Supplementary Material for the Manuscript "An Efficient Iterative Approach for Uniformly Representing Pareto Fronts"

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1 Performance profile plots

Let us consider the performance profile plots Uniformity level value, shown in Figure 1. Methods that use ASF pruning consistently achieve better uniformity levels compared to methods that do not use ASF pruning. This is due to the difference of densities of solutions at the boundary of the PF representations compared to the interior, as explained in the previous subsection. Among `Representer` and `Random Pruned`, `Representer` performs better at higher iterations. This is due to the usage of DSS algorithm, which leads to a more uniform representation compared to random sampling.

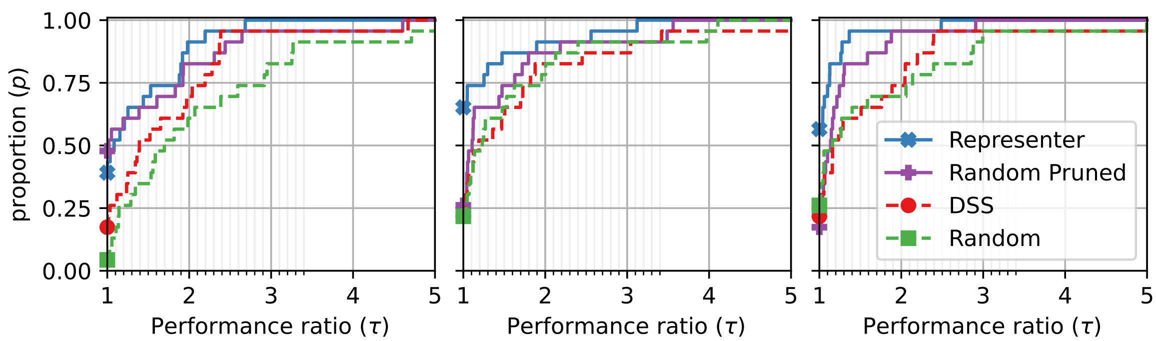

Figure 1: Performance profile plots of the Uniformity level values of the compared approaches in all problems at 50 (left), 200 (center) and 500 (right) iterations. The shared legend is shown in the right plot.

We show the performance profile plots for the IGD indicator values in Figure 2. As the solutions found by the four methods are exact, the IGD values do not reflect convergance to the PF. Instead in this study, IGD is a measure of both coverage and uniformity achieved by the PF representation. At 50 iterations, the `Representer` method achieves significantly better IGD performance than the other three methods. At such a low number of iterations, good coverage is difficult to attain and, therefore, has a significant impact on the attained IGD values. The good performance attained by `Representer` indicates again that it is good at attaining good coverage with few iterations. At higher iterations, most parts of the PF are well covered. Therefore, uniformity has a higher impact on the attained IGD values. At 500 iterations, similar to the performance profile of uniformity levels, methods that utilize ASF pruning achieve better performance than methods that do not.

2 Scatter plot visualizations

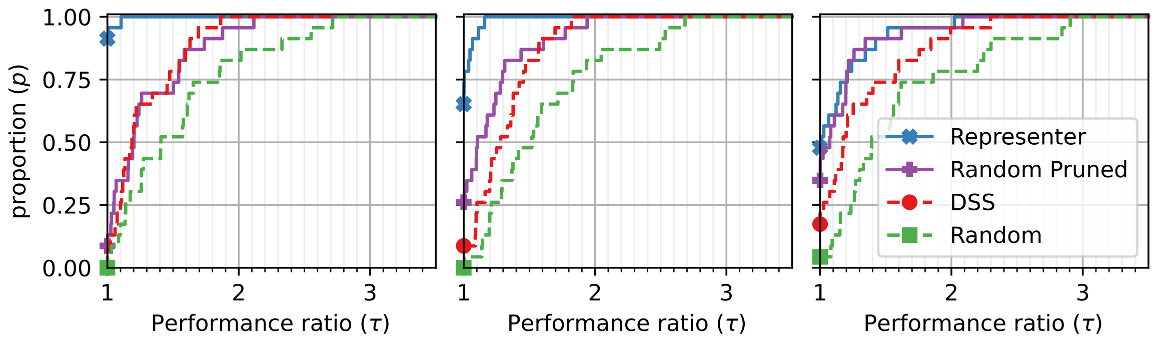

Figure 2: Performance profile plots of the IGD indicator values of the compared approaches in all problems at 50 (left), 200 (center) and 500 (right) iterations. The shared legend is shown in the right plot.

