left(N_H\right)}{\theta\left(N_L\right)} - 1\right)\rho\delta_0^R}_{\Delta_1} \\ & + \underbrace{\operatorname{Corr}\left(d_{j,1}^{in}, \varepsilon_1^j\right)\left(\theta\left(N_H\right)\sqrt{\operatorname{Var}\left(d_{j,1}^{in}; N_H, \xi\right)} - \theta\left(N_L\right)\sqrt{\operatorname{Var}\left(d_{j,1}^{in}; N_L, \xi\right)}\right)\sigma_\varepsilon}_{\Delta_2}\end{aligned} \tag{57}$$

$\Delta_1 = 0.771$ is the component due to persistence of aggregate retail sentiment, and $\Delta_2 = 0.206$ is the component due to aggregation of idiosyncratic sentiment shocks. Figure 28 plots the two components. The second component alone would be sufficient to squeeze short sellers, which suggests that the skewed influence distribution on social network has an economically large impact on asset price.

6.2 Why did short sellers exit the market after January 2021?

In the model, there are three mechanisms that can help explain why short sellers stayed out of the market and price remained high after January 2021: (1) Short sellers updated their perceptions about retail sentiment risk post the GameStop frenzy; (2) The market for GameStop became price inelastic due to financial constraints and wealth redistribution; (3) Short sellers lost wealth and were forced to exit the market.

Change in short sellers' risk perceptions After observing a large influx of retail investors to WSB in January 2021, short sellers may have updated their perceptions about the sentiment risk distribution, and thus would trade less aggressively. This can explain why price stayed high and short interest stayed low after January 2021.

In the model, the short institution's risk perception depends on his perception about the growth of the social network. In the numerical example of Section 5.3, short sellers believe that the network size remains constant from time 0 to time 1. Let $\tilde{N}_1$ denote short sellers' time-0 perception about the network size at time 1, then $\tilde{N}_1 = N_L$. Their perception of the risk distribution corresponding to $\tilde{N}_1$ is plotted in the solid blue line of Figure 29.

Now consider a counterfactual scenario where short sellers perfectly anticipate the growth of the network from time 0 to time 1, i.e., their perception about time-1 network size is $\tilde{N}_1 = N_H$. This corresponds to a different perception of the risk distribution, which is plotted in the dashed red line of Figure 29. I solve the time-0 equilibrium under this counterfactual risk perception. Table 3 compares the time-0 price under these two different risk perceptions. The time-0 price under risk perception $\tilde{N}_1 = N_H$ (column 4) is higher than that under

$\tilde{N}_1 = N_L$ (column 3). This is primarily because the expected payoff of the risky asset is higher under the new risk perception. Table 3 in the Internet Appendix compares other equilibrium outcomes at time 0.

Table 3
Compare Time-0 Equilibria under Different Risk Perceptions

		Value	
Description	Notation	$\tilde{N}_1 = N_L$	$\tilde{N}_1 = N_H$
(1)	(2)	(3)	(4)
Expected log payoff (risk-adjusted)	$\mathbb{E}_0[p_1] + \frac{1}{2}\sigma_0^2$	4.658	5.664
Price of time-0 realized sentiment shock	$\frac{\sum_i \alpha_0^i(p_0)\tau^i\delta_0^i}{\tau_0(p_0)}$	0.039	0.163
Price of time-1 risk	$-\frac{1}{\tau_0(p_0)}\sigma_0^2$	-0.449	-1.215
Sum	p_0	4.249	4.612

Change in aggregate demand elasticity in the market for GameStop After the January 2021 short squeeze episode, the market for GameStop became may have become price-inelastic for two reasons. First, price-elastic institutions hit their constraints and effectively became price-inelastic. Second, retail investors' wealth share increases and they are less price-elastic than (unconstrained) institutions. Since aggregate price elasticity is a weighted average of individual elasticity. This implies that the market for GameStop may have become price-inelastic after January 2021, and a moderate retail sentiment shock can have a large price impact.

Capital loss Short sellers like Melvin Capital lost a large fraction of wealth and exited the market.

7 Conclusion

In this paper, I provide new evidence on the price impact of retail sentiment, and demonstrate that it can trigger a subsequent investor composition change. This investor composition change is crucial for understanding the price surge in the GameStop short squeeze episode. I present a model that reconciles the findings on price, quantity and retail sentiment dynamics. In particular, I microfound aggregate sentiment fluctuations using a network model of belief formation, and show that the changing social network structure can lead to extreme outcomes in the asset market. The changing social dynamics may be "a new risk for fund managers to adapt to".[17]

[17] Berengere Sim (January 17, 2022), Hedge funds scour Reddit a year after GameStop: 'It's the tip of the iceberg of generational change', Financial News.

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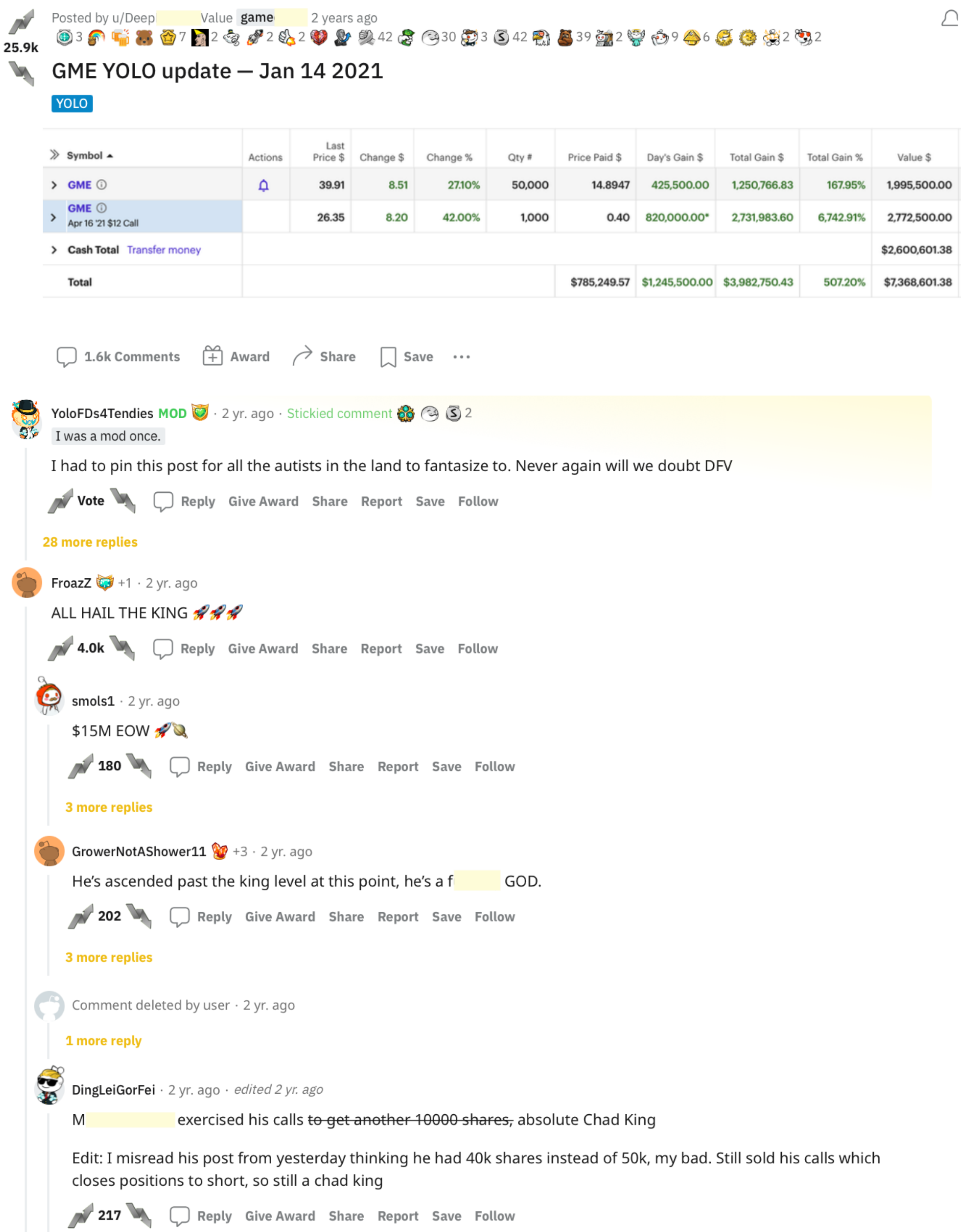


25.9k

Posted by u/Deep Value game 2 years ago

GME YOLO update — Jan 14 2021

YOLO

Symbol	Actions	Last Price $	Change $	Change %	Qty #	Price Paid $	Day's Gain $	Total Gain $	Total Gain %	Value $
GME		39.91	8.51	27.10%	50,000	14.8947	425,500.00	1,250,766.83	167.95%	1,995,500.00
GME Apr 16 '21 $12 Call		26.35	8.20	42.00%	1,000	0.40	820,000.00*	2,731,983.60	6,742.91%	2,772,500.00
Cash Total Transfer money										$2,600,601.38
Total						$785,249.57	$1,245,500.00	$3,982,750.43	507.20%	$7,368,601.38

1.6k Comments Award Share Save ...

YoloFDs4Tendies MOD · 2 yr. ago · Stickied comment 2

I was a mod once.

I had to pin this post for all the autists in the land to fantasize to. Never again will we doubt DFV

Vote Reply Give Award Share Report Save Follow

28 more replies

FroazZ +1 · 2 yr. ago

ALL HAIL THE KING 🚀🚀🚀

4.0k Reply Give Award Share Report Save Follow

smols1 · 2 yr. ago

$15M EOW 🚀

180 Reply Give Award Share Report Save Follow

3 more replies

GrowerNotAShower11 +3 · 2 yr. ago

He's ascended past the king level at this point, he's a f GOD.

202 Reply Give Award Share Report Save Follow

3 more replies

Comment deleted by user · 2 yr. ago

1 more reply

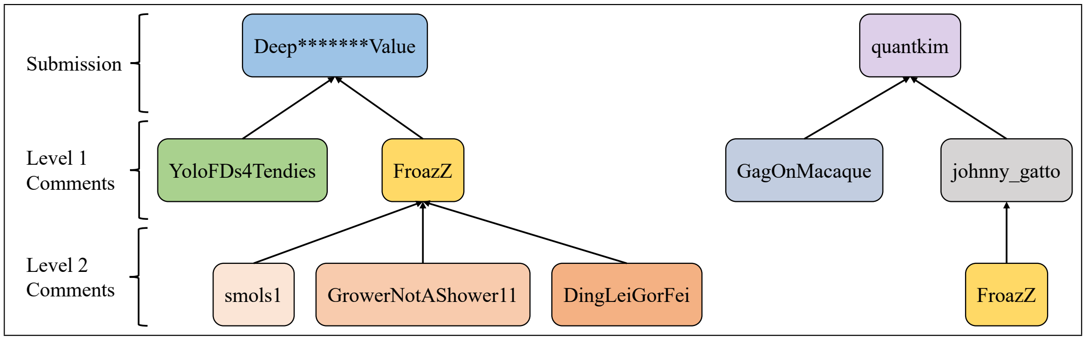
DingLeiGorFei · 2 yr. ago · *edited 2 yr. ago*

M exercised his calls ~~to get another 10000 shares,~~ absolute Chad King

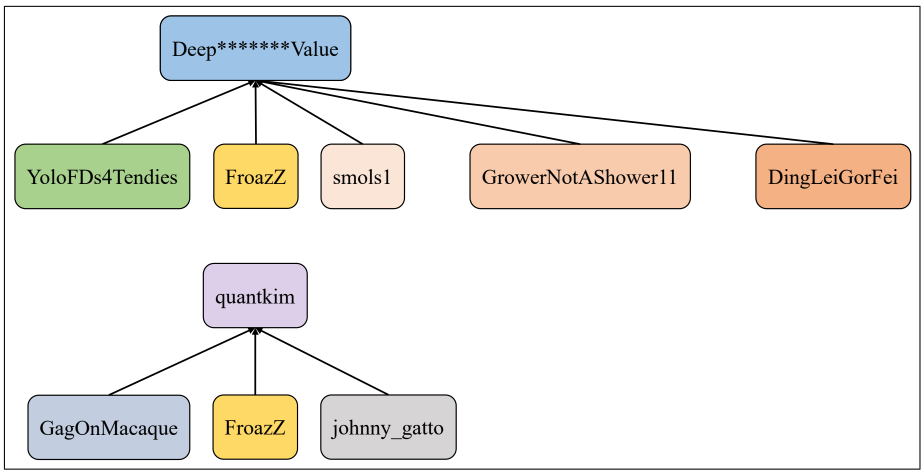
Edit: I misread his post from yesterday thinking he had 40k shares instead of 50k, my bad. Still sold his calls which closes positions to short, so still a chad king

217 Reply Give Award Share Report Save Follow

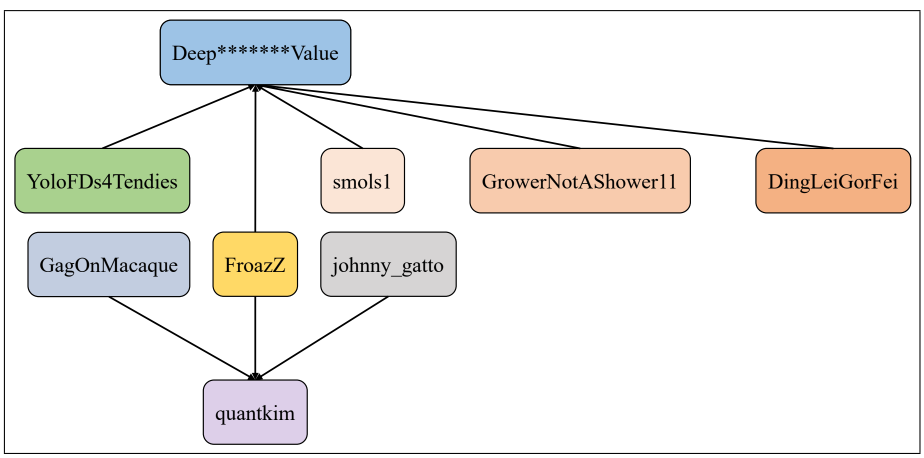
Figure 2. Example of a conversation tree. This figure shows an example of the conversation tree on WSB, retrieved from https://www.reddit.com/r/wallstreetbets/comments/kxeq23/gme_yolo_update_jan_14_2021/.



(a) Comment trees



(b) Simplified comment trees



(c) User network

Figure 3. Generic representations of comment trees and user network. This figures presents the comment trees and the corresponding user network. Panel (a) plots two trees, one of which corresponds to the conversation in Figure 2. Panel (b) plots the simplified trees. Panel (c) plots the user network constructed from these two trees.

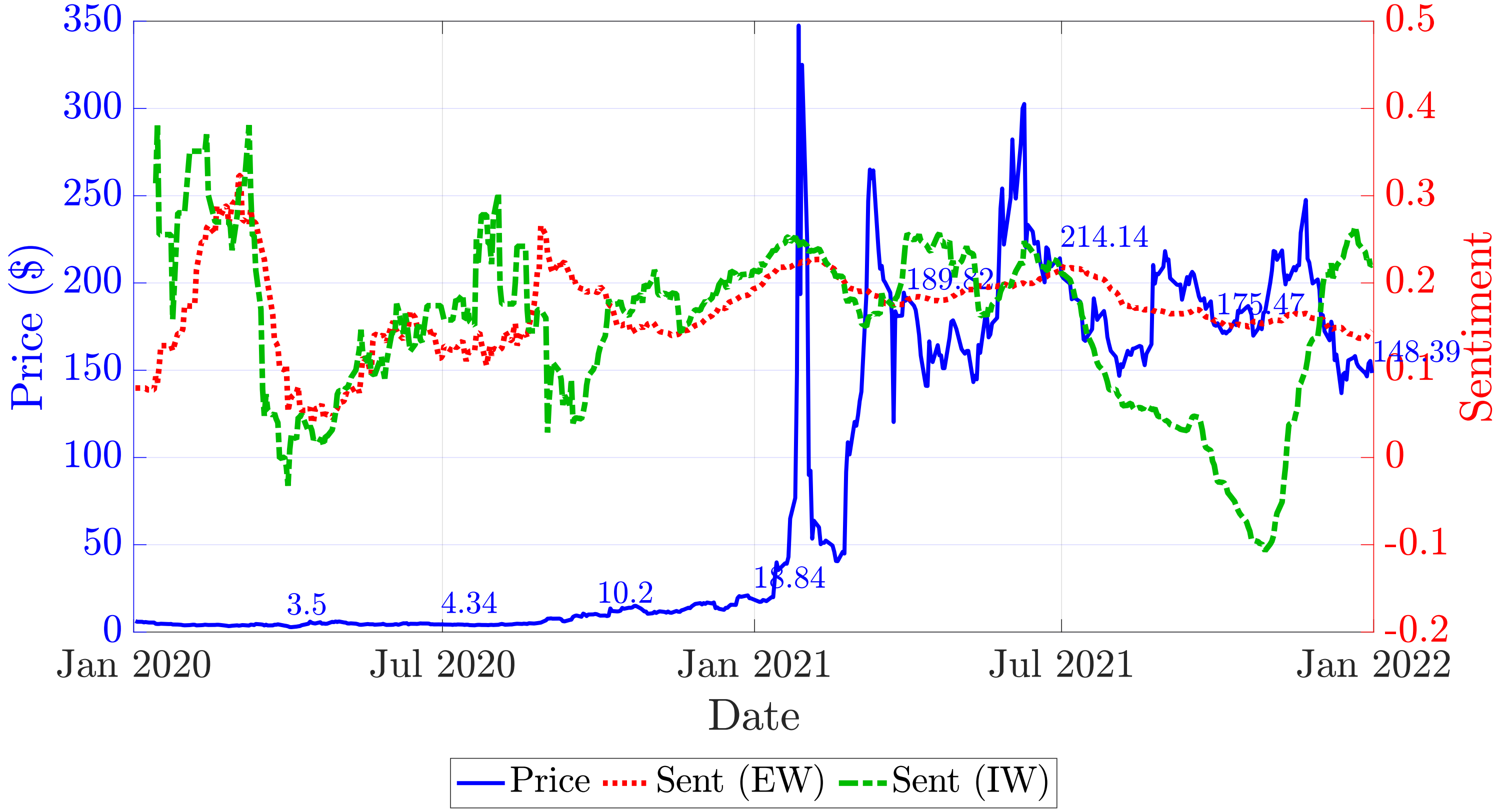
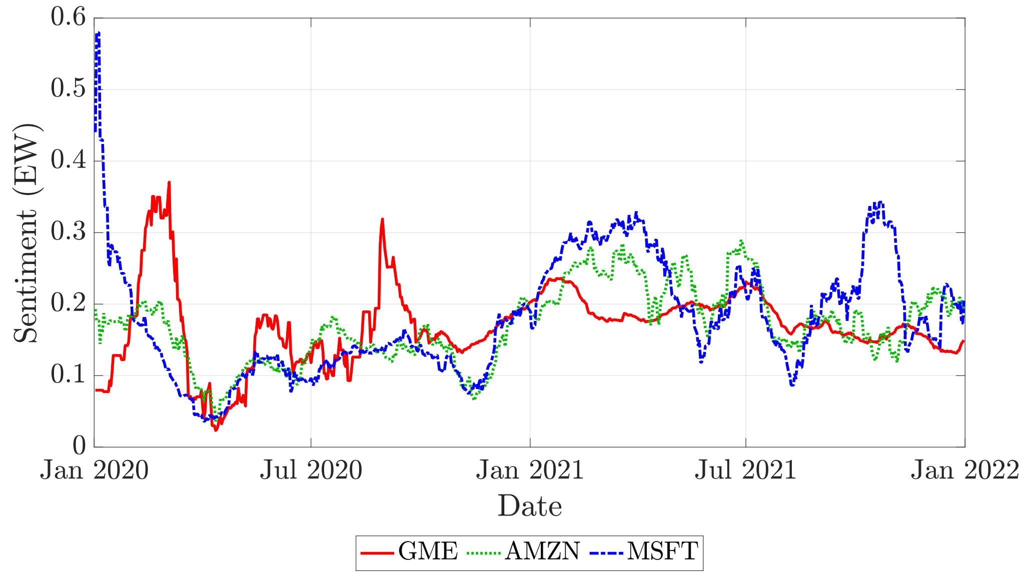
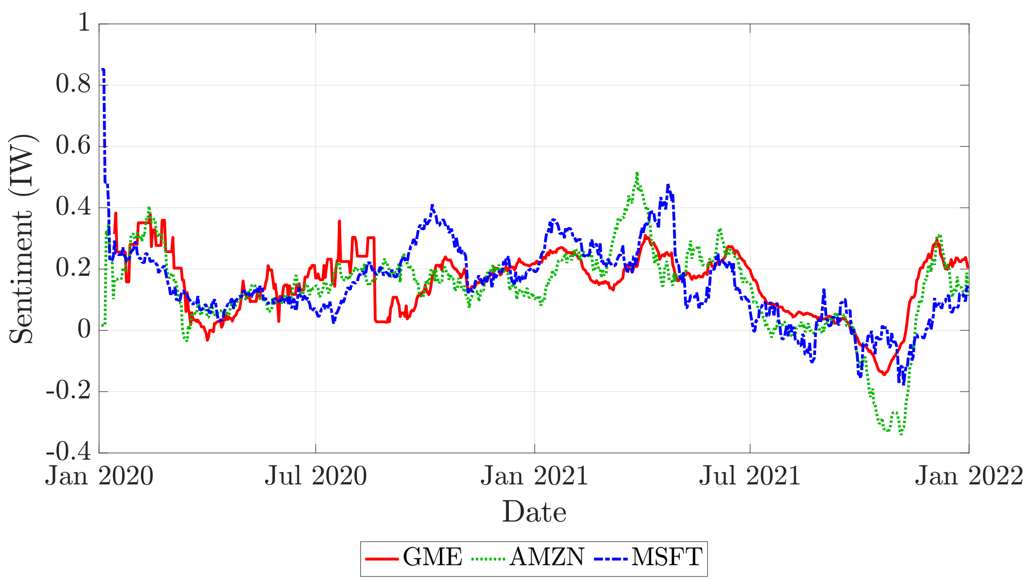


Figure 4. Price and sentiment of GameStop. This figure plots the daily close price (solid blue line), equal-weighted sentiment (dotted red line), and influence-weighted sentiment (dash-dotted green line) of GameStop. The sentiment series are 30-day moving averages.



(a) Equal-weighted sentiment



(b) Influence-weighted sentiment

Figure 5. Sentiment of GameStop versus tech stocks. This figure plots the daily sentiment of GameStop versus Amazon and Microsoft. Panel (a) plots the equal-weighted sentiment of the three stocks, while panel (b) plots the influence-weighted sentiment. The sentiment series are 30-day moving averages.

