HW 4: Advanced SIMO and MISO Communication

ECE 5680 – Wireless Communication

- **Due date:** Friday, October 23, 2015 at 11:59pm
- **Submission instructions:** Upload your solutions (solutions and Matlab code) to Cornell Blackboard as a *single* `zip`-file; name the file ⟨netid⟩-hw⟨number⟩.zip. Include all simulation plots in either `spdf` or `eps` format in your archive. If you want to get detailed comments on your homework solutions, create a *single* `pdf`-file that contains all answers. I strongly suggest the use of LaTeX to typeset your responses, but a scanned version of your handwritten answers is also OK.
- **Total points:** 100pts

Problem 1: Higher-order constellations and Gray mapping (10pts)

So far, we have only considered BPSK and QPSK constellation sets, which allowed for the transmission of 1 bit or 2 bit per time slot, respectively. In this problem, we simulate the performance of higher-order constellations, which allow us to transmit more bits per time slot. We will furthermore compare the symbol error rate (SER) versus the so-called *bit error rate* (BER). For simplicity, we consider a non-fading single-input single-output (SISO) additive white Gaussian noise (AWGN) channel, which we model as $y = x + n$, where $y \in \mathbb{C}$ is the received vector and $n \sim \mathcal{CN}(0, N_0)$ is the additive Gaussian noise.

Part 1 In practice, if the signal-to-noise ratio (SNR) of the channel is high (e.g., if you are close to the base station), we can transmit more than just 2 bits (e.g., with QPSK) over the wireless channel per time slot. Fig. 1 summarizes the most prominent higher-order constellation sets. The goal of this part is to simulate the symbol error-rate (SER) performance for BPSK (not shown), QPSK, 16-QAM, 64-QAM, and 8-PSK as done in previous homework assignments. Assume equally likely transmit symbols and declare an error whenever $x \neq \hat{x}$, where $\hat{x}$ is the estimate, $x \in \mathcal{X}$ the originally transmitted symbol, and $\mathcal{X}$ the constellation set. Combine all SER vs. SNR curves in a single MATLAB plot. *Hint: Be careful with your SNR calculations.*

Remark: You should see that for higher-order constellations, you also need a higher SNR to achieve the same symbol error rate. This means that in practice to use, for example, 64-QAM, you need less noise to enable reliable communication. At low SNR, you typically have to use BPSK.

Part 2 So far, we have always compared the SER performance. In practice, however, one is often interested in the so-called bit error rate (BER), which corresponds to the rate of bits that were decoded in error. To obtain the BER, one needs to know the mapping between bit labels and constellation points. Example bit mappings are shown in Fig. 1. All these mappings are so-called Gray-mappings, for which the labels of neighboring constellation points only differ in one bit. This has the advantage that if you confuse one constellation with its neighbor (which is the typical error at high SNR), only one bit error is generated.

Simulate the BER for all constellation sets. Combined all BER vs. SNR curves in a single MATLAB plot.

Remark: While the SER does not depend on the bit mapping, the BER does. For the reason explained above, most practical communication systems use Gray mappings. One could, however, come up with other mappings as well. For example, so-called anti-Gray mappings try to flip as many bits as possible for neighboring constellation points. If one confuses two neighboring constellation points for such anti-Gray mappings, then more bits would be in error than for the Gray-mapping case.

Part 3 In a large number of papers, you may find the quantity E_b/N_0 on the x-axis of an error-rate performance plot instead of the average receive SNR as defined in class. For the considered AWGN system, the E_b/N_0 is the "energy per bit to noise power" and is defined as follows:

$$\frac{E_b}{N_0} = \frac{\mathbb{E}[|x|^2]/\rho}{\mathbb{E}[|n|^2]} = \frac{E_s/\rho}{N_0} = \frac{SNR}{\rho},$$

where ρ is the number of bits per constellation symbol (e.g., for 8-PSK, we have $\rho = 3\,\text{bit}$). Plot the BER performance of Part 2 against E_b/N_0 in decibel (dB). Explain the difference of this plot to the one obtained in Part 2. *Hint: You don't have to re-simulate the curves, you just need to redefine the x-axis for each constellation set.*

Problem 2: SIMO wireless system with channel estimation (15pts)

In previous homework assignments, we have always assumed that for coherent detection, the receiver knows the channel perfectly. In practice, however, the receiver must acquire an estimate of the channel and then, use this estimate for data detection. In this problem, we will study the error-rate performance of a single-input single output (SIMO) wireless communication system that includes channel estimation and coherent detection.