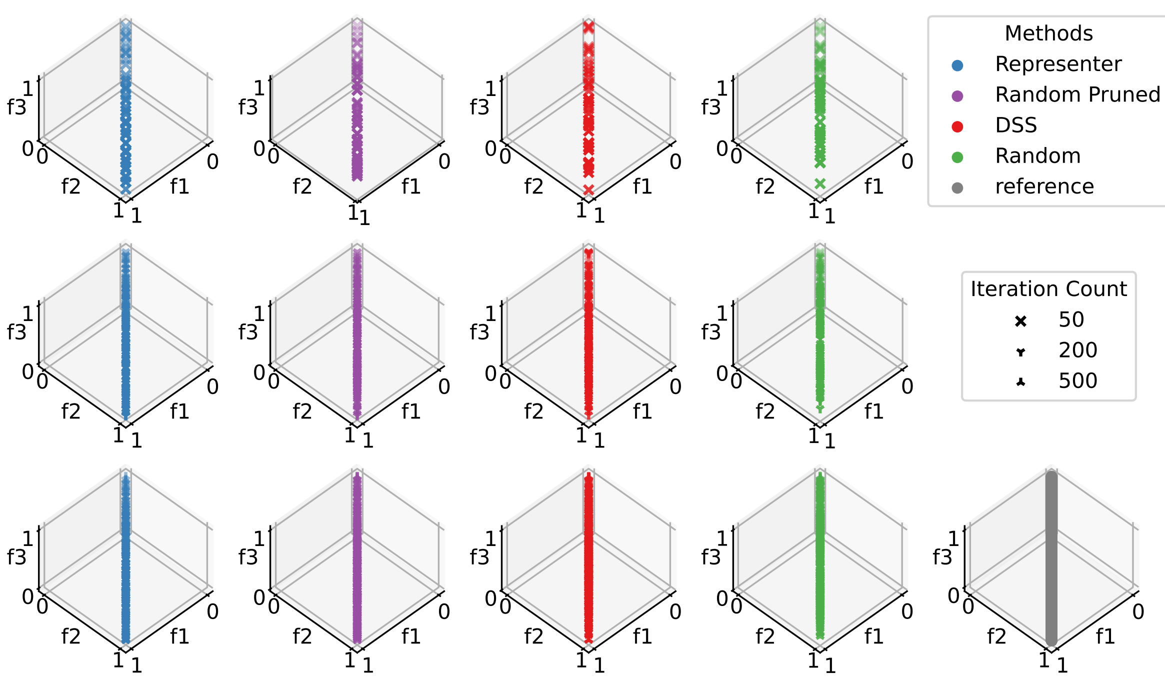

Figure 3: Representations of PFs for a three-objective DTLZ5 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

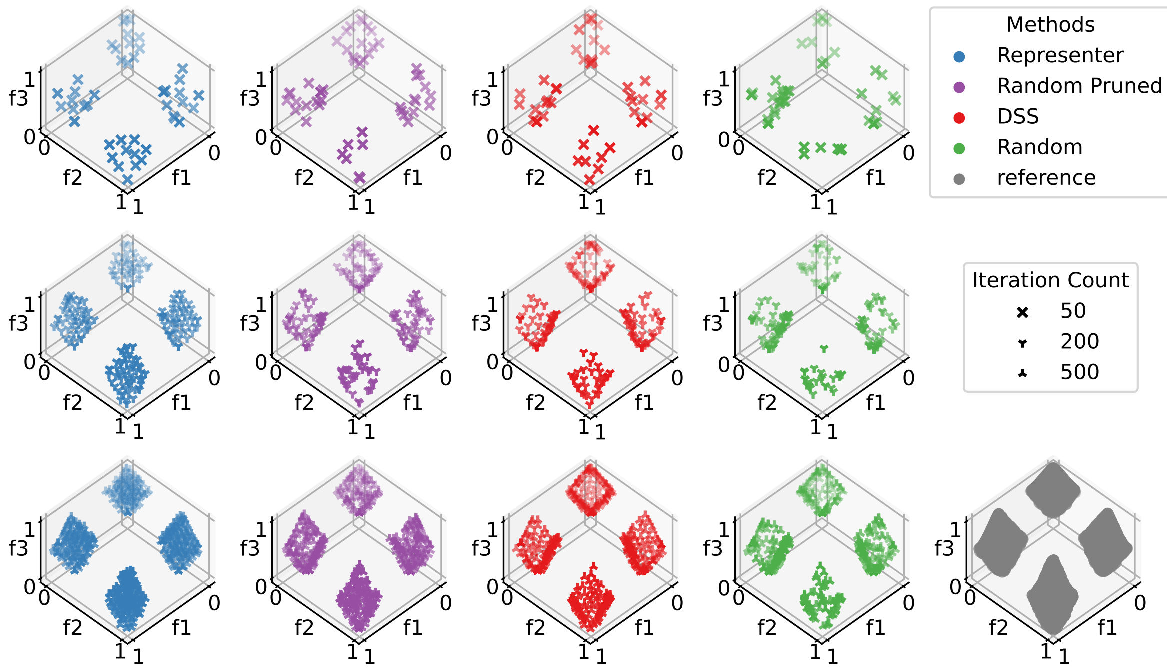

Figure 4: Representations of PFs for a three-objective DTLZ7 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

