HW 2: Real-World Channels and Fading

ECE 5680 – Wireless Communication

- **Due date:** Thursday, September 24, 2015 at 11:59pm
- **Submission instructions:** Upload your solutions (solutions and Matlab code) to Cornell Blackboard as a *single* zip-file; name the file ⟨netid⟩-hw⟨number⟩.zip. If you want to get detailed comments on your homework olutions, create a *single* pdf file that contains all answers. I strongly suggest the use of LaTeX to typeset your responses, but a scanned version of your handwritten answers is also OK.
- **Total points:** 100pts

Problem 1: Complex random vectors (20pts)

Let $\mathbf{x} \in \mathbb{C}^N$ be an N-dimensional zero-mean complex-valued random vector.

Part 1 Define the following real-valued $2N$-dimensional zero-mean random vector

$$\mathbf{z} = \begin{bmatrix} \mathbf{x}_R \\ \mathbf{x}_I \end{bmatrix},$$

where $\mathbf{x}_R$ and $\mathbf{x}_I$ denote the real and imaginary parts of the complex-valued vector $\mathbf{x}$, respectively. Express the covariance matrix $\mathbf{K}_\mathbf{z} = \mathbb{E}\left[\mathbf{z}\mathbf{z}^T\right]$ of the real-valued vector $\mathbf{z}$ as a function of the covariance matrix $\mathbf{K}_\mathbf{x} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right]$ and the pseudo-covariance matrix $\mathbf{J}_\mathbf{x} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^T\right]$ of the complex-valued vector $\mathbf{x}$.

Remark: This result implies that the covariance matrix $\mathbf{K}_\mathbf{x}$ and the pseudo-covariance matrix $\mathbf{J}_\mathbf{x}$ completely characterize the second-order statistics of the complex-valued random vector $\mathbf{x}$!

Part 2 Assume now that the complex-valued random vector $\mathbf{x}$ is circularly symmetric, i.e., that for any $\phi \in [0, 2\pi)$ the distribution of $\mathbf{x}e^{j\phi}$ is equal to the distribution of $\mathbf{x}$. Show that the covariance matrix $\mathbf{K}_\mathbf{z}$ is completely characterized by $\mathbf{K}_\mathbf{x}$ only. *Hint: First show that* $\mathbf{J}_\mathbf{x} = \mathbf{0}$.

Part 3 What does the result obtained in Part 2 tell you about the correlation between the real and the imaginary parts of the circularly-symmetric complex-valued vector $\mathbf{x}$? *Hint: Are they dependent?*

Part 4 Assume that each entry x_k, $k = 1, \ldots, N$, in the N-dimensional complex-valued vector $\mathbf{x}$ is modeled as $x_k = x_R + jx_I$ with $x_R, x_I \sim \mathcal{N}(0, N_0/2)$; we also assume that all entries are independent. Compute the covariance matrix $\mathbf{K}_\mathbf{x}$ and pseudo-covariance matrix $\mathbf{J}_\mathbf{x}$. Is the vector $\mathbf{x}$ circularly symmetric?

Problem 2: Categorizing real-world fading channels (20pts)

Tables 1 and 2 list typical (measured) physical-layer parameters of common wireless systems. Use the information provided in the tables to classify the radio channel of the following wireless communication systems into the different channel types learned in class. *Hint: The delay spread is determined by the environment; the Doppler spread depends on the velocity of the terminal and scattering objects. Use common sense to come up with a suitable estimate for the Doppler spread and select appropriate delay-spread numbers from Table 1.*

Example We consider a 900 MHz GSM system operating in a hilly environment, where the mobile terminal moves on a freeway with about 75 mph (which is roughly 33.3 m/s). The corresponding maximum Doppler spread for the 900 MHz carrier frequency of GSM can be calculated as follows:

$$D_s = \frac{vf}{c} = \frac{33.3 \times 900 \cdot 10^6}{3 \cdot 10^8} \approx 100\,\text{Hz}.$$

