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AJS-26

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P.1 SNR of OFDM system:

$$y_w = \underbrace{\lambda_w x_w}_{x_w} + n_w \quad ; \quad w = 0, 1, \dots, W-1$$

$$\lambda_w = \frac{1}{\sqrt{W}} \sum_{l=1}^{L-1} h_l \exp\left(-\frac{j 2\pi l w}{W}\right)$$

$$h_l \sim \mathcal{CN}(0, 1)$$

$$n_w \sim \mathcal{CN}(0, N_0)$$

$$W = 64$$

$$\therefore \text{SNR} = \frac{\mathbb{E}[\|x\|^2]}{\mathbb{E}[\|n\|^2]}$$

$$x = \begin{bmatrix} x_1 & x_1 \\ x_2 & x_2 \\ \vdots & \vdots \\ x_{W-1} & x_{W-1} \end{bmatrix}$$

$$\therefore \mathbb{E}[\|n\|^2] =$$

$$n = \begin{bmatrix} n_w \\ 1 \end{bmatrix}$$

$$= \mathbb{E}\left[\sum_{w=0}^{W-1} |\lambda_w x_w|^2\right]$$

$$= \sum_{w=0}^{W-1} \mathbb{E}[|\lambda_w x_w|^2]$$

$$= \sum_{w=0}^{W-1} \mathbb{E}|\lambda_w|^2 \cdot \mathbb{E}|x_w|^2 \quad \xrightarrow{E_s}$$

$$= E_s \cdot \sum_{w=0}^{W-1} \mathbb{E}|\lambda_w|^2$$

$$= E_s \cdot \sum_{w=0}^{W-1} \mathbb{E}\left[\frac{1}{W} \sum_{l=0}^{L-1} |h_l \exp(-\frac{j 2\pi l w}{W})|^2\right]$$

$$= E_s \cdot \frac{1}{W} \sum_{w=0}^{W-1} \mathbb{E}\left[\sum_{l=0}^{L-1} |h_l|^2\right]$$

$$= E_s \cdot \frac{1}{W} \cdot (L \cdot W) \quad \because \mathbb{E}[|h_l|^2] = 1$$

$$= L \cdot E_s$$

$$4 E [||n||^2] = W \cdot N_0$$

$$\therefore \text{SNR} = \frac{L \cdot E_s}{W \cdot N_0} \quad ; \quad E_s \rightarrow \text{energy per constellation point}$$

From plot diversity is 1

P.2. Yes, the error rate performance 2-3 dB better than part 1

Diversity is still 1

P.3. Yes, <sup>considerable</sup> performance gain is seen

Diversity becomes 2

## Problem 2

$$\text{P.1} \quad \mathcal{X} = \left\{ \underset{00}{-1-j}, \underset{01}{-1+j}, \underset{10}{+1-j}, \underset{11}{+1+j} \right\}$$

$$\text{then } L([s, 1]) = \log \left( \frac{P([s]_1 = 1 | y, h)}{P([s]_1 = 0 | y, h)} \right)$$

$$= \log \left( \frac{\sum_{s \in \mathcal{X}_1} f(y|s, h) p(s)}{\sum_{s \in \mathcal{X}_0} f(y|s', h) p(s')} \right) \quad \text{--- (1)}$$

$$\mathcal{X}_1 = \{2, 3\} \quad ; \quad \mathcal{X}_0 = \{0, 1\}$$

$$\leftarrow p(s) = p(s') = 0.5$$



$$\therefore y_i = h_i s_i + n_i$$

$$\therefore f(y_i | s_i, h_i) \sim \mathcal{CN}(h_i s_i, N_0)$$

$$= \frac{1}{2\pi N_0} \exp\left(-\frac{|y - h s|^2}{2N_0}\right)$$

Plugging this in eq ① & upon simplification we get

$$L([s]_1) = \log \left( \frac{\sum_{s \in \{2,3\}} \exp\left(-\frac{|y - h s|^2}{2N_0}\right)}{\sum_{s' \in \{0,1\}} \exp\left(-\frac{|y - h s'|^2}{2N_0}\right)} \right)$$

Similarly

$$L([s]_2) = \log \left( \frac{\sum_{s \in \{1,3\}} \exp\left(-\frac{|y - h s|^2}{2N_0}\right)}{\sum_{s' \in \{0,2\}} \exp\left(-\frac{|y - h s'|^2}{2N_0}\right)} \right)$$

P.2. Performance is same as Part 3 problem 1

P.3 Yes, interleaving helps, without it the diversity reduces to 1 as both the copy of information bits goes through correlated channel & hence we lose diversity.