The principle of channel estimation-based coherent detection is as follows. Communication is divided into two phases. In the first phase, the transmitter sends so-called *pilot symbols*, which are known to the receiver. The receiver can then compute an estimate of the wireless channel. In the second phase, the transmitter sends data symbols (the so-called payload). The receiver uses the estimated channel to detect the transmitted data symbols in a coherent fashion. To ensure that this two-phase approach works in practice, the coherence time of the channel must be larger than the duration of both phases. Put simply, the channel must remain constant over both phases. Note that this transmission scheme is used in virtually all practical wireless systems, including IEEE 802.11 (wireless LAN) or 3GPP LTE.

Consider a SIMO wireless systems with $M_T = 1$ transmit antenna and $M_R \geq 1$ receive antennas. Consider a flat Rayleigh fading channel (i.e., a channel which is frequency flat) and assume that the coherence time is larger than 2 time slots. Furthermore, assume that we are transmitting equally-likely BPSK signals $x \in \{+1, -1\}$. As usual, the input-output relation of this system can be modeled as

$$\mathbf{y}[t] = \mathbf{h}x[t] + \mathbf{n}[t], \; t \in \{1,2\}, \tag{1}$$

where $t = 1$ corresponds to the training phase in time slot 1, and $t = 2$ to the data phase in time slot 2. In (1), $\mathbf{y} \in \mathbb{C}^{M_R}$ is the M_R-dimensional receive vector (each entry corresponds to the signal obtained at each receive antenna), $\mathbf{h} \in \mathbb{C}^{M_R}$ models the individual paths from the transmit antenna to the M_R receive antennas (note that the channel vector $\mathbf{h} \in \mathbb{C}^{M_R}$ remains constant over both time slots), and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_{M_R})$ models additive Gaussian receive noise. Assume that the receive antennas are spaced