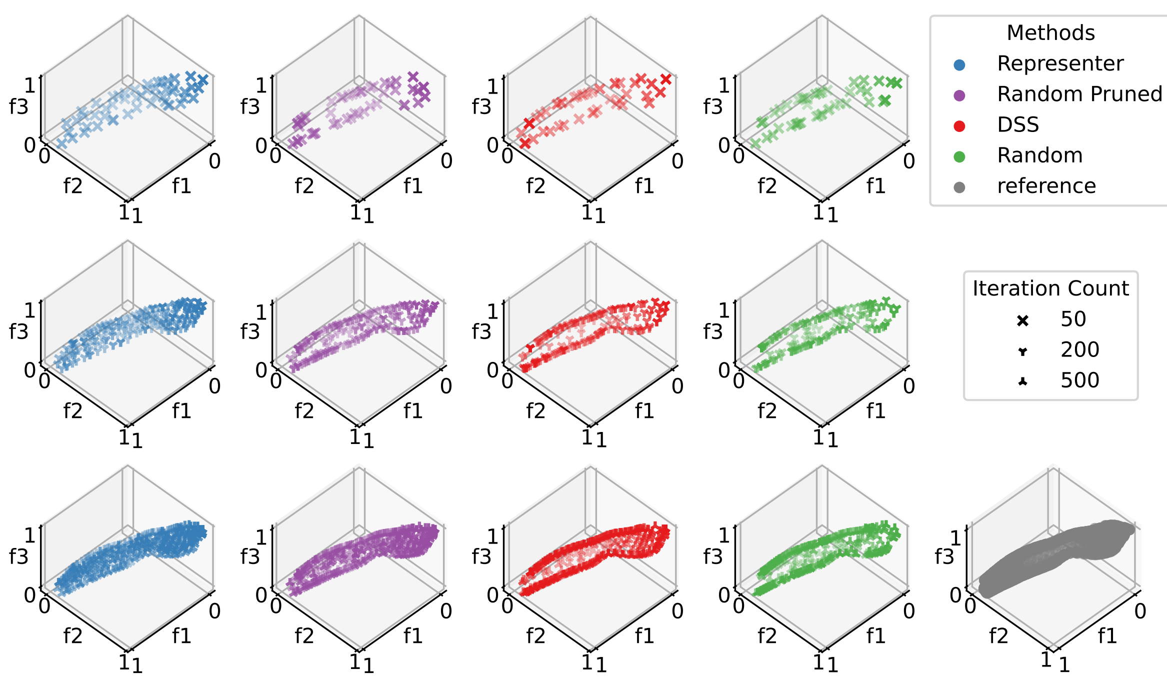

Figure 5: Representations of PFs for a three-objective forest problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

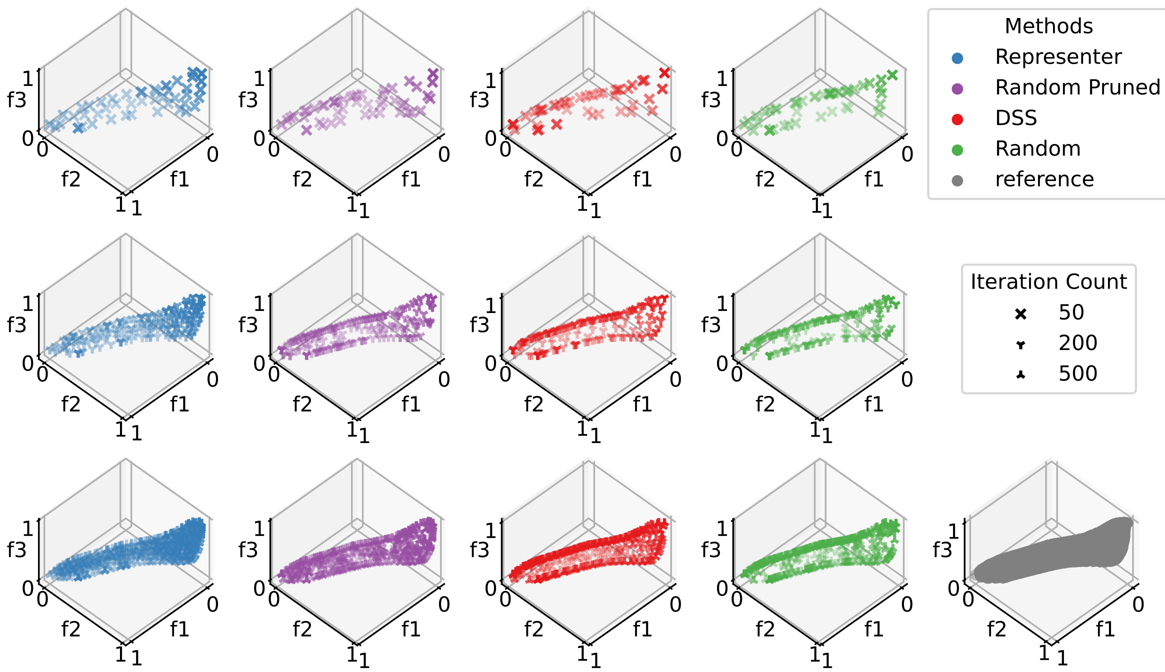

Figure 6: Representations of PFs for a three-objective forest problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