### Problem 3

P.1 Yes, the obtained diversity makes sense

$$M_T = M_R = 2$$

$$\therefore M_L = \overset{\text{Diversity}}{M_R} = 2$$

$$Z.F. = M_R - M_T + 1 = 1$$

$$MMSE = M_R - M_T + 1 = 1$$

P.2 For  $4 \times 4$  QPSK config, sphere decoding is slower than simple ML

But as # of antennas or constellation size increases, simple ML becomes very slow but sphere decoding marginally increases marginally at high SNRs.

P.3 SIC is only better than MMSE is very high SNR  
i.e.  $> 35\text{dB}$

But it takes more time (complexity) than MMSE.

### Problem 4:

Performance of both Eigen Beamforming & Zero Forcing Precoding is almost similar

But both have <sup>application in</sup> different scenario

Eigen B.F. is useful in point to point MIMO  
& Z.F. Precoding is useful in MU-MIMO.

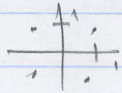


B.F.

SNR

In normal QPSK

$$s = 1 + ji$$



$$\therefore E_s = 2$$

$$; SNR = \frac{E_s \cdot M_T}{N_0}$$

← per antenna

$$\text{If } E[\|x\|^2] = 1 \quad (\text{B.F. cov.})$$

$$x = [s_1, s_2, s_3, \dots, s_{M_T}]$$

$$\therefore E[\|x\|^2] = E[s_1^2 + s_2^2 + \dots] = M_T \cdot 2$$

$$\therefore \text{to make it } 1 \quad \bullet \quad \boxed{\frac{x}{\sqrt{M_T \cdot 2}}}$$

$$\text{then } E[\|x\|^2] = \frac{1}{2 \cdot M_T} \times 2 \cdot M_T = 1$$

$$\therefore E_s = E[\|s\|^2] = \frac{2}{2 \cdot M_T} = \frac{1}{M_T}$$

$$\therefore SNR_{BF} = \left(\frac{1}{M_T}\right)^{\leftarrow E_s} \cdot \frac{M_T}{N_0} \quad (\text{per receive antenna})$$

