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P1 Part 1

Call the matlab script: HW4_Q1 inside HW4_Q1 folder

Syntax: HW4_Q1('all')

Result in Fig 1

Part2: Same as above (Above command plots both SER & BER

Result in Fig 1

Part 3: Same command as above

Result in Fig 2

In part 2, we plotted BER against SNR, which is signal power per symbol / Noise power

While in part 3, we plotted BER against Eb/No which is transmit signal power per bit / Noise power

that's the reason we get similar plot for BPSK & QPSK in part 3 since for both energy per bit is same

$$SNR\_bpsk = \frac{a^2}{N_0} \rightarrow 1 \text{ bit sent}$$

$$SNR\_qpsk = \frac{2a^2}{N_0} \rightarrow 2 \text{ bits sent}$$

$\therefore$ Per bit energy is same.

P2.1 $\underline{y}[t] = \underline{h}\, x[t] + \underline{n}[t]$; $t \in 1, 2$

$y \in \mathbb{C}^{M_R}$; $h \in \mathbb{C}^{M_R}$

$h \sim \mathcal{CN}(0, I_{M_R})$

$n \sim \mathcal{CN}(0, N_0 I_{M_R})$

Training phase i.e. at $t=1$

$x[1] = 1$

$\therefore \underline{y}[1] = \underline{h} + \underline{n}[1]$

$\therefore$ MAP estimation:

$$\hat{h} = \underset{h \in \mathbb{C}^{M_R}}{\operatorname{argmax}} \Pr(\underline{h} \mid \underline{y}[1])$$

$$\hat{h} = \underset{h \in \mathbb{C}^{M_R}}{\operatorname{argmax}} \frac{\Pr(\underline{y}[1] \mid \underline{h}) \cdot P(\underline{h})}{\underbrace{P(\underline{y}[1])}}$$

$\longrightarrow$ Not relevant for this optimization $\therefore$ dropping

$(\underline{y}[1] \mid h)$ is $\sim \mathcal{CN}(\underline{h}, N_0)$

& $h \sim \mathcal{CN}(0, 1)$

$\therefore$ Above objective function can be simplified into

$$C_1 \exp\left(\frac{-\|y[1] - \underline{h}\|^2}{N_0}\right) \times C_2 \cdot \exp\left(\frac{-\|h\|^2}{1}\right)$$

$\because$ C_1 & C_2 does not depend on $\underline{h}$ $\therefore$ dropping them & taking log, gives

$$-\left(\frac{\|y[1] - h\|^2}{N_0} + \|h\|^2\right)$$

$$\therefore \hat{h} = \underset{h \in \mathbb{C}^{M_K}}{\arg\max} \; -\left(\frac{\|y[i] - h\|^2}{N_0} + \|h\|^2 \right)$$

$$= \underset{h \in \mathbb{C}^{M_K}}{\arg\min} \; \underbrace{\frac{\|y[i] - h\|^2}{N_0} + \|h\|^2}_{f(\underline{h})}$$

Differentiating $f(\underline{h})$ with $\underline{h}$ & equating to 0

$$f(\underline{h}) = \frac{\|y[i] - h\|^2}{N_0} + \|h\|^2$$

$$= \frac{(y[i] - h)^H (y[i] - h)}{N_0} + h^H h$$

$$= \underbrace{\frac{y[i]^H y[i] - 2\,\mathrm{Re}\{y[i]^H \underline{h}\}}{N_0}}_{①} + \underbrace{\frac{h^H h}{N_0} + h^H h}_{②}$$