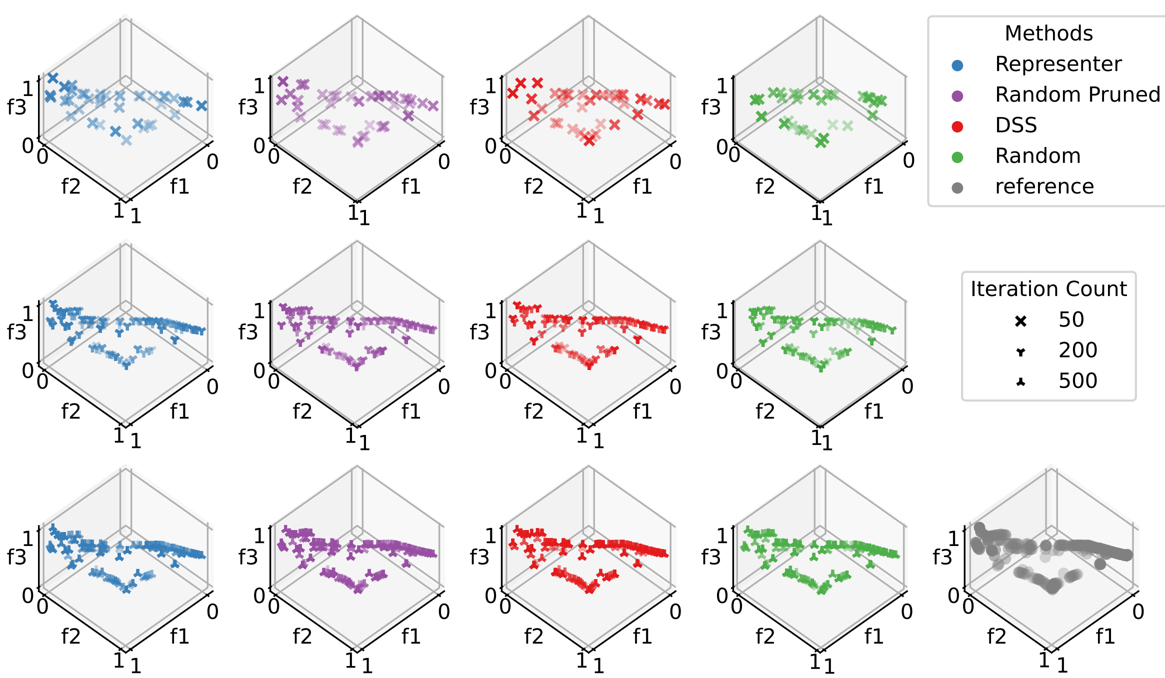

Figure 7: Representations of PFs for a three-objective forest problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

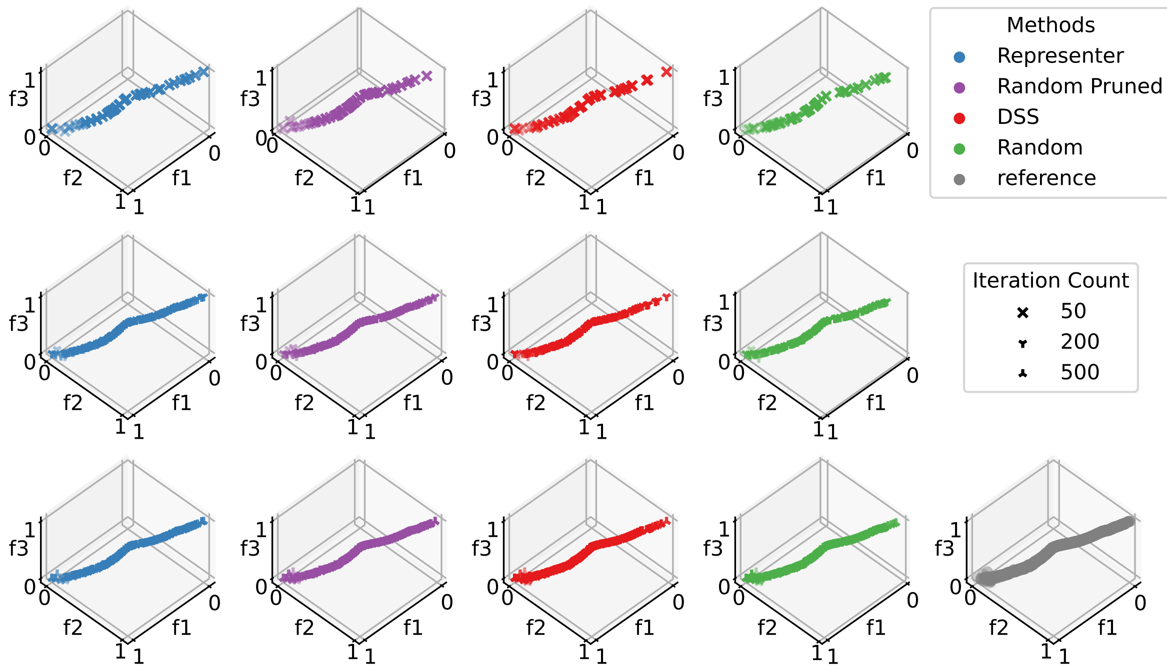

Figure 8: Representations of PFs for a three-objective forest problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