ZFPC

$$H \tilde{x} = x$$

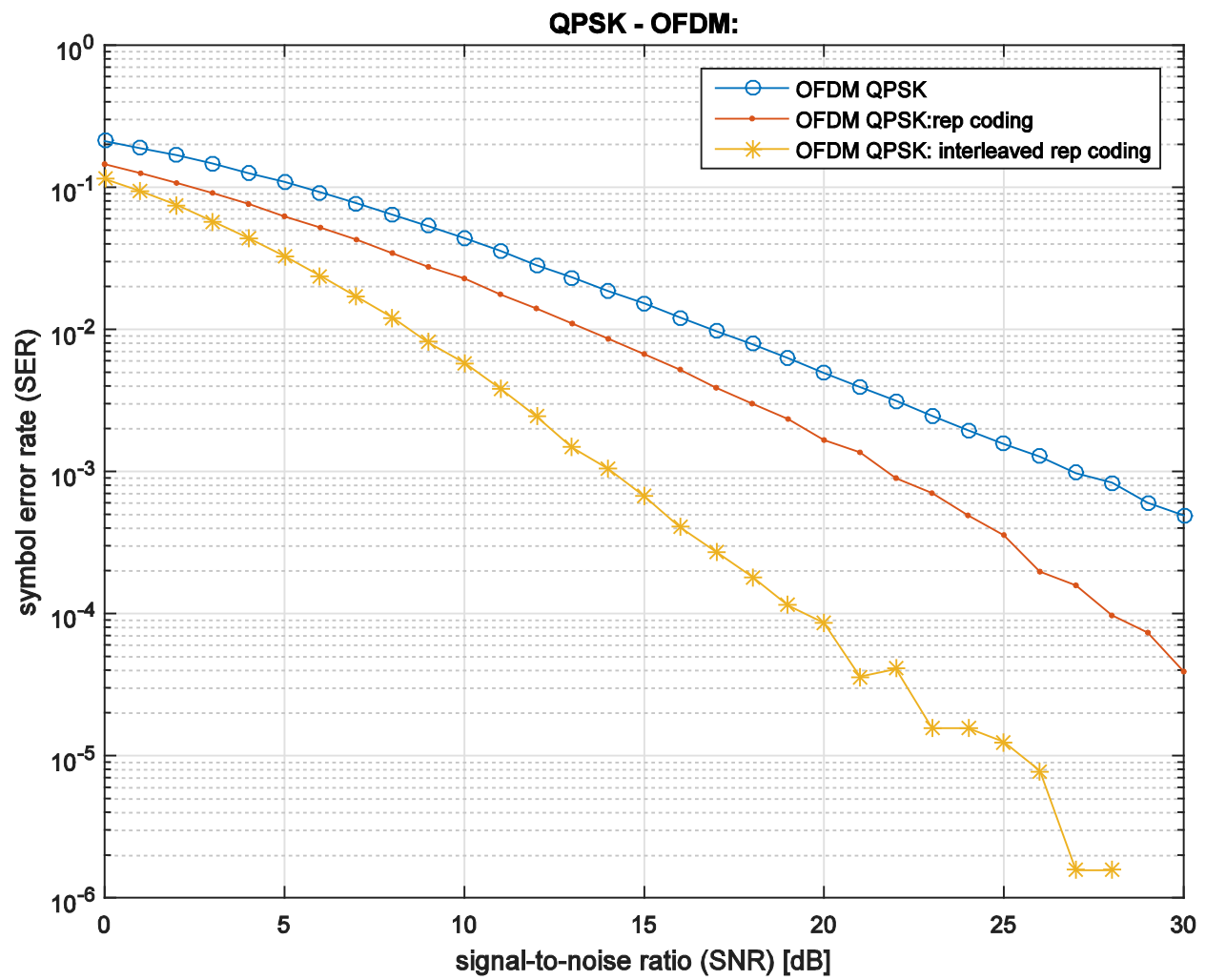
$$\therefore \tilde{x} = H^{-1} x$$

$$E[\|\tilde{x}\|^2] = \|H^H (H H^H)^{-1} x\|^2 = A$$