$=$

$$\therefore \nabla_h f(\underline{h}) = \nabla_h ① + \nabla_h ②$$

$$\nabla_h ② = \frac{1+N_0}{N_0} \nabla_{\underline{h}} (\underline{h}^H \underline{h}) = \frac{1+N_0}{N_0} \Big(\nabla_{\mathrm{Re}(\underline{h})} (h^H h) + i \, \nabla_{\mathrm{Im}(\underline{h})} (h^H h) \Big)$$

$$= \frac{1+N_0}{N_0} \left(2\,\mathrm{Re}(\underline{h}) + i\,2 \cdot \mathrm{Im}(\underline{h}) \right)$$

$$= \left(\frac{1+N_0}{N} \right) \cdot 2 \cdot (\underline{h})$$

$$\nabla_{\underline{h}} ① = \frac{1}{N_0} \Big(0 - 2 \big(\nabla_{\mathrm{Re}(\underline{h})} (\mathrm{Re}(y[i]^H \underline{h}) + i \, \nabla_{\mathrm{Im}(\underline{h})} (\mathrm{Re}(y[i]^H \underline{h}) \Big)$$

$$\mathrm{Re}(y[i]^H \underline{h}) = \mathrm{Re}\Big(\sum_i (y_i^R - i\, y_i^I) . (h^R + i\, h^I) \Big)$$

$$= \sum_i (y_i^R h_i^R + y_i^I h_i^I) = \mathrm{Re}(\underline{y})^T \mathrm{Re}(\underline{h}) + \mathrm{Im}(\underline{y})^T \mathrm{Im}(\underline{h})$$

$\therefore \nabla_{Re(\underline{h})}\left(Re(\underline{y})^T Re(\underline{h}) + Im(\underline{y})^T Im(\underline{h})\right)$

$= Re(\underline{y})$

Similarly

$\nabla_{Im(\underline{h})}\left(Re(\underline{y})^T Re(\underline{h}) + Im(\underline{y})^T Im(\underline{h})\right)$

$= Im(\underline{y})$

$\therefore \nabla_h (1) = \frac{-2}{N_0}\left(Re(\underline{y}) + i\, Im(\underline{y})\right) = \frac{-2}{N_0}\underline{y}$

$\therefore \frac{-2}{N_0}\underline{y} + 2\frac{(1+N_0)}{N_0}\underline{h} = 0$

$\therefore \hat{\underline{h}} = \frac{\underline{y}}{1+N_0}$

P.2.2 if $f(\underline{h}) = 1$

then $\hat{h} = \underset{h \in \mathbb{C}^{M_R}}{\operatorname{argmax}} -\left(\frac{\|y[i] - h\|^2}{N_0} + 0\right)$

↑ follows from previous Ques. 2.1

$\therefore \hat{h} = \underset{\underline{h} \in \mathbb{C}^{M_R}}{\operatorname{argmin}} \underbrace{\left(\frac{\|y[i] - h\|^2}{N_0}\right)}_{f(\underline{h})}$

$\therefore$ It can be clearly seen that this function is minimized when $\underline{h} = y[i]$ as $\|y[i] - h\|^2$ is always positive.

Mathematically

$\hat{h} = \nabla_{\underline{h}} f(\underline{h}) = \frac{-2}{N_0}\underline{y} + \frac{2}{N_0}\underline{h} \Rightarrow 0 \quad \therefore \underline{\hat{h}} = \underline{y}$

P2.3 $\underline{y}[2] = \underline{\hat{h}}\, x[2] + \underline{n}[2]$, doing coherent detection

MRC combining

$$\tilde{y}[2] = \frac{\hat{h}^H}{\|\hat{h}\|} \cdot \underline{y}[2] = \frac{\hat{h}^H \hat{h}}{\|\hat{h}\|} x[2] + \frac{\hat{h}^H}{\|\hat{h}\|} \underline{n}[2] = \|\hat{h}\| x[2] + \underset{\uparrow\ \mathcal{CN}(0,N}{w}$$

$\because x[2]$ is Real $\tilde{y}[2] = Re(\tilde{y}[2]) = \|\hat{h}\| x[2] + Re(w)$

MAP detection Rule

Equal for +1 & -1

$$\hat{x}[2] = \underset{x \in (+1,-1)}{\text{argmax}} \; Pr(\tilde{y}[2] \mid x[2]) \cancel{Pr(x[2])}$$

$$= \underset{x \in (+1,-1)}{\text{argmax}} \; Pr(\tilde{y}[2] \mid x[2])$$

$$\because \tilde{y}[2] \sim N\left(\|\hat{h}\| x[2], \frac{N_0}{2}\right)$$

$$Pr(\tilde{y}[2] \mid x = +1) \underset{-1}{\overset{+1}{\gtrless}} Pr(\tilde{y}[2] \mid x = -1)$$

$$\left(\tilde{y}[2] - \|\hat{h}\|.(+1)\right)^2 \underset{-1}{\overset{+1}{\lessgtr}} \left(\tilde{y}[2] - \|\hat{h}\|(-1)\right)^2$$