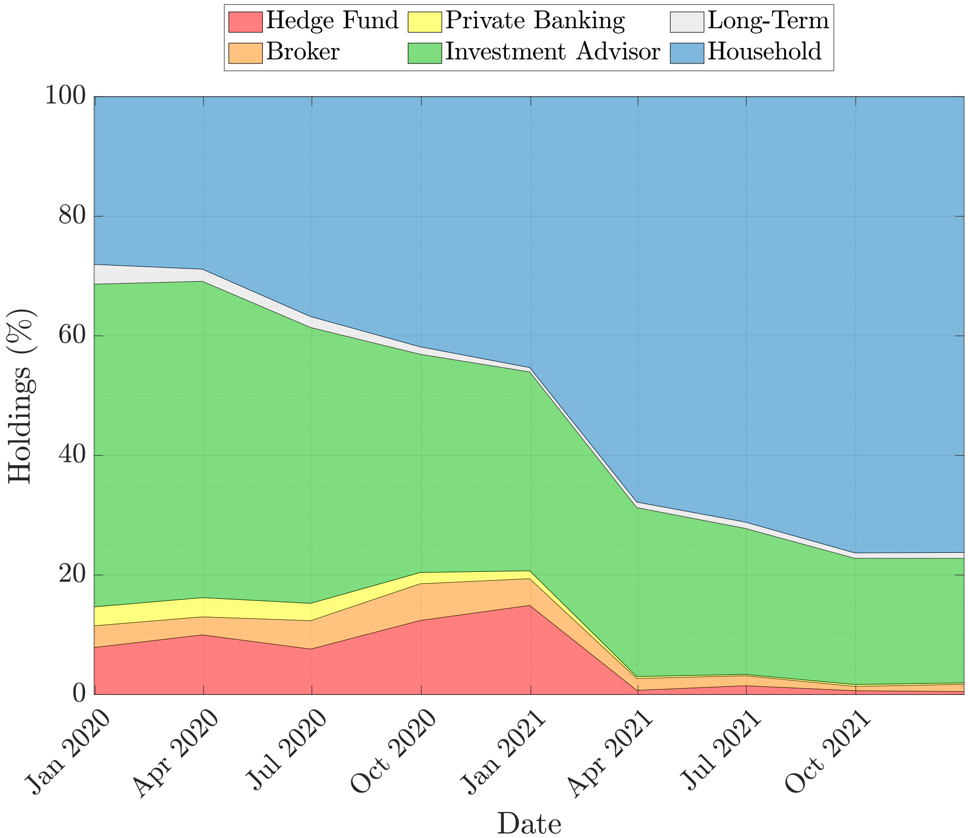
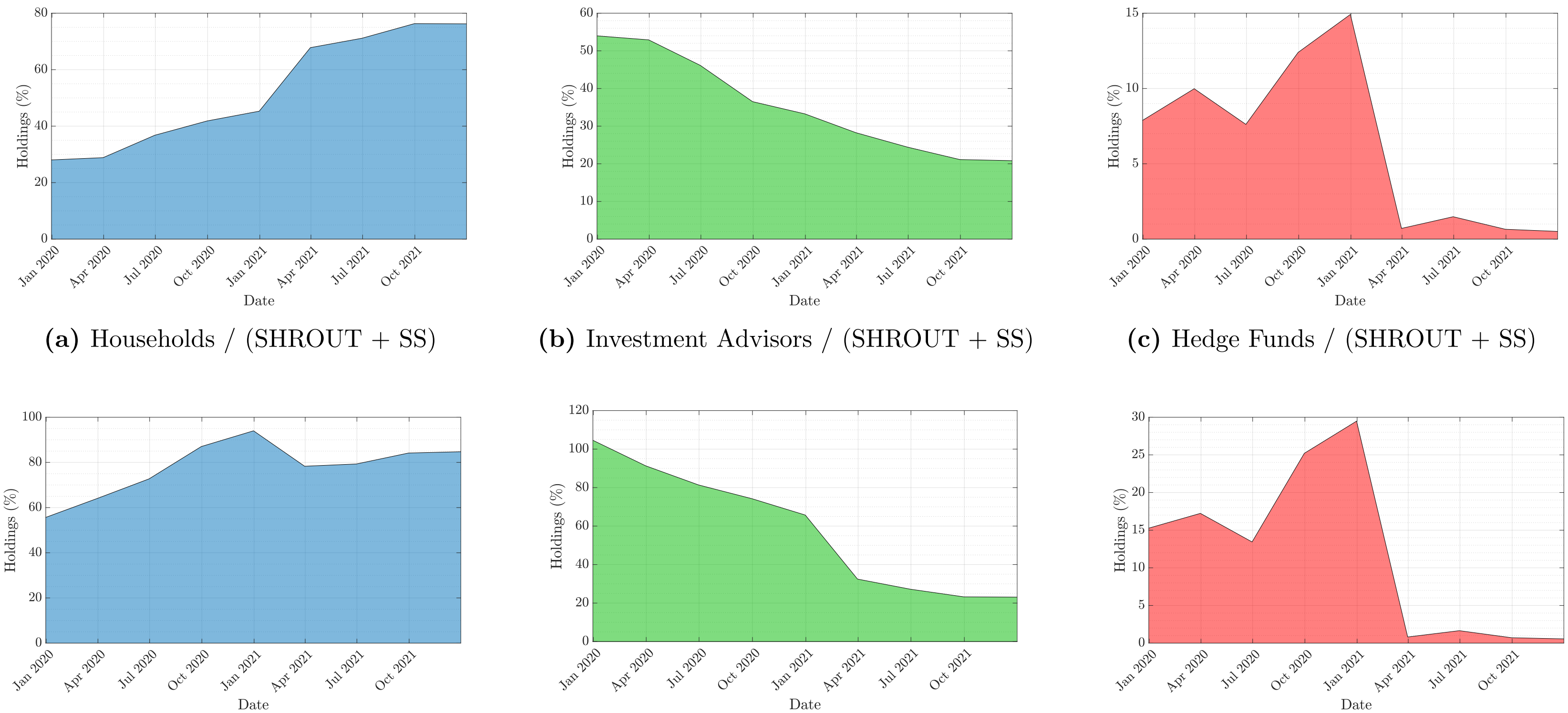


Figure 6. Holdings of long investors in GameStop. This figure plots the holdings of long investors in GameStop. The y axis is the number of shares held divided by number of shares outstanding plus number of shares sold short.



(a) Households / (SHROUT + SS)

(b) Investment Advisors / (SHROUT + SS)

(c) Hedge Funds / (SHROUT + SS)

(d) Households / SHROUT

(e) Investment Advisors / SHROUT

(f) Hedge Funds / SHROUT

Figure 7. Holdings of GameStop by investor group. This figure plots the holdings of Households, Investment Advisors and Hedge Funds in GameStop. For panel (a), (b), (c), the denominator is the number of shares outstanding plus number of shares sold short. For panel (d), (e), (f), the denominator is the number of shares outstanding.

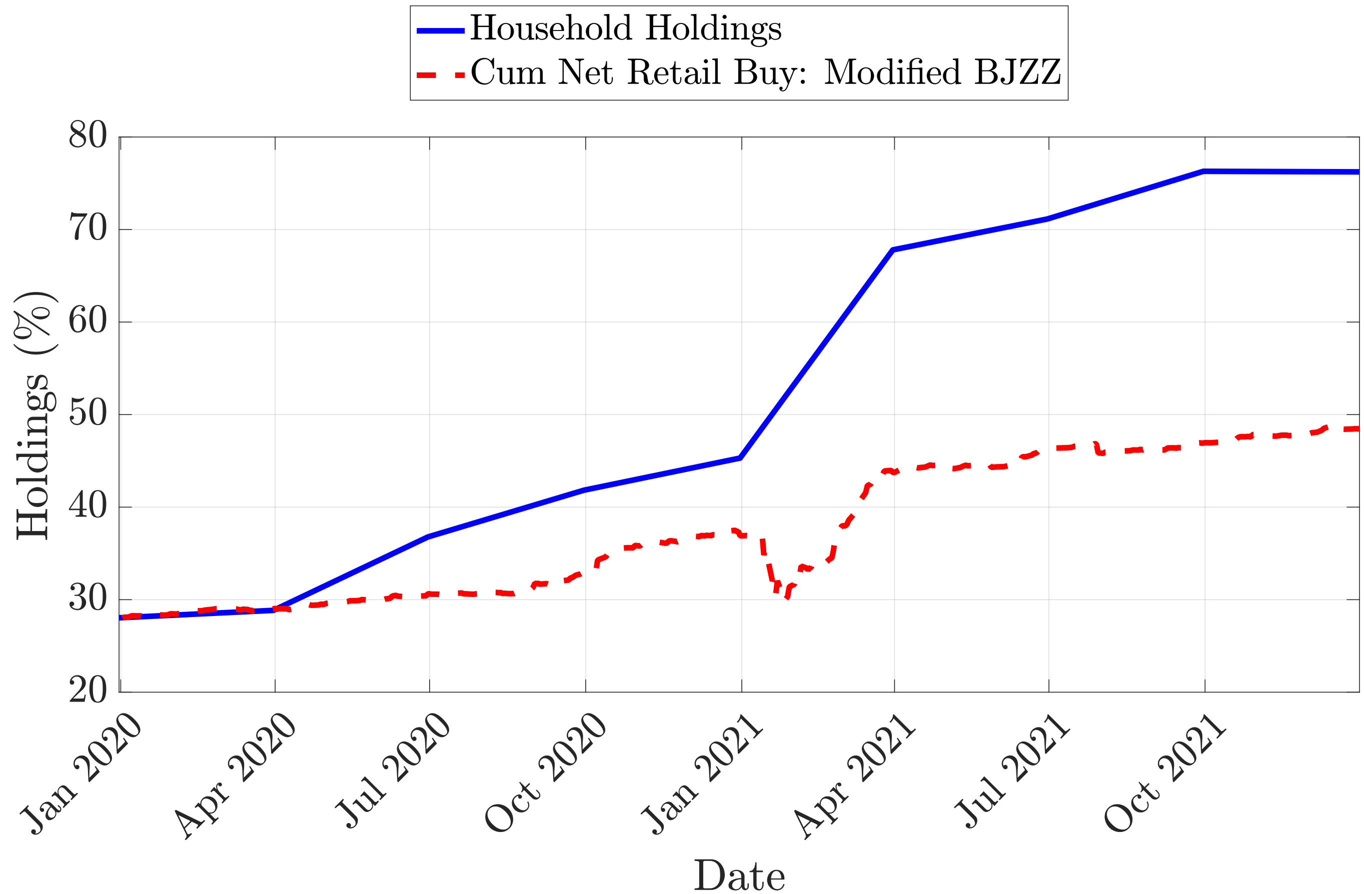


Figure 8. Household holdings versus cumulative net retail buy volume for GameStop. This figure plots the quarterly household holdings of GameStop (solid blue line) versus the daily cumulative net retail buy volume (dotted red line). The denominator for both series is number of shares outstanding plus number of shares sold short.

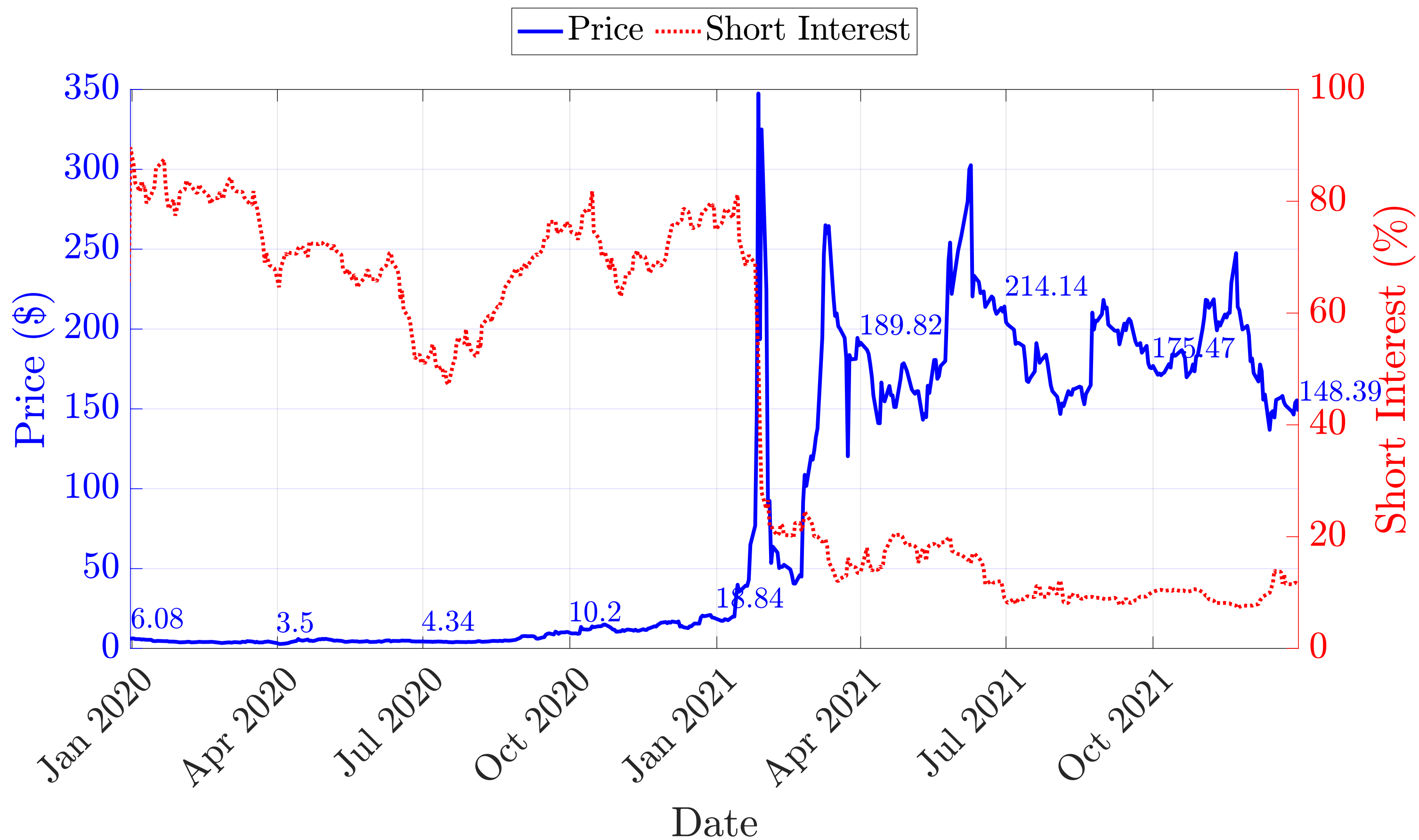
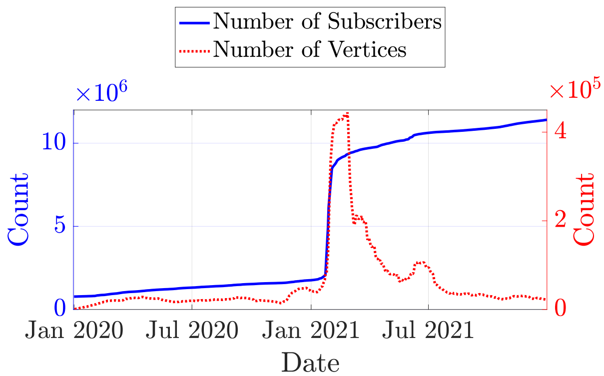
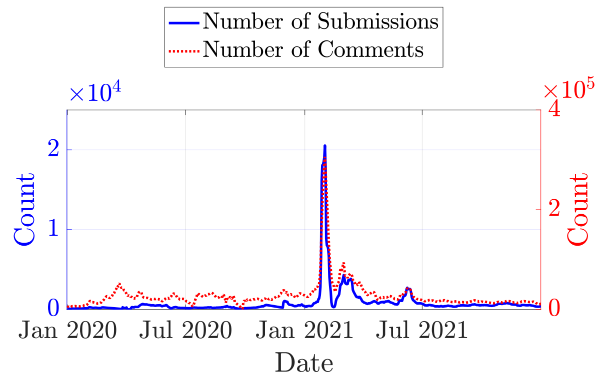


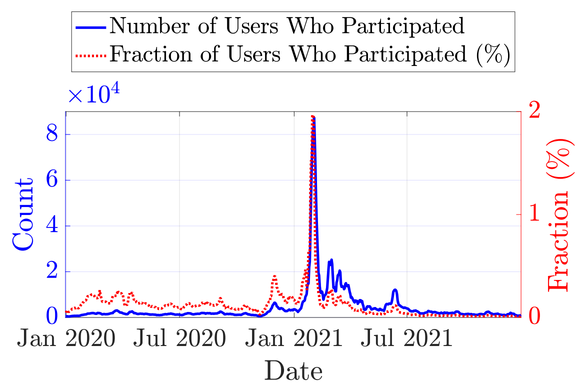
Figure 9. Price and short interest of GameStop. This figure plots the daily close price of GameStop (solid blue line), and the daily short interest (dotted red line). The short interest is the number of shares sold short divided by number of shares outstanding.



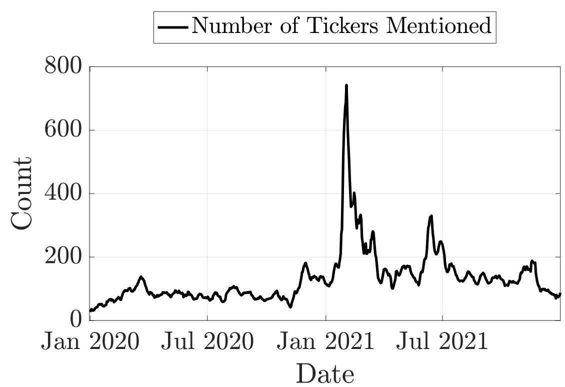
(a) Number of users



(b) Number of submissions and comments

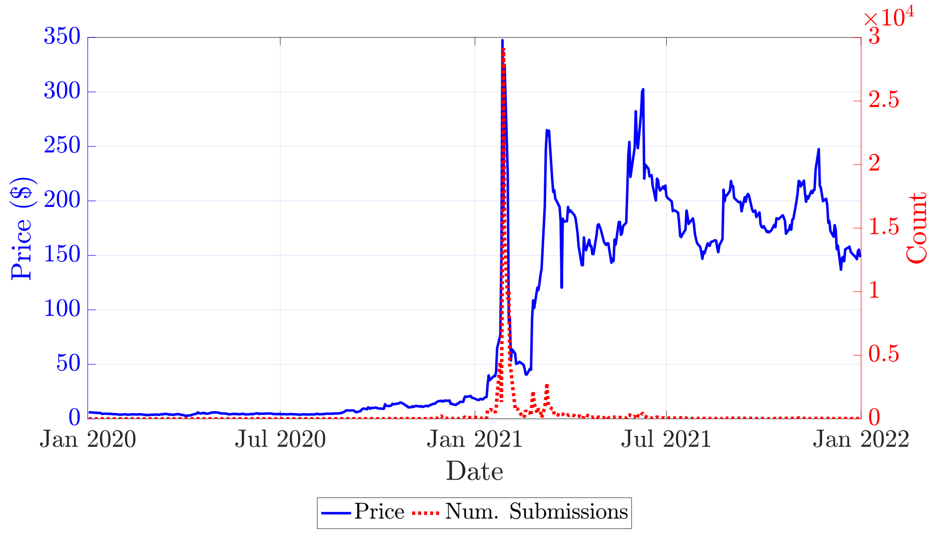


(c) Number of users who participated

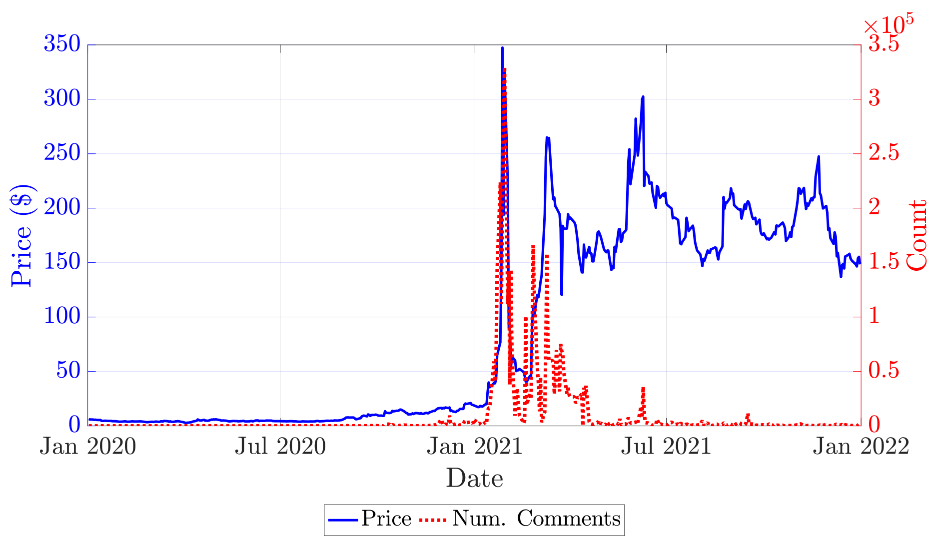


(d) Number of tickers mentioned

Figure 10. WSB statistics. This figure shows the time variation in the WSB statistics. Each line is a daily time series. Panel (a) plots the total number of subscribers to WSB (solid blue line), and the number of vertices (nodes) of the constructed network (dotted red line), on each day. When computing the latter statistics, I use the network constructed from the sample of submissions and comments about CRSP common stocks, over a 30-day rolling window (methods described in Section 2.1.2). Panel (b) plots the number of new submissions (solid blue line), and the number of new comments (dotted red line) made on each day. Panel (c) plots the number of users who participated in the discussion of CRSP common stocks (solid blue line), and the fraction of WSB subsribers who participated in these discussions (dotted red line), on each day. Panel (d) plots the number of stock tickers mentioned on WSB on each day. The series in panel (b)-(d) are 7-day moving averages.

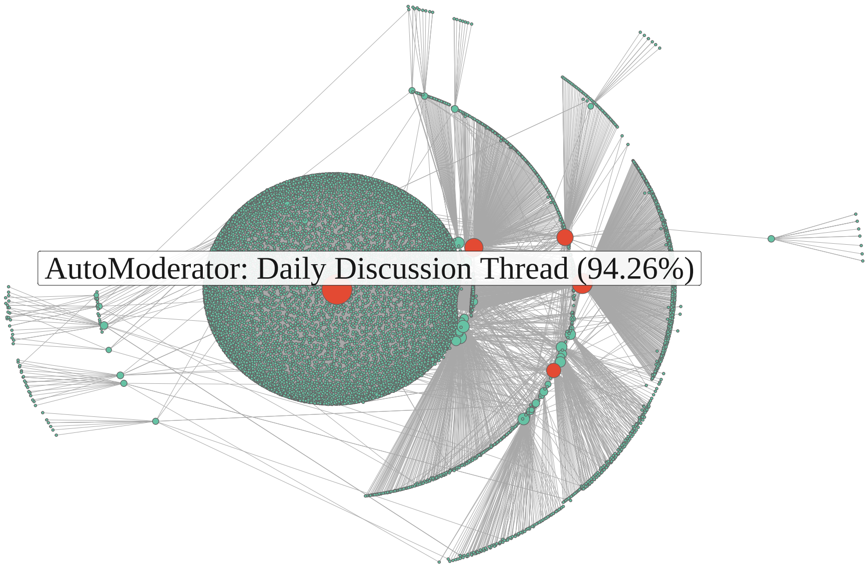


(a) Number of submissions

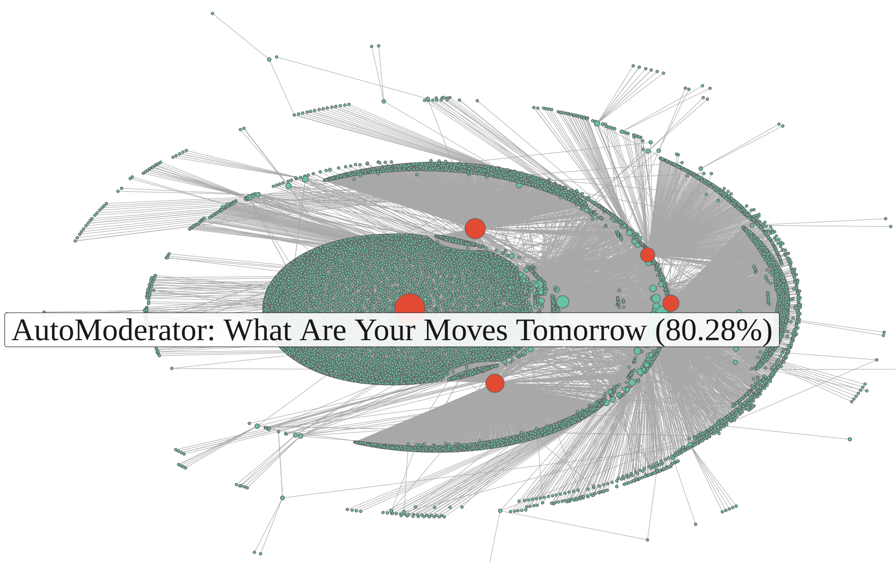


(b) Number of comments

Figure 11. WSB discussion volume on GameStop. This figure shows the time variation in WSB discussion volume on GameStop. Each line is a daily time series. Panel (a) plots the price of GameStop (solid blue line), and the number of submissions on GameStop (dotted red line), on each day. Panel (b) plots the price of GameStop (solid blue line), and the number of comments on GameStop (dotted red line), on each day.



(a) Discussion from 6-9am



(b) Discussion from 4-7pm

Figure 12. User discussions on January 14, 2022. Panel (a) plots the discussions from 6-9am, while panel (b) plots the discussions from 4-7pm.