$$E\left[\left\|\frac{\tilde{x}}{\sqrt{A}}\right\|^2\right] = 1$$

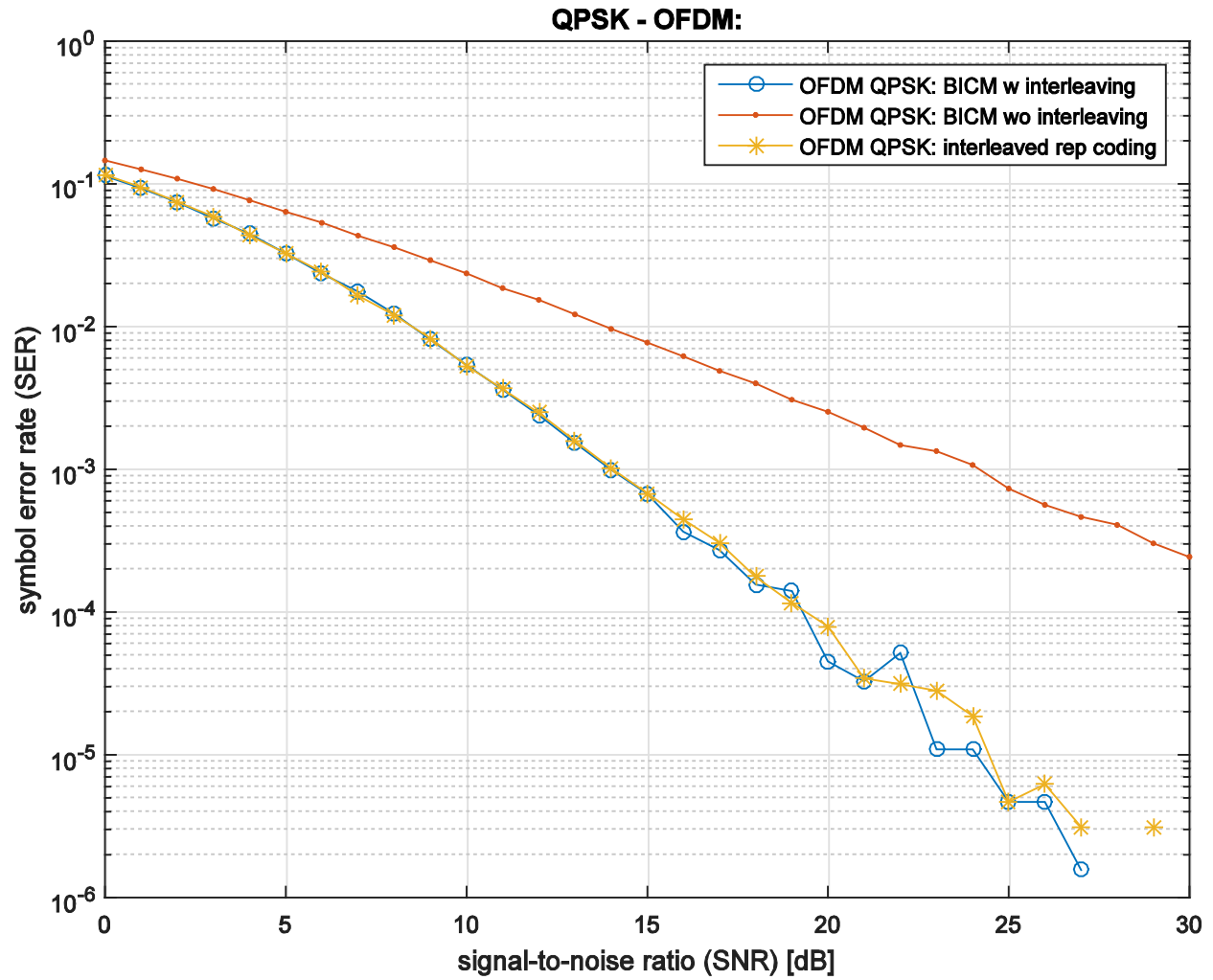
$$\therefore \text{Transmit } \frac{\tilde{x}}{\sqrt{A}}$$