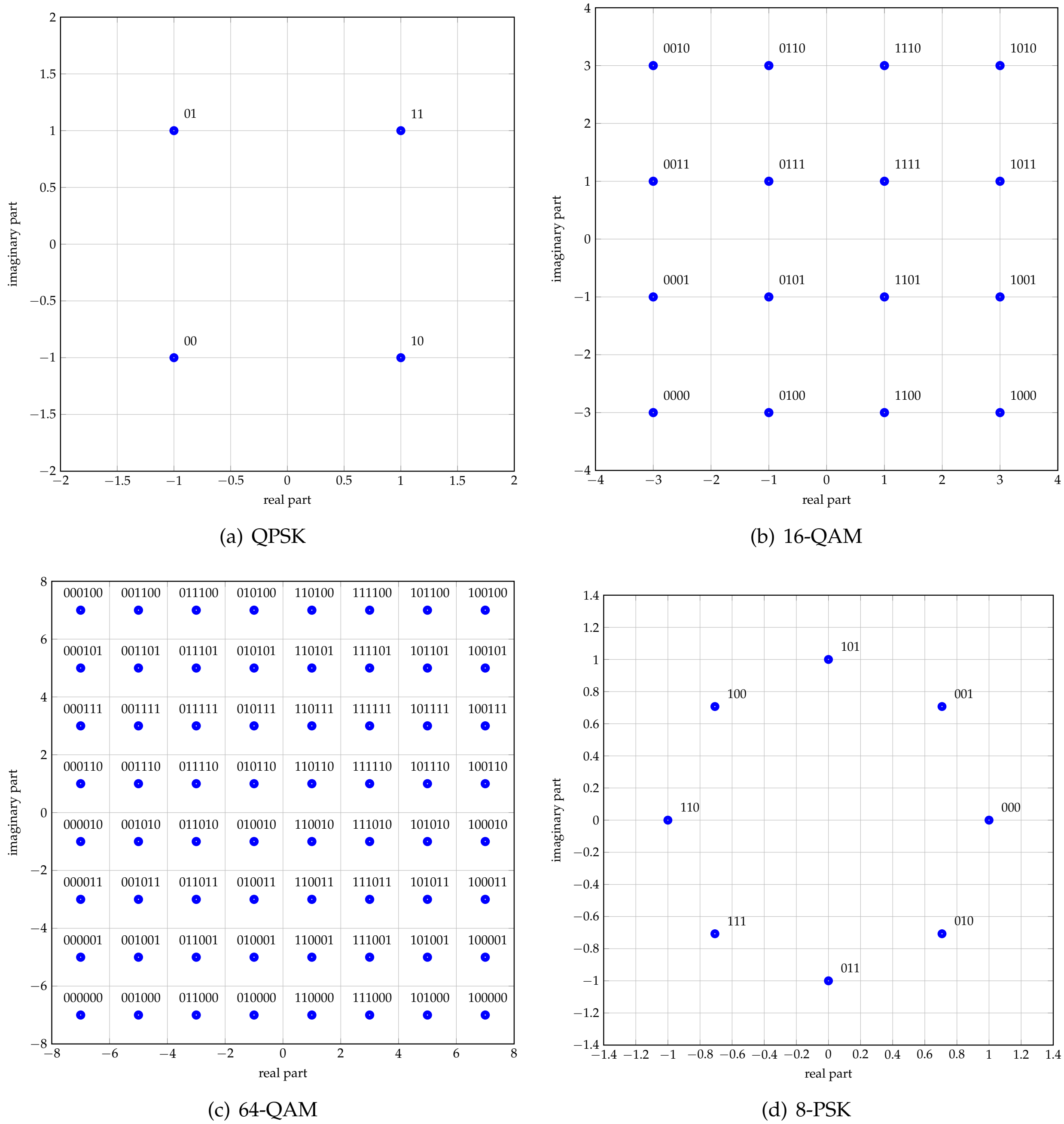

Figure 1: Higher-order modulation schemes with Gray mapping, where the bit labels of neighboring constellation points only differ in one bit. QAM stands for quadrature amplitude modulation and PSK for phase-shift keying (all constellation points are maximally separated and on the unit-circle).

sufficiently far apart. This assumption together with the flat Rayleigh fading assumption implies that the channel gains are distributed as $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_R})$.

Part 1 During the training phase, i.e., at $t = 1$, assume that the transmitter sends a pilot signal $x[1] = 1$ and the receiver is aware of this convention. We now derive the optimal estimate for the channel vector $\mathbf{h}$. To this end, consider the input-output relation $\mathbf{y}[1] = \mathbf{h}x[1] + \mathbf{n}[1]$ with $x[1] = 1$ and compute the maximum a-posteriori (MAP) estimate of $\mathbf{h}$. More specifically, the MAP estimate for $\mathbf{h}$ corresponds to the following optimization problem:

$$\hat{\mathbf{h}} = \arg\max_{\mathbf{h}\in\mathbb{C}^{M_R}} f_{\mathbf{h}}(\mathbf{h}|\mathbf{y}[1]). \tag{2}$$

Here, $f_{\mathbf{h}}(\mathbf{h}|\mathbf{y}[1])$ is the probability density function of the channel vector $\mathbf{h}$ given the received signal $\mathbf{y}[1]$ during time slot 1. Simplify the MAP estimate (2) using Bayes' rule, i.e., provide a closed-form expression. *Hint: You may need matrix/vector calculus to obtain a closed form expression for the MAP estimator. In case you are not familiar with these concepts, please consult Sections 2 and 4 of "The Matrix Cookbook" for more details [1].*

Part 2 In practice, the prior distribution $f(\mathbf{h})$ of the channel vector $\mathbf{h}$ is often unknown (we always assumed $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_T})$, but this might be not the case in real-world systems). In this case, the best approach is to ignore the prior distribution in the MAP rule derived in Part 1. Hence, derive the so-called maximum likelihood (ML) channel estimation rule. *Hint: Just assume that $f(\mathbf{h}) = 1$.*

Remark: Both the MAP and ML rule are common ways for estimating the channel at the receiver. As we will see in Problem 2, the performance of both estimators differs in practice.

