Linear Algebra 2019 (autumn) – Exercise Sheet 1

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Solving linear equations

1. Solve the following systems of linear equations.[1]

$$a)\ \begin{cases} x+y &= 5 \\ 2x-5y &= 4 \end{cases} \qquad b)\ \begin{cases} x+y+z &= 3 \\ x-y &= 0 \\ x &= -46 \end{cases} \qquad c)\ \begin{cases} 2x+3y+z &= 0 \\ x-y+z &= 1 \\ 3x+2y+2z &= 1 \end{cases}$$

2. Solve the following systems.

$$a)\ \begin{cases} w-x+z &= 1 \\ x-2y-z &= 0 \\ w+3y &= 2 \end{cases} \qquad b)\ \begin{cases} 2x-5y+4z &= 0 \\ x+y+z &= 0 \\ 4x-3y+6z &= 1 \end{cases} \qquad c)\ \begin{cases} x-2y+3z &= 1 \\ 2x-4y+6z &= 2 \\ -x+2y-3z &= -1 \end{cases}$$

3. Solve the following systems.

$$a)\ \begin{cases} w-x+y-z &= 1 \\ w+z &= 2 \end{cases} \qquad b)\ \begin{cases} x+y &= 0 \\ 3x+5y &= 2 \\ 2x+4y &= 2 \end{cases}$$

4. Determine for which $a \in \mathbb{R}$ the following system has no solution, a unique solution, or infinitely many solutions.

$$\begin{cases} x-2y+3z &= 2 \\ x+3y-2z &= 5 \\ 2x-y+az &= 1 \end{cases}$$

Geometry

5. Find the intersection point of the following two lines in $\mathbb{R}^2$: the line that goes through the points $(2,1)$ and $(-1,4)$, and the line through the origin with slope 2.

6. Solve the following two systems, and describe the solution sets geometrically.

$$a)\ \begin{cases} x+y+1 &= 0 \\ 2x+2y-2 &= 0 \end{cases} \qquad b)\ \begin{cases} x+2y+z &= 3 \\ x+y+3z &= 0 \end{cases}$$

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[1] *solving a system* means: Either determine that the system has no solutions, or determine that it has a unique solution and find it, or determine that it has infinitely many solutions and find a parametric description that gives all of them.

Vector and matrix equations

7. For the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 11 \\ 16 \\ 21 \end{pmatrix}$, find $l, m \in \mathbb{R}$ such that $\mathbf{c} = l\mathbf{a} + m\mathbf{b}$.

8. For $A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ -2 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, solve $A\mathbf{x} = \mathbf{b}$.

9. Find a such that $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \in \text{Span}\{ \begin{pmatrix} 1 \\ 0 \\ a \end{pmatrix}, \begin{pmatrix} a \\ 1 \\ 2 \end{pmatrix} \}$.

True/False

10. Determine if the following statements are true or false.

    (a) A vector $\mathbf{b}$ is a linear combination of the columns of the matrix A if and only if $A\mathbf{x} = \mathbf{b}$ is consistent.

    (b) If $u \in \text{Span}\{v_1, \ldots, v_n\}$, then $v_1 \in \text{Span}\{u, v_2, \ldots, v_n\}$.

    (c) For $u, v \in \mathbb{R}^3$, $\text{Span}\{u, v\}$ is always a plane.

    (d) If $A = [\mathbf{a}_1 \cdots \mathbf{a}_n]$ is an $m \times n$ matrix and $\text{Span}\{\mathbf{a}_1, \ldots, \mathbf{a}_n\} \neq \mathbb{R}^m$, then there is a $\mathbf{b} \in \mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ is inconsistent.