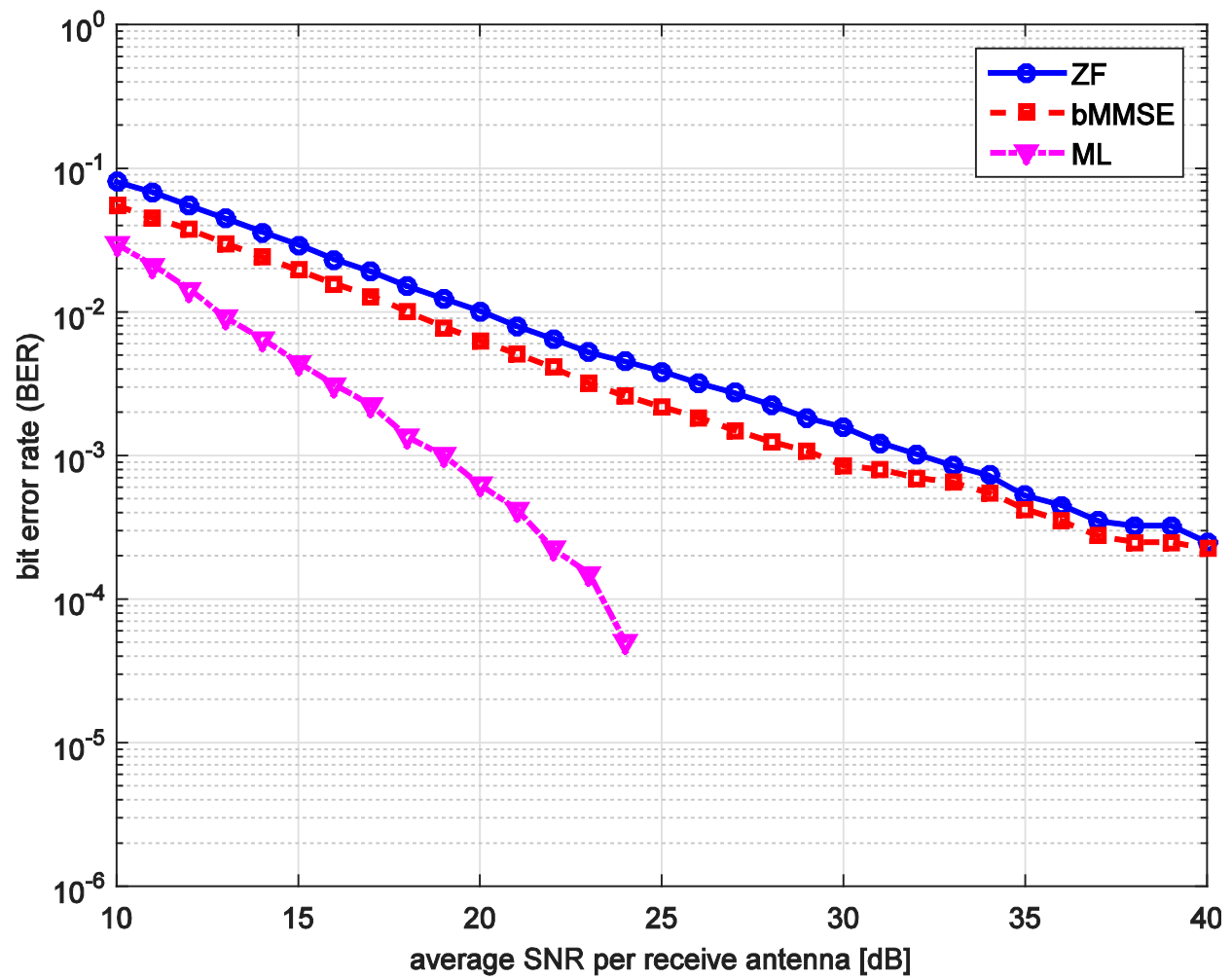
$$\text{then } SNR_{ZFPC} = \frac{1}{N_0} \quad (\text{per receive antenna})$$