Part 3 Coherent detection in phase 2 works as follows. One typically assumes that the MAP or ML estimate $\hat{\mathbf{h}}$ of the channel is correct and just plugs it into the MAP data detection rule, i.e., one considers the following input-output relation during the second time slot: $\mathbf{y}[2] = \hat{\mathbf{h}}x[2] + \mathbf{n}[2]$. Since $x[2]$ is unknown to the receiver, we have to perform coherent data detection. Write down the MAP detection rule by assuming that $\hat{\mathbf{h}}$ is exact. *Hint: We have done this at least 10 times by now.*

Remark: Since the channel estimate $\hat{\mathbf{h}}$ is not exactly the real channel $\mathbf{h}$, the above MAP data detection rule is not optimal. We will study an improved approach in Problems 4 and 5.

Problem 3: Simulation of a SIMO system with channel estimation (15pts)

We now simulate the symbol error rate (SER) of the system in Problem 2, and compare the performance of perfect channel knowledge, as well as MAP and ML channel estimation. As done in previous homework assignments, write a MATLAB script that performs Monte-Carlo simulations to compute the average symbol error rate (SER). Perform $T = 100,000$ trials (we compute $10\times$ more to get nicer curves), sweep the SNR between 0 dB and 30 dB, and plot the resulting average SER vs. SNR performance in a doubly-logarithmic plot. Generate results for $M_R \in \{1, 2\}$ receive antennas. *Hint: Once again, be extremely careful with the SNR definition and the normalization of your vectors...*

Part 1 Perform transmission in two phases. First, in the training phase transmit $x[1] = 1$ and estimate the channel vector $\mathbf{h}$ using MAP estimation. Then, use the estimate $\hat{\mathbf{h}}$ and perform MAP data detection in the second phase, i.e., at $t = 2$. Repeat this scheme for T trials and compute the SER performance. As usual, generate a nice MATLAB plot.

Part 2 Perform transmission in two phases. First, in the training phase transmit $x[1] = 1$ and estimate the channel vector $\mathbf{h}$ using ML estimation. Then, use the estimate $\hat{\mathbf{h}}$ and perform MAP data detection in the second phase, i.e., at $t = 2$. Repeat this scheme for T trials and compute the SER performance. As usual, generate a nice MATLAB plot.

Part 3 Generate a plot that includes the simulated SER curves from Part 1 (MAP channel estimation) and Part 2 (ML channel estimation), and also include an SER curve where you use the exact channel vector in the detection stage (remember that this is not practical, but it is useful to include this curve as a reference). Answer the following questions:

- Are the slopes of all simulated SER curves correct?
- Do you expect MAP channel estimation to perform better/worse than ML channel estimation?
- Do you have an idea of how one could modify the system to obtain improved channel estimates?

Problem 4: Joint channel estimation and data detection in SIMO wireless systems (15pts)

In Problems 2 and 3, we have considered a simple two-phase approach to the problem of acquiring channel knowledge at the receiver and data detection, i.e., we first estimate the channel vector and then, perform coherent data detection by assuming that the estimate is correct. However, the optimal approach is to *simultaneously* estimate the channel and perform data detection. We now derive such a joint channel estimation and data detection approach for the SIMO system studied in Problem 2. To this end, it is often convenient to rewrite the input-output relation in (1) in matrix-vector form as

$$\mathbf{Y} = \mathbf{h}\mathbf{x}^T + \mathbf{N},$$

where, $\mathbf{Y} = [\mathbf{y}[1], \mathbf{y}[2]]$, $\mathbf{x}^T = [x[1], x[2]]$, and $\mathbf{N} = [\mathbf{n}[1], \mathbf{n}[2]]$.