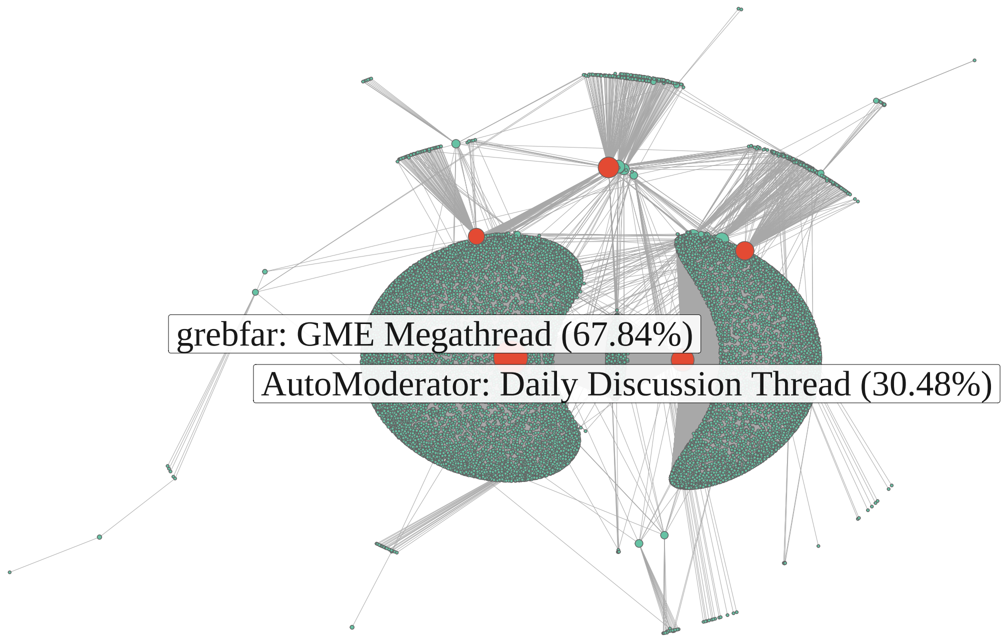


Figure 13. Megathreads on January 21, 2021. This figure plots the user discussions on January 21, 2021, from 6-8am.

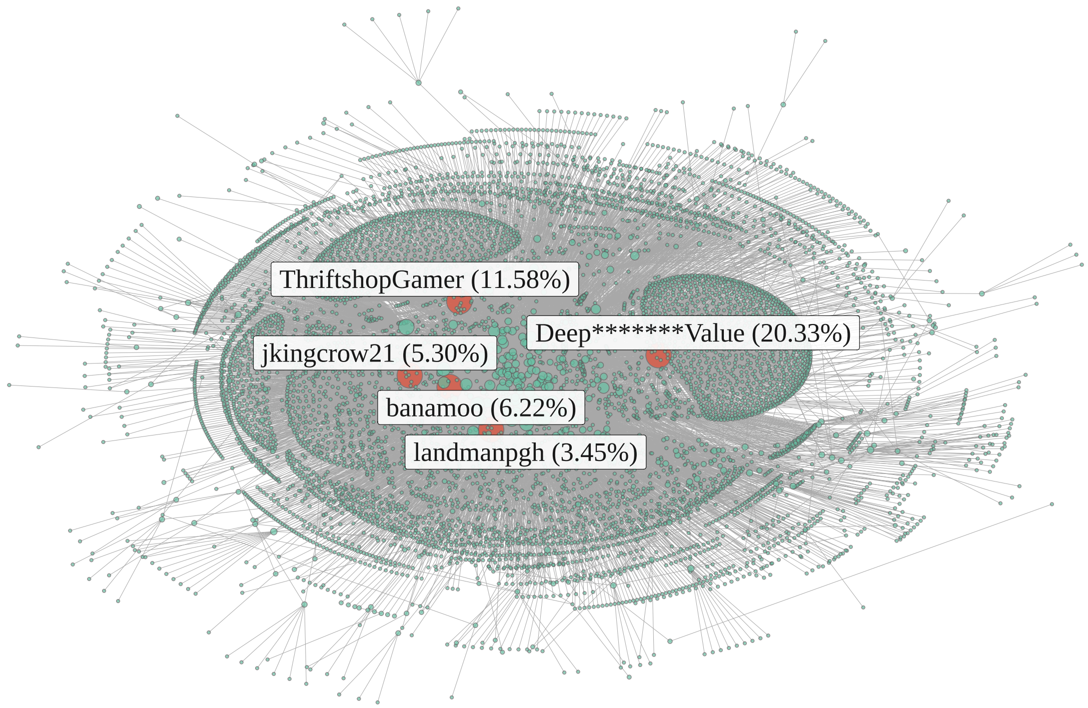


Figure 14. User network for GameStop on January 14, 2021. This figure plots the WSB user network for GameStop, on January 14, 2021.

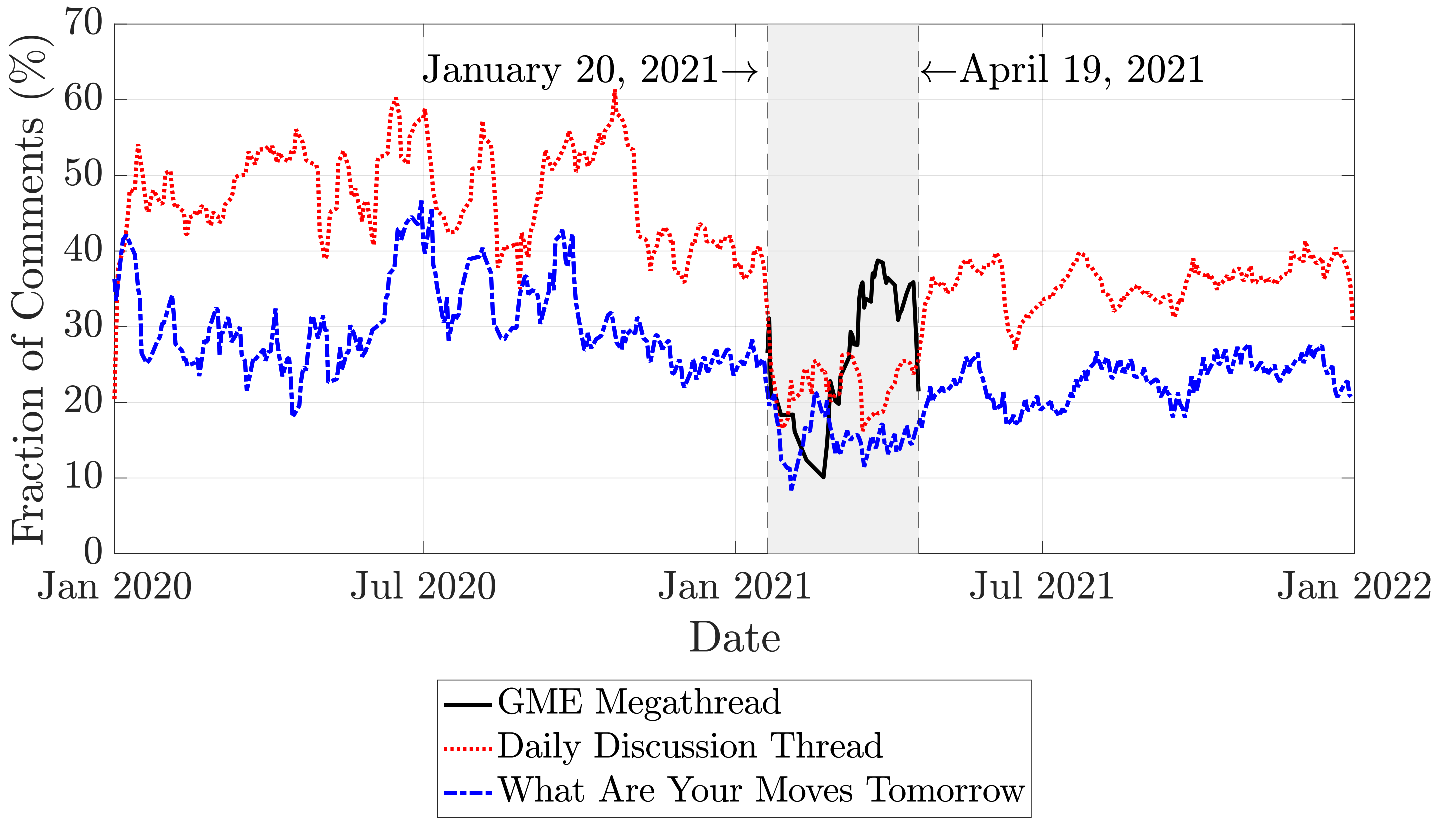


Figure 15. Comments received by megathreads. This figure plots the daily fraction of comments received by GME Megathread (solid black line), Daily Discussion Thread (dotted red line), and What Are Your Moves Tomorrow (dash-dotted blue line).

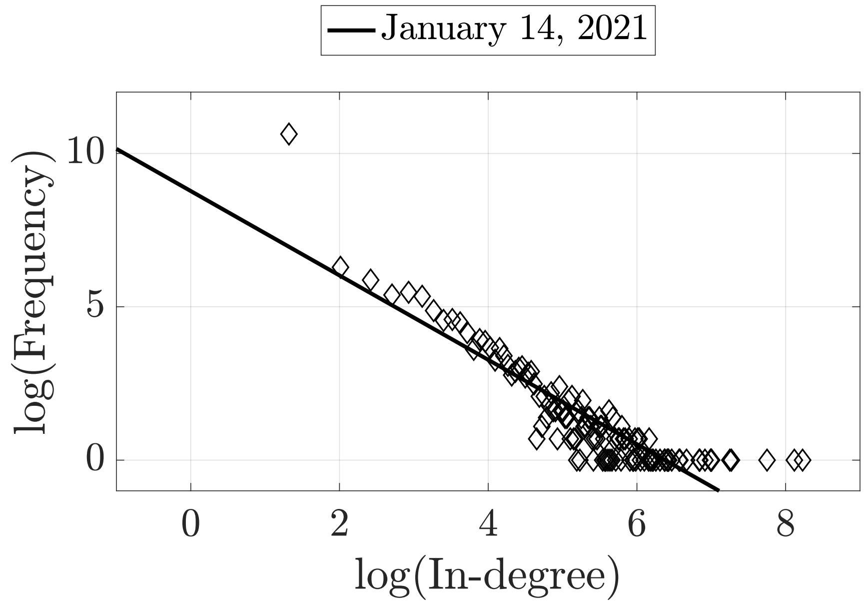


Figure 16. Log-log plot of in-degree distribution, January 14, 2021 This figure plots the log of in-degree on the x-axis, and the log empirical frequency on the y axis. The vector of in-degrees is from the network of January 14, 2021.

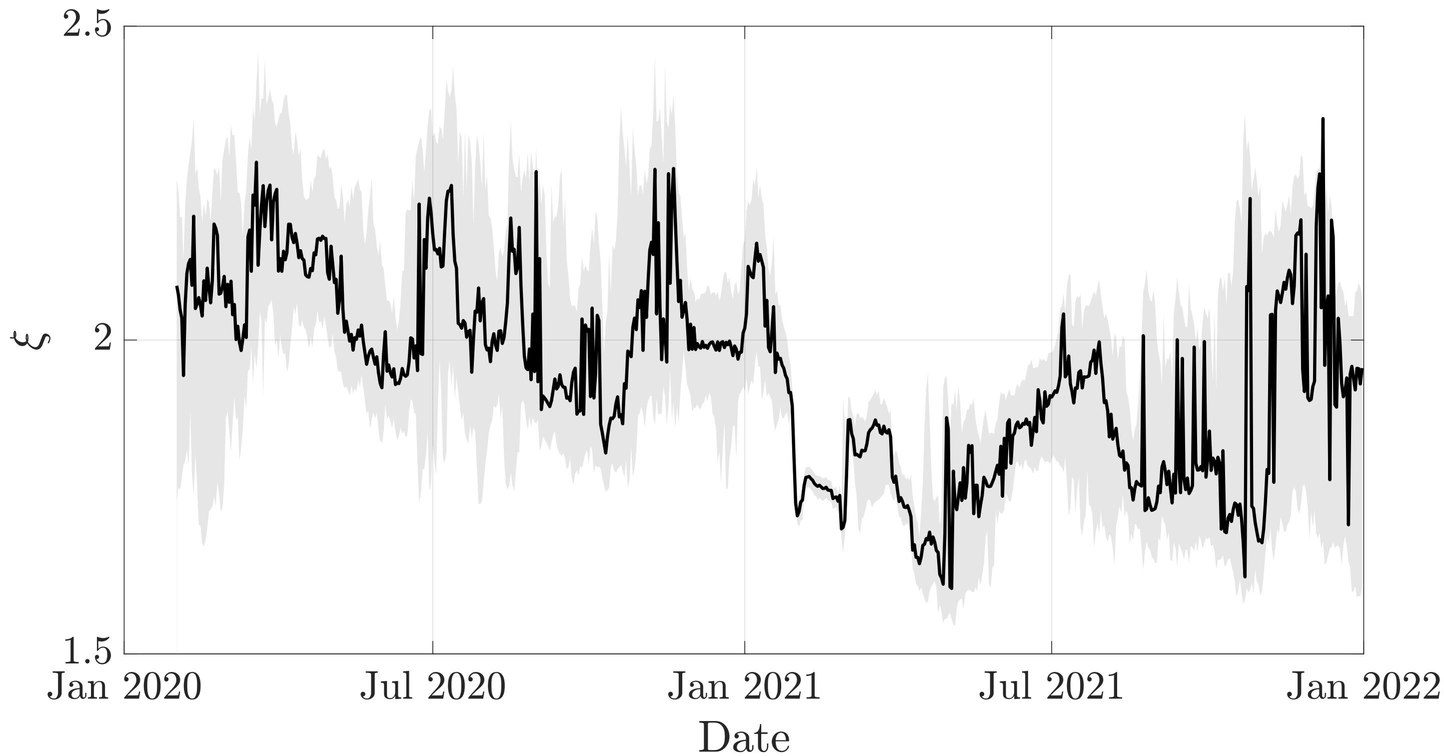


Figure 17. $\hat{\xi}_t$ **estimates.** This figure plots the daily estimate of $\hat{\xi}_t$.

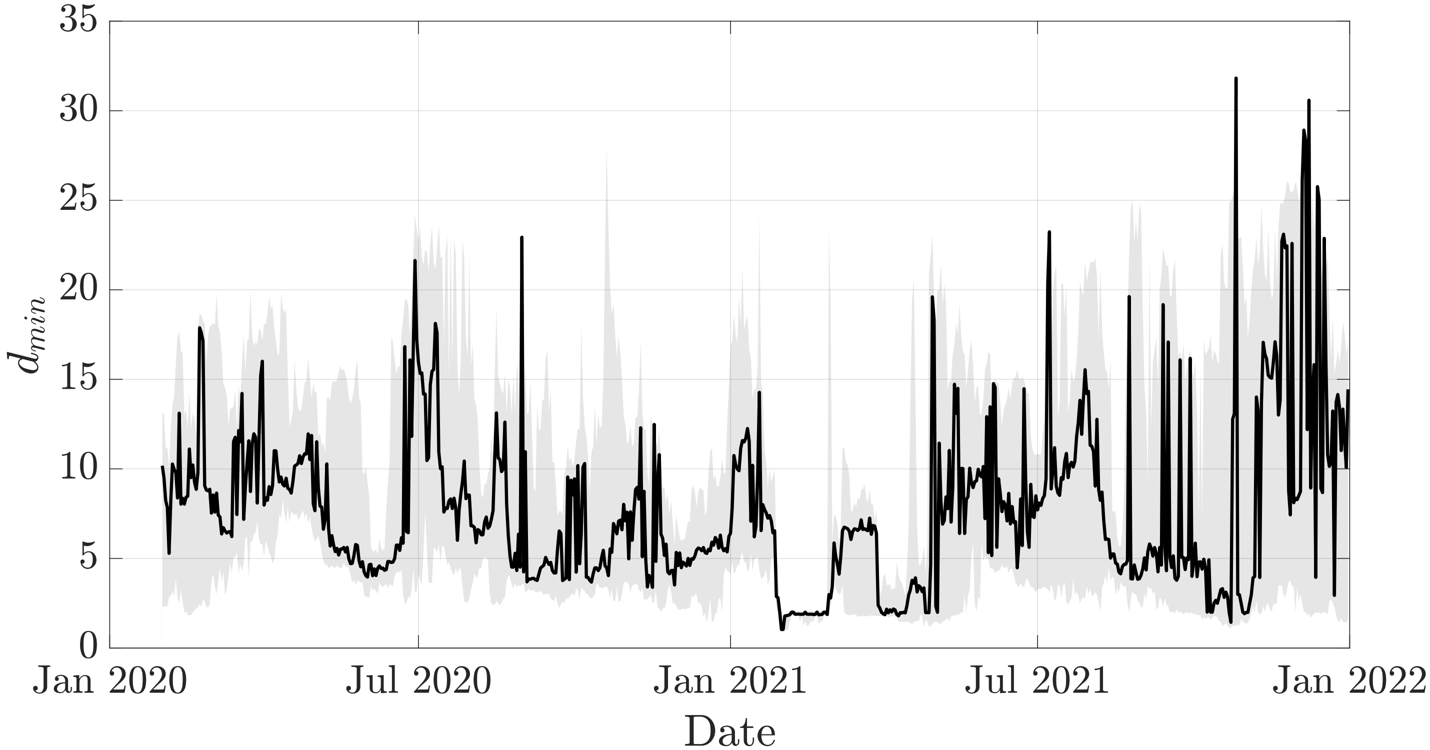


Figure 18. $d_{\min,t}$ **estimates.** This figure plots the daily estimate of $d_{\min,t}$.

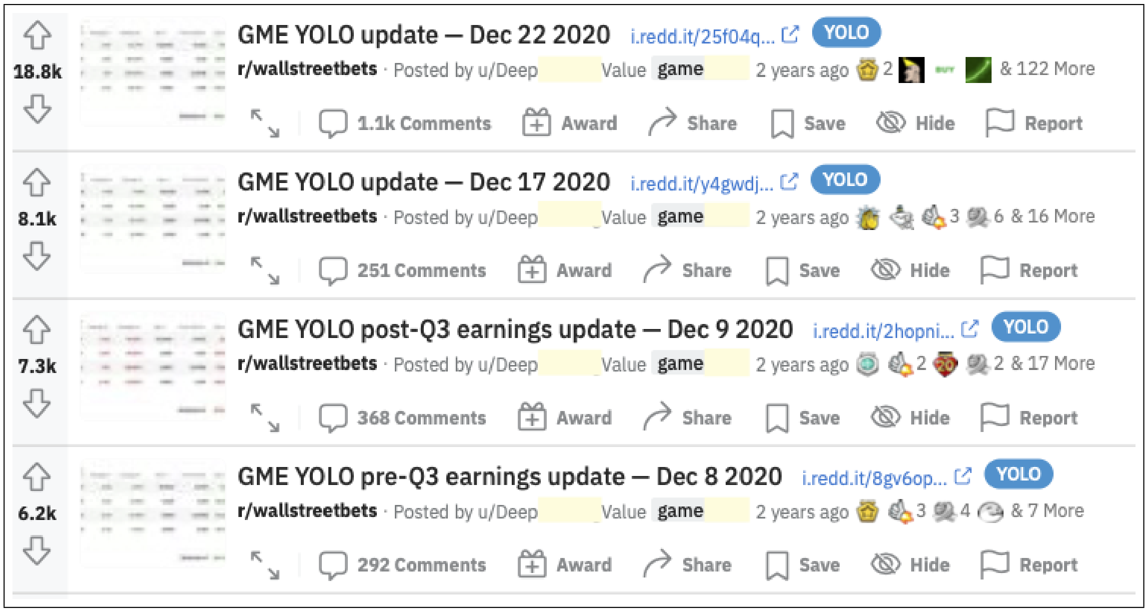


Figure 19. Example of Deep*****Value's posts.**

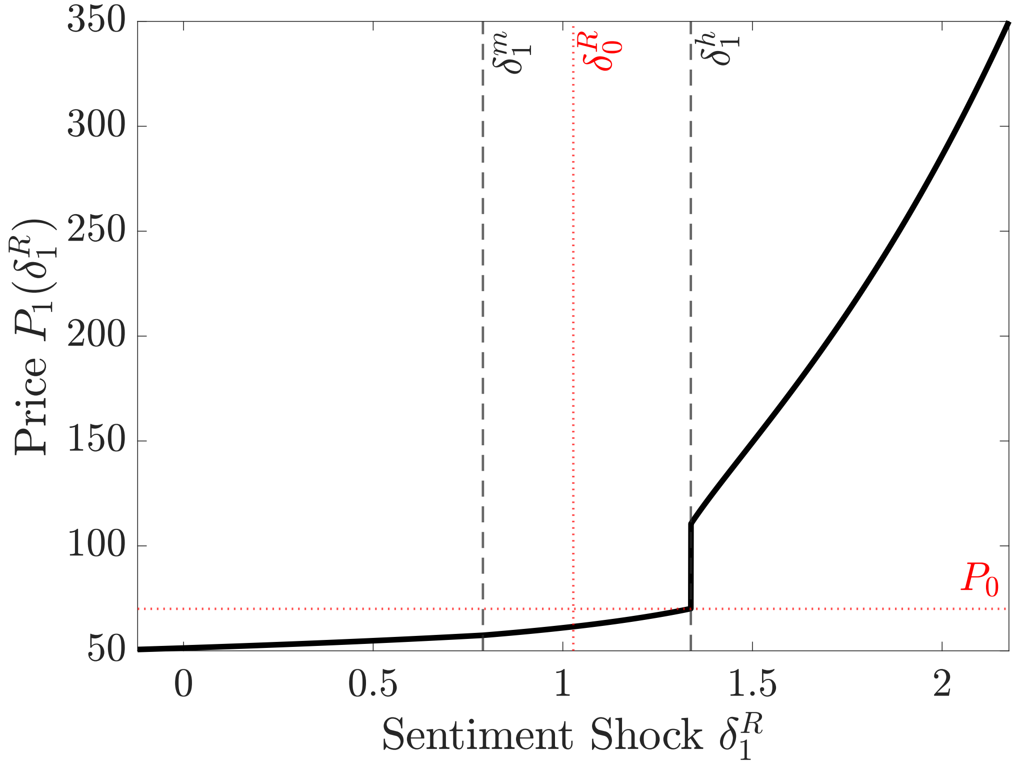


Figure 20. Time-1 price impact. This figure plots the time-1 price $P_1\left(\delta_1^R\right)$ as a function of the retail sentiment shock δ_1^R.

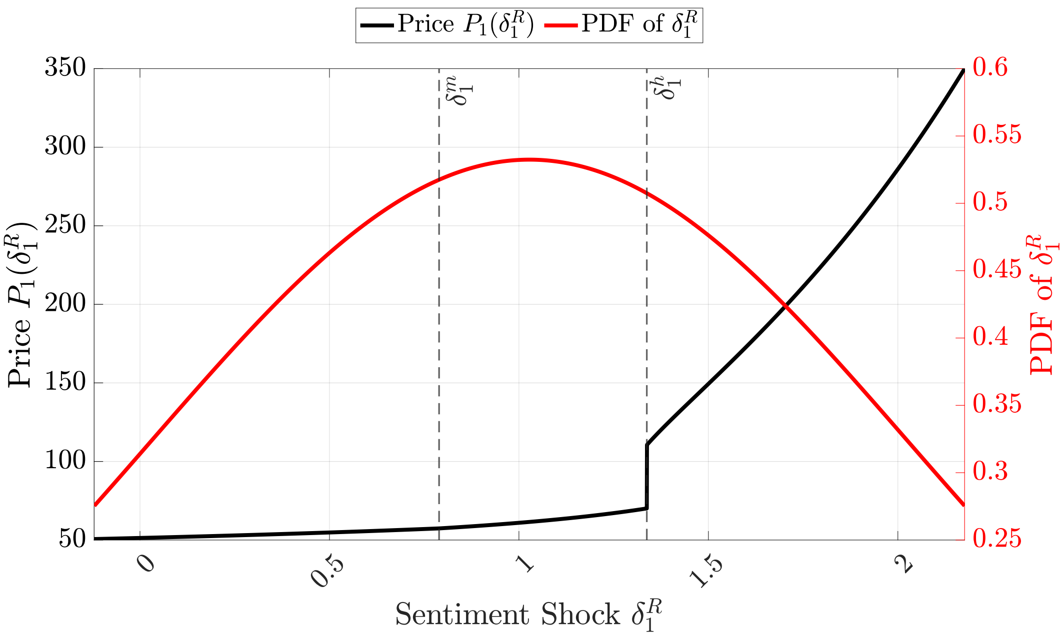
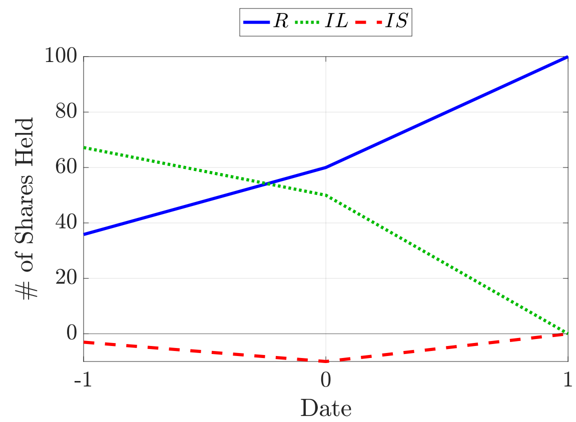
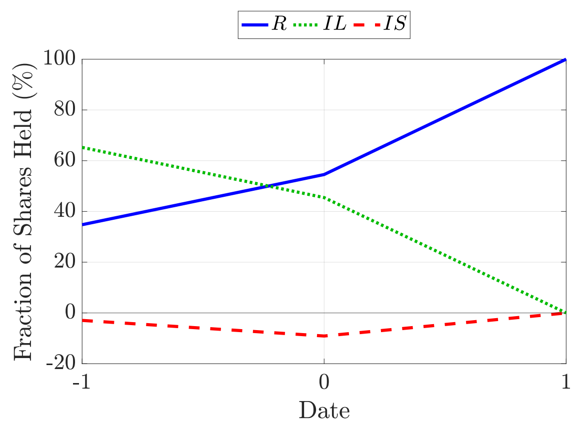


Figure 21. Time-1 price impact and shock distribution. This figure plots the time-1 price $P_1\left(\delta_1^R\right)$ (solid black line), and the PDF of the sentiment shock distribution (solid red line).