**Problem 1**

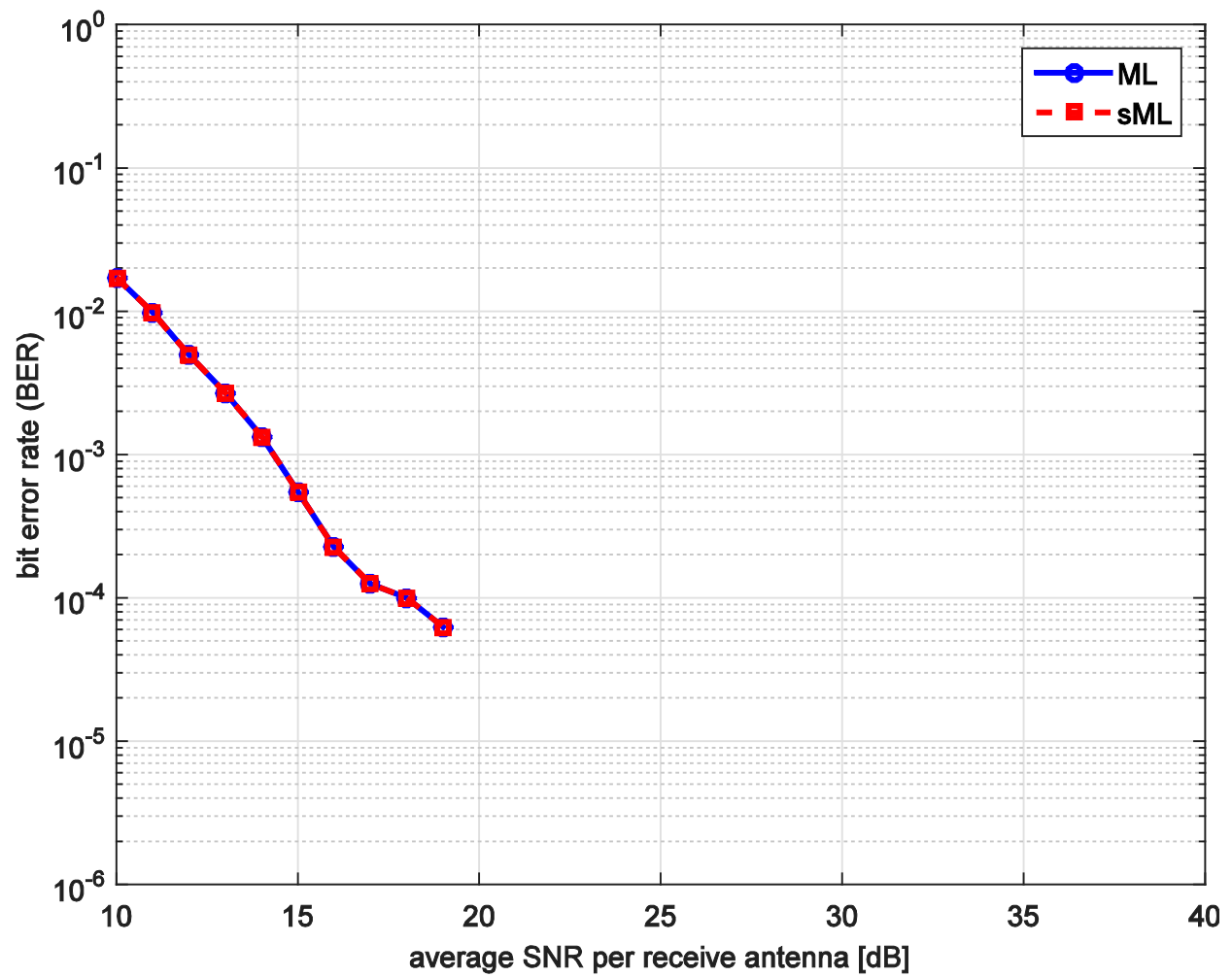
## Problem 2



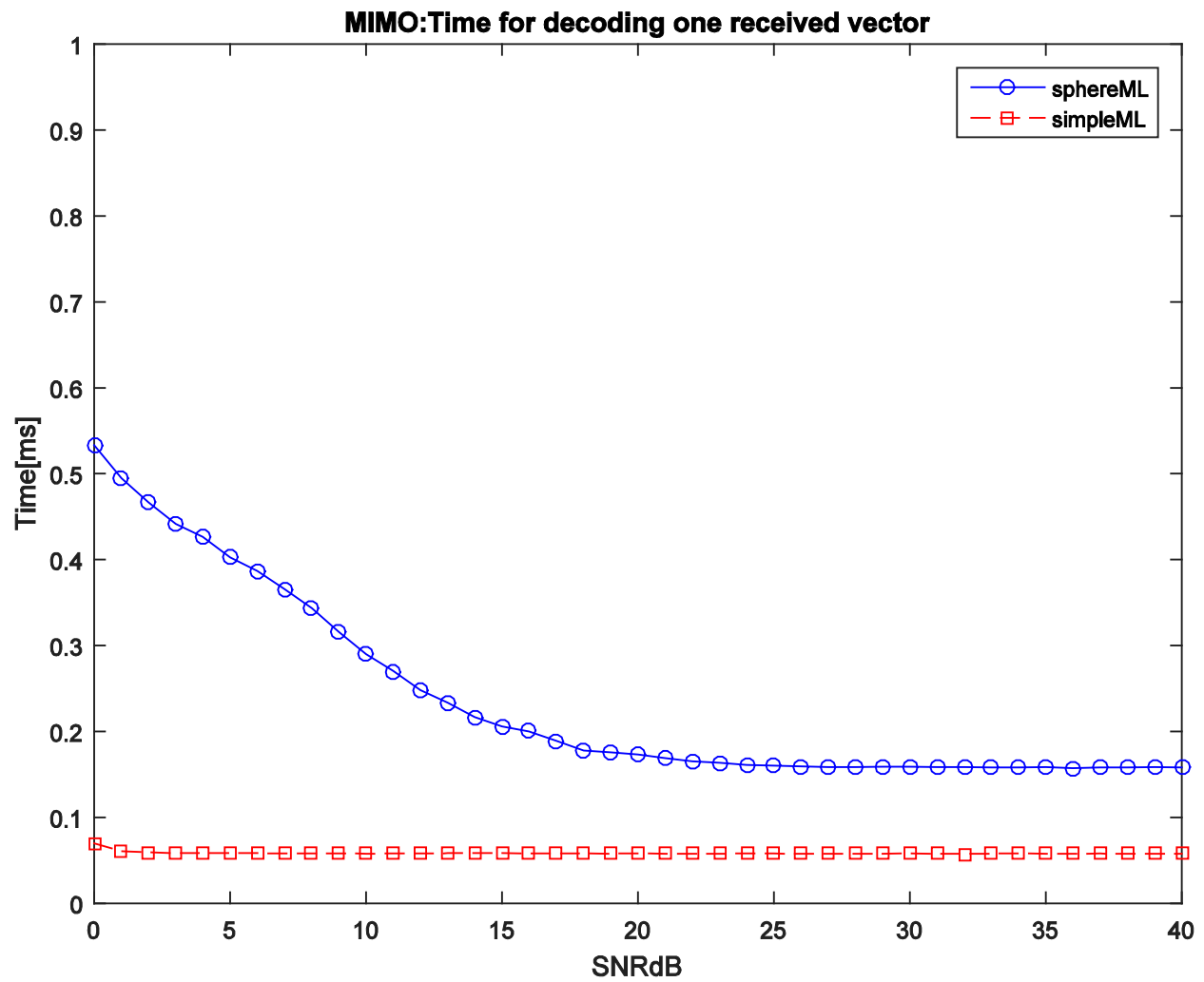