Part 1 The idea of joint channel estimation and data detection is to solve the following joint MAP detection and estimation problem:

$$\{\hat{\mathbf{x}}^T, \hat{\mathbf{h}}\} = \underset{\mathbf{x}^T \in \{-1,+1\}^2,\, \mathbf{h} \in \mathbb{C}^{M_R}}{\arg\max} \; f_{\mathbf{x}^T,\mathbf{h}}(\mathbf{x}^T, \mathbf{h} | \mathbf{Y}). \tag{3}$$

Here, $f_{\mathbf{x}^T,\mathbf{h}}(\mathbf{x}^T, \mathbf{h}|\mathbf{Y})$ is the probability density function of the data vector $\mathbf{x}^T$ and the channel vector $\mathbf{h}$, given the received signals contained in the matrix $\mathbf{Y}$. Simplify (3) and provide a closed form expression. *Hint: This problem is quite difficult. Use Bayes' rule and apply the logarithm to get rid of constants. Then, separate the maximization (or minimization) into an optimization over $\mathbf{x}^T$ and an optimization in $\mathbf{h}$. Interestingly, if $\mathbf{x}^T$ is held constant, the optimization problem for $\mathbf{h}$ has a closed form expression. Use the facts that the Frobenius norm is $\|\mathbf{A}\|_F^2 = trace(\mathbf{A}^H\mathbf{A})$ and the trace satisfies $trace(\mathbf{AB}) = trace(\mathbf{BA})$, as well as matrix/vector calculus to obtain a closed form expression for this optimization problem [1]. Then, plug the optimal estimate for $\mathbf{h}$ into your expression so that the problem will only depend on $\mathbf{x}^T$. Do not forget the fact that $x[1] = 1$.*

Part 2 Analogously to Problem 2 Part 2, it is often unrealistic to know the prior distribution of the channel vector $f(\mathbf{h})$. Perform the same steps as in Part 1 but derive the ML estimate by ignoring the prior $f(\mathbf{h})$.

Remark: Joint channel estimation and data detection achieves the lowest possible error rate and provides more accurate channel estimates than the simple two-phase approach studied in Problems 1 and 2. In practice, however, joint channel estimation and data detection is not used often due to the rather high computational complexity that is required to solve the underlying optimization problem.

Problem 5: Simulation of joint channel estimation and data detection (15pts)

We now simulate the symbol error rate (SER) of the approach derived in Problem 4, and compare the performance of perfect channel knowledge and coherent detection with joint MAP or ML channel estimation and data detection. Write a MATLAB script that performs Monte-Carlo simulations to compute the average symbol error rate (SER). Perform $T = 100,000$ trials, sweep the SNR between 0 dB and 30 dB, and plot the resulting average SER vs. SNR performance in a doubly-logarithmic plot. Generate results for $M_R \in \{1, 2\}$ receive antennas.

Part 1 Use the joint MAP channel estimation and data detection method derived in Problem 4 Part 1 to compute an estimate of the data symbol $x[2]$. Repeat this scheme for T trials and compute the SER performance. As usual, generate a nice MATLAB plot for $M_R \in \{1, 2\}$ receive antennas.

Part 2 Use the joint ML channel estimation and data detection method derived in Problem 4 Part 2 to compute an estimate of the data symbol $x[2]$. Repeat this scheme for T trials and compute the SER performance. As usual, generate a nice MATLAB plot for $M_R \in \{1, 2\}$ receive antennas.

Part 3 Generate a plot that includes the simulated SER curves from Part 1 and Part 2, and include an SER curve where you use the exact channel vector in the detection stage (remember that this scheme is not practical, but it is useful to include this curve as a reference). Also include the two curves obtained in Problem 3, i.e., for MAP and ML channel estimation followed by data detection. Answer the following questions:

- Are the slopes of all simulated SER curves correct?
- Does the performance of all schemes make sense, i.e., are the schemes you expect to be better also better in your simulations?
- Write down two ideas that could improve the SER performance of your system.