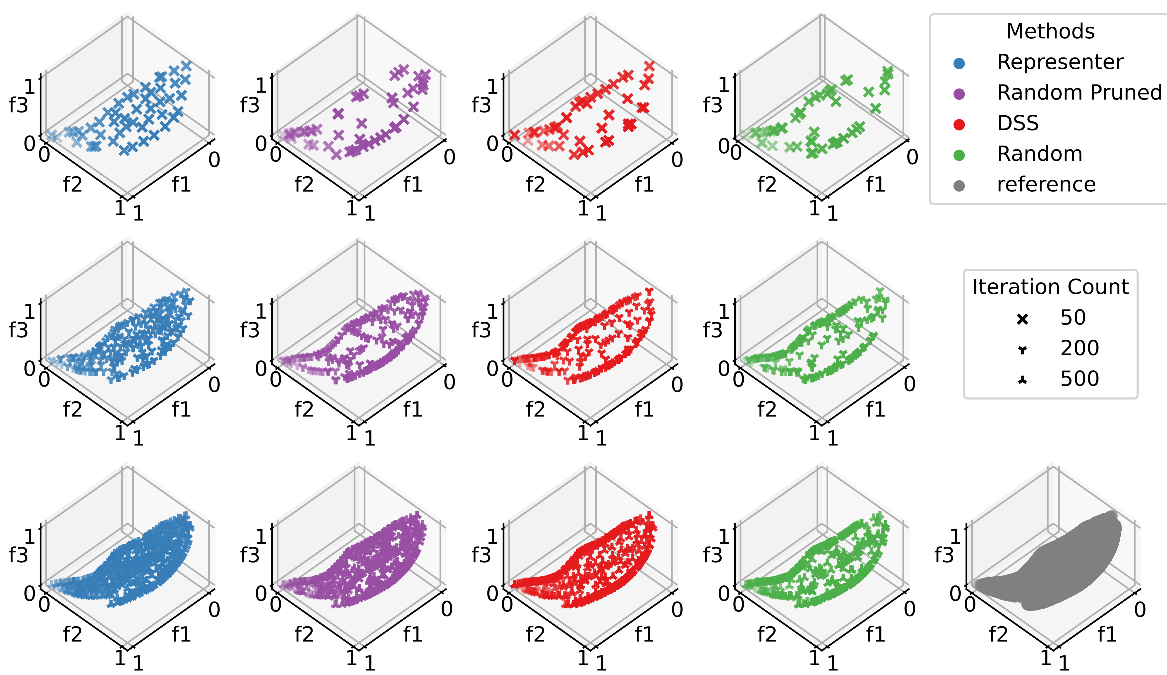

Figure 9: Representations of PFs for a three-objective forest problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

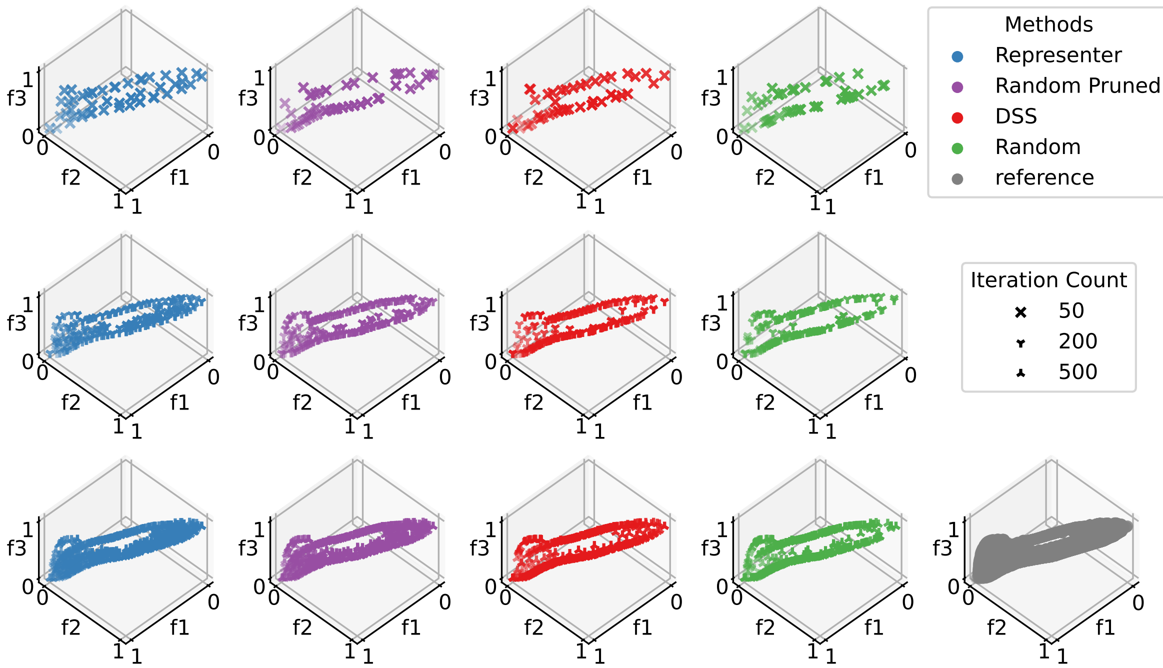

Figure 10: Representations of PFs for a three-objective forest problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

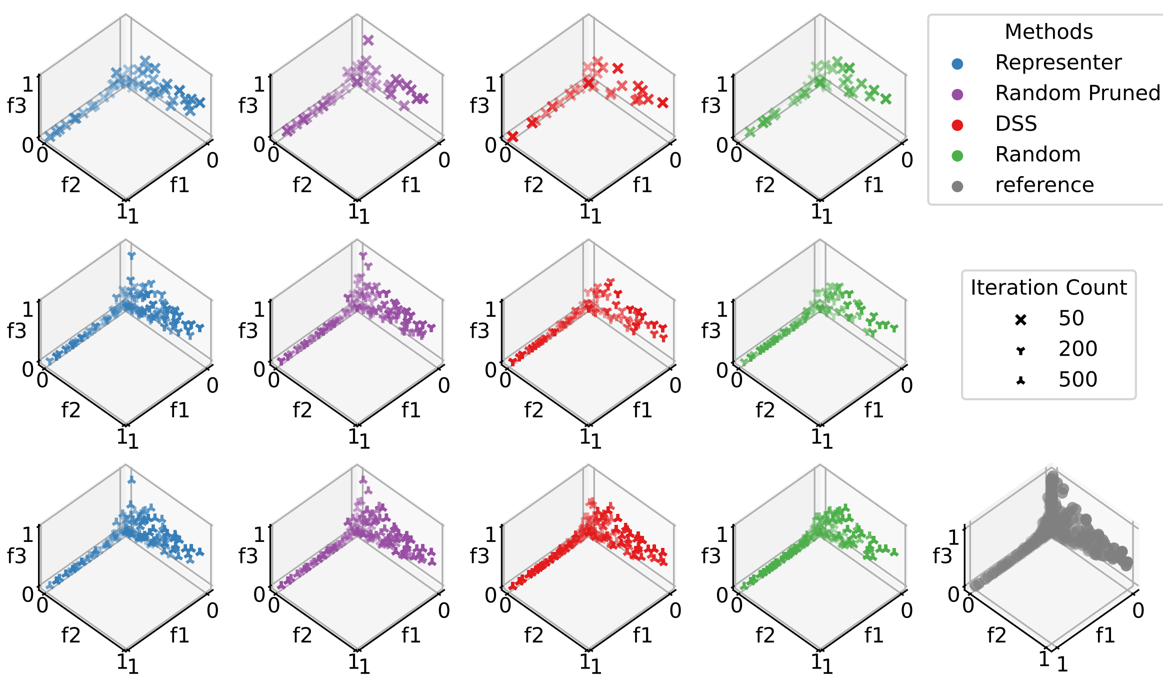

Figure 11: Representations of PFs for a three-objective RE31 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