**Problem 3 part1**

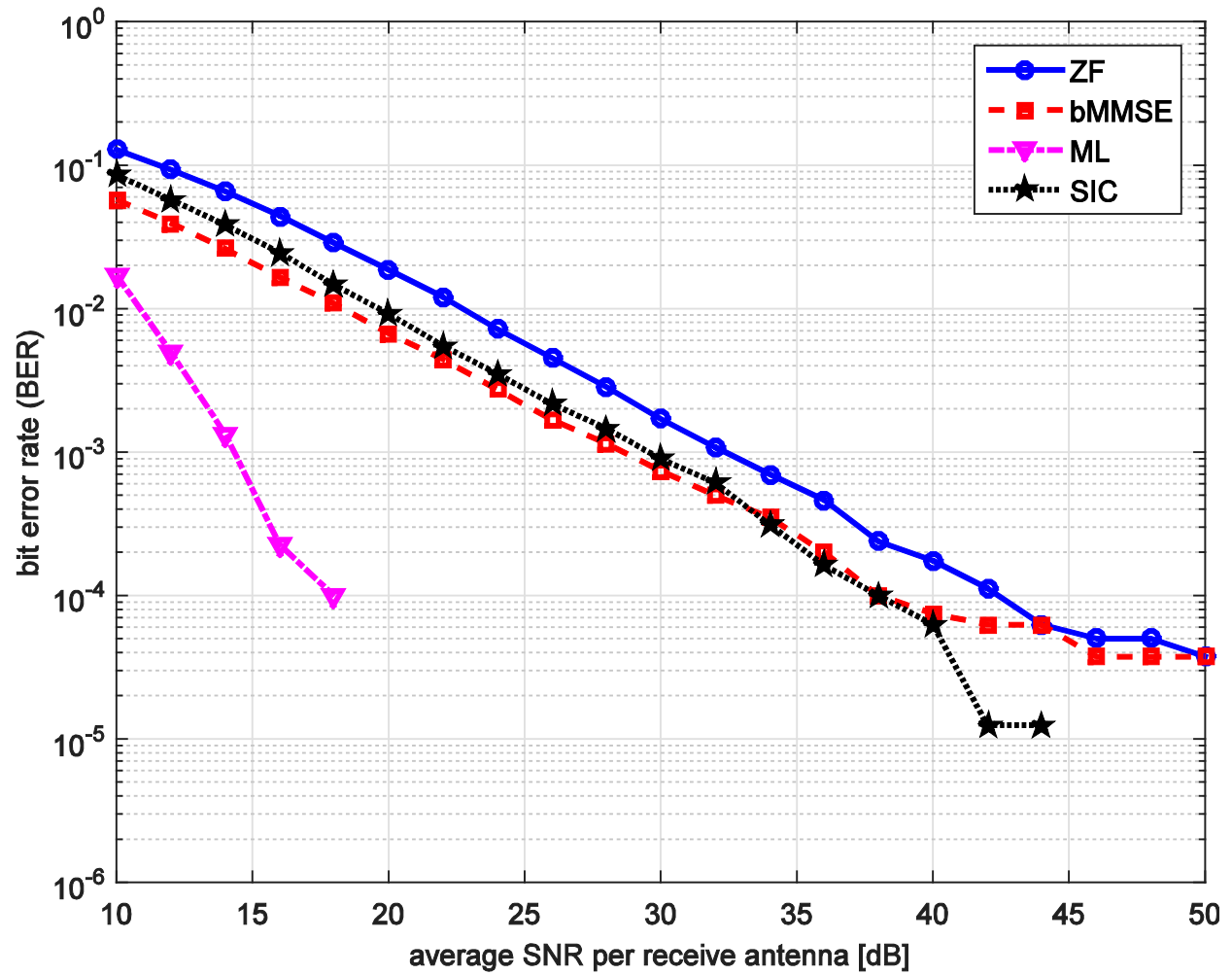




**Problem 3 part2 performance**

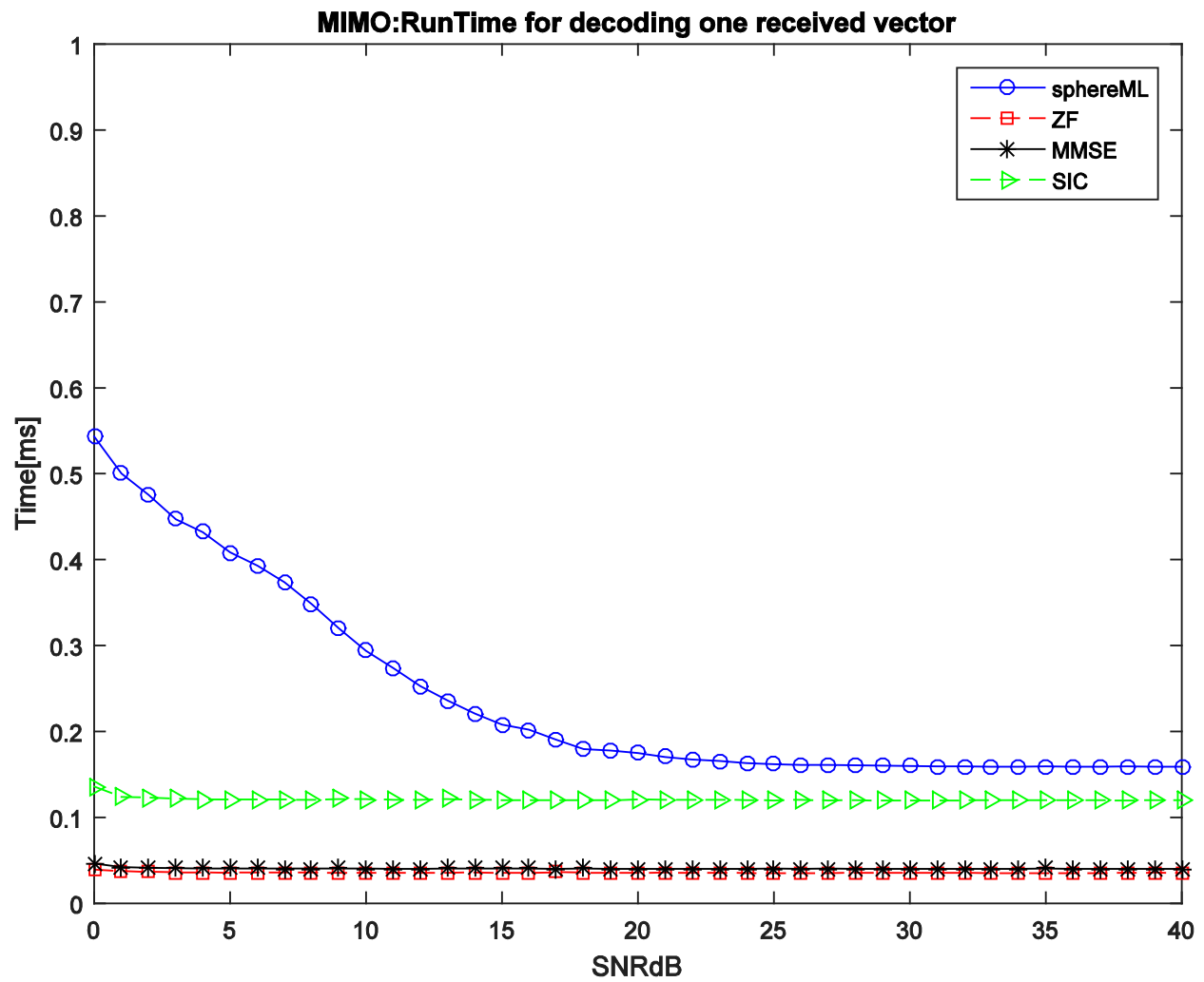


**Problem 3 part2 runtime**



**Problem 3 part3 performance**





**Problem 3 part3 runtime**