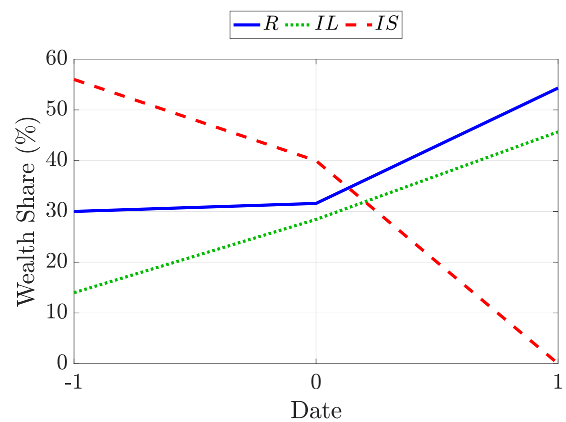
(a) Price and short interest



(b) Holdings (shares)

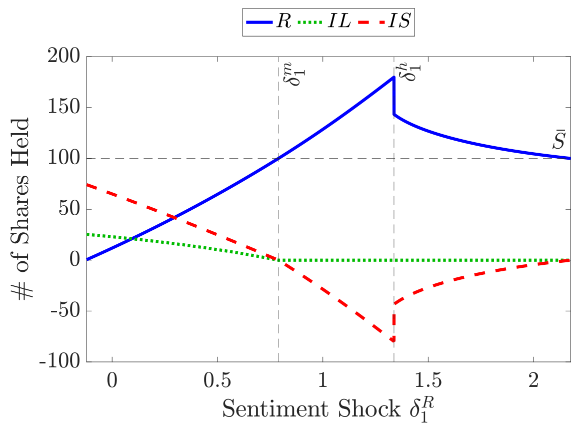


(c) Holdings (%)

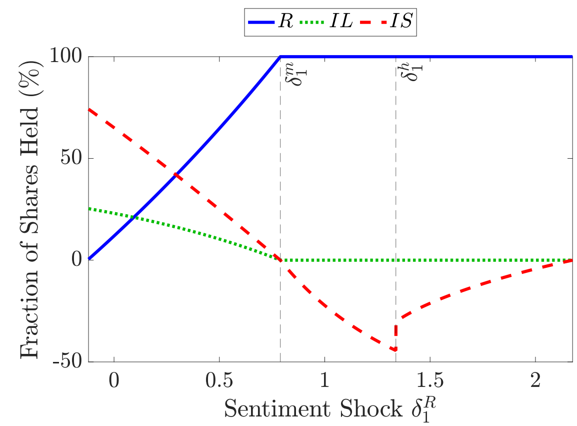


(d) Wealth shares

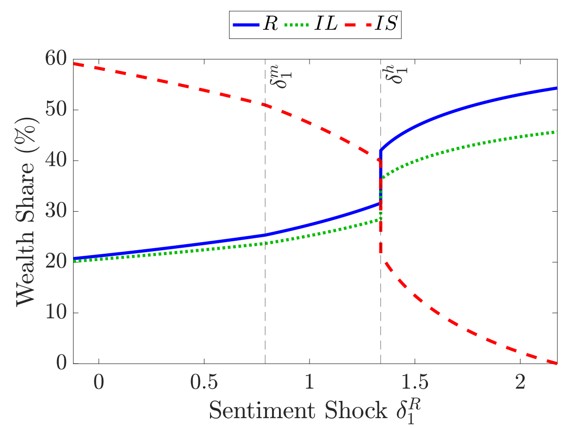
Figure 22. Time series predictions from the model.



(a) Holdings (shares)



(b) Holdings (%)



(c) Wealth shares

Figure 23. Time-1 holdings and wealth shares as functions of sentiment shock.

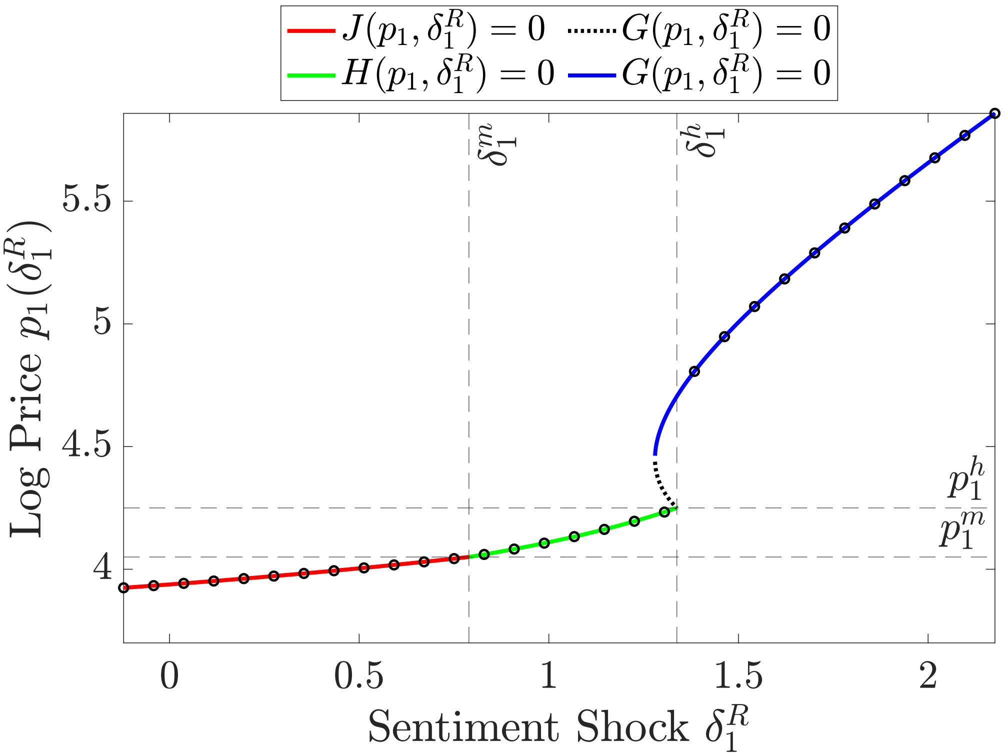


Figure 24. Multiple equilibria. This figure plots all the time-1 equilibria.

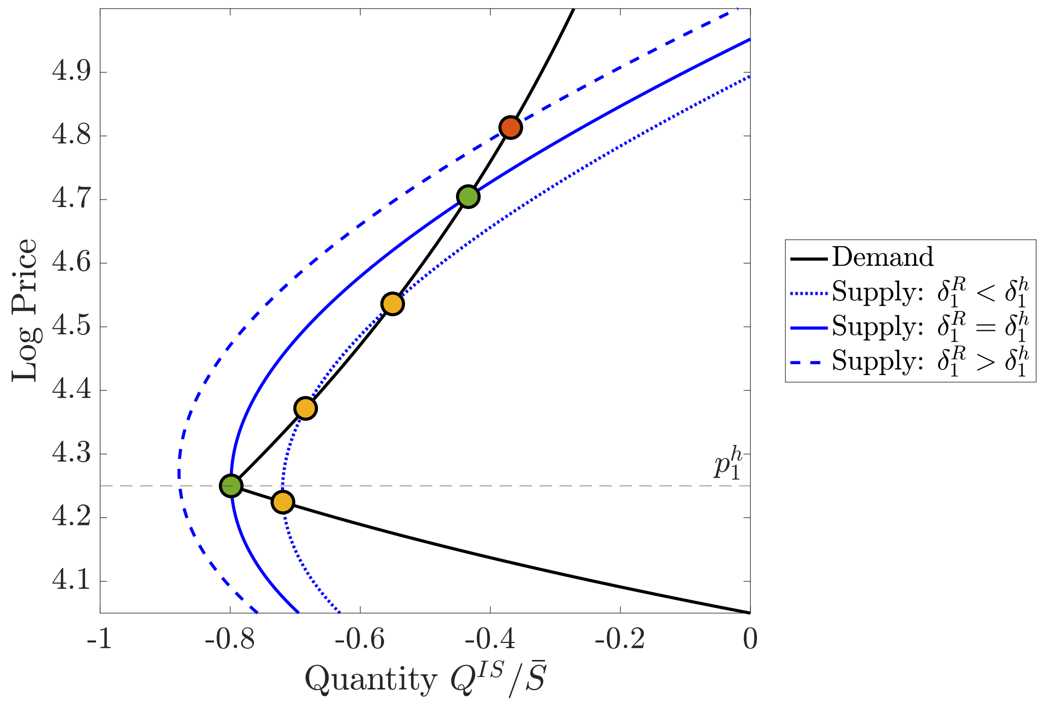


Figure 25. Demand and supply from short institution's perspective.

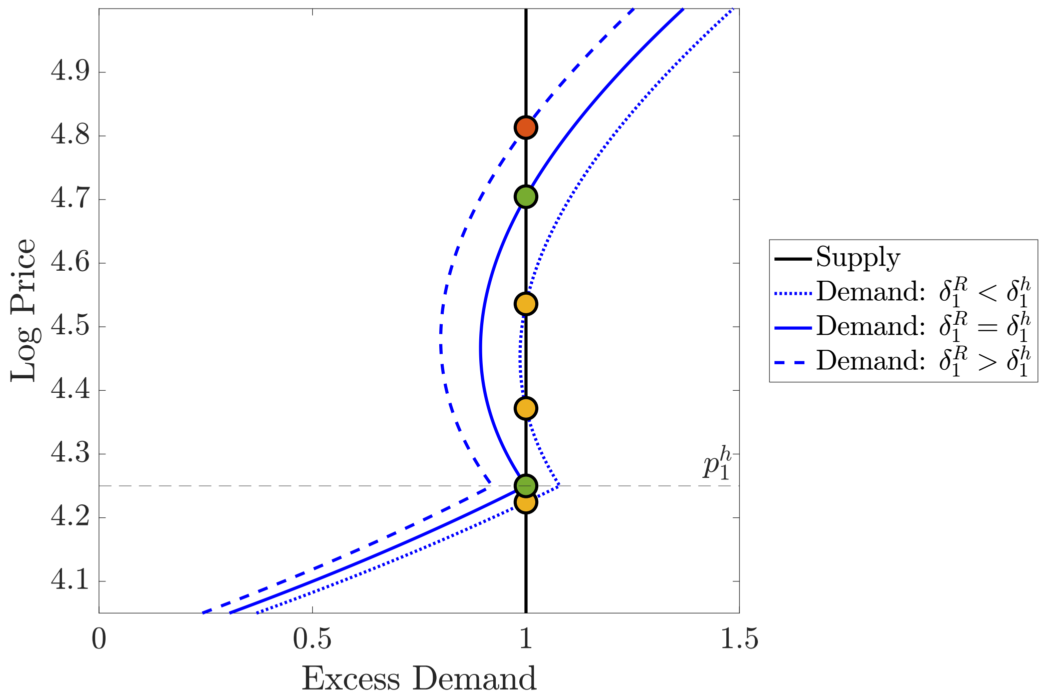


Figure 26. Excess demand from short institution's perspective.

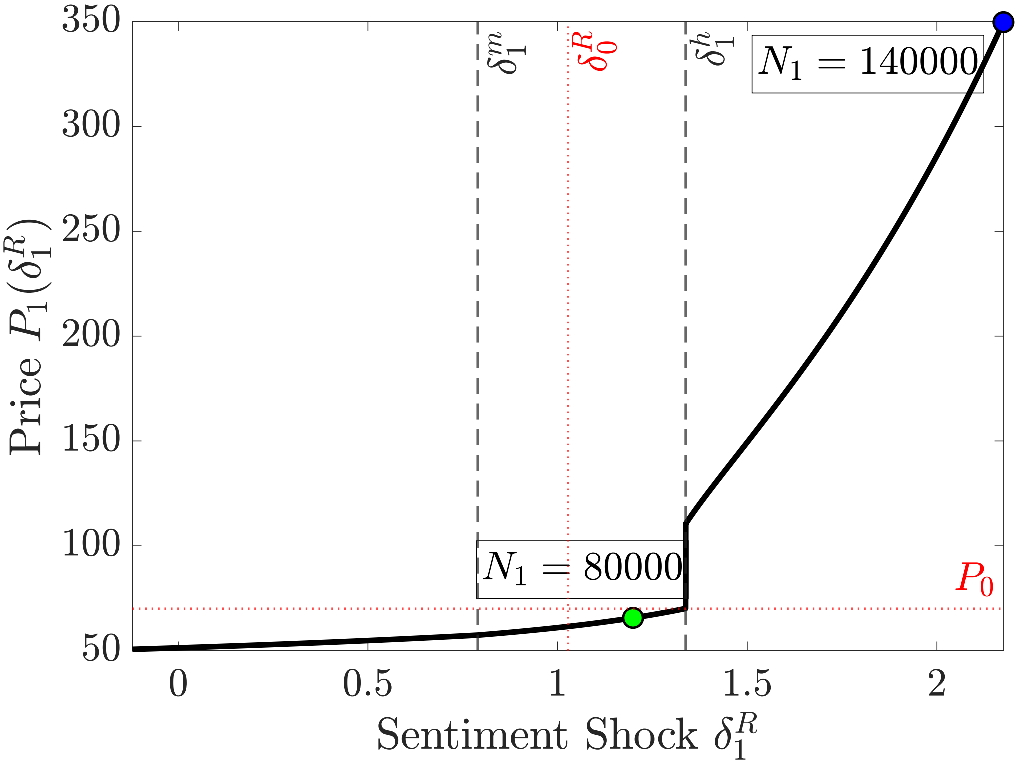


Figure 27. Time-1 realized price versus counterfactual price.

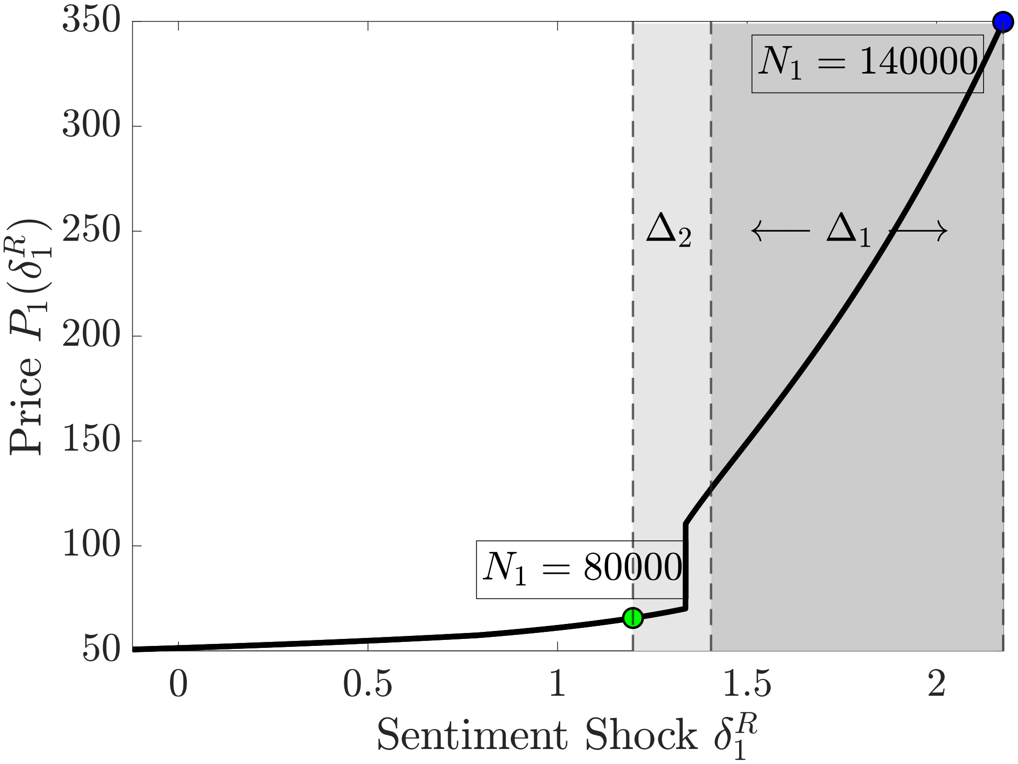


Figure 28. Decomposition of the gap between time-1 realized sentiment shock and counterfactual sentiment shock.

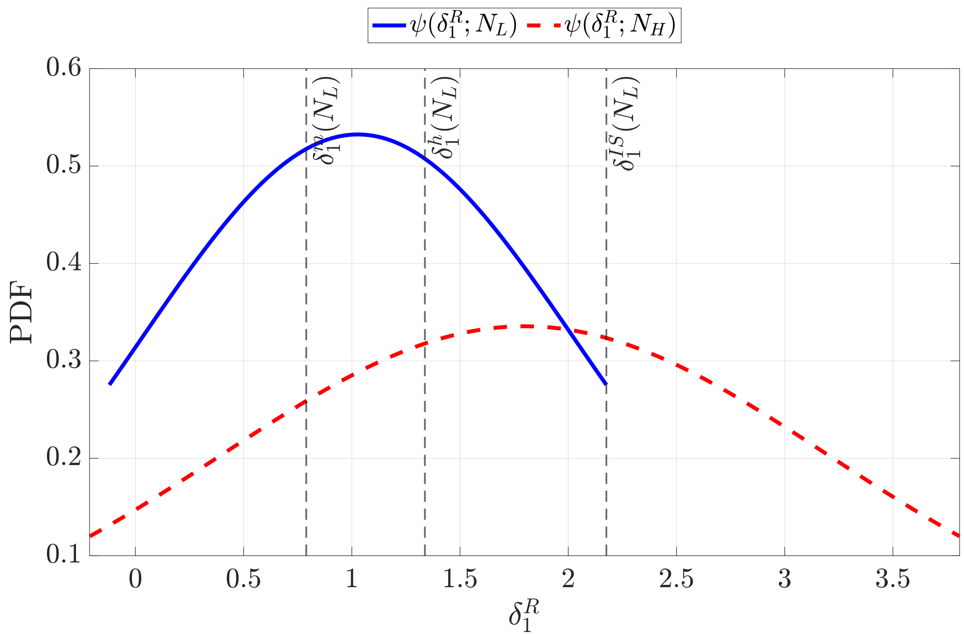
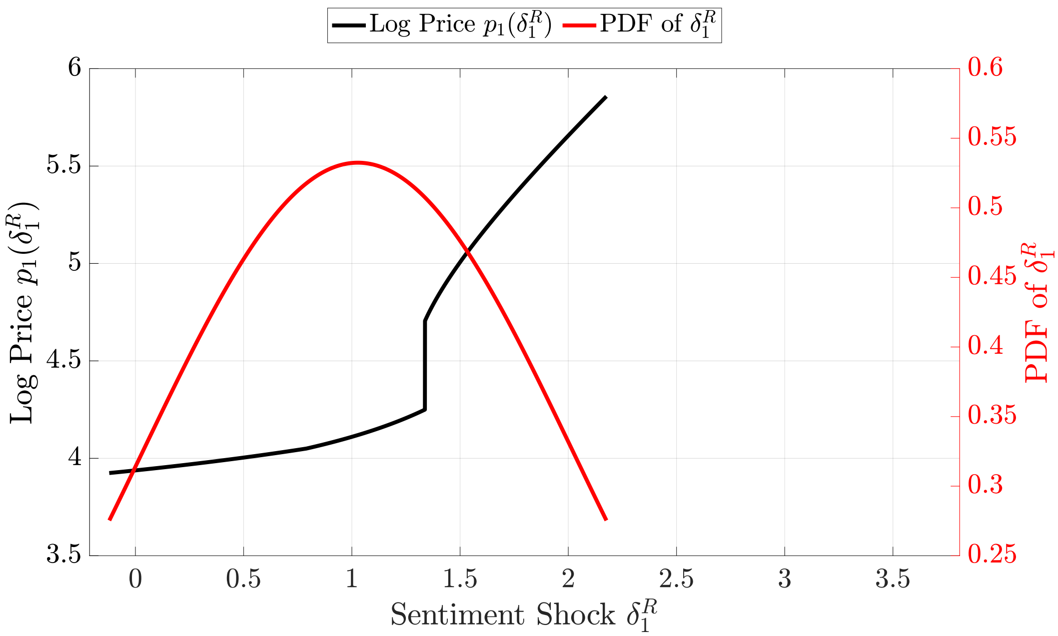
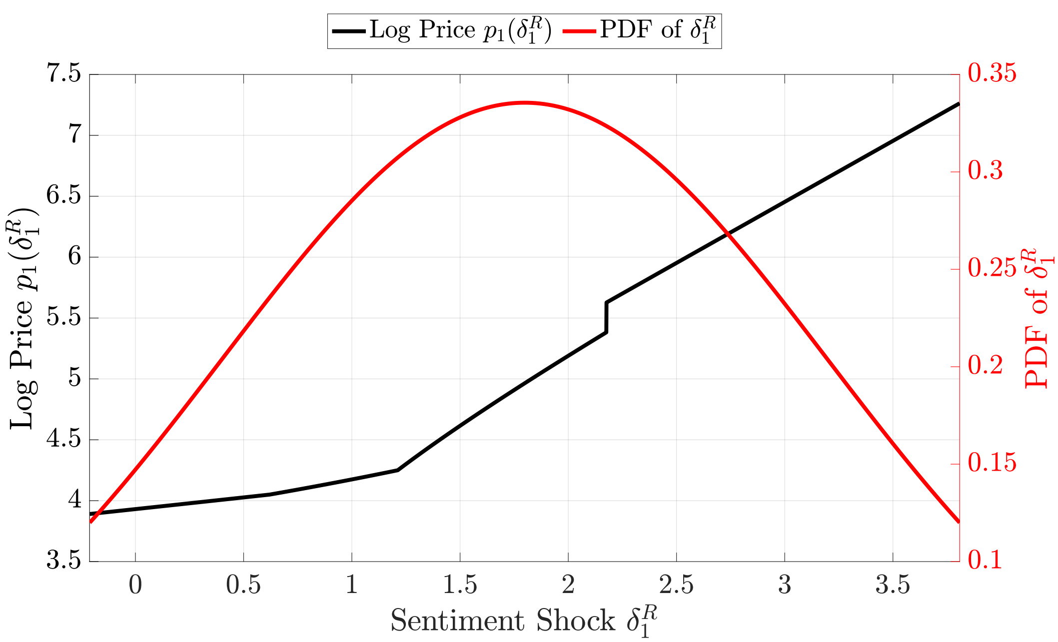


Figure 29. Time-1 retail sentiment risk distribution under different network sizes.



(a) Time-1 (log) price and risk distribution under risk perception $\tilde{N}_1 = N_L$



(b) Time-1 (log) price and risk distribution under risk perception $\tilde{N}_1 = N_H$

Figure 30. Time-1 (log) price and risk distribution under different risk perceptions at time 0.

Table 4
Modified VADER Lexicon

This table shows the modification to the VADER lexicon.

Positive			Negative		
Word	Emoji	Score	Word	Emoji	Score
rocket	🚀	4.0	bear(s)		-2.0
moon(ing)		4.0	paper		-4.0
diamond		4.0			
gem (stone)	💎	4.0			
hold(ing)		4.0			
tendie(s)		4.0			
yolo		4.0			
retard(s-ed)		2.0			
autist(s)		2.0			
degenerate(s)		2.0			
ape(s)		2.0			
gorilla(s)	🦍	2.0			

Table 5
Top Institutions that Long GME in 2020 Q4

This table shows the top 2 institutions that held GameStop in 2020 Q4 within each category.