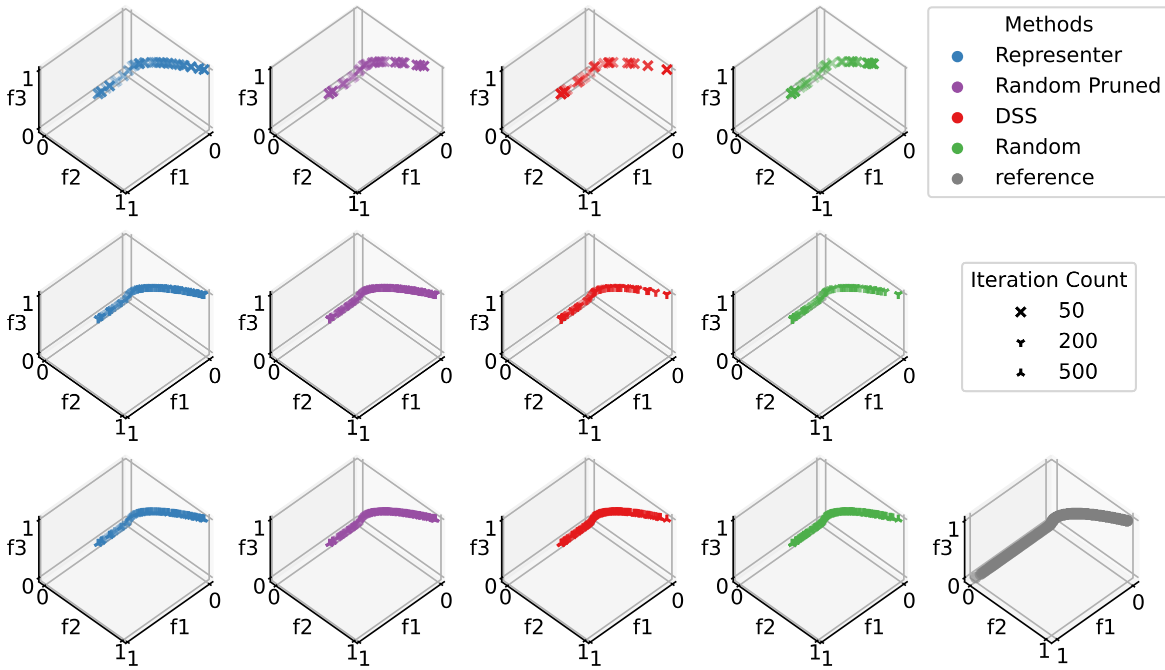

Figure 12: Representations of PFs for a three-objective RE32 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

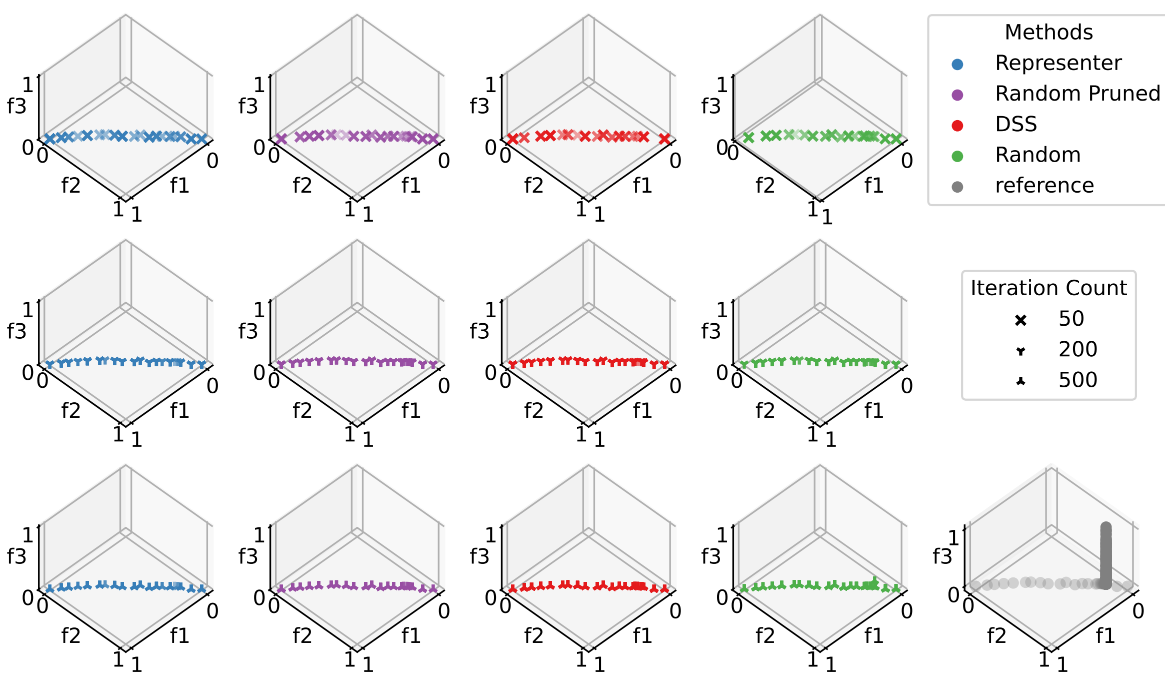

Figure 13: Representations of PFs for a three-objective RE33 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