Here, v is the velocity, f is the carrier frequency, and c the speed of light. As discussed in class, the Doppler spread D_s and the coherence time T_c are inversely proportional. In practice, several different proportionality constants are used; for the sake of simplicity, we use $T_c = 1/D_s$. Hence, we can compute the coherence time for the scenario at hand:

$$T_c = \frac{1}{100\,\text{Hz}} \approx 10\,\text{ms}.$$

From Table 2, we note that the frame/slot length[1] of the 900 MHz GSM system is 0.577 ms. As a consequence, roughly 10ms/0.577ms = 17 frames experience the same channel. Hence, we can assume the channel to be *slow fading* for the duration of a frame/slow in our scenario. We next assume that the system is operating in a hilly environment, possibly rural (one is usually not driving through an office with 75 mph). Thus, comparing the system with the measurements for the residential areas of San Francisco, CA (see Table 1), we can assume a maximum delay spread of $T_d = 25\,\mu\text{s}$, which corresponds to a minimum coherence bandwidth of

$$B_c = \frac{1}{25 \times 10^{-6}\,\text{s}} = 40\,\text{kHz},$$

where we assumed $B_c = 1/T_d$. From Table 2, we note that the channel bandwidth for the considered 900 MHz GSM system is 200 kHz and hence, the channel is *frequency selective* for the considered scenario. In summary, the channel can be classified as *slow fading* and *frequency selective*—now it's your turn.

Part 1 Characterize the channel of an IEEE 802.11b wireless local area network (WLAN) in an office. Clearly state all your assumptions.

Part 2 Characterize the channel of an UMTS terminal operated on board of a high-speed train moving at 155 mph and passing through a rural environment. Clearly state all your assumptions.

Remark: The parameter description of wireless channels reduces all the information contained in the scattering function to just a few numbers. Consequently, the resulting channel characterization is rather imprecise. The relation between delay spread and coherence bandwidth, and Doppler spread and coherence time is only approximate, so that some systems cannot be classified unambiguously.

[1] Communication is performed in bursts called frames or slots, i.e., transmission only happens over short amounts of time.

Table 1: Measured channel parameters

Environment	Carrier freq. [MHz]	Bandwidth [MHz]	Delay spread [μs]	Notes
urban	910	10	2	Manhattan
suburban 1	910	10	0.24	5 km Tx–Rx separation
suburban 2			2	NLOS worst case
urban	910	10 (?)	0.73	Toronto
suburban			0.59	
urban	892	4	1.69	Washington, D.C.
urban			1–3	Oakland, CA
residential			10–25	San Francisco, CA
urban	942.225	4	<2.7	Hamburg
urban			<2	Stuttgart
urban			<3	Düsseldorf
urban			<2.9	Frankfurt
urban	910	20	0.3–0.7	mircrocell
urban	1718	<8	0.9–4.7	Bergen,
rural			2.8–14.4	Norway
factory	1300	200	0.03–0.30	several distances
office	910	?	0.036–0.080	LOS & NLOS
office 1	1100	400	0.01–0.05	cubicle offices
office 2			0.02–0.03	
office	1500	200	0.05	clustered arrivals
office	2400	200	0.025–0.074	LOS & NLOS
supermarket			0.035–0.083	
office	2400	500	0.005–0.015	small Tx–Rx separation
office	5200	120	0.0065	
foyer			0.0092	
lab			0.0108	
office	5250	100	0.025–0.05	MIMO

Table 2: Parameters of common wireless systems

System	Frequency band uplink [MHz]	Frequency band downlink [MHz]	Channel BW [MHz]	Frame/slot length [ms]	Access	Duplex
GSM	890–915 1710–1785 1850–1910	935–960 1805–1880 1930–1990	0.2	0.577	TDMA	FDD
IS-136	824–849	869–894	0.03	6.667	TDMA	FDD
IS-95	824–849	869–894	1.25	20	CDMA	FDD
UMTS	1885–2025	2110-2200	5	10	CDMA	FDD/TDD
DECT	1880–1900	1880–1900	1.728	0.417	TDMA	TDD
IEEE 802.11b	2400–2484	2400–2484	22	<18.96	CSMA/CA	TDD
IEEE 802.11a	5150–5350 5725–5825	5150–5350 5725–5825	20	<5.476	CSMA/CA	TDD
IEEE 802.11g	2400–2484	2400–2484				

Problem 3: Error-rate performance of QPSK signaling in an AWGN channel (30pts)

In practice, one is often interested in transmitting multiple bits at a time. While BPSK only transmits a single bit per constellation symbol, quadrature phase-shift keying (QPSK) transmits two bits per symbol. The goal is to characterize the symbol error rate (SER) of QPSK in a complex additive white Gaussian noise (AWGN) channel. Consider the input-output relation $y = x + n$, where $y \in \mathbb{C}$ is the received signal,

$$x \in \mathcal{X} = \{+1 + j, +1 - j, -1 + j, -1 - j\} \tag{1}$$

is the QPSK transmit signal (assume that all symbols are transmitted with probability 0.25), and n is circularly symmetric complex Gaussian noise with variance N_0, i.e., $n \sim \mathcal{CN}(0, N_0)$. Let $E_s = \mathbb{E}\left[|x|^2\right]$ be the transmit signal power and $N_0 = \mathbb{E}\left[|n|^2\right]$, and define the signal-to-noise ratio (SNR) as $SNR = E_s/N_0$.