Hedge Funds	Maverick Capital Ltd. Senvest Management LLC
Brokers	Goldman Sachs & Co. LLC Morgan Stanley & Co. LLC
Private Banking	Aperio Group LLC Permit Capital LLC (Private Equity)
Investment Advisors	Fidelity Management & Research Co. LLC BlackRock Fund Advisors
Long-Term Investors	The Public Sector Pension Investment Board The California Public Employees Retirement System

Internet Appendix for “Retail Trading and Asset Prices: The Role of Changing Social Dynamics”

Fulin Li*

October 19, 2022

Contents

*The University of Chicago, fli3@chicagobooth.edu.

A1 Omitted derivations and proofs

A1.1 Dynamics of wealth shares

Since the risk-free asset is in zero net supply, the time-t aggregate wealth is equal to the market value of the risky asset, $P_t\bar{S}$.

Investor i's wealth share at time $t+1$ is thus

$$\begin{aligned}
\alpha_{t+1}^i &\equiv \frac{A_{t+1}^i}{P_{t+1}\bar{S}} \\
&= \frac{A_t^i\left(w_t^i\frac{P_{t+1}}{P_t}+1-w_t^i\right)}{P_{t+1}\bar{S}} \\
&= \frac{\alpha_t^i P_t\bar{S}\left(w_t^i\frac{P_{t+1}}{P_t}+1-w_t^i\right)}{P_{t+1}\bar{S}} \\
&= \alpha_t^i\left(\left(1-w_t^i\right)\exp\left(p_t-p_{t+1}\right)+w_t^i\right),
\end{aligned}$$

where the second line uses the budget constraint (18) together with the assumption of constant risk-free rate $R_{f,t}=1$.

A1.2 Market clearing

Market clearing for the risk-free asset holds if and only if the aggregate wealth is equal to the market value of the risky asset, i.e.,

$$\sum_i A_t^i = P_t\bar{S}.$$

Market clearing condition for the risky asset is

$$\sum_i Q_t^i = \bar{S} \iff \sum_i \frac{w_t^i A_t^i}{P_t} = \bar{S} \iff \sum_i w_t^i A_t^i = P_t \bar{S}.$$

Hence, the two market clearing conditions reduce to

$$\sum_i A_t^i = \sum_i w_t^i A_t^i = P_t \bar{S}.$$

This is equivalent to the following condition

$$\sum_i \alpha_t^i w_t^i = 1, \alpha_t^i = \frac{A_t^i}{P_t \bar{S}}. \tag{A1}$$

From equation (A1), we can solve for the equilibrium price.

A1.3 Optimal portfolio choice

A1.3.1 Retail investors

Retail investor j solves the following problem

$$\begin{aligned} U_t^j\left(A_t^j\right) &= \max_{w_t^j} w_t^j\left(\mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t}\right) + \frac{1}{2} w_t^j\left(1 - w_t^j\right) \mathrm{Var}_t^j\left(r_{t+1}\right) \\ &\quad + \frac{1}{2}\left(1 - \gamma^R\right)\left(w_t^j\right)^2 \mathrm{Var}_t^i\left(r_{t+1}\right). \end{aligned}$$

The F.O.C. is

$$\begin{aligned} & \mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^j\left(r_{t+1}\right) - \gamma^R w_t^j \mathrm{Var}_t^j\left(r_{t+1}\right) = 0 \\ \Longrightarrow \quad & w_t^j = \frac{\mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^j\left(r_{t+1}\right)}{\gamma^R \mathrm{Var}_t^j\left(r_{t+1}\right)} = \tau^R \frac{\mathbb{E}_t^j\left[r_{t+1}\right] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^j\left(r_{t+1}\right)}{\mathrm{Var}_t^j\left(r_{t+1}\right)}. \end{aligned}$$

Substitute retail investors' subjective beliefs into the above expression, we get their demands for the risky asset.

- For a type-1 retail investor j, his time-0 and time-1 demands for the risky asset are

$$w_0^j = \tau^R \left(\frac{\mathbb{E}_0[p_1] + y_0^j - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{A2}$$

$$w_1^j = \tau^R \left(\frac{\mu_d + y_1^j - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{A3}$$

- For a type-2 retail investor j', his time-0 and time-1 demands for the risky asset are

$$w_0^{j'} = \tau^R \left(\frac{\mathbb{E}_0[p_1] - p_0}{\sigma_0^2} + \frac{1}{2} \right), \tag{A4}$$

$$w_1^{j'} = \tau^R \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right). \tag{A5}$$

A1.3.2 Long institution

The long institution IL solves the following problem

$$\begin{aligned} U_t^{IL}\left(A_t^{IL}\right) &= \max_{w_t^{IL}} w_t^{IL}\left(\mathbb{E}_t^{IL}[r_{t+1}] - r_{f,t}\right) + \frac{1}{2} w_t^{IL}\left(1 - w_t^{IL}\right)\mathrm{Var}_t^{IL}(r_{t+1}) \\ &\quad + \frac{1}{2}\left(1-\gamma^I\right)\left(w_t^{IL}\right)^2 \mathrm{Var}_t^{IL}(r_{t+1}) \\ \text{s.t.} &\quad w_t^{IL} \geq 0. \end{aligned}$$

Since the objective function is quadratic in portfolio weight w_t^{IL} and has a global maximum, the solution to this constrained problem is

$$w_t^{IL} = \max\left\{0, \tau^I \frac{\mathbb{E}_t^{IL}[r_{t+1}] - r_{f,t} + \frac{1}{2}\mathrm{Var}_t^{IL}(r_{t+1})}{\mathrm{Var}_t^{IL}(r_{t+1})}\right\}.$$

Substitute IL's beliefs (equations (11), (12), and (14)) into the above expression, we get his time-0 and time-1 demands for the risky asset

$$\begin{aligned} w_0^{IL} &= \max\left\{0, \tau^I \left(\frac{\mathbb{E}_0[p_1] + \delta_0^{IL} - p_0}{\sigma_0^2} + \frac{1}{2} \right)\right\}, \\ w_1^{IL} &= \max\left\{0, \tau^I \left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2} \right)\right\}. \end{aligned}$$

A1.3.3 Short institution

The short institution IS solves the following problem

$$\begin{aligned} U_t^{IS}\left(A_t^{IS}\right) &= \max_{w_t^{IS}} w_t^{IS}\left(\mathbb{E}_t^{IS}\left[r_{t+1}\right]-r_{f,t}\right)+\frac{1}{2}w_t^{IS}\left(1-w_t^{IS}\right)\mathrm{Var}_t^{IS}\left(r_{t+1}\right) \\ &\quad +\frac{1}{2}\left(1-\gamma^I\right)\left(w_t^{IS}\right)^2\mathrm{Var}_t^{IS}\left(r_{t+1}\right) \\ &\text{s.t.}\quad w_t^{IS}\geq-\frac{1}{m}. \end{aligned}$$

The solution is

$$w_t^{IS} = \max\left\{-\frac{1}{m},\tau^I\frac{\mathbb{E}_t^{IS}\left[r_{t+1}\right]-r_{f,t}+\frac{1}{2}\mathrm{Var}_t^{IS}\left(r_{t+1}\right)}{\mathrm{Var}_t^{IS}\left(r_{t+1}\right)}\right\}.$$

Substitute IS's beliefs (equations (11), (13), and (14)) into the above expression, we get his time-0 and time-1 demands for the risky asset

$$\begin{aligned} w_0^{IS} &= \max\left\{-\frac{1}{m},\tau^I\left(\frac{\mathbb{E}_0\left[p_1\right]+\delta_0^{IS}-p_0}{\sigma_0^2}+\frac{1}{2}\right)\right\}, \\ w_1^{IS} &= \max\left\{-\frac{1}{m},\tau^I\left(\frac{\mu_d-p_1}{\sigma_d^2}+\frac{1}{2}\right)\right\}. \end{aligned}$$

A1.4 Proof of Lemma 1

Proof. I prove the existence result in two steps. First, I show that the aggregate demand of the $\bar{N}$ retail investors is equal to the demand of the aggregate retail investor specified in equations (35) and (36), and thus the equilibrium price can be solved from the market clearing condition (38). Then I derive the wealth share dynamics of the aggregate retail investor in equation (37).

I begin by restating the timeline and the wealth share dynamics of individual retail investors. At time $t-1$ after trading, retail investor j has dollar wealth A_t^j and wealth share α_t^j. At time t before trading, the $\bar{N}$ retail investors first split their aggregate wealth $\sum_{k=1}^{\bar{N}} A_t^k$ equally. In particular, they split their aggregate stock position and aggregate bond position equally. After that, retail investor j has wealth $\hat{A}_t^j=\frac{1}{\bar{N}}\sum_{k=1}^{\bar{N}} A_t^k$ and wealth share

$$\hat{\alpha}_t^j \equiv \frac{\hat{A}_t^j}{A_t}=\frac{\frac{1}{\bar{N}}\sum_{k=1}^{\bar{N}} A_t^k}{A_t}=\frac{1}{\bar{N}}\sum_{k=1}^{\bar{N}}\alpha_t^k. \tag{A6}$$

Retail investors then trade with each other. Specifically, retail investor j allocates his wealth

$\hat{A}_t^j$ into the risky asset and the risk-free asset. His demand for the risky asset (in terms of the number of shares) is $Q_t^j = \frac{w_t^j \hat{A}_t^j}{P_t}$, where w_t^j is his optimal portfolio weight. After trading, his end-of-period wealth share becomes

$$\alpha_{t+1}^j = \hat{\alpha}_t^j \left(\left(1 - w_t^j\right) \exp\left(p_t - p_{t+1}\right) + w_t^j \right). \tag{A7}$$

Next, I show that the equilibrium price of the risky asset is the same as that in an economy with three investors – an aggregate retail investor, the long institution, and the short institution. And the demand of the aggregate retail investor is the sum of the demand of the $\bar{N}$ retail investors.

At time t, market clearing for the risky asset implies that

$$\begin{aligned}
& \sum_{j=1}^{\bar{N}} Q_t^j + Q_t^{IL} + Q_t^{IS} = \bar{S} \\
\Longrightarrow \quad & \sum_{j=1}^{\bar{N}} \frac{w_t^j \hat{A}_t^j}{P_t} + \frac{w_t^{IL} A_t^{IL}}{P_t} + \frac{w_t^{IS} A_t^{IS}}{P_t} = \bar{S} \\
\Longrightarrow \quad & \sum_{j=1}^{\bar{N}} w_t^j \hat{\alpha}_t^j + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \sum_{j=1}^{\bar{N}} w_t^j \left(\frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} \alpha_t^k \right) + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \frac{1}{\bar{N}} \sum_{j=1}^{N_t} \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1}\right] + y_t^j - p_t}{\sigma_t^2} + \frac{1}{2} \right) \\
& + \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \frac{1}{\bar{N}} \sum_{j=N_t+1}^{\bar{N}} \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1}\right] - p_t}{\sigma_t^2} + \frac{1}{2} \right) \\
& + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1}\right] - p_t}{\sigma_t^2} + \frac{1}{2} \right) + \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} \tau^R \frac{y_t^j}{\sigma_t^2} \\
& + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1 \\
\Longrightarrow \quad & \left(\sum_{k=1}^{\bar{N}} \alpha_t^k \right) \tau^R \left(\frac{\mathbb{E}_t \left[p_{t+1}\right] + \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} y_t^j - p_t}{\sigma_t^2} + \frac{1}{2} \right) + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1,
\end{aligned}$$

where the fourth line uses the definition of $\hat{\alpha}_t^j$ in equation (A6), and the fifth line uses the optimal portfolio weights of retail investors in equations (25)-(28).

Define

$$A_t^R \equiv \sum_{j=1}^{\bar{N}} A_t^j, \alpha_t^R \equiv \sum_{j=1}^{\bar{N}} \alpha_t^j, \tag{A8}$$

$$\delta_t^R \equiv \theta\left(N_t\right) \frac{1}{N_t} \sum_{j=1}^{N_t} y_t^j, \tag{A9}$$

$$w_t^R \equiv \tau^R \left(\frac{\mathbb{E}_t\left[p_{t+1}\right] + \delta_t^R - p_t}{\sigma_t^2} + \frac{1}{2} \right) = \frac{1}{\bar{N}} \sum_{j=1}^{\bar{N}} w_t^j. \tag{A10}$$

Then the market clearing condition can be written as

$$w_t^R \alpha_t^R + w_t^{IL} \alpha_t^{IL} + w_t^{IS} \alpha_t^{IS} = 1,$$

with $\alpha_t^R + \alpha_t^{IL} + \alpha_t^{IS} = \sum_{j=1}^{\bar{N}} \alpha_t^j + \alpha_t^{IL} + \alpha_t^{IS} = 1$.

Hence, the equilibrium price of the risky asset is the same as that in an economy with three investors – an aggregate retail investor R, the long institution IL, and the short institution IS, where the three investors have demand $\left(w_t^R, w_t^{IL}, w_t^{IS}\right)$, and wealth shares $\left(\alpha_t^R, \alpha_t^{IL}, \alpha_t^{IS}\right)$. In other words, there exists an aggregate retail investor whose demand for the risky asset is given by equation (A10). The aggregate retail investor has constant relative risk tolerance $\tau^R = \frac{1}{\gamma^R}$ and subjective beliefs

$$\begin{aligned} \mathbb{E}_0^R\left[p_1\right] &= \mathbb{E}_0\left[p_1\right] + \delta_0^R, \mathrm{Var}_0^R\left(p_1\right) = \sigma_0^2, \\ \mathbb{E}_1^R\left[\tilde{d}\right] &= \mu_d + \delta_1^R, \mathrm{Var}_1^R\left(\tilde{d}\right) = \sigma_d^2. \end{aligned}$$

Finally, I derive the wealth share dynamics of the aggregate retail investor. From the definition of α_{t+1}^R in (A8),

$$\begin{aligned}
\alpha_{t+1}^R &\equiv \sum_{j=1}^{\bar{N}} \alpha_{t+1}^j \\
&= \sum_{j=1}^{\bar{N}} \hat{\alpha}_t^j \left(\left(1-w_t^j\right)\exp\left(p_t - p_{t+1}\right) + w_t^j\right) \\
&= \left(\frac{1}{\bar{N}}\sum_{k=1}^{\bar{N}} \alpha_t^k\right) \sum_{j=1}^{\bar{N}} \left(\left(1-w_t^j\right)\exp\left(p_t - p_{t+1}\right) + w_t^j\right) \\
&= \alpha_t^R \left(\left(1 - \frac{1}{\bar{N}}\sum_{j=1}^{\bar{N}} w_t^j\right)\exp\left(p_t - p_{t+1}\right) + \frac{1}{\bar{N}}\sum_{j=1}^{\bar{N}} w_t^j\right) \\
&= \alpha_t^R \left(\left(1-w_t^R\right)\exp\left(p_t - p_{t+1}\right) + w_t^R\right),
\end{aligned}$$

where the second line uses investor j's wealth share dynamics in equation (A7), and the last line uses the aggregate retail investor's demand in equation (A10). $\square$

A1.5 Proof of Proposition 1

Proof. I focus on monotone equilibrium of Definition 1, with sentiment cutoffs δ_1^m, δ_1^h satisfying $\underline{\delta}_1 < \delta_1^m < \delta_1^h < \bar{\delta}_1$. Hence, $\forall \delta_1^R \in [\underline{\delta}_1, \delta_1^m)$, the equilibrium price $p_1\left(\delta_1^R\right) < p_1^m$. Similarly, $\forall \delta_1^R \in \left[\delta_1^m, \delta_1^h\right)$, the price $p_1\left(\delta_1^R\right) \in \left[p_1^m, p_1^h\right)$. And $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the price $p_1\left(\delta_1^R\right) \geq p_1^h$.

Next, I solve the equilibrium price from the market clearing condition in equation (38).

- For $\delta_1^R \in [\underline{\delta}_1, \delta_1^m)$, I look for an equilibrium price $p_1 < p_1^m$. Substitute the optimal portfolio choices of the three investors, (36), (30), and (32) into the market clearing condition (38), I get

$$\begin{aligned}
& \frac{\alpha_1^R\left(p_1\right)\tau^R}{\sigma_d^2}\delta_1^R + \sum_i \alpha_1^i\left(p_1\right)\tau^i\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad & p_1 = \mu_d + \left(\frac{\frac{\alpha_1^R(p_1)\tau^R}{\sigma_d^2}\delta_1^R - 1}{\sum_i \alpha_1^i\left(p_1\right)\tau^i} + \frac{1}{2}\right)\sigma_d^2 \\
\Longrightarrow \quad & p_1 = \mu_d + \left(\frac{1}{2} + \frac{\frac{\alpha_1^R(p_1)\tau^R}{\sigma_d^2}\delta_1^R - 1}{\tau_1\left(p_1\right)}\right)\sigma_d^2
\end{aligned}$$

where

$$\tau_1(p_1) \equiv \sum_i \alpha_1^i(p_1)\,\tau^i = \alpha_1^R(p_1)\,\tau^R + \left(1-\alpha_1^R(p_1)\right)\tau^I$$

Define the function

$$J\left(p_1,\delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R(p_1)\,\tau^R\delta_1^R - \sigma_d^2}{\tau_1(p_1)}\right) - p_1$$

Then the equilibrium price p_1 solves $J\left(p_1,\delta_1^R\right)=0$.

The cutoff sentiment shock δ_1^m solves $J\left(p_1^m,\delta_1^m\right)=0$, which yields

$$\delta_1^m = \frac{\left(p_1^m - \mu_d - \frac{1}{2}\sigma_d^2\right)\tau_1(p_1^m) + \sigma_d^2}{\alpha_1^R(p_1^m)\,\tau^R} = \frac{\sigma_d^2}{\alpha_1^R(p_1^m)\,\tau^R}$$

- For $\delta_1^R \in \left[\delta_1^m,\delta_1^h\right)$, I look for an equilibrium price $p_1 \in \left[p_1^m,p_1^h\right)$. Substitute the optimal portfolio choices of the three investors, (36), (30), and (32) into the market clearing condition (38), I get

$$\begin{aligned}
& \alpha_1^R(p_1)\,\tau^R\left(\frac{\mu_d+\delta_1^R-p_1}{\sigma_d^2}+\frac{1}{2}\right) + \alpha_1^{IS}(p_1)\,\tau^I\left(\frac{\mu_d-p_1}{\sigma_d^2}+\frac{1}{2}\right) = 1 \\
\Longrightarrow\quad & \frac{\alpha_1^R(p_1)\,\tau^R}{\sigma_d^2}\delta_1^R + \left(\alpha_1^R(p_1)\,\tau^R + \alpha_1^{IS}(p_1)\,\tau^I\right)\left(\frac{\mu_d-p_1}{\sigma_d^2}+\frac{1}{2}\right) = 1 \\
\Longrightarrow\quad & p_1 = \mu_d + \left(\frac{1}{2} + \frac{\frac{1}{\sigma_d^2}\alpha_1^R(p_1)\,\tau^R\delta_1^R - 1}{\alpha_1^R(p_1)\,\tau^R + \alpha_1^{IS}(p_1)\,\tau^I}\right)\sigma_d^2 \\
\Longrightarrow\quad & p_1 = \mu_d + \left(\frac{1}{2} + \frac{\frac{1}{\sigma_d^2}\alpha_1^R(p_1)\,\tau^R\delta_1^R - 1}{\hat{\tau}_1(p_1)}\right)\sigma_d^2
\end{aligned}$$

where

$$\hat{\tau}_1(p_1) \equiv \alpha_1^R(p_1)\,\tau^R + \alpha_1^{IS}(p_1)\,\tau^I$$

Define the function

$$H\left(p_1,\delta_1^R\right) \equiv \mu_d + \left(\frac{1}{2}\sigma_d^2 + \frac{\alpha_1^R(p_1)\,\tau^R\delta_1^R - \sigma_d^2}{\hat{\tau}_1(p_1)}\right) - p_1$$

Then the equilibrium price p_1 solves $H\left(p_1,\delta_1^R\right)=0$.

The cutoff sentiment shock δ_1^h solves $H\left(p_1^h, \delta_1^h\right) = 0$, which yields

$$\delta_1^h = \frac{\left(p_1^h - \mu_d - \frac{1}{2}\sigma_d^2\right)\hat{\tau}_1\left(p_1^h\right) + \sigma_d^2}{\alpha_1^R\left(p_1^h\right)\tau^R} = \frac{\frac{1}{m\tau^I}\hat{\tau}_1\left(p_1^h\right) + 1}{\alpha_1^R\left(p_1^h\right)\tau^R}\sigma_d^2$$

- For $\delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, I look for an equilibrium price $p_1 \geq p_1^h$. Substitute the optimal portfolio choices of the three investors, (36), (30), and (32) into the market clearing condition (38), I get

$$\alpha_1^R(p_1)\tau^R\left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right) - \alpha_1^{IS}(p_1)\frac{1}{m} = 1$$
$$\Longrightarrow \quad p_1 = \mu_d + \delta_1^R + \left(\frac{1}{2} - \frac{1 + \alpha_1^{IS}(p_1)\frac{1}{m}}{\alpha_1^R(p_1)\tau^R}\right)\sigma_d^2$$

Define the function

$$G\left(p_1, \delta_1^R\right) = \mu_d + \delta_1^R + \left(\frac{1}{2} - \frac{1 + \alpha_1^{IS}(p_1)\frac{1}{m}}{\alpha_1^R(p_1)\tau^R}\right)\sigma_d^2 - p_1$$

Then the equilibrium price p_1 solves $G\left(p_1, \delta_1^R\right) = 0$.

□

A1.6 Lemma A1 and proof

Lemma A1 (Properties of the implicit function $G\left(p_1, \delta_1^R\right)$). *Consider a monotone equilibrium of Definition 1, where the time-0 portfolios satisfy $w_0^R > 1$, $w_0^{IS} < 0$, $w_0^R > w_0^{IL} > w_0^{IS}$, and investors always have strictly positive wealth $\forall \delta_1 \in \left(\underline{\delta}_1, \bar{\delta}_1\right)$. Let p_1^R denote the price at which the retail investor's time-1 wealth is zero,*

$$p_1^R \equiv p_0 + \log\left(1 - \frac{1}{w_0^R}\right)$$

Then the implicit function $G\left(p_1, \delta_1^R\right)$ has the following properties on $p_1 \in \left(p_1^R, +\infty\right)$:

1. *$G\left(p_1, \delta_1^R\right)$ is continuous and strictly increasing in δ_1^R: $\frac{\partial G\left(p_1, \delta_1^R\right)}{\partial \delta_1} = 1 > 0$.*
2. *$G\left(p_1, \delta_1^R\right)$ is continuous and strictly concave in p_1: $\frac{\partial^2 G\left(p_1, \delta_1^R\right)}{\partial p_1^2} < 0$.*
3. *$\frac{\partial G\left(p_1, \delta_1^R\right)}{\partial p_1}$ does not depend on δ_1^R: $\frac{\partial^2 G\left(p_1, \delta_1^R\right)}{\partial p_1 \partial \delta_1^R} = 0$.*
4. *$G\left(p_1, \delta_1^R\right)$, as a function of p_1, has at most two distinct roots on $p_1 \in \left(p_1^R, +\infty\right)$.*

Proof. First, I derive p_1^R from

$$\begin{aligned}
& \alpha_1^R\left(p_1^R\right)=0 \\
\Longrightarrow \quad & 0=\alpha_0^R\left(\left(1-w_0^R\right)\exp\left(p_0-p_1^R\right)+w_0^R\right) \\
\Longrightarrow \quad & p_1^R=p_0+\log\left(1-\frac{1}{w_0^R}\right)
\end{aligned}$$

Then $\forall p_1>p_1^R$, $\alpha_1\left(p_1\right)>0$. And thus $G\left(p_1,\delta_1^R\right)$ is continuous and twice differentiable, $\forall p_1>p_1^R$, $\forall\delta_1^R$.