Problem 6: Improved 2×1 MISO communication using Alamouti space-time coding (20pts)

In Homework 2, we studied the performance of MISO (multiple-input multiple-output) communication systems. The schemes we studied there were wasteful in terms of degrees of freedom. A more elegant solution exists for the case of two transmit antennas, the so-called *Alamouti scheme* [2]. This schemes is used in several communication standards for third-generation wireless systems and is an example of so-called *space-time codes*. The Alamouti scheme transmits two complex symbols s_1 and s_2 (e.g., each chosen from a higher-order constellation set) over two time slots from two transmit antennas A and B. In time slot 1, we send $x_A[1] = s_1$ and $x_B[1] = s_2$, where * denotes complex conjugation. In time slot 2, we send $x_A[2] = -s_2^*$ and $x_B = s_1^*$. We assume a Rayleigh flat fading MISO system with $M_T = 2$ transmit antennas and $M_R = 1$ receive antenna, and assume that the channel remains constant over both time slots. This system can be modeled as follows:

$$y[1] = h_A s_1 + h_B s_2 + n[1]$$
$$y[2] = -h_A s_2^* + h_B s_1^* + n[2],$$

where $h_A, h_B \sim \mathcal{CN}(0,1)$ and $n[1], n[2] \sim \mathcal{CN}(0, N_0)$. For simplicity, we assume BPSK transmission for both symbols, i.e., $s_1, s_2 \in \{-1, 1\}$. The goal is now to derive the optimal data detector and to study the associated diversity.

Part 1 Rewrite the input-output relation of the Alamouti scheme using the following matrix-vector form:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where $\mathbf{y} = [y[1]; y^*[2]]$, $\mathbf{x} = [s_1; s_2]$, and $\mathbf{n} = [n[1]; n^*[2]]$ are all two-dimensional column vectors. In particular, provide $\mathbf{H}$. *Hint: Find the entries of the* 2×2 *matrix depending on* h_A *and* h_B*.*

Part 2 Compute $\mathbf{H}^H\mathbf{H}$. What do you conclude from the result?

Part 3 The result in Part 2 shows that optimal detection for the Alamouti scheme is straightforward. Derive the optimal detector for the transmit symbols s_1 and s_2.

Part 4 Derive the error probability. What is the diversity of the Alamouti scheme? *Hint: Use the facts that (i) we consider BSPK transmission and (ii) that we derived the error probability for similar expressions before.*

Remark: Compared with other MISO schemes, such as repetition coding in space, the Alamouti scheme doubles the transmission rate, while enabling second order diversity. Note that this could be achieved without knowing the channel at the transmitter!

Problem 7: Simulation of the Alamouti scheme (10pts)

We now simulate the symbol error rate (SER) of the Alamouti scheme derived in Problem 6. Write a MATLAB script that performs Monte-Carlo simulations to compute the average symbol error rate (SER). Perform $T = 100{,}000$ trials, sweep the SNR between 0 dB and 30 dB, and plot the resulting average SER vs. SNR performance in a doubly-logarithmic plot.

Part 1 Simulate the 2×1 MISO scheme and generate a nice MATLAB plot. Does the Alamouti scheme really achieve second order diversity?

Remark: The Alamouti scheme is the only complex full-rate (two symbols over two time slots) orthogonal space-time block code [3]. Generalizations to more transmit antennas exist, but their rate

is lower (e.g., three symbols over four time slots). Alternatively, there are full-rate, full-diversity space-time codes for more time slots, but they are not orthogonal, i.e., the receiver needs to do more work than in the orthogonal case to separate the data symbols transmitted via the different antennas.

References

[1] K. B. Petersen and M. S. Pedersen, "The Matrix Cookbook," Technical University of Denmark, November 2012

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE J. Sel. Areas Commun., vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," IEEE Trans. Inf. Theory, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.