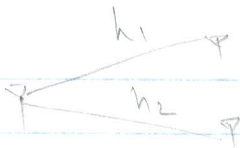
Optimal detector for $\mathcal{X} = \{+1, -1\}$

* End of Ques 2

P.3. - Yes

- MAP would work better if our assumption on the prior of h (channel) is correct otherwise it can be worse than ML.
- We could increase the # of time slots over which we do training or we can do incremental update of h as we do reception of data.

P4.1



$$\underset{\sim}{Y} = \underline{h}\ \underline{x}^T + \underset{\sim}{N}$$

If $M_R = 2$

$$\underset{\sim}{Y} = \begin{bmatrix} y^1[1] & y^1[2] \\ y^2[1] & y^2[2] \end{bmatrix} \quad ; \quad \underline{x}^T = [\,x[1] \ \ x[2]\,]$$

$$\underset{\sim}{N} = \begin{bmatrix} n^1[1] & n^1[2] \\ n^2[1] & n^2[2] \end{bmatrix} \quad ; \quad \underline{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$\{\hat{\underline{x}}^T, \hat{\underline{h}}\} = \underset{x^T \in \{-1,+1\}^2,\ \underline{h} \in \mathbb{C}^{M_R}}{\operatorname{argmax}} f_{x^T,h}(x^T, h \mid \underset{\sim}{Y})$$

$$f_{x^T,h}(\underline{x}^T, \underline{h} \mid \underset{\sim}{Y}) = f_{x^T,h}(x^T \mid h, Y)\, f_h(h \mid Y)$$

$$= f_{x^T}(x^T \mid y)\, f_h(h \mid y)$$

$$= \frac{f_{x^T}(y \mid x^T)\, f(x^T)}{\cancel{f(Y)}}\ \frac{f(y \mid h)\, f(h)}{f(Y)}$$

don't care in optimization problem.

0.5 as only two possibilities $\{1,+1\}, \{1,-1\}$

$$= \underset{x^T,\ \underline{h}}{\operatorname{argmax}}\ f_{x^T}(y \mid x^T)\, f(x^T)\, f(y \mid h)\, f(h)$$

$f_{x^T}(y \mid x^T) \sim \mathcal{CN}(0, 1+N_0)$; $f(y \mid h) \sim \mathcal{CN}(\underline{h}\,\underline{x}^T, N_0 I)$; $f(h) \sim \mathcal{CN}(0,1)$

$\therefore$ Doing optimization over $\underline{h}$, after taking log we get

$$\left(\frac{-\|Y - 0\|_F^2}{(1+N_0)} - \frac{\|Y - h x^T\|_F^2}{N_0} - \frac{\|h\|_2^2}{1} \right) \leftarrow \text{optimization function}$$

$$= \operatorname{argmin} \left(\frac{\|Y - 0\|_F^2}{1+N_0} + \frac{\|Y - h x^T\|_F^2}{N_0} \ \frac{\|h\|_2^2}{1} \right)$$

Differentiating w.r.t. to $\underline{h}$

$$\frac{1}{N_0} \text{trace}\left[(\underset{\sim}{y} - hx^T)^H (y - hx^T) \right] + \underline{h}^H \underline{h}$$

$$\frac{1}{N_0} \text{trace}\Big[y^H y \underbrace{- 2\, Re\left((hx^T)^H y \right)}_{1} + \underbrace{(hx^T)^H (hx^T)}_{2} \Big] + \underbrace{h^H h}_{3}$$

$$\nabla_h (3) = 2\, \underline{h}$$

$$\nabla_h (2) = \nabla_h \text{ Trace}\left((hx^T)^H (hx^T) \right)$$

$$= \text{Trace}\left((x^T)^H . x^T \right) \nabla_h (h^H h)$$

$$= \text{Trace}\left(\underline{x} . \underline{x}^T \right) . 2\, \underline{h}$$