To show Properties 1-3, compute the following derivatives

$$\begin{aligned}
\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial\delta_1^R} &= 1 \\
\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1} &= -\left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2} \\
&\quad\cdot\left(\frac{d\alpha_1^{IS}\left(p_1\right)}{dp_1}\frac{1}{m}\alpha_1^R\left(p_1\right)\tau^R-\frac{d\alpha_1^R\left(p_1\right)}{dp_1}\tau^R\left(1+\alpha_1^{IS}\left(p_1\right)\frac{1}{m}\right)\right)\sigma_d^2-1 \\
&= \left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2}\exp\left(p_0-p_1\right) \\
&\quad\cdot\tau^R\left(\alpha_0^{IS}\left(1-w_0^{IS}\right)\frac{1}{m}\alpha_1^R\left(p_1\right)-\alpha_0^R\left(1-w_0^R\right)\left(1+\alpha_1^{IS}\left(p_1\right)\frac{1}{m}\right)\right)\sigma_d^2 \\
&\quad -1 \\
&= \left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2}\exp\left(p_0-p_1\right) \\
&\quad\cdot\alpha_0^R\tau^R\left(w_0^R-1+\frac{1}{m}\alpha_0^{IS}\left(w_0^R-w_0^{IS}\right)\right)\sigma_d^2-1 \\
\frac{\partial^2 G\left(p_1,\delta_1^R\right)}{\partial p_1\partial\delta_1^R} &= 0 \\
\frac{\partial^2 G\left(p_1,\delta_1^R\right)}{\partial p_1^2} &= -\left(\alpha_1^R\left(p_1\right)\tau^R\right)^{-2}\sigma_d^2 \\
&\quad\cdot\left(\frac{d\alpha_1^R\left(p_1\right)}{dp_1}\tau^R\left(1+\alpha_1^{IS}\left(p_1\right)\frac{1}{m}\right)-\frac{d\alpha_1^{IS}\left(p_1\right)}{dp_1}\frac{1}{m}\alpha_1^R\left(p_1\right)\tau^R\right) \\
&\quad\cdot\left(1+\frac{2}{\alpha_1^R\left(p_1\right)}\frac{d\alpha_1^R\left(p_1\right)}{dp_1}\right)
\end{aligned}$$

From the wealth share dynamics, we get

$$\begin{aligned}
& \alpha_{t+1}^i\left(p_{t+1}\right)=\alpha_t^i\left(\left(1-w_t^i\right)\left(p_t-p_{t+1}\right)+w_t^i\right) \\
\Longrightarrow \quad & \frac{d\alpha_{t+1}^i\left(p_{t+1}\right)}{dp_{t+1}}=-\alpha_t^i\left(1-w_t^i\right)\exp\left(p_t-p_{t+1}\right)
\end{aligned}$$

Since $w_0^R > 1$ and $w_0^{IS} < 0$, we have

$$\frac{d\alpha_1^R(p_1)}{dp_1} > 0, \frac{d\alpha_1^{IS}(p_1)}{dp_1} < 0$$

Hence, $\frac{\partial^2 G\left(p_1,\delta_1^R\right)}{\partial p_1^2} < 0$, i.e. $G\left(p_1, \delta_1^R\right)$ is strictly concave in p_1, $\forall p_1 \in \left(\delta_1^R, +\infty\right)$.

Next, I show property 4. For a given δ_1^R, suppose $G\left(p_1, \delta_1^R\right)$ has more than two roots. Let x_1, x_2, x_3 denote three of the roots, with $x_1 < x_2 < x_3$. Then $\exists \lambda \in (0,1)$, such that $x_2 = \lambda x_1 + (1-\lambda)x_3$. Since $G\left(p_1, \delta_1^R\right)$ is continuous and strictly concave in p_1,

$$0 = \lambda G\left(x_1, \delta_1^R\right) + (1-\lambda) G\left(x_3, \delta_1^R\right) = G\left(\lambda x_1 + (1-\lambda)x_3, \delta_1^R\right) < G\left(x_2, \delta_1^R\right) = 0$$

A contradiction. Hence, $\forall p_1 \in \left(p_1^R, +\infty\right)$, $G\left(p_1, \delta_1^R\right)$ (as a function of p_1) has at most two distinct roots. $\square$

A1.7 Proof of Proposition 2

Proof. I first show that $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, $G\left(p_1, \delta_1^R\right) = 0$ has exactly one root that satisfies $p_1 > p_1^h$. Suppose otherwise, then from Lemma A1, there are two roots x_1 and x_2 which satisfy $p_1^h < x_1 < x_2$, and $G\left(x_1, \delta_1^R\right) = G\left(x_2, \delta_1^R\right) = 0$. Since $G\left(p_1^h, \delta_1^h\right) = 0$ and $\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial \delta_1^R} = 1 > 0$, then $G\left(p_1^h, \delta_1^R\right) > G\left(p_1^h, \delta_1^h\right) = 0$, $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$. $p_1^h < x_1 < x_2 \to \exists \lambda \in (0,1)$ such that $x_1 = \lambda p_1^h + (1-\lambda)x_2$. And since $G\left(p_1, \delta_1^R\right)$ is strictly concave in p_1, we have

$$0 < \lambda G\left(p_1^h, \delta_1^R\right) + (1-\lambda) G\left(x_2, \delta_1^R\right) < G\left(\lambda p_1^h + (1-\lambda)x_2, \delta_1^R\right) = G\left(x_1, \delta_1^R\right) = 0$$

A contradiction. Hence, $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, $G\left(p_1, \delta_1^R\right)$ has exactly one root that satisfies $p_1 > p_1^h$. In a monotone equilibrium of Definition 1, this is the unique equilibrium price in the high sentiment region $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$.

Next, I derive conditions for discontinuity in price. Consider the following two cases:

- Case 1: $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0$.

  From the strict concavity of $G\left(p_1, \delta_1^R\right)$ in Lemma A1, $\forall p_1 > p_1^h$, $\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1} < \left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0$. This implies that $G\left(p_1, \delta_1^h\right) < G\left(p_1^h, \delta_1^h\right) = 0, \forall p_1 > p_1^h$. Hence, p_1^h is the largest root of $G\left(p_1, \delta_1^h\right) = 0$.

From Lemma A1, $\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1 \partial \delta_1^R} = 0$ and $\frac{\partial^2 G\left(p_1,\delta_1^R\right)}{\partial p_1^2} < 0$. Then

$$\begin{aligned}
& \left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0 \\
\Longrightarrow \quad & \left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=p_1^h} \leq 0, \forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right] \\
\Longrightarrow \quad & \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1} < 0, \forall p_1 > p_1^h, \forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]
\end{aligned}$$

Moreover, if $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} = 0$, then $\left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=p_1^h} = 0$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$. Otherwise, $\left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=p_1^h} < 0$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$.

Using the implicit function theorem, $\forall p_1 > p_1^h$, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$,

$$\begin{aligned}
& \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} + \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial \delta_1^R} = 0 \\
\Longrightarrow \quad & \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} + 1 = 0 \\
\Longrightarrow \quad & \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R} = -\frac{1}{\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}} > 0
\end{aligned}$$

Hence, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the equilibrium price $p_1\left(\delta_1^R\right)$ is strictly increasing in δ_1^R. Furthermore, $p_1\left(\delta_1^R\right)$ is continuous in δ_1^R on $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, and is right-continuous at $\delta_1^R = \delta_1^h$.

- Case 2: $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h} > 0$.

  First, I prove that $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, $G\left(p_1,\delta_1^R\right) = 0$ has two distinct roots, denoted as $x_1\left(\delta_1^R\right)$ and $x_2\left(\delta_1^R\right)$, with $x_1\left(\delta_1^R\right) \leq p_1^h < x_2\left(\delta_1^R\right)$. And $x_1\left(\delta_1^R\right) = p_1^h$ if and only if $\delta_1^R = \delta_1^h$.

  - $\forall \delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, we have $G\left(p_1^h,\delta_1^R\right) > G\left(p_1^h,\delta_1^h\right) = 0$, and $G\left(+\infty,\delta_1^R\right) = -\infty$. Let p_1^R denote the price at which the retail investor's time-1 wealth share is exactly zero, then p_1^R satisfies

$$\begin{aligned} & \alpha_1^R\left(p_1^R\right)=0 \\ \Longrightarrow\quad & 0=\alpha_0^R\left(\left(1-w_0^R\right)\exp\left(p_0-p_1^R\right)+w_0^R\right) \\ \Longrightarrow\quad & p_1^R=p_0+\log\left(1-\frac{1}{w_0^R}\right) \end{aligned}$$

And we have $G\left(p_1^R,\delta_1^R\right)=-\infty$. Then $G\left(p_1^R,\delta_1^R\right)=G\left(+\infty,\delta_1^R\right)=-\infty<0<G\left(p_1^h,\delta_1^R\right)$. By the intermediate value theorem, $G\left(p_1,\delta_1^R\right)=0$ has two distinct roots $x_1\left(\delta_1^R\right),x_2\left(\delta_1^R\right)$ such that $p_1^R<x_1\left(\delta_1^R\right)<p_1^h<x_2\left(\delta_1^R\right)$, $\forall\delta_1^R\in\left(\delta_1^h,\bar{\delta}_1\right]$. In a monotone equilibrium of Definition 1, $x_2\left(\delta_1^R\right)$ is the unique equilibrium price.

Next, I show that $\forall\delta_1^R\in\left(\delta_1^h,\bar{\delta}_1\right]$, $\left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=x_2\left(\delta_1^R\right)}<0$. Suppose otherwise, then $\left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=x_2\left(\delta_1^R\right)}\geq 0\implies\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}>0,\forall p_1<x_2\left(\delta_1^R\right)$. This implies $0=G\left(p_1^h,\delta_1^h\right)<G\left(p_1^h,\delta_1^R\right)<G\left(x_2\left(\delta_1^R\right),\delta_1^R\right)=0$, a contradiction.

– At the cutoff $\delta_1^R=\delta_1^h$, $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=p_1^h}>0$ implies that, $\exists\varepsilon>0$ and small, $G\left(p_1^h+\varepsilon,\delta_1^h\right)>G\left(p_1^h,\delta_1^h\right)=0$. Together with $G\left(+\infty,\delta_1^h\right)=-\infty<0$, this implies that $G\left(p_1,\delta_1^h\right)$ has two distinct roots $x_1\left(\delta_1^h\right),x_2\left(\delta_1^h\right)$ such that $x_1\left(\delta_1^h\right)=p_1^h<x_2\left(\delta_1^h\right)$.

  Next, I show that $\left.\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}\right|_{p_1=x_2\left(\delta_1^h\right)}<0$. Suppose otherwise, then $\left.\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\right|_{p_1=x_2\left(\delta_1^h\right)}\geq 0\implies\frac{\partial G\left(p_1,\delta_1^h\right)}{\partial p_1}>0,\forall p_1<x_2\left(\delta_1^h\right)$. This implies $0=G\left(p_1^h,\delta_1^h\right)<G\left(x_2\left(\delta_1^h\right),\delta_1^h\right)=0$, a contradiction.

In a monotone equilibrium of Definition 1, $\forall\delta_1^R\in\left(\delta_1^h,\bar{\delta}_1\right]$, the equilibrium price has to be greater than p_1^h. Hence, $x_2\left(\delta_1^R\right)$ is the unique equilibrium price on $\delta_1^R\in\left(\delta_1^h,\bar{\delta}_1\right]$. And since $p_1^h<x_2\left(\delta_1^h\right)$, the pricing function $p_1\left(\delta_1^R\right)$ is discontinuous at $\delta_1^R=\delta_1^h$.

Using the implicit function theorem, $\forall p_1>x_2\left(\delta_1^h\right)$, $\forall\delta_1^R\in\left[\delta_1^h,\bar{\delta}_1\right]$,

$$\begin{aligned} & \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R}+\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial\delta_1^R}=0 \\ \Longrightarrow\quad & \frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}\frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R}+1=0 \\ \Longrightarrow\quad & \frac{dp_1\left(\delta_1^R\right)}{d\delta_1^R}=-\frac{1}{\frac{\partial G\left(p_1,\delta_1^R\right)}{\partial p_1}}>0 \end{aligned}$$

Hence, $\forall \delta_1^R \in \left[\delta_1^h, \bar{\delta}_1\right]$, the equilibrium price $p_1\left(\delta_1^R\right)$ is strictly increasing in δ_1^R. Furthermore, $p_1\left(\delta_1^R\right)$ is continuous in δ_1^R on $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$, and is discontinuous at $\delta_1^R = \delta_1^h$.

$\square$

A1.8 Proof of Proposition 3

Proof.

- Low sentiment $\delta_1^R \in [\underline{\delta}_1, \delta_1^m)$: from the optimal portfolio choices of the three investors, (36), (30), (32), and the market clearing condition (38), we get

$$
\begin{aligned}
& \frac{\alpha_1^R\left(p_1\right)\tau^R}{\sigma_d^2}\delta_1^R + \sum_i \alpha_1^i\left(p_1\right)\tau^i\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad & \frac{\alpha_1^R\left(p_1\right)\tau^R}{\sigma_d^2}\delta_1^R + \tau_1\left(p_1\right)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad & \alpha_1^R\left(p_1\right)\tau^R\delta_1^R + \tau_1\left(p_1\right)\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) = \sigma_d^2
\end{aligned}
$$

  Using the implicit function theorem,

$$
\begin{aligned}
& \alpha_1^R\left(p_1\right)\tau^R + \frac{d\left(\alpha_1^R\left(p_1\right)\tau^R\delta_1^R\right)}{dp_1}\frac{dp_1}{d\delta_1^R} + \frac{d\tau_1\left(p_1\right)}{dp_1}\frac{dp_1}{d\delta_1^R}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \tau_1\left(p_1\right)\frac{dp_1}{d\delta_1^R} = 0 \\
\Longrightarrow \quad & \frac{dp_1}{d\delta_1^R} = \frac{\frac{\alpha_1^R(p_1)\tau^R}{\tau_1(p_1)}}{1 - \frac{1}{\tau_1(p_1)}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R + \frac{d\tau_1(p_1)}{dp_1}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)\right)}
\end{aligned}
$$

- Medium sentiment $\delta_1^R \in \left(\delta_1^m, \delta_1^h\right)$: from the optimal portfolio choices of the three investors, (36), (30), (32), and the market clearing condition (38), we get

$$
\begin{aligned}
& \frac{\alpha_1^R\left(p_1\right)\tau^R}{\sigma_d^2}\delta_1^R + \left(\alpha_1^R\left(p_1\right)\tau^R + \alpha_1^{IS}\left(p_1\right)\tau^I\right)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad & \frac{\alpha_1^R\left(p_1\right)\tau^R}{\sigma_d^2}\delta_1^R + \hat{\tau}_1\left(p_1\right)\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) = 1 \\
\Longrightarrow \quad & \alpha_1^R\left(p_1\right)\tau^R\delta_1^R + \hat{\tau}_1\left(p_1\right)\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) = \sigma_d^2
\end{aligned}
$$

Using the implicit function theorem,

$$\alpha_1^R(p_1)\tau^R + \frac{d\left(\alpha_1^R(p_1)\tau^R\delta_1^R\right)}{dp_1}\frac{dp_1}{d\delta_1^R} + \frac{d\hat{\tau}_1(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \hat{\tau}_1(p_1)\frac{dp_1}{d\delta_1^R} = 0$$

$$\Longrightarrow \quad \frac{dp_1}{d\delta_1^R} = \frac{\frac{\alpha_1^R(p_1)\tau^R}{\hat{\tau}_1(p_1)}}{1 - \frac{1}{\hat{\tau}_1(p_1)}\left(\frac{d\left(\alpha_1^R(p_1)\right)}{dp_1}\tau^R\delta_1^R + \frac{d\hat{\tau}_1(p_1)}{dp_1}\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right)\right)}$$

- High sentiment $\delta_1^R \in \left(\delta_1^h, \bar{\delta}_1\right]$: from the optimal portfolio choices of the three investors, (36), (30), (32), and the market clearing condition (38), we get

$$\alpha_1^R(p_1)\tau^R\left(\frac{\mu_d + \delta_1^R - p_1}{\sigma_d^2} + \frac{1}{2}\right) - \alpha_1^{IS}(p_1)\frac{1}{m} = 1$$

$$\Longrightarrow \quad \frac{\alpha_1^R(p_1)\tau^R}{\sigma_d^2}\delta_1^R + \alpha_1^R(p_1)\tau^R\left(\frac{\mu_d - p_1}{\sigma_d^2} + \frac{1}{2}\right) - \alpha_1^{IS}(p_1)\frac{1}{m} = 1$$

$$\Longrightarrow \quad \alpha_1^R(p_1)\tau^R\delta_1^R + \alpha_1^R(p_1)\tau^R\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \alpha_1^{IS}(p_1)\frac{1}{m}\sigma_d^2 = \sigma_d^2$$

Using the implicit function theorem,

$$\alpha_1^R(p_1)\tau^R + \frac{d\left(\alpha_1^R(p_1)\tau^R\delta_1^R\right)}{dp_1}\frac{dp_1}{d\delta_1^R} + \frac{d\alpha_1^R(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\tau^R\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \alpha_1^R(p_1)\tau^R\frac{dp_1}{d\delta_1^R}$$
$$-\frac{d\alpha_1^{IS}(p_1)}{dp_1}\frac{dp_1}{d\delta_1^R}\frac{1}{m}\sigma_d^2 = 0$$

$$\Longrightarrow \quad \frac{dp_1}{d\delta_1^R} = \frac{1}{1 - \frac{1}{\alpha_1^R(p_1)\tau^R}\left(\frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\delta_1^R + \frac{d\alpha_1^R(p_1)}{dp_1}\tau^R\left(\mu_d + \frac{1}{2}\sigma_d^2 - p_1\right) - \frac{d\alpha_1^{IS}(p_1)}{dp_1}\frac{1}{m}\sigma_d^2\right)}$$

$\square$

A1.9 Proof of Proposition 4

Proof. To derive the time-0 equilibrium price, substitute the optimal portfolio choices of the three investors, (35), (29), and (31) into the market clearing condition (38),

$$\left(\alpha_0^R(p_0)\tau^R + \left(1 - \alpha_0^R(p_0)\right)\tau^I\right)\left(\frac{\mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] - p_0}{\sigma_0^2} + \frac{1}{2}\right) + \sum_i \frac{\alpha_0^i(p_0)\tau^i\delta_0^i}{\sigma_0^2} = 1$$

$$\Longrightarrow \quad \tau_0(p_0)\left(\mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] - p_0 + \frac{1}{2}\sigma_0^2\right) + \sum_i \alpha_0^i(p_0)\tau^i\delta_0^i = \sigma_0^2$$

$$\Longrightarrow \quad p_0 = \mathbb{E}_0\left[p_1\left(\delta_1^R\right)\right] + \left(\frac{1}{2}\sigma_0^2 + \frac{\sum_i \alpha_0^i(p_0)\tau^i\delta_0^i - \sigma_0^2}{\tau_0(p_0)}\right)$$

where

$$\tau_0(p_0) \equiv \sum_i \alpha_0^i(p_0)\tau^i = \alpha_0^R(p_0)\tau^R + \left(1-\alpha_0^R(p_0)\right)\tau^I$$

The rest of the proof follows Proposition 1. □

A1.10 Implicit price at time -1

I assume that, at time -1, investors do not anticipate future sentiment shocks. They believe that the prices at time 0 and 1 will reflect the present value of the terminal dividend, and the prices are deterministic. Hence, from time -1 to 0 and from time 0 to 1, the risky asset should have the same one-period return as the risky-free asset. This implies that $p_{-1} = \tilde{p}_0 = \tilde{p}_1$, where $\tilde{p}_0$ and $\tilde{p}_1$ denote investors' beliefs about time-0 and time-1 prices, respectively.

The implicit price p_{-1} is such that investors do not want to trade at time -1. Since $p_{-1} = \tilde{p}_0 = \tilde{p}_1$, investors believe that they will not have incentives to trade at time 0 and 1, and thus they believe their asset positions and wealth shares remain constant from time -1 to time 1. In this case, the aggregate risk tolerance remains constant from time -1 to time 1, and is equal to

$$\tau_{-1} = \alpha_{-1}^R\tau^R + \left(1-\alpha_{-1}^R\right)\tau^I.$$

Impose the market clearing condition in equation (38), we can solve for the implicit price

$$p_{-1} = \tilde{p}_0 = \tilde{p}_1 = \mu_d + \left(\frac{1}{2} - \frac{1}{\tau_{-1}}\right)\sigma_d^2.$$

Note that at time -1, investors do not want to trade, because they believe that the risky asset has the same return as the risk-free asset.

A1.11 Proof of Lemma 2

Proof. I first compute the m-th moment of $d^{in}_{j,t}$ in the cross section of retail investors, using the PDF specified in equation (49) with support $[d_{\min}, d_{\max}(N_t)]$.

$$\begin{aligned}
\mathbb{E}^{CS}\left[\left(d^{in}_{j,t}\right)^m\right] &= \int_{d_{\min}}^{d_{\max}(N_t)} x^m \frac{\xi-1}{d_{\min}}\left(\frac{x}{d_{\min}}\right)^{-\xi} dx \\
&= \frac{\xi-1}{d_{\min}^{1-\xi}} \int_{d_{\min}}^{d_{\max}(N_t)} x^{m-\xi} dx \\
&= \frac{\xi-1}{d_{\min}^{1-\xi}} \frac{1}{m+1-\xi} x^{m+1-\xi}\Big|_{d_{\min}}^{d_{\max}(N_t)} \\
&= \frac{\xi-1}{\xi-m-1} \frac{1}{d_{\min}^{1-\xi}} \left(d_{\min}^{m+1-\xi} - \left(d_{\max}(N_t)\right)^{m+1-\xi}\right).
\end{aligned}$$

The cross-sectional variance of $d^{in}_{j,t}$ is thus

$$\begin{aligned}
\operatorname{Var}^{CS}\left(d^{in}_{j,t}\right) &= \mathbb{E}\left[\left(d^{in}_{j,t}\right)^2\right] - \left(\mathbb{E}\left[d^{in}_{j,t}\right]\right)^2 \\
&= \frac{\xi-1}{3-\xi}\frac{1}{d_{\min}^{1-\xi}}\left(\left(d_{\max}(N_t)\right)^{3-\xi} - d_{\min}^{3-\xi}\right) - \left(\frac{\xi-1}{\xi-2}\right)^2 \frac{1}{d_{\min}^{2-2\xi}}\left(d_{\min}^{2-\xi} - \left(d_{\max}(N_t)\right)^{2-\xi}\right)^2.
\end{aligned}$$

□

A1.12 Proof of Proposition 5

Proof. The proof follows from Acemoglu et al. (2012).

