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P.1 SNR of OFDM system:

$$y_w = \underbrace{\lambda_w x_w}_{\gamma_w} + n_w \quad ; \quad w = 0, 1, \ldots, W-1$$

$$\& \ \lambda_w = \frac{1}{\sqrt{W}} \sum_{l=1}^{L-1} h_l \exp\left(\frac{-j2\pi}{W} l w\right)$$

$h_l \sim CN(0, 1)$

$n_w \sim CN(0, N_0)$

$W = 64$

$$\therefore SNR = \frac{\mathbb{E}[\|\gamma\|^2]}{\mathbb{E}[\|n\|^2]} \qquad \gamma = \begin{bmatrix} \lambda_1 x_1 \\ \lambda_2 x_2 \\ \vdots \\ \lambda_{W-1} x_{W-1} \end{bmatrix}$$

$$\underline{n} = \begin{bmatrix} \vdots \\ n_w \\ \vdots \end{bmatrix}$$

$$\therefore \mathbb{E}[\|\gamma\|^2] =$$

$$= \mathbb{E}\left[\sum_{w=0}^{W-1} |\lambda_w x_w|^2\right]$$

$$= \sum_{w=0}^{W-1} \mathbb{E}[|\lambda_w x_w|^2]$$

$$= \sum_{w=0}^{W-1} \mathbb{E}|\lambda_w|^2 \cdot \underbrace{\mathbb{E}[|x_w|^2]}_{E_s}$$

$$= E_s \cdot \sum_{w=0}^{W-1} \mathbb{E}|\lambda_w|^2$$

$$= E_s \cdot \sum_{w=0}^{W-1} \mathbb{E}\left[\frac{1}{W} \sum_{l=0}^{L-1} \left|h_l \exp\left(-\frac{j2\pi}{W} l w\right)\right|^2\right]$$

$$= E_s \cdot \frac{1}{W} \sum_{w=0}^{W-1} \mathbb{E}\left[\sum_{l=0}^{L-1} |h_l|^2\right]$$

$$= E_s \cdot \frac{1}{W} \cdot (L \cdot W) \quad \because \mathbb{E}[|h_l|^2] = 1$$

$$= L \cdot E_s$$

$\& \ \mathbb{E}\left[\|\underline{n}\|^2\right] = W \cdot N_0$

$\therefore \ SNR = \dfrac{L \cdot E_s}{W \cdot N_0}$; $E_s \rightarrow$ energy per constellation point

From plot diversity is 1

P.2. Yes, the error rate performance 2-3 dB better than part 1

Diversity is still 1

P.3 Yes, considerable performance gain is seen

Diversity becomes 2

<u>Problem 2</u>

P.1 $\mathcal{X} = \{-1-j, -1+j, +1-j, +1+j\}$
$\quad\quad\ \ \overset{0}{00}, \ \overset{1}{01}, \ \overset{2}{10}, \ \overset{3}{11}$

then $L([s]_1) = \log\left(\dfrac{P([s]_1 = 1 \mid y, h)}{P([s]_1 = 0 \mid y, h)}\right)$

$$= \log\left(\frac{\sum_{s \in \mathcal{X}_1^1} f(y \mid s, h)\, p(s)}{\sum_{s' \in \mathcal{X}_1^0} f(y \mid s', h)\, p(s')}\right) \quad \text{—} (1)$$

$\mathcal{X}_1^1 = \{2, 3\}$; $\mathcal{X}_1^0 = \{0, 1\}$

$\& \ p(s) = p(s') = 0.5$

$\because y_i = h_i s_i + n_i$

$$\therefore f(y_i \mid s_i, h_i) \sim CN(h_i s_i, N_0)$$

$$= \frac{1}{2\pi N_0} \exp\left(\frac{-|y - hs|^2}{2N_0}\right)$$

Plugging this in eq ① & upon simplification we get

$$L([s]_1) = \log\left(\frac{\sum_{s \in 2,3} \exp\left(\frac{-|y-hs|^2}{2N_0}\right)}{\sum_{s' \in 0,1} \exp\left(\frac{-|y-hs|^2}{2N_0}\right)}\right)$$

Similarly

$$L([s]_2) = \log\left(\frac{\sum_{s \in 1,3} \exp\left(\frac{-|y-hs|^2}{2N_0}\right)}{\sum_{s \in 0,2} \exp\left(\frac{-|y-hs|^2}{2N_0}\right)}\right)$$

P.2. Performance is same as Part 3 problem 1

P.3 Yes, interleaving helps, without it the diversity reduces to 1 as both the copy of Information bits goes through correlated channel & hence we lose diversity.

Problem 3

P.1 Yes, the obtained diversity makes sense

$M_T = M_R = 2$

		Diversity
ML	=	$M_R = 2$
ZF	=	$M_R - M_T + 1 = 1$
MMSE	=	$M_R - M_T + 1 = 1$

P.2 For 4x4 QPSK config, sphere docoding is slower than simple ML
But as # of antennaes or constellation size increases simple ML becomes very slow but sphere decoding runtime increases marginally at high SNRs.

P.3 SIC is only better than MMSE is very high SNR i.e. > 35 dB

But it takes more time (complexity) than MMSE.

Problem 4:

Performance of both Eigen Beamforming & Zero Forcing Precoding is almost similar

But both have application in different scenario

Eigen B.F. is useful in point to point MIMO & Z.F. Precoding is useful in MU-MIMO.

B.F.

SNR

In normal QPSK

$s = 1+i$

$\therefore E_s = 2$; $SNR = \dfrac{\overset{\text{Per antenna one}}{E_s} \cdot M_T}{N_0}$

If $\mathbb{E}[\|x\|^2] = 1$ (B.F. Con.

$$\underline{x} = [s_1, s_2, s_3 \cdots s_{M_T}]$$

$\therefore \mathbb{E}[\|x\|^2] = \mathbb{E}[s_1^2 + s_2^2 + \cdots] = M_T \cdot 2$

$\therefore$ to make it 1 $\boxed{\underline{x} / \sqrt{M_T \cdot 2}}$

then $\mathbb{E}[\|x\|^2] = \dfrac{1}{2 \cdot M_T} \times 2 \cdot M_T = 1$

$\therefore E_s = \mathbb{E}[|s|^2] = \dfrac{2}{2 \cdot M_T} = \dfrac{1}{M_T}$

$\therefore SNR_{BF} = \overset{E_s}{\left(\dfrac{1}{M_T}\right)} \cdot \dfrac{M_T}{N_0}$ (per receive antenna)

ZFPC

$H \tilde{x} = x$ $\therefore$ $\tilde{x} = H \backslash x$

$\mathbb{E}[\|\tilde{x}\|^2] = \|H^H (H H^H)^{-1} x\| = A$

$\mathbb{E}\left[\left\| \dfrac{\tilde{x}}{\sqrt{A}} \right\|^2\right] = 1$

$\therefore$ Transmit $\dfrac{\tilde{x}}{\|\tilde{x}\|}$

then $SNR_{ZFPC} = \dfrac{1}{N_0}$ (Per receive antenna)