$$= 2\underline{h} . Tr(\underline{x} . x^T)$$

$$\nabla_h (1) = -2 \nabla_h Tr\left\{ Re\left((hx^T)^H y \right) \right\}$$

$$= -2 \nabla_h Tr \left\{ Re\left\{ (x^T)^H h^H . y \right\} \right\}$$

$$= -2 \nabla_h Tr \left\{ Re\left\{ (x^*) \underline{h}^H . \underset{\sim}{y} \right\} \right\}$$

$$= -2 \left(\underset{\sim}{y} . \underline{x}^* \right)$$

Combining all 3

$$\frac{-2\, \underset{\sim}{y} . \underline{x}^*}{N_0} + \frac{2\underline{h} . Tr(x . x^T)}{N_0} + 2\underline{h} = 0$$

$$\therefore \hat{h} = \frac{\underset{\sim}{y} . \underline{x}^*}{N_0 + Tr(x . x^T)} = \frac{\underset{\sim}{y} . \underline{x}^*}{N_0 + \underline{x}^T \underline{x}} = \frac{\underset{\sim}{y} . x^*}{N_0 + \|\underline{x}\|^2} = \frac{\underset{\sim}{y} . \underline{x}^*}{N_0 + (1 + x^2[2])}$$

$$\because \underline{x} = \begin{bmatrix} 1 \\ x[2] \end{bmatrix}$$

$$\hat{h} = \frac{\underset{\sim}{Y} \cdot x^*}{N_0 + x^T x} \qquad \because \quad x = \begin{bmatrix} 1 \\ x[2] \end{bmatrix}$$

$$\hat{h} = \frac{\underset{\sim}{Y} \cdot x^*}{N_0 + 1 + x^2[2]}$$

Putting this back in our optimization problem

$$= \arg\min \left(\frac{\|Y\|_F^2}{1+N_0} + \frac{\left\| Y - \frac{Y \cdot x^* x^T}{N_0 + x^T x} \right\|_F^2}{N_0} + \left\| \frac{Y \quad x^*}{N_0 + x^T x} \right\|_2^2 \right.$$

$$x = \begin{bmatrix} 1 \\ x_2 \end{bmatrix} \quad \therefore \quad x^* x^T = \begin{bmatrix} 1 \\ x_2 \end{bmatrix} \begin{bmatrix} 1 & x_2 \end{bmatrix} = \begin{bmatrix} 1 & x_2 \\ x_2 & x_2^2 \end{bmatrix}$$

$\because x_2$ is real $x_2^* = x_2$

$$\& \quad x^T x = 1 + x_2^2$$

$$Y \cdot x^* = \begin{bmatrix} \underset{\sim}{y}[1] & \underset{\sim}{y}[2] \\ & \end{bmatrix}_{m \times 2} \begin{bmatrix} 1 \\ x_2 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \underset{\sim}{y}[1] + x_2 \underset{\sim}{y}[2] \end{bmatrix}_{m \times 1}$$

$$\therefore \hat{h} = \frac{\underset{\sim}{y}[1] + x_2 \underset{\sim}{y}[2]}{N_0 + 1 + x_2^2}$$

$$= \underset{x_2 \in +1,-1}{\arg\min} \left(\frac{tr(Y^H Y)}{1+N_0} + \frac{tr\left(\left(Y - \frac{Y x^* x^T}{N_0 + x^T x}\right)^H \left(Y - \frac{Y x^* x^T}{N_0 + x^T x}\right) \right)}{N_0} + \left(\frac{Y x^*}{N_0 + x^T x}\right)^H \left(\frac{Y x^*}{N_0 + x^T x}\right) \right)$$

$$= \underset{x_2 \in +1,-1}{\arg\min} \left(\frac{y^H[1]\, y[1] + y^H[2]\, y[2]}{1 + N_0} + \frac{\|y[1](N_0 + x_2^2) - y_2 x_2\|^2 + \|y(2)(N_0+1) - y(1) x_2\|^2}{N_0(N_0 + 1 + x_2^2)} + \left\| \frac{y(1) + x_2(y(2))}{N_0 + 1 + x_2^2} \right\|^2 \right)$$

Decision Rule.