Using the PDF of $d^{in}_{j,t}$ in equation (49), I first derive the counter-CDF

$$P_{N_t}(x) \equiv \Pr\left(d^{in}_{j,t} > x\right) = \int_x^{+\infty} \frac{\xi-1}{d_{\min}}\left(\frac{y}{d_{\min}}\right)^{-\xi} dy = \left(\frac{x}{d_{\min}}\right)^{1-\xi} \tag{A11}$$

Define the empirical counterpart as

$$\hat{P}_{N_t}(x) = \frac{1}{N_t}\left|\left\{j \in \mathcal{I}_{N_t} : d^{in}_{j,t} > x\right\}\right| = \frac{1}{N_t}\sum_{j=1}^{N_t} \mathbf{1}\left\{d^{in}_{j,t} > x\right\}$$

Let $\boldsymbol{B}_t = \{b_{1,t}, b_{2,t}, \cdots, b_{m_t,t}\}$ denote the set of values $d^{in}_{j,t}$ takes, with $b_{1,t} < b_{2,t} < \cdots < b_{m_t,t}$, and the convention that $b_{0,t} = 0$. Then

$$\begin{aligned}
& \sum_{j=1}^{N_t} \left(d_{j,t}^{in}\right)^2 = N_t \sum_{k=1}^{m_t} \left(b_{k,t}\right)^2 \left(\hat{P}_{N_t}\left(b_{k-1,t}\right) - \hat{P}_{N_t}\left(b_{k,t}\right)\right) \\
= \; & N_t \left(b_{1,t}^2 \left(\hat{P}_{N_t}\left(b_{0,t}\right) - \hat{P}_{N_t}\left(b_{1,t}\right)\right) + \cdots + b_{m_t}^2 \left(\hat{P}_{N_t}\left(b_{m_t-1,t}\right) - \hat{P}_{N_t}\left(b_{m_t,t}\right)\right)\right) \\
= \; & N_t \left(\left(b_{1,t}^2 - b_{0,t}^2\right) \hat{P}_{N_t}\left(b_{0,t}\right) + \cdots + \left(b_{m_t,t}^2 - b_{m_t-1,t}^2\right) \hat{P}_{N_t}\left(b_{m_t-1,t}\right) - b_{m_t,t}^2 \hat{P}_{N_t}\left(b_{m_t,t}\right)\right) \\
= \; & N_t \sum_{k=0}^{m_t-1} \left(b_{k+1,t}^2 - b_{k,t}^2\right) \hat{P}_{N_t}\left(b_{k,t}\right) \\
= \; & N_t \sum_{k=0}^{m_t-1} \left(b_{k+1,t} + b_{k,t}\right)\left(b_{k+1,t} - b_{k,t}\right) \hat{P}_{N_t}\left(b_{k,t}\right) \\
= \; & 2N_t \sum_{k=0}^{m_t-1} \left(\frac{b_{k,t} + b_{k+1,t}}{2}\right)\left(b_{k+1,t} - b_{k,t}\right) \hat{P}_{N_t}\left(b_{k,t}\right)
\end{aligned}$$

Replace the empirical counter-CDF $\hat{P}_{N_t}\left(b_{k,t}\right)$ with the continuous function in (A11).

$$\begin{aligned}
\sum_{j=1}^{N_t} \left(d_{j,t}^{in}\right)^2 &= 2N_t \int_{d_{\min}}^{d_{\max}(N_t)} x \left(\frac{x}{d_{\min}}\right)^{1-\xi} dx \\
&= 2N_t \int_{d_{\min}}^{d_{\max}(N_t)} x \frac{d_{\min}}{2-\xi} d\left(\frac{x}{d_{\min}}\right)^{2-\xi} \\
&= 2N_t \frac{d_{\min}}{2-\xi} \left(x \left(\frac{x}{d_{\min}}\right)^{2-\xi} \Bigg|_{d_{\min}}^{d_{\max}(N_t)} - \int_{d_{\min}}^{d_{\max}(N_t)} \left(\frac{x}{d_{\min}}\right)^{2-\xi} dx \right) \\
&= 2N_t \frac{d_{\min}}{2-\xi} \left(x \left(\frac{x}{d_{\min}}\right)^{2-\xi} \Bigg|_{d_{\min}}^{d_{\max}(N_t)} - \frac{d_{\min}}{3-\xi} \left(\frac{x}{d_{\min}}\right)^{3-\xi} \Bigg|_{d_{\min}}^{d_{\max}(N_t)} \right) \\
&= 2N_t \frac{d_{\min}}{2-\xi} \left(d_{\max}\left(N_t\right) \left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi} - d_{\min} - \frac{d_{\min}}{3-\xi} \left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{3-\xi} + \frac{d_{\min}}{3-\xi} \right) \\
&= 2N_t \frac{d_{\min}}{2-\xi} \left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi} \left(d_{\max}\left(N_t\right) - \frac{1}{3-\xi} d_{\max}\left(N_t\right)\right) - \left(d_{\min} - \frac{d_{\min}}{3-\xi}\right) \right) \\
&= 2N_t \frac{d_{\min}}{2-\xi} \left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi} \frac{2-\xi}{3-\xi} d_{\max}\left(N_t\right) - \frac{2-\xi}{3-\xi} d_{\min} \right)
\end{aligned}$$

Using the dynamics of aggregate retail sentiment δ_t^R in equation (48), we can compute the conditional mean of δ_t^R

$$\mathbb{E}_{t-1}\left[\delta_t^R\right] = \frac{\theta\left(N_t\right)}{\theta\left(N_{t-1}\right)} \rho \delta_{t-1}^R,$$

and the conditional variance

$$\begin{aligned}
\mathrm{Var}_{t-1}\left(\delta_t^R\right) &= \left(\theta\left(N_t\right)\right)^2 \frac{1}{N_t^2} \sum_{j=1}^{N_t} \left(d_{j,t}^{in}\right)^2 \sigma_\varepsilon^2 \\
&= \left(\theta\left(N_t\right)\right)^2 \frac{1}{N_t} \frac{2d_{\min}}{2-\xi} \sigma_\varepsilon^2 \left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi} \frac{2-\xi}{3-\xi} d_{\max}\left(N_t\right) - \frac{2-\xi}{3-\xi} d_{\min}\right) \\
&= \left(\theta\left(N_t\right)\right)^2 \frac{2d_{\min}}{N_t} \frac{1}{3-\xi} \left(\left(\frac{d_{\max}\left(N_t\right)}{d_{\min}}\right)^{2-\xi} d_{\max}\left(N_t\right) - d_{\min}\right) \sigma_\varepsilon^2 \\
&= \left(\theta\left(N_t\right)\right)^2 \frac{2d_{\min}^{\xi-1}}{N_t} \frac{1}{3-\xi} \left(\left(d_{\max}\left(N_t\right)\right)^{3-\xi} - d_{\min}^{3-\xi}\right) \sigma_\varepsilon^2 \\
&= O\left(N_t^{\frac{4-2\xi}{\xi-1}}\right).
\end{aligned}$$

The last equality uses $d_{\max}\left(N_t\right) = O\left(N_t^{\frac{1}{\xi-1}}\right)$. Hence, the conditional volatility of aggregate retail sentiment is

$$\sqrt{\mathrm{Var}_{t-1}\left(\delta_t^R\right)} = O\left(N_t^{\frac{2-\xi}{\xi-1}}\right).$$

□

A1.13 Distribution of time-1 aggregate retail sentiment shock

Define $c_j \equiv \frac{1}{N} d_j^{in}$, and the random variable $X_j = \mu + \varepsilon_1^j$, $\mu = \delta_0^R$. Let σ^2 denote the pre-truncation variance of ε_1^j, then X_j follows a truncated normal distribution on $[-\bar{\varepsilon}, \bar{\varepsilon}]$ with pre-truncation mean μ and variance σ^2, and X_j is i.i.d. in the cross section. Further define $\rho \equiv \frac{\bar{\varepsilon}}{\sigma}$, $a = \mu - \rho\sigma$, $b = \mu + \rho\sigma$. Then the PDF of X_j is

$$f_{X_j}(x) = \frac{1}{\sigma} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} = \frac{1}{\sigma} \frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi\left(\rho\right) - 1}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the PDF and CDF of a standard normal random variable, respectively.

The time-1 aggregate retail sentiment shock δ_1^R can be written as

$$\delta_1^R = \sum_{j=1}^{N} c_j X_j$$

Hence, the characteristic function of δ_1^R is

$$\begin{aligned}
\varphi_{\delta_1^R}(t) &= \varphi_{X_1}(c_1 t)\,\varphi_{X_2}(c_2 t)\cdots\varphi_{X_N}(c_N t) \\
&= \prod_{j=1}^{N}\varphi_{X_j}(c_j t) = \prod_{j=1}^{N}\mathbb{E}\left[e^{itc_jX_j}\right] \\
&= \prod_{j=1}^{N}\left[\int_a^b e^{itc_jx}\frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1}dx\right]
\end{aligned}$$

Note that

$$\begin{aligned}
&\int_a^b e^{itc_jx}\frac{1}{\sigma_j}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1}dx \\
=\ & \frac{1}{2\Phi(\rho)-1}\int_a^b \frac{1}{\sqrt{2\pi}\sigma}\exp\left(itc_jx-\frac{(x-\mu)^2}{2\sigma^2}\right)dx \\
=\ & \frac{1}{2\Phi(\rho)-1}\int_a^b \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2-2\mu x+\mu^2-2itc_jx\sigma^2}{2\sigma^2}\right)dx \\
=\ & \frac{1}{2\Phi(\rho)-1}\exp\left(\frac{\left(\mu+itc_j\sigma^2\right)^2-\mu^2}{2\sigma^2}\right)\int_a^b \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(x-\left(\mu+itc_j\sigma^2\right)\right)^2}{2\sigma^2}\right)dx \\
=\ & \frac{1}{2\Phi(\rho)-1}\exp\left(c_j\mu it-\frac{1}{2}c_j^2\sigma^2t^2\right)\int_a^b \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(x-\left(\mu+c_j\sigma^2it\right)\right)^2}{2\sigma^2}\right)dx
\end{aligned}$$

Define $y \equiv \frac{x-\left(\mu+c_j\sigma^2it\right)}{\sigma} \implies x = \sigma y + \left(\mu+c_j\sigma^2it\right) \implies dx = \sigma dy$. And note that $\frac{a-\left(\mu+c_j\sigma^2it\right)}{\sigma} = -\rho-c_j\sigma it$, $\frac{b-\left(\mu+c_j\sigma^2it\right)}{\sigma} = \rho-c_j\sigma it$. Then

$$\begin{aligned}
&\int_a^b e^{itc_jx}\frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1}dx \\
=\ & \frac{1}{2\Phi(\rho)-1}\exp\left(c_j\mu it-\frac{1}{2}c_j^2\sigma^2t^2\right)\int_a^b \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(x-\left(\mu+c_j\sigma^2it\right)\right)^2}{2\sigma^2}\right)dx \\
=\ & \frac{1}{2\Phi(\rho)-1}\exp\left(c_j\mu it-\frac{1}{2}c_j^2\sigma^2t^2\right)\int_{\frac{a-\left(\mu+c_j\sigma^2it\right)}{\sigma}}^{\frac{b-\left(\mu+c_j\sigma_j^2it\right)}{\sigma}} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)dy \\
=\ & \frac{1}{2\Phi(\rho)-1}\exp\left(c_j\mu it-\frac{1}{2}c_j^2\sigma^2t^2\right)\int_{-\rho-c_j\sigma it}^{\rho-c_j\sigma it} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)dy \\
=\ & \exp\left(c_j\mu it-\frac{1}{2}c_j^2\sigma^2t^2\right)\frac{\Phi(\rho-c_j\sigma it)-\Phi(-\rho-c_j\sigma it)}{2\Phi(\rho)-1} \\
=\ & \exp\left(c_j\mu it-\frac{1}{2}c_j^2\sigma^2t^2\right)\frac{\Phi(\rho-c_j\sigma it)+\Phi(\rho+c_j\sigma it)-1}{2\Phi(\rho)-1}
\end{aligned}$$

Hence,

$$
\begin{aligned}
\varphi_{S_n}(t) &= \prod_{j=1}^{n}\left[\int_a^b e^{itc_jx}\frac{1}{\sigma}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{2\Phi(\rho)-1}dx\right] \\
&= \exp\left(\left(\sum_{j=1}^{n}c_j\mu\right)it-\frac{1}{2}\left(\sum_{j=1}^{n}c_j^2\sigma^2\right)t^2\right)\prod_{j=1}^{n}\frac{\Phi(\rho-c_j\sigma it)+\Phi(\rho+c_j\sigma_j it)-1}{2\Phi(\rho)-1}
\end{aligned}
$$

The characteristic function of δ_1^R is

$$
\begin{aligned}
&\varphi_{S_n}(t)=\exp\left(\left(\sum_{j=1}^{N}c_j\mu\right)it-\frac{1}{2}\left(\sum_{j=1}^{N}c_j^2\sigma^2\right)t^2\right)\prod_{j=1}^{N}\frac{\Phi(\rho-c_j\sigma it)+\Phi(\rho+c_j\sigma it)-1}{2\Phi(\rho)-1} \\
\Longrightarrow\quad &\varphi_{\delta_1}(t)=\exp\left(\left(\sum_{j=1}^{N}c_j\right)\mu it-\frac{1}{2}\left(\sum_{j=1}^{N}c_j^2\right)\sigma_\varepsilon^2t^2\right)\prod_{j=1}^{N}\frac{\Phi\left(\frac{\bar{\varepsilon}}{\sigma_\varepsilon}-c_j\sigma_\varepsilon it\right)+\Phi\left(\frac{\bar{\varepsilon}}{\sigma_\varepsilon}+c_j\sigma_\varepsilon it\right)-1}{2\Phi\left(\frac{\bar{\varepsilon}}{\sigma_\varepsilon}\right)-1} \\
\Longrightarrow\quad &\varphi_{\delta_1}(t)=\exp\left(\mu it-\frac{1}{2}\left(\sum_{j=1}^{N}c_j^2\right)\sigma_\varepsilon^2t^2\right)\prod_{j=1}^{N}\frac{\Phi\left(\frac{\bar{\varepsilon}}{\sigma_\varepsilon}-c_j\sigma_\varepsilon it\right)+\Phi\left(\frac{\bar{\varepsilon}}{\sigma_\varepsilon}+c_j\sigma_\varepsilon it\right)-1}{2\Phi\left(\frac{\bar{\varepsilon}}{\sigma_\varepsilon}\right)-1}
\end{aligned}
$$

Compare the characteristic function of δ_1^R with another random variable $\tilde{\delta}_1$, which follows a truncated normal distribution on $[\mu-\bar{\varepsilon},\mu+\bar{\varepsilon}]$, with mean $\sum_{j=1}^{N}c_j\mu=\mu$ and variance $\sum_{j=1}^{N}c_j^2\sigma_\varepsilon^2$.

$$
\begin{aligned}
\varphi_{\tilde{\delta}_1}(t) &= \exp\left(\mu it-\frac{1}{2}\left(\sum_{j=1}^{N}c_j^2\right)\sigma_\varepsilon^2t^2\right) \\
&\quad\cdot\frac{\Phi\left(\frac{\bar{\varepsilon}}{\sqrt{\sum_{j=1}^{N}c_j^2}\sigma_\varepsilon}-\sqrt{\sum_{j=1}^{N}c_j^2}\sigma_\varepsilon it\right)+\Phi\left(\frac{\bar{\varepsilon}}{\sqrt{\sum_{j=1}^{N}c_j^2}\sigma_\varepsilon}+\sqrt{\sum_{j=1}^{N}c_j^2}\sigma_\varepsilon it\right)-1}{2\Phi\left(\frac{\bar{\varepsilon}}{\sqrt{\sum_{j=1}^{N}c_j^2}\sigma_\varepsilon}\right)-1}
\end{aligned}
$$

Hence, the distribution of δ_1^R can be approximated by a truncated normal distribution, if the cross sectional distribution of c_j is skewed.

A2 Reddit data

A2.1 Variable definitions

I construct two data frames following the steps in Section 2.1.1 – one includes all the submissions, and the other includes all the comments.

In the data frame of submissions, each row is a unique submission. And it has the following fields:

- `id`: the unique id of the submission, e.g., "eifjq5". I add the prefix "t3_" to the submission `id` to facilitate the mapping between the submission and its associated comments.
- `author`: the name of the author of the submission, e.g., "Ituglobal".
- `author_fullname`: the unique user id of the author of the submission, prefixed by "t2_", e.g., "t2_6rjw5".
- `created_utc`: the UTC date and time at which the submission was created.
- `title`: the textual content of the title of the submission.
- `selftext`: the textual content of the body text of the submission.

In the data frame of comments, each row is a unique comment. And it has the following fields:

- `id`: the unique id of the comment, e.g., "fctzgly". I add the prefix "t1_" to the `id` to facilitate the mapping between the comment in question and its parent comment.
- `link_id`: the unique id of the submission that the comment in question replies to, e.g., "t3_eiwx9h".
- `parent_id`: the unique id of the parent comment (or submission) of the comment in question. If the comment is a reply to another comment, then the is prefixed by "t1_". Otherwise, it is a reply to a submission, and it's prefixed by "t3_".
- `created_utc`: the UTC date and time at which the comment was created.
- `author`: the name of the author of the comment, e.g., "urfriendosvendo".
- `author_fullname`: the unique user id of the author of the comment, prefixed by "t2_", e.g., "t2_12ol3k".
- `body`: the textual content of the comment.

A2.2 Constructing the sample of submissions and comments

I first run the following algorithm to tag submissions and comments with stock tickers, and then select samples of submissions and comments.

1. Retrieve the list of tickers of CRSP common stocks.

2. Search for stock tickers in the text of the submission.[1]

   (a) First pass search: search for CRSP stock tickers in the augmented body text[2].

      i. Preprocess the augmented body text in the following order:

         - Replace ‘’ / - with space.
         - Replace & with space if it appears between words.
         - Replace . with space.
         - Remove all other punctuation marks.
         - Tokenize augmented body text and only keep non-empty tokens.

      ii. Search for CRSP stock tickers in the augmented body text in a case-insensitive way. A submission is tagged with a ticker if the ticker can be found in the list of tokens.

   (b) Manually go over the matched tickers, add $ sign in front of those tickers that are common words, and use this updated list of tickers in the second pass search.

   (c) Second pass search: repeat the procedures in the first pass search, but using the updated list of tickers from the previous step.

3. Drop submissions where `author_fullname` is empty, or "[deleted]", or "[removed]". I also drop those where `id` is empty, or "[deleted]", or "[removed]".

4. Drop submissions where `author` is one of the bots in Table A1.

5. Only keep submissions tagged with at least one CRSP common stock ticker, and only keep the comments associated with these selected submissions (see Appendix A2.3 below for the procedure of matching submissions with comments).

   If a submission is tagged with a ticker, then the associated comments are also tagged with the same ticker. A submission or comment can be tagged with multiple stock tickers.

[1]For GameStop, I search for both its ticker "GME" and the company name "GameStop".

[2]A submission has its title and body text. I obtain the augmented body text by appending the body text to the title, separated by a white space.

Finally, I construct the following two samples of submissions and comments:

- Sample of submissions and comments for CRSP common stocks, by performing steps 1-5 above.
- Sample of all submissions and comments, by performing steps 1-4 above.

For each of the sample, I keep one data frame for submissions and another data frame for comments, with the structure described in Appendix A2.1. And I construct the network using these two data frames.

A2.3 Constructing the network

As is described in Appendix A2.1, the submission data frame and the comment data frame has a common field – the field `id` in the submission data frame corresponds to the `link_id` in the comment data frame. This allows me to recover the comment tree described in.

For each of the sample described in Appendix A2.2, I merge the submission data frame and comment data frame by the common field described above, and only keep submissions with at least one comment. In the merged dataset, each row corresponds to a comment, with information on the author of the comment, and the author of the submission that the comment replies to. This allows me to construct the network of users from the commenting relationship.

A3 FactSet data

I following the procedure in Gabaix and Koijen (2022) and Koijen et al. (2022):

1. Merge the holdings data (`[own_v5].[own_inst_eq_v5].[own_inst_13f_detail]`) with the entity sub type data
   (`[own_v5].[own_hub_ent_v5].[own_ent_institutions]`),
   by `factset_entity_id`.

   Each record in this merged dataset corresponds to a filer entity (with unique id `factset_entity_id`).

2. For those filer entities with missing entity sub type (from the previous step), find the corresponding roll-up entity
   (from `[own_v5].[own_hub_ent_v5].[own_ent_13f_combined_inst]`), and assign the sub type of the roll-up entity to the filer entity.

- To identify the sub type of the roll-up entity: merge the roll-up entity data ([own_v5].[own_hub_ent_v5].[own_ent_13f_combined_inst]) with the entity sub type data ([own_v5].[own_hub_ent_v5].[own_ent_institutions]), by factset_rollup_entity_id in the former (factset_entity_id in the latter).
$\Longrightarrow$ 12,276 out of the 12,295 roll-up entities have non-missing entity sub type.

3. Classify institutions into six types using entity_sub_type:

- Hedge Funds: entity_sub_type = "AR", "FH", "FF", "FU", "FS", "HF".
- Brokers: entity_sub_type = "BM", "IB", "ST", "MM", "BR".
- Private Banking: entity_sub_type = "CP", "FY", "VC", "PB".
- Investment Advisors: entity_sub_type = "IC", "RE", "PP", "SB", "MF", "IA".
- Long-Term Investors: entity_sub_type = "FO", "SV", "IN", "PF".

A4 Modified BJZZ algorithm to identify retail trades

1. Start with any trade with price not at the midpoint of bid and ask.
2. Match the NBBO to the timestamp of the trade, and then compute bid-ask spread quoted before the trade.
3. If the spread quoted before the trade is one cent, use the original BJZZ algorithm to sign the trade.
4. If the trade price is outside the bid-ask spread, use the original BJZZ algorithm to sign the trade.
5. Otherwise, if the trade is below the midpoint, label the trade as a sell. If the trade is above the midpoint, label the trade as a buy.

   I also implement the $[0.4, 0.6]$ "donut" in this step, as in the original BJZZ algorithm.

A5 Fitting power-law distribution

For each calendar day t, I fit a power-law distribution to the vector of user influence, $\left(d^{in}_{1,t}, d^{in}_{2,t}, \cdots, d^{in}_{N_t,t}\right)^\intercal$ computed in Section 2.1.3, and estimate the exponent $\hat{\xi}_t$ and the threshold value $\hat{d}^{in}_{\min,t}$. Following Rantala (2019), I use maximum likelihood method to estimate these parameters. Specifically, I use the power.law.fit function of the igraph package in R, with the "plfit" implementation.

I use bootstrap methods to compute the confidence intervals. The steps are:

1. Generate a bootstrap sample $\left\{d_{k,t}^{in}(b)\right\}_{k=1}^{N_t}$ by sampling the original data $\left(d_{1,t}^{in}, d_{2,t}^{in}, \cdots, d_{N_t,t}^{in}\right)^{\intercal}$ randomly with replacement.

2. Estimate the parameters $\xi_t(b)$ and $d_{\min,t}(b)$ for this bootstrapped sample, using the maximum likelihood method described above.

3. Repeat steps 1 and 2 for $B = 5000$ times, and obtain the vector of estimates $\{\xi_t(b)\}_{b=1}^{B}$, $\{d_{\min,t}(b)\}_{b=1}^{B}$.

4. For the $\hat{\xi}_t$ estimate, the lower (upper) bound of the 95% confidence interval is the 2.5th (97.5th) percentile of the empirical distribution $\{\xi_t(b)\}_{b=1}^{B}$. Similarly, for the $\hat{d}_{min,t}$ estimate, the lower (upper) bound of the 95% confidence interval is the 2.5th (97.5th) percentile of the empirical distribution $\{d_{\min,t}(b)\}_{b=1}^{B}$.

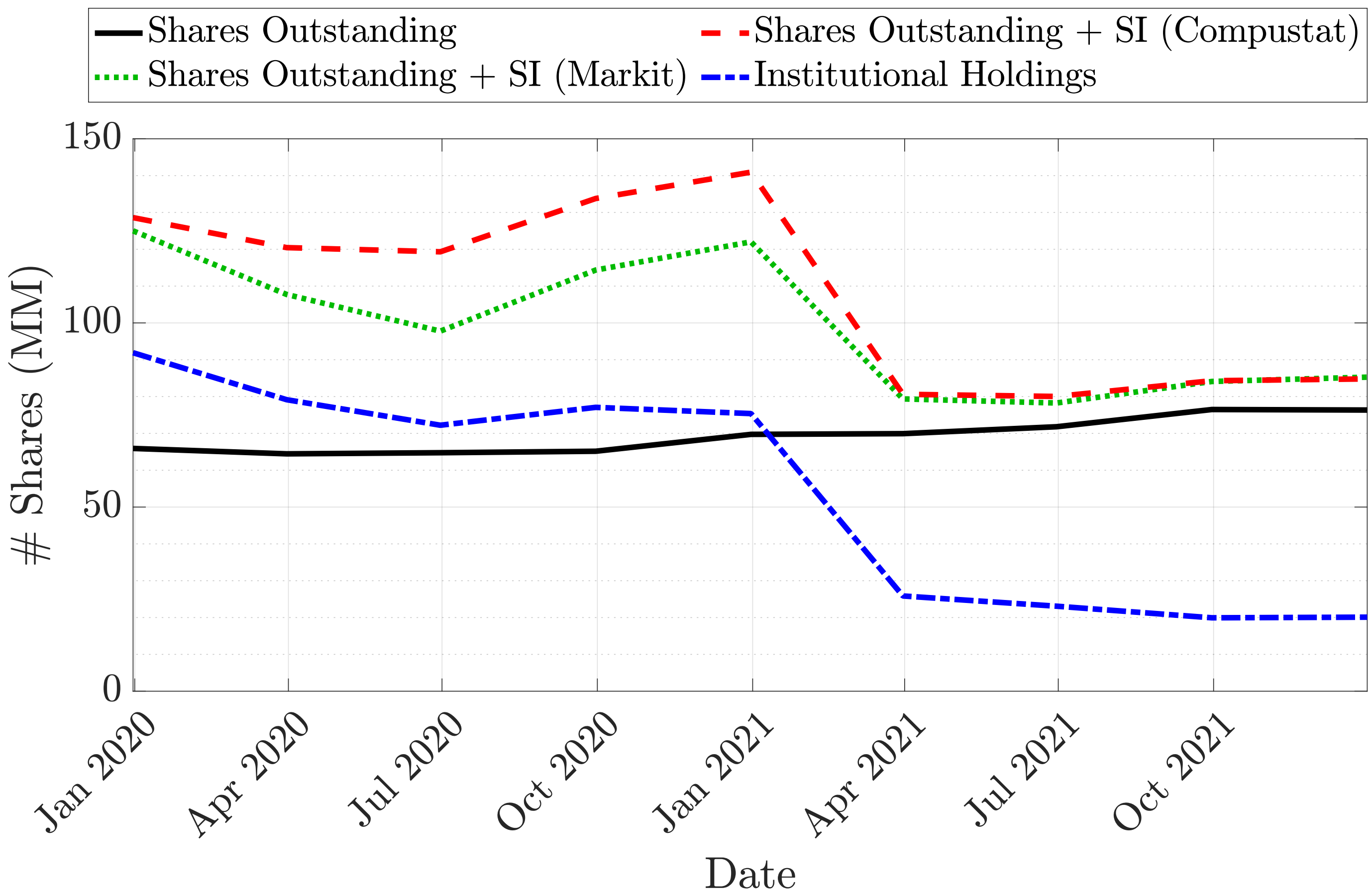


Figure A1. Shares outstanding and institutional ownership of GameStop. This figure compares the number of shares outstanding with institutional ownership of GameStop.