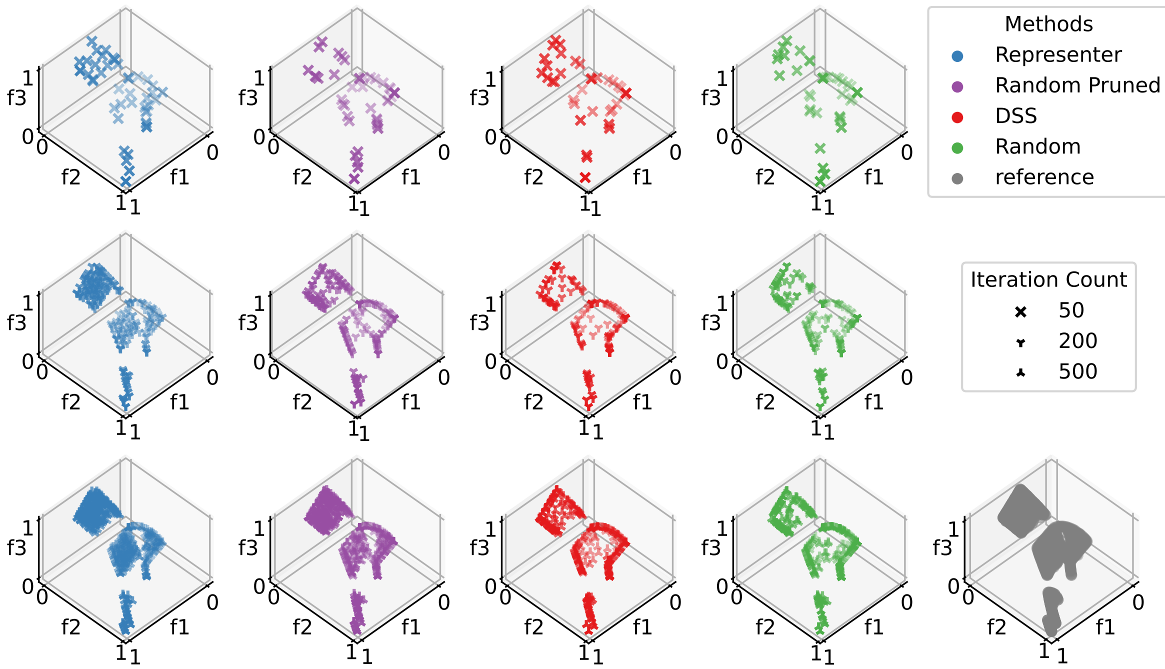

Figure 14: Representations of PFs for a three-objective RE34 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

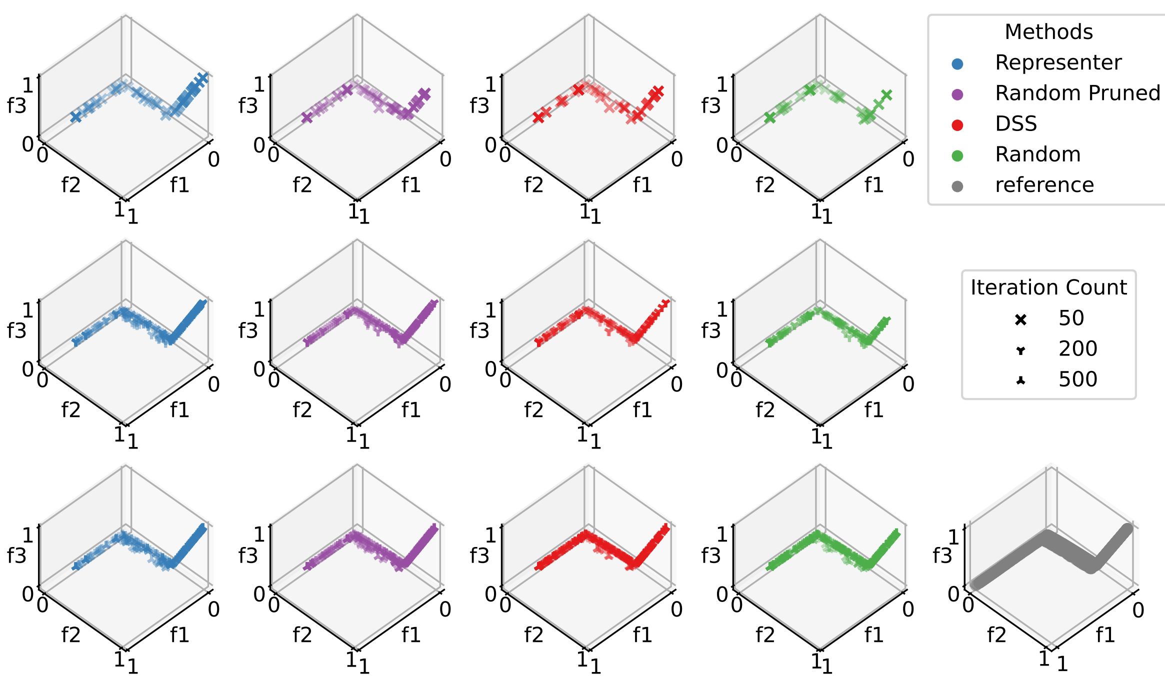

Figure 15: Representations of PFs for a three-objective RE35 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