P4.2 From 4.1 we have

$$\{\hat{x}^T, \hat{h}\} = \underset{x^T, h}{\text{argmax}}\ f_{x^T}(y|x^T)\ \cancel{f(x^T)}^{\,0.5}\ f(y|h)\ \cancel{f(h)}^{\,1}$$

ignoring $f(h)$

i.e. $f(h)=1$

Problem becomes

$$= \underset{x^T, h}{\text{argmax}}\ \underbrace{f(y|x^T)}_{\sim CN(0,\, 1+N_0)}\ \underbrace{f(y|h)}_{\sim CN(hx^T,\, N_0)}$$

Doing optimization over h, & after taking log we get

$$\underset{h}{\text{argmax}}\ -\left(\frac{\|y-0\|_F^2}{1+N_0} + \frac{\|y-hx^T\|_F^2}{N_0}\right)$$

$$= \underset{h}{\text{argmin}}\ \left(\frac{\|y\|_F^2}{1+N_0} + \frac{\|y-hx^T\|_F^2}{N_0}\right)$$

Differentiating w.r.t. to h & as it follows from prob 4.1

$$\frac{-2\, y \cdot x^*}{N_0} + \frac{2h \cdot Tr(x \cdot x^T)}{N_0} = 0$$

$$\therefore\ \hat{h} = \frac{y \cdot x^*}{Tr(x \cdot x^T)} = \frac{y \cdot x^*}{x^T x} = \frac{y \cdot x^*}{\|x\|^2}$$

$$\because\ x = \begin{bmatrix} 1 \\ x_2 \end{bmatrix}$$

$$\hat{h} = \frac{y[1] + x_2 y[2]}{1 + x_2^2}$$

$\therefore$ Decision Rule after putting this $\hat{h}$ back

$$= \underset{x_2 \in +1,-1}{\arg\min} \left(\frac{y^H(1)\,y(1) + y^H(2)\,y(2)}{1+N_0} + \frac{\|y(1)\,x_2^2 - y(2)x_2\|^2}{N_0(1+x_2^2)} \right.$$

$$\left. + \frac{\|y(2)(1) - y(1)x_2\|^2}{N_0(1+x_2^2)} \right)$$

* End of Ques 4

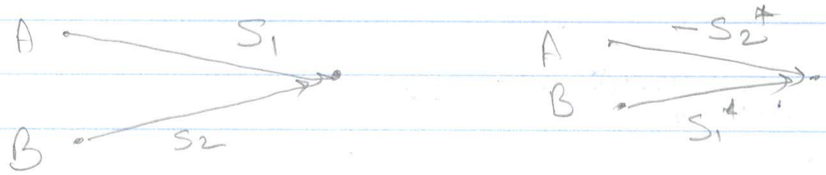
P.5

- Yes

- Yes, the scheme with perfect channel estimates performs ~3dB better than ML or MAP channel estimate which makes sense as in MAP & ML the estimates have a component of noise in them, so essentially the noise doubles causing 3dB performance loss.

- Use more pilot symbols & also keep improving the estimates as time progress.
- Also advance techniques like channel tracking or channel estimation on preamble can be used to get better estimate.

(6) MISO



$$y[1] = h_A s_1 + h_B s_2 + n[1]$$

$$y[2] = -h_A s_2^* + h_B s_1^* + n[2]$$

$$\begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix} = \underbrace{H}_{\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{2\times 1} + \begin{bmatrix} n[1] \\ n^*[2] \end{bmatrix}$$