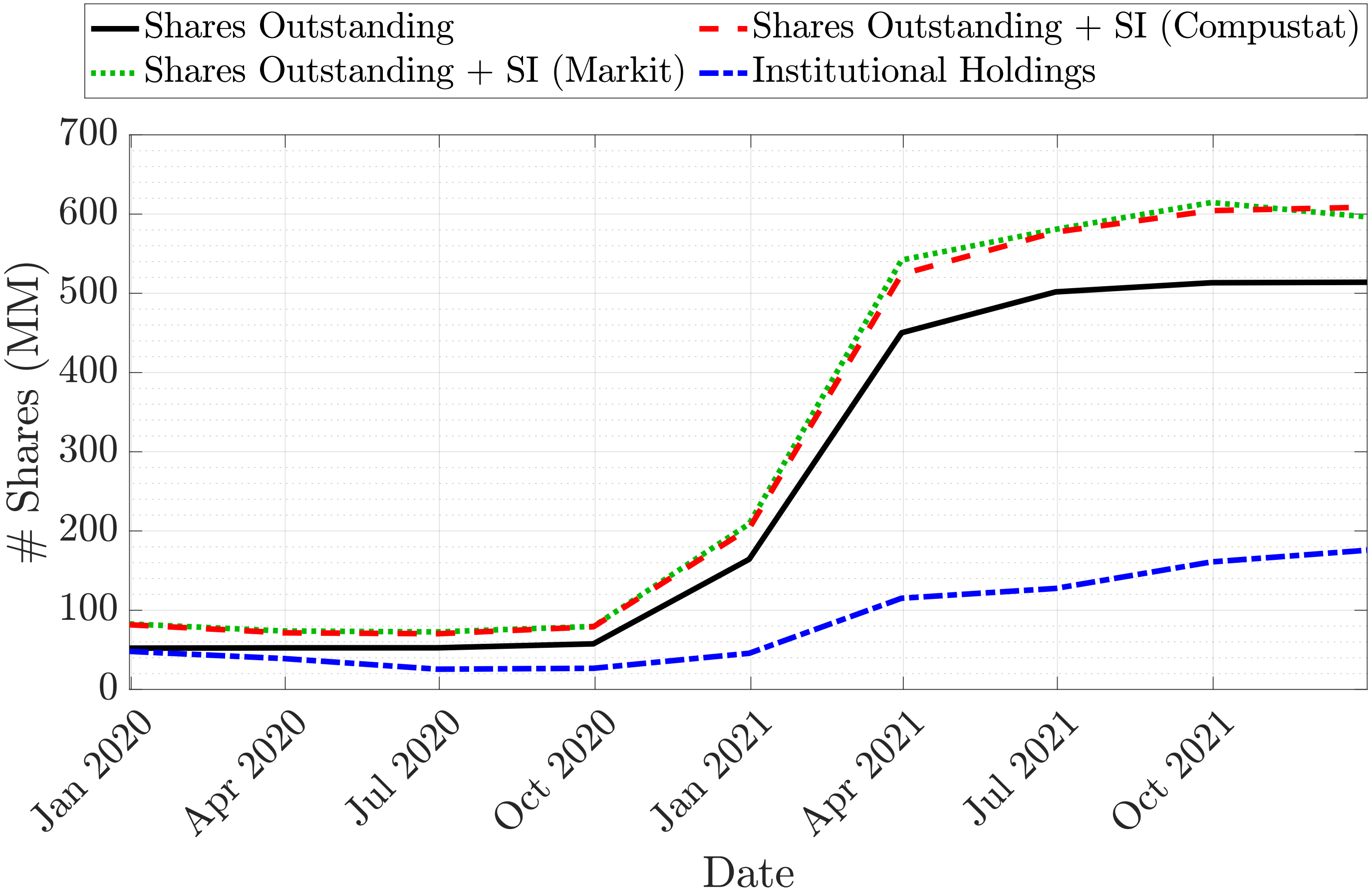


Figure A2. Shares outstanding and institutional ownership of AMC. This figure compares the number of shares outstanding with institutional ownership of AMC.

Figure A3. Price and sentiment of Amazon. This figure plots the daily close price (solid blue line), equal-weighted sentiment (dotted red line), and influence-weighted sentiment (dash-dotted green line) of Amazon. The sentiment series are 30-day moving averages.

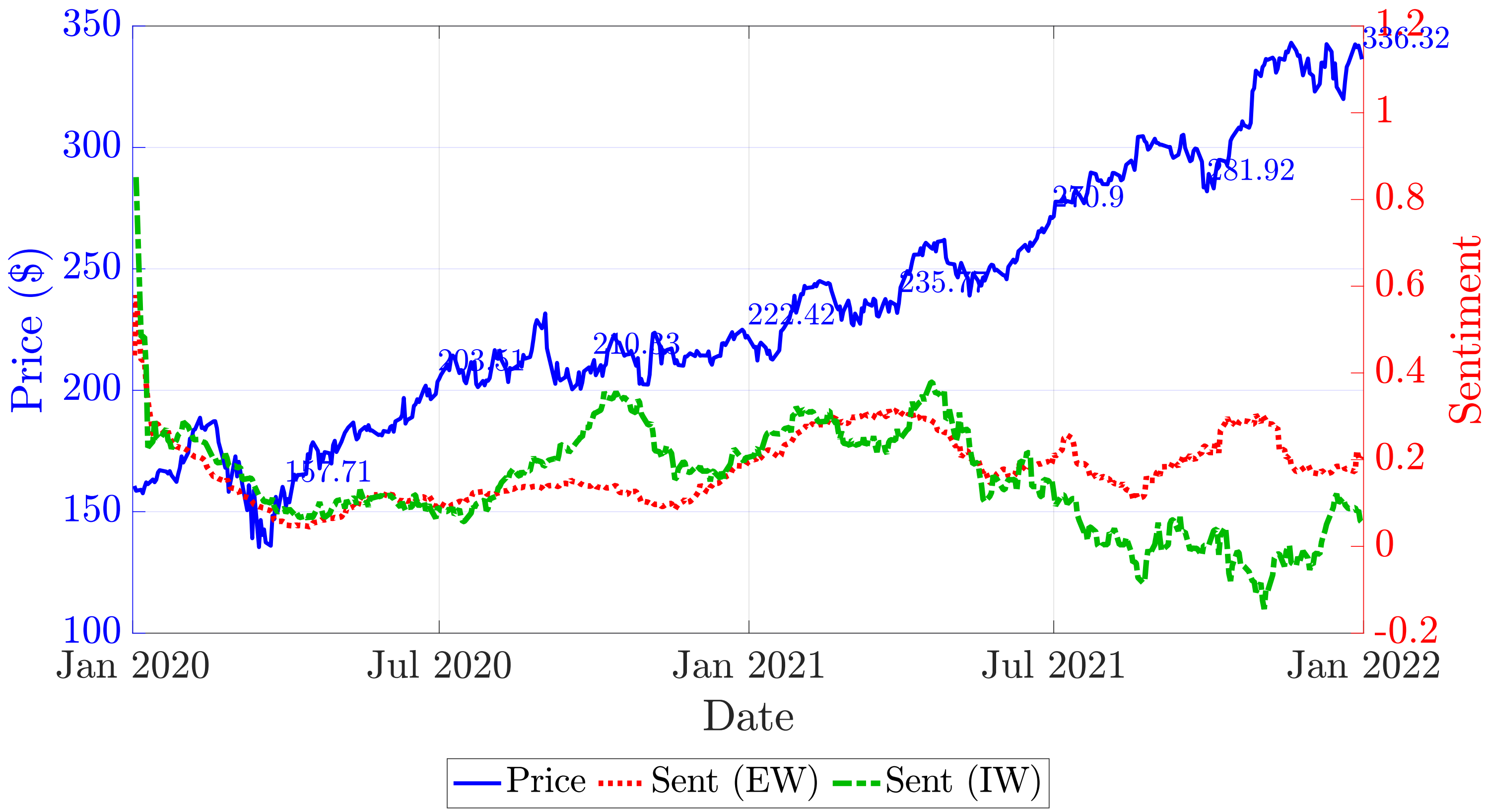


Figure A4. Price and sentiment of Microsoft. This figure plots the daily close price (solid blue line), equal-weighted sentiment (dotted red line), and influence-weighted sentiment (dash-dotted green line) of Microsoft. The sentiment series are 30-day moving averages.

Figure A5. Price and sentiment of AMC. This figure plots the daily close price (solid blue line), equal-weighted sentiment (dotted red line), and influence-weighted sentiment (dash-dotted green line) of AMC. The sentiment series are 30-day moving averages.

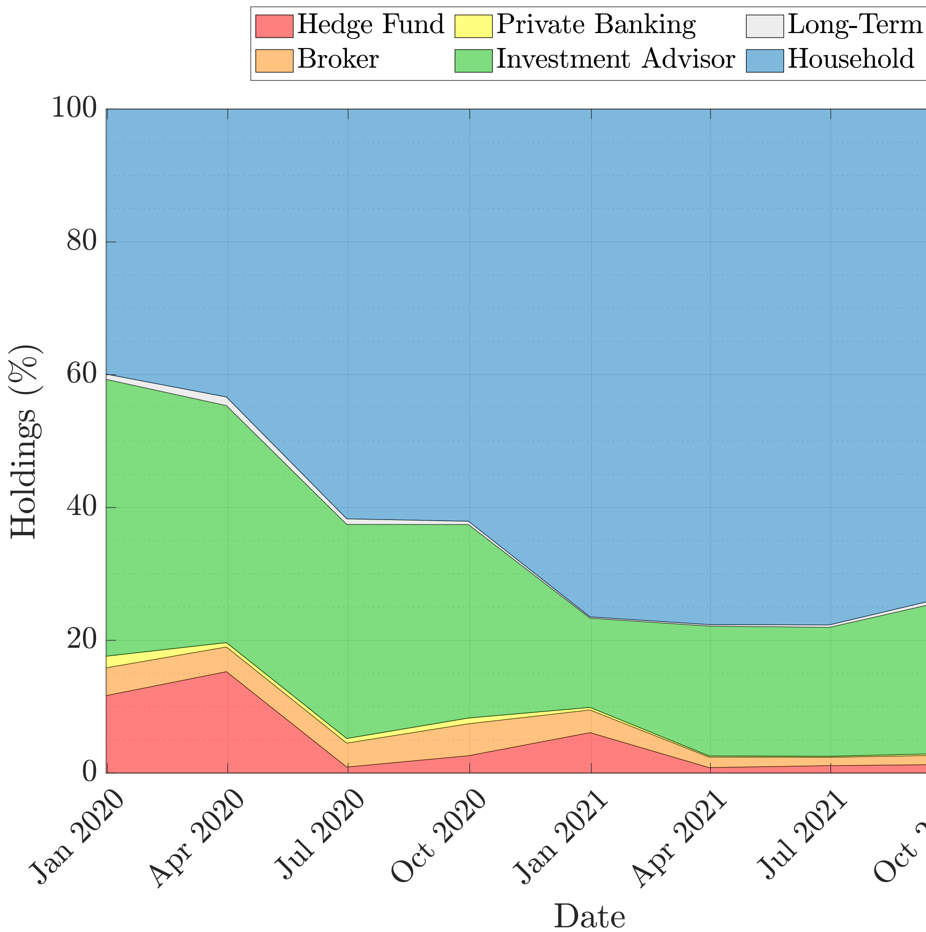
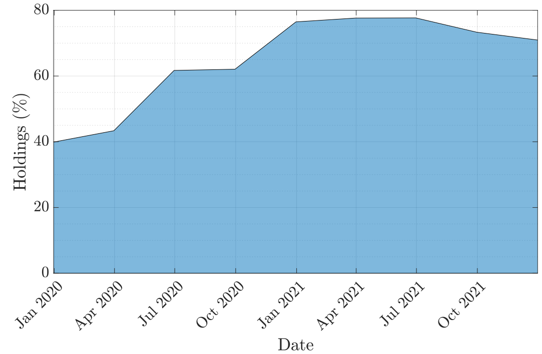
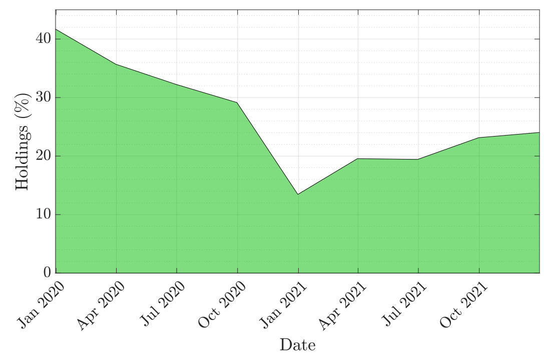


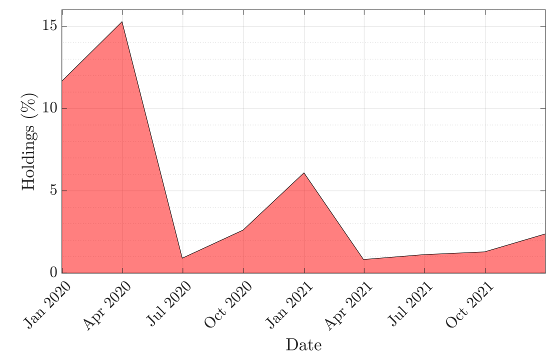
Figure A6. Holdings of long investors in AMC. This figure plots the holdings of long investors in AMC. The y axis is the number of shares held divided by number of shares outstanding plus number of shares sold short.



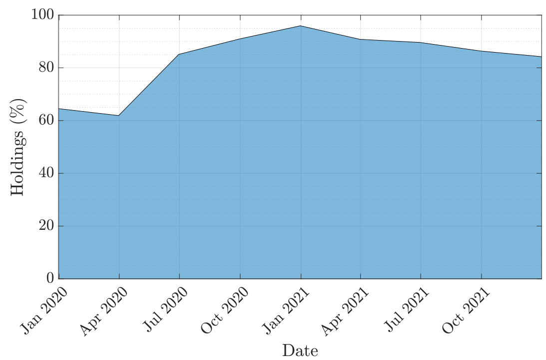
(a) Households / (SHROUT + SS)



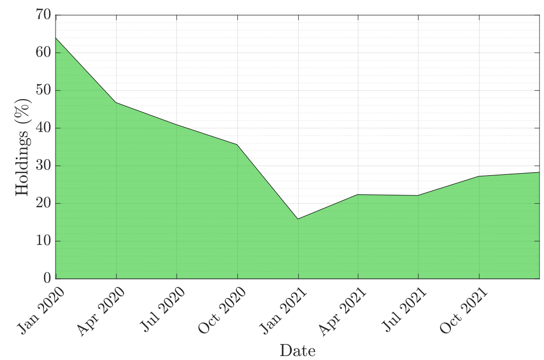
(b) Investment Advisors / (SHROUT + SS)



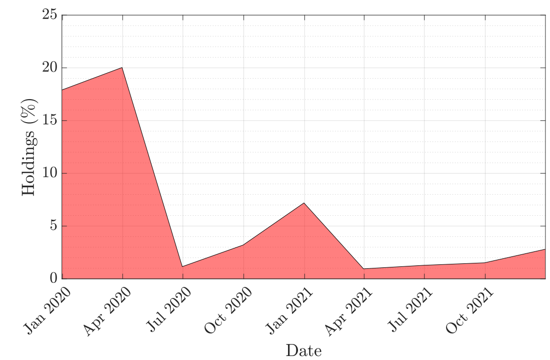
(c) Hedge Funds / (SHROUT + SS)



(d) Households / SHROUT



(e) Investment Advisors / SHROUT



(f) Hedge Funds / SHROUT

Figure A7. Holdings of AMC by investor group. This figure plots the holdings of Households, Investment Advisors and Hedge Funds in AMC. For panel (a), (b), (c), the denominator is the number of shares outstanding plus number of shares sold short. For panel (d), (e), (f), the denominator is the number of shares outstanding.

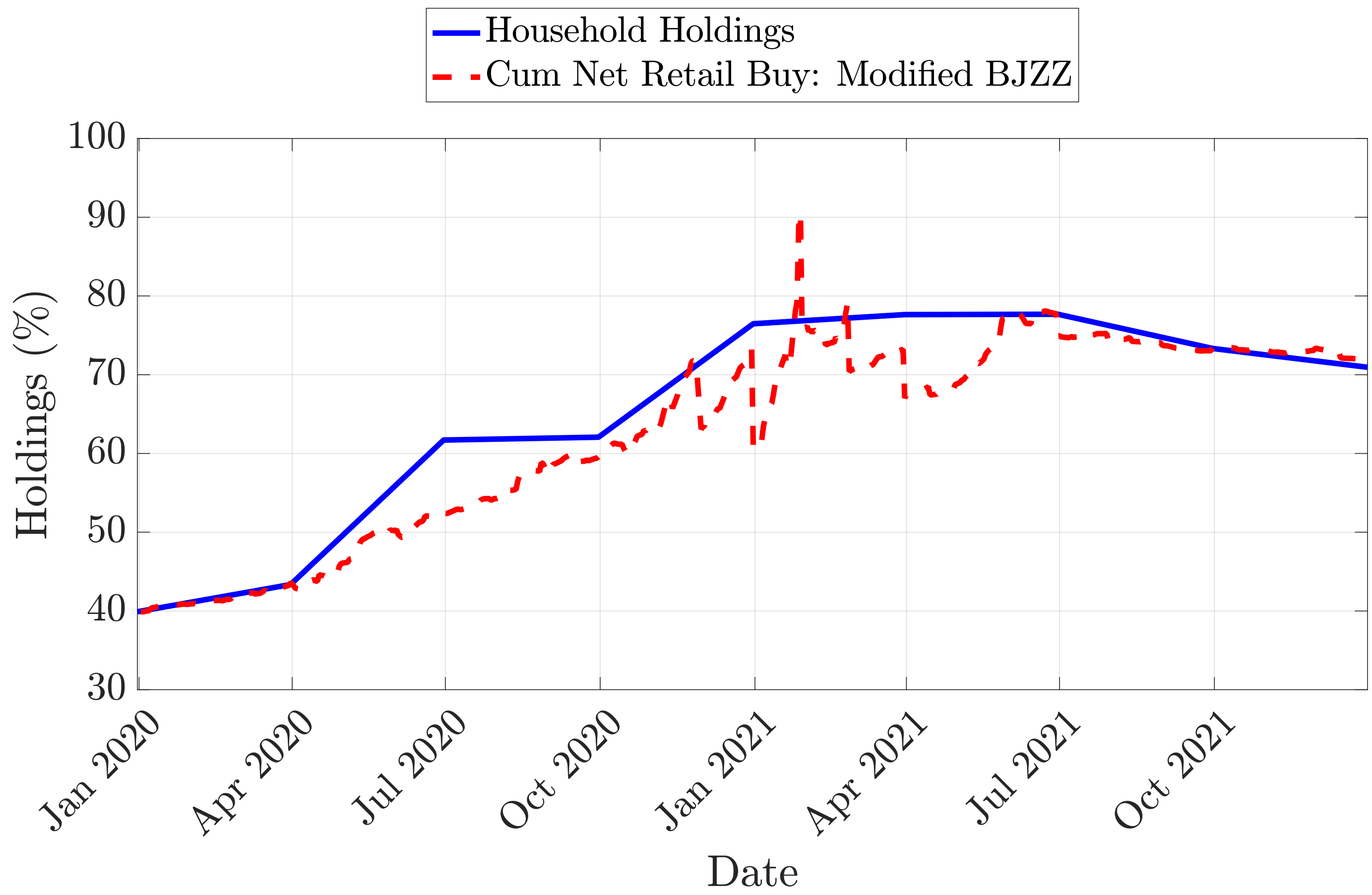


Figure A8. Household holdings versus cumulative net retail buy volume for AMC. This figure plots the quarterly household holdings of AMC (solid blue line) versus the daily cumulative net retail buy volume (dotted red line). The denominator for both series is number of shares outstanding plus number of shares sold short.

Figure A9. Price and short interest of AMC. This figure plots the daily close price of AMC (solid blue line), and the daily short interest (dotted red line). The short interest is the number of shares sold short divided by number of shares outstanding.

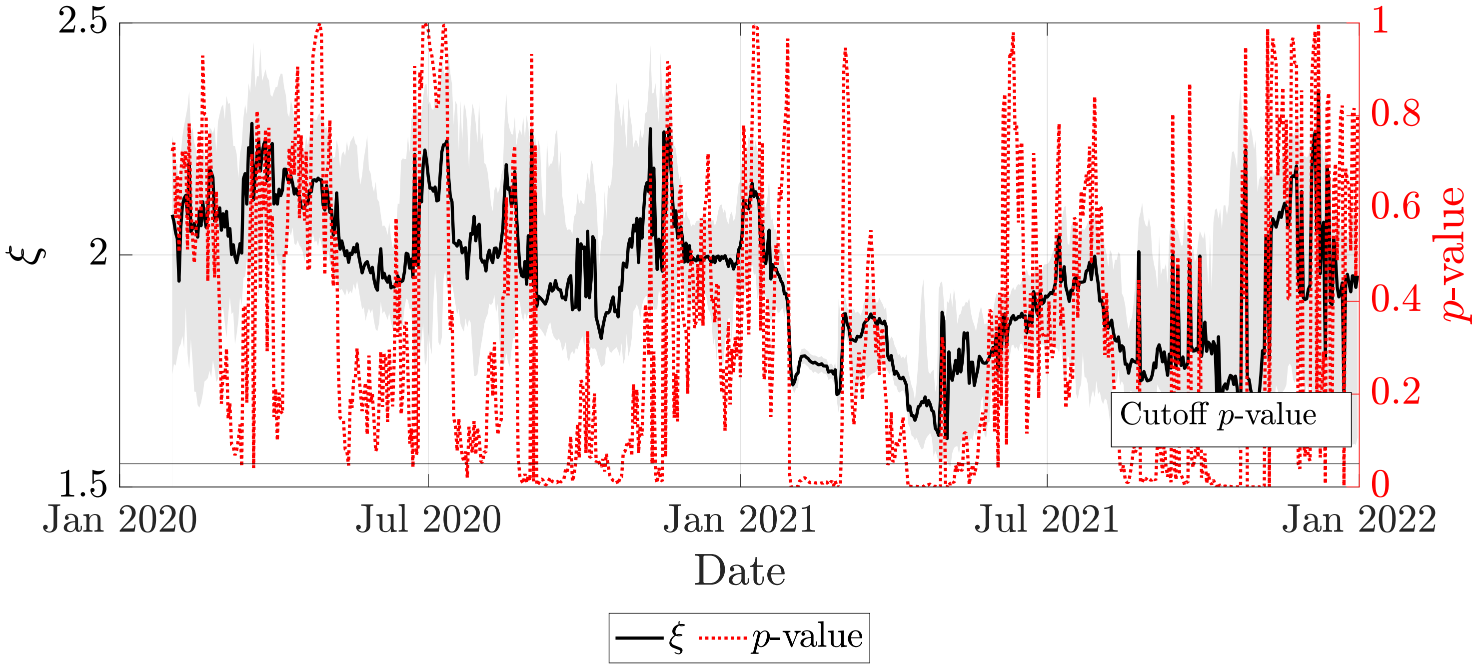


Figure A10. p-value for power-law fitting.

Table A1
Reddit Bots

This table shows the Reddit bots whose submissions are removed from the sample.

Bot Name
WSBVoteBot
RemindMeBot
Generic_Reddit_Bot
ReverseCaptioningBot
LimbRetrieval-Bot
NoGoogleAMPBot
RepostSleuthBot
GetVideoBot
CouldWouldShouldBot

Table A2
Compare Time-0 Equilibria under Different Risk Perceptions

		Value	
Description	Notation	$\tilde{N}_1 = N_L$	$\tilde{N}_1 = N_H$
Log price	p_0	4.249	4.612
Portfolio weights	w_0^R	1.900	1.024
	w_0^{IL}	1.759	1.288
	w_0^{IS}	-0.250	0.539
Num. shares held	Q_0^R	60	34
	Q_0^{IL}	50	52
	Q_0^{IS}	-10	14
Wealth shares	α_0^R	0.316	0.329
	α_0^{IL}	0.284	0.403
	α_0^{IS}	0.400	0.269
Expected log payoff	$\mathbb{E}_0[p_1]$	4.469	5.157
Variance of log return	σ_0^2	0.378	1.015