Advanced information, computation, communication I
EPFL - Fall semester 2018-2019

Week 1

September 21, 2018

Exercise 1. Solve for x:

1. $x = \log_{10} 10000000$
2. $x = \log_3 81$
3. $x = \log_2 \frac{1}{4}$
4. $x = \log_8 32$
5. $\log_x 125 = 3$
6. $\log_x 16 = \frac{4}{3}$
7. $\log_2 \frac{1}{x} = 5$
8. $2 \log_9 x = 1$
9. $\log_4 64^x = 12$
10. $x^{\log_3 x} = 81$
11. $\log_2(x^4 - 4) - \log_2(x^2 + 2) = -2$

Exercise 2. Let a, b and c be positive real numbers. Show that $a^{\log_b c} = c^{\log_b a}$.

Exercise 3. Show that the number of decimal digits of N is $\lfloor \log_{10} N \rfloor + 1$.

Exercise 4. Considering the previous exercise how would you compute the bit-length of a number, i.e., the number of digits (or, actually, "bits") in its binary representation?

Exercise 5. Number of decimal digits of 2^{2018} is:

○ 607

○ 608

○ 2018

○ 2019

Exercise 6. Charlotte, Giulia and Patrick are starting university next year. They have applied to EPFL, ETHZ and USI, and their preferences are listed as follows

Student	Most preferable	$\longrightarrow$	least preferable
Charlotte	USI	ETHZ	EPFL
Giulia	EPFL	USI	ETHZ
Patrick	ETHZ	EPFL	USI

The universities, on the other hand, have their own lists of preferred students

University	Most preferable	$\longrightarrow$	least preferable
EPFL	Giulia	Charlotte	Patrick
ETHZ	Giulia	Patrick	Charlotte
USI	Patrick	Charlotte	Giulia

In how many ways can we match students with universities, so that there is no pair of a student and a university who both prefer each other more than the university/student they have been matched up with?

Exercise 7. The stable perfect matching problem is a generalisation of the previous exercise in the following way: we are given two equally sized sets A and B and to each element of the sets is assigned an ordering of preferences for the elements of the other set. A matching between two sets is a pairing between the elements of the sets, i.e., a bijective function from A to B. A matching is called unstable if there exist two elements $a \in A$, $b \in B$ that are not paired and both of them prefer each other to the element they have been paired with. A matching that is not unstable is called stable.

Is a stable perfect matching unique? Either prove that every stable perfect matching is unique, or disprove it with a counterexample.

Exercise 8. Let $\{A, B, C, D\}$ be a set of men, and $\{a, b, c, d\}$ a set of women. We want to match up men and women using the Gale-Shapley algorithm in two different ways. The preferences of men and women are given in the following lists, going from most preferable on the left to least preferable on the right.

Men	1st	2nd	3rd	4th
A	c	d	b	a
B	d	c	a	b
C	a	c	b	d
D	b	d	a	c

Women	1st	2nd	3rd	4th
a	D	A	B	C
b	C	B	A	D
c	C	B	A	D
d	D	A	B	C

1. If the men propose, and women accept/reject, what is the matching after the algorithm terminates?
2. If the women propose, and men accept/reject, what is the matching after the algorithm terminates?
3. Who is the best possible (stable) valid partner for "a"?

Exercise 9. Let $k \in \mathbb{N}$ be such that $k > 1$ and recall that $n! = 1 \cdot 2 \cdot \ldots \cdot (n-1) \cdot n$ for $n \in \mathbb{N}$.
Tick the correct answer:

1. $\lim_{n\to\infty} \frac{n^k}{k^n}$
   - ○ ∞
   - ○ 0
   - ○ 1
   - ○ $k!/n!$

2. $\lim_{n\to\infty} \frac{(k+n)^n}{k^n}$
   - ○ ∞
   - ○ 0
   - ○ 1
   - ○ $(k+1)/k$

3. $\lim_{n\to\infty} \frac{n!}{n^n}$
   - ○ ∞
   - ○ 0
   - ○ 1
   - ○ e^{-1}

4. $\lim_{n\to\infty} \frac{2^{(n!)^k}}{2^{k^n}}$
   - ○ ∞
   - ○ 0
   - ○ 1
   - ○ $e^{-k\ln(2)}$

5. $\lim_{n\to\infty} \frac{\log((n!)^k)}{\log(n^n)}$
   - ○ ∞
   - ○ 0
   - ○ 1
   - ○ k