$$y[1] = a s_1 + b s_2 + n[1] \Rightarrow a = h_A \; ; b = h_B$$

$$y^*[2] = c s_1 + d s_2 + n^*[2]$$

$$y^*[2] = \left((-h_A s_2^*) + h_B s_1^* + n[2] \right)^*$$

$$= \overline{-h_A s_2^*} + \overline{h_B s_1^*} + \overline{n[2]}$$

$$= -h_A^* s_2 + h_B^* s_1 + n^*[2]$$

$$\Rightarrow c = h_B^*$$

$$d = -h_A^*$$

$$H = \begin{bmatrix} h_A & h_B \\ h_B^* & -h_A^* \end{bmatrix}$$

6.2 $H^H H$; $H = \begin{bmatrix} h_A & h_B \\ h_B^* & -h_A^* \end{bmatrix}$

$$\therefore H^H = \begin{bmatrix} h_A^* & h_B \\ h_B^* & -h_A \end{bmatrix}$$

$$\therefore H^H H = \begin{bmatrix} h_A^* & h_B \\ h_B^* & -h_A \end{bmatrix} \begin{bmatrix} h_A & h_B \\ h_B^* & -h_A^* \end{bmatrix}$$

$$= \begin{bmatrix} h_A^* h_A + h_B h_B^* & h_A^* h_B - h_B h_A^* \\ h_B^* h_A - h_A h_B^* & h_B^* h_B + h_A h_A^* \end{bmatrix}$$

$$= \begin{bmatrix} |h_A|^2 + |h_B|^2 & 0 \\ 0 & |h_A|^2 + |h_B|^2 \end{bmatrix}$$

$H^H H = (|h_A|^2 + |h_B|^2)\, I_{2\times2}$ → Diagonal matrix

$\therefore H$ is orthogonal matrix

6.3 $\underline{y} = \underset{\sim}{H}\,\underline{x} + \underline{n}$

$$y = \begin{bmatrix} y[1] \\ y^*[2] \end{bmatrix} ; \quad H = \begin{bmatrix} h_A & h_B \\ h_B^* & -h_A^* \end{bmatrix} ; \quad x = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} ; \quad n = \begin{bmatrix} n[1] \\ n^*[2] \end{bmatrix}$$

Detection:

For detection we combine the two received signals at two time slot by multiplying above eq^n with H^H

$$\therefore \; H^H \underline{y} = H^H H \underline{x} + H^H \underline{n}$$

$$H^H H = (|h_A|^2 + |h_B|^2)\, I_{2\times 2} \quad \text{from } 6.2.$$

$$\therefore \; \underline{\tilde{y}} = (|h_A|^2 + |h_B|^2)\, \underline{x} + \underline{\tilde{n}}$$

$$\underline{\tilde{y}} = \begin{bmatrix} h_A^* y[1] + h_B y^*[2] \\ h_B^* y[1] - h_A y^*[2] \end{bmatrix}_{2\times 1} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix}$$

$$\underline{\tilde{n}} = \begin{bmatrix} h_A^* n[1] + h_B n^*[2] \\ h_B^* n[1] - h_A n^*[2] \end{bmatrix}_{2\times 1}$$

$$\therefore \; \tilde{y}_1 = (|h_A|^2 + |h_B|^2)\, s_1 + h_A^* n[1] + h_B n^*[2]$$

$$\tilde{y}_2 = (|h_A|^2 + |h_B|^2)\, s_2 + h_B^* n[1] - h_A n^*[2]$$

Doing ML detection ~~on~~ on $Re\{\tilde{y}_1\}$ since s_1 is real $\in\{-1,+1\}$

$$\therefore \underbrace{Re\{\tilde{y}_1\}}_{\text{new } \tilde{y}_1} \sim \mathcal{N}\left((|h_A|^2 + |h_B|^2)\, s_1 ,\; ((h_A^*)^2 + (h_B)^2)\frac{N_0}{2}\right)$$

$\because \quad s_1 \in \{-1, +1\} \quad \& \quad p(-1) = p(+1) = 0.5$

ML. detector

$$\hat{s}_1 = \underset{s_1 \in \{-1,+1\}}{\operatorname{argmax}} \Pr\{s_1 \mid \tilde{y}_1\}$$

$$= \underset{s_1 \in (+1,-1)}{\operatorname{argmax}} \frac{\Pr(\tilde{y}_1 \mid s_1)\, P(s_1)}{P(\tilde{y}_1)}$$

$$\Rightarrow \quad \Pr(\tilde{y}_1 \mid s_1 = +1) \underset{-1}{\overset{+1}{\gtrless}} P(\tilde{y}_1 \mid s_1 = -1)$$