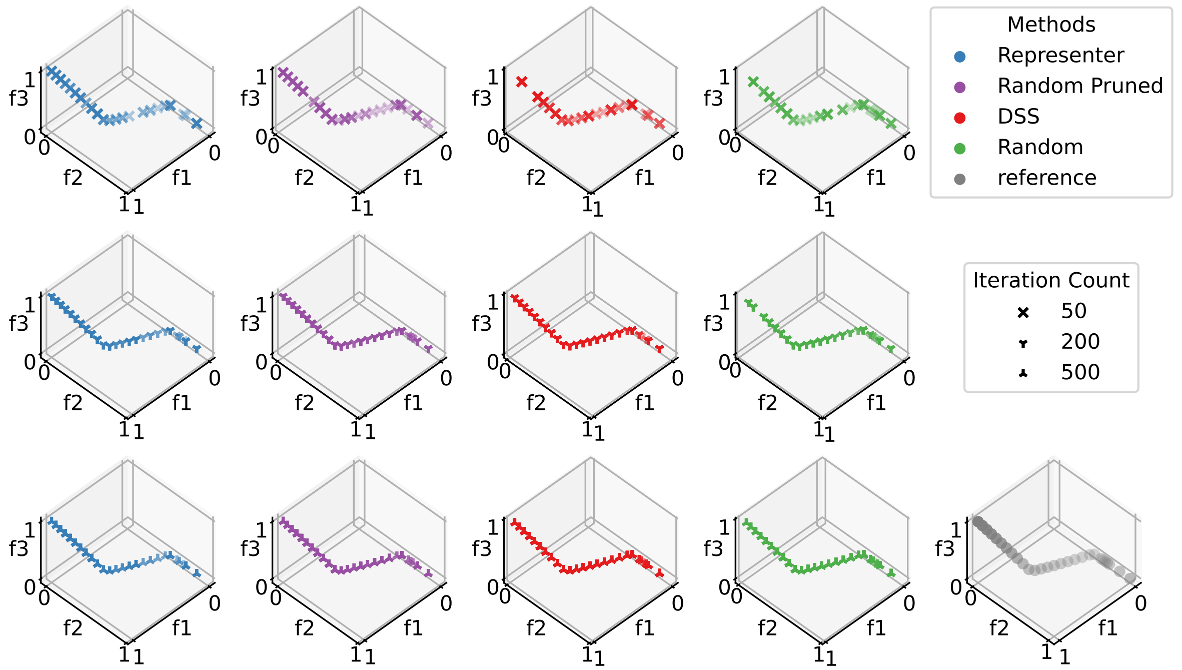

Figure 16: Representations of PFs for a three-objective RE36 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.

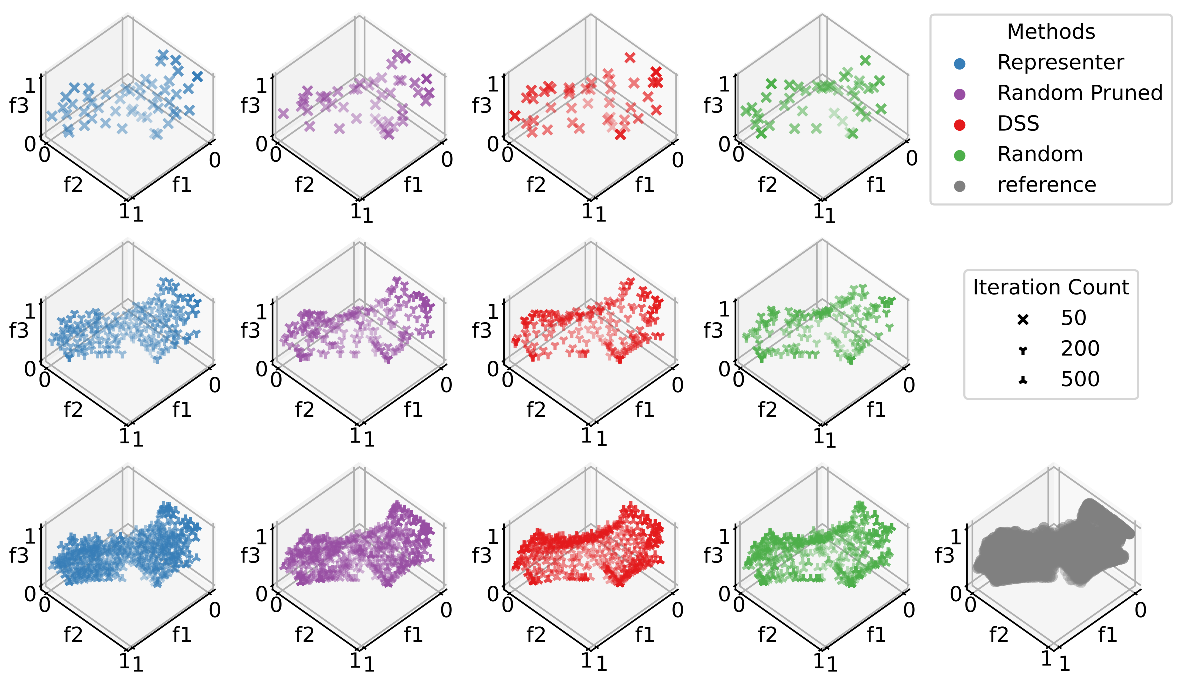

Figure 17: Representations of PFs for a three-objective RE37 problem generated by four approaches at 50, 200, and 500 iterations. A reference PF is also shown.