Remark: Always remember that your SER results (from simulations or analytical expressions) will be completely wrong if your definition of the SNR is incorrect.

Part 1 Derive the optimal decision rule for the above QPSK system, which uses the received signal y to generate an estimate $\hat{x} \in \mathcal{X}$ for which the symbol error rate $\Pr\{x \neq \hat{x}\}$ is minimized.

Part 2 As in Problem 4 of Homework 1, write a MATLAB script that performs Monte-Carlo simulations to compute the symbol error rate (SER). Perform $T = 10,000$ trials and sweep the SNR between -10 dB and $+20$ dB. Plot the resulting SER vs. SNR performance as in Homework 1.

Part 3 Extend your complex-valued simulator from Part 2 with BPSK transmission, i.e., use the constellation set $\mathcal{X} = \{-1, +1\}$, and compute the SER for the same number of trials and SNR range as in Part 2. Generate a plot that combines the SER performance of QPSK and BPSK. Explain the difference between both SER curves? *Hint: Always use intuition to check whether the results you get actually make sense.*

Problem 4: Error-rate performance of QPSK signaling in a fading channel (30pts)

Virtually all wireless channels experience fading. The goal of this problem is to characterize the SER of a system with a simple flat-fading channel model, where the wireless channel can be represented by a single discrete-time filter tap $h[k] \sim \mathcal{CN}(0, 1)$; this simple fading model is called Rayleigh fading. For every transmission k, the discrete-time input-output relation of the flat-fading channel is

$$y[k] = h[k]x[k] + n[k],$$

and we assume the channel gain (or filter tap) $h[k]$ to be known at the receiver and independent of the transmit signal and the noise. The other quantities $y[k] \in \mathbb{C}$, $x[k] \in \mathcal{X}$, and $n[k] \sim \mathcal{CN}(0, N_0)$, are—as usual—the receive signal, transmit signal, and noise, respectively. In the following, assume QPSK signaling (1) and ignore the transmission (or sample) index k to simplify notation. Furthermore, assume a coherent detector, i.e., that the receiver perfectly knows the channel gain $h[k]$.

Remark: In practical systems, the channel coefficient $h[k]$ cannot be known perfectly. Nevertheless, there exist sophisticated channel-estimation methods that acquire accurate estimates of $h[k]$.

Part 1 The common SNR definition used in fading wireless systems is the so-called "average receive SNR," which is defined as $SNR = \mathbb{E}_{h,x}\left[|hx|^2\right]/\mathbb{E}\left[|n|^2\right]$. Compute the average receive SNR for the above fading model and QPSK signaling (1). Also derive the maximum a-posteriori (MAP) detection rule.

Part 2 Similarly to Problem 3 of this homework, write a MATLAB script that performs Monte-Carlo simulations to compute the *average SER*, which is given by $P_e = \mathbb{E}_h[\Pr\{x \neq \hat{x}|h\}]$. Perform $T = 10,000$ trials and sweep the SNR between $-10\,\text{dB}$ and $+20\,\text{dB}$. Plot the resulting average SER vs. SNR performance in a doubly-logarithmic plot as in Problem 3. *Hint: To obtain the average SER, you have to generate a new channel coefficient $h[k]$, transmit symbol $s[k]$, and noise realization $n[k]$, for every transmission (sample k).*

Part 3 Combine the SER results from Problem 3, Part 2, with the average SER results of the above Part 2 in a single MATLAB plot. Can you intuitively explain the striking differences between data transmission in a non-fading AWGN channel (Problem 3) and that of a wireless system with fading (this Problem 4)?

Remark: Your simple MATLAB simulation script already has a variety of parameters, such as the channel model (fading or AWGN) or modulation scheme (BPSK or QPSK). Since you are going to add more and more parameters and implement a variety of capabilities in future homework assignments, you should think about how to make your simulator modular and, most importantly, fast.