$\because \tilde{y}_1$ is gaussian with mean $(|h_A|^2 + |h_B|^2) s_1$

$$\Rightarrow -\left[\tilde{y}_1 - (|h_A|^2 + |h_B|^2)(+1)\right]^2 \underset{-1}{\overset{+1}{\gtrless}} -\left[\tilde{y}_1 - (|h_A|^2 + |h_B|^2)(-1)\right]^2$$

$$\Rightarrow \left[\tilde{y}_1 - (|h_A|^2 + |h_B|^2)\right]^2 \underset{-1}{\overset{+1}{\lessgtr}} \left[\tilde{y}_1 + (|h_A|^2 + |h_B|^2)\right]^2$$

Optimal detector. for s_1

Similarly for s_2 also

Optimal detector

$$\left[\tilde{y}_2 - (|h_A|^2 + |h_B|^2)\right]^2 \underset{-1}{\overset{+1}{\lessgtr}} \left[\tilde{y}_2 + (|h_A|^2 + |h_B|^2)\right]^2$$

6.4 Prob. of error

$$Pe(h_A, h_B) = P\left[\left[\tilde{y}_1 - (|h_A|^2 + |h_B|^2)\right]^2 > \left[\tilde{y}_1 + (|h_A|^2 + |h_B|^2)\right]^2 \Big| S_1 = +1\right]$$

$$= P\left[\left(Re\left(h_A^* n[1] + h_B n^*[2]\right)\right)^2 > \left[2(|h_A|^2 + |h_B|^2) + Re\left(h_A^* n[1] + h_B n^*[2]\right)\right]^2\right]$$

$$= P\Big(0 > \left[2(|h_A|^2 + |h_B|^2)\right]^2 + 2 \cdot 2(|h_A|^2 + |h_B|^2) \cdot \underbrace{Re\{h_A^* n[1] + h_B n^*[2]\}}$$

$$= P\left((|h_A|^2 + |h_B|^2)^2 < + (|h_A|^2 + |h_B|^2) \cdot R_n \right]$$

$$R_n = -Re\{h_A^* n[1] + h_B n^*[2]\}$$

$$\sim N\left(0, (|h_A^*|^2 + |h_B|^2)\frac{N_0}{2}\right)$$

$$\therefore P\left((|h_A|^2 + |h_B|^2) < R_n \right)$$

Divide by $\sqrt{(|h_A|^2 + |h_B|^2 \frac{N_0}{2}}$ on both side

$$\therefore P\left(\sqrt{\frac{(|h_A|^2 + |h_B|^2) \cdot 2}{N_0}} < R'_n \right)$$

$$R'_n \sim N(0,1)$$

$$Pe(h_A, h_B) = Q\left(\sqrt{\frac{(|h_A|^2 + |h_B|^2) \cdot 2}{N_0}}\right) = Q\left(\sqrt{2(|h_A|^2 + |h_B|^2) \cdot SNR}\right)$$

$, SNR = 1/N_0$

$$Pe = E_{(h_A, h_B)}\, Pe(h_A, h_B)$$

and since $|h_A|^2 + |h_B|^2$ is chi-squared distributed with $2 \cdot 2 = 4$ degrees of freedom

$$Pe = \left(\frac{1}{2}(1-\mu)\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k ; \quad \mu = \sqrt{\frac{SNR}{1+SNR}}$$

$$P_e \xrightarrow[\text{SNR}]{\text{large}} (4\,\text{SNR})^{-L} \binom{2L-1}{L}$$

where $L = 2$ in this case

$$\therefore P_e = (4\,\text{SNR})^{-2} \binom{3}{2} = 3.(4\,\text{SNR})^{-2}$$

$\therefore$ Diversity order is 2 in this case.

*End of Ques 6.

P.7 Yes alamouti scheme achieves 2nd order diversity as it can be seen in the plot that Pe decreases 1 decade per 5 dB. (slope).