Q4

(a)

n	Optimal sequence	M(n, 0)
1	[+ -]	2
2	[+ o -]	3
3	[+ o o -]	4
4	[+ + - -]	4
5	[+ + - o -]	5
6	[+ + o - -]	5
7	[+ + o - o -]	6
8	[+ + o o - -]	6
9	[+ + + - - -]	6
10	[+ + + - - o -]	7
11	[+ + + - o - -]	7
12	[+ + + o - - -]	7
13	[+ + + o - - o -]	8
14	[+ + + o - o - -]	8
15	[+ + + o o - - -]	8
16	[+ + + + - - - -]	8
17	[+ + + + - - - o -]	9
18	[+ + + + - - o - -]	9
19	[+ + + + - o - - -]	9
20	[+ + + + o - - - -]	9
21	[+ + + + o - - - o -]	10

(b)

Base case ($n = s^2$, $s \in Z$)

Each item represent the moving distance during each time steps.

The number of items represent number of time steps, that is M.

$1 + 2 + 3 + \ldots + s + (s-1) + (s-2) + \ldots + 1 + 0 = s^2$

'+' * s　　　　　　'–' * s

Let $s^2 = n$, so $s = \sqrt{n}$, and $M(n, 0) = 2\sqrt{n} = 2\sqrt{n}$

if $s(s+1) < n <= (s+1)^2$, $s \in Z$:

we change from the base case formula and avoid breaking the continuity.

formula	**n**
$1 + 2 + 3 + \ldots + (s+1) + s + (s-1) + (s-2) + \ldots + 1 + 0 = s^2$	$(s+1)^2$
-1	$(s+1)^2 -1$
-1　-1	$(s+1)^2 -2$
-1　-1　-1	$(s+1)^2 -3$
…	…
-1　-1　-1　-1　-1　…　-1	$(s+1)^2 -s$

-1 * s　　　　　　　That is s(s+1) +1

The number of items is still 2(s+1), means M(n,0) = 2s+2

if $s^2 < n <= s(s+1)$, $s \in Z$:

formula	**n**
$1 + 2 + 3 + \ldots + (s+1) + s + (s-1) + (s-2) + \ldots + 1 + 0 = s^2$	$(s+1)^2$
-(s+1)	$(s+1)^2 – (s+1)$
-(s+1)　-1	$(s+1)^2 – (s+2)$
-(s+1)　-1　-1	$(s+1)^2 – (s+3)$
…	…
-(s+1)　-1　-1　-1　-1　…　-1	$(s+1)^2 – (s+s)$

That is s^2 +1

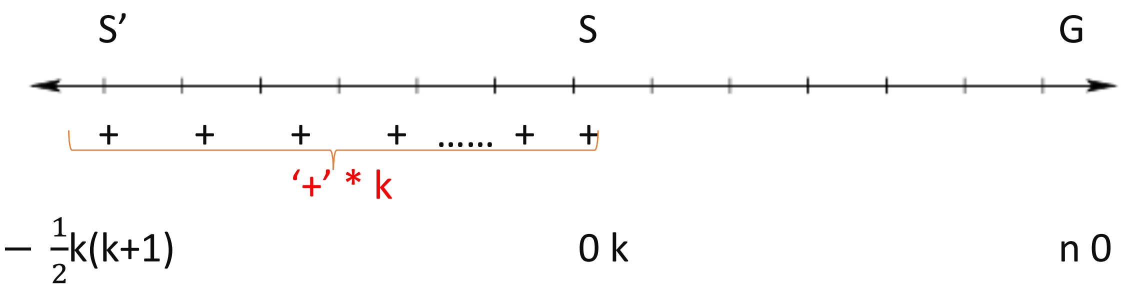
Remove this item !

The number of items is 2s+1, means M(n,0) = 2s+1

We conclude that:

$$M(n, 0) = \left\lceil 2\sqrt{n} \right\rceil = \begin{cases} 2s+1, & \text{if} \quad s^2 < n \le s(s+1) \\ 2s+2, & \text{if } s(s+1) < n \le (s+1)^2 \end{cases}$$

(c)



We assume that the object start from S’ with a velocity of 0, and keep speeding up to S,when it arrived S,it has the speed of k, the location of S’ should be (1 + 2 + 3 +…+ k) = $-\frac{1}{2}$k(k+1) far away from S.

Let n’ = S’ to G, starts with 0 speed and ends with 0 speed.

M(n’, 0) = M(n, k) + k

We use the formula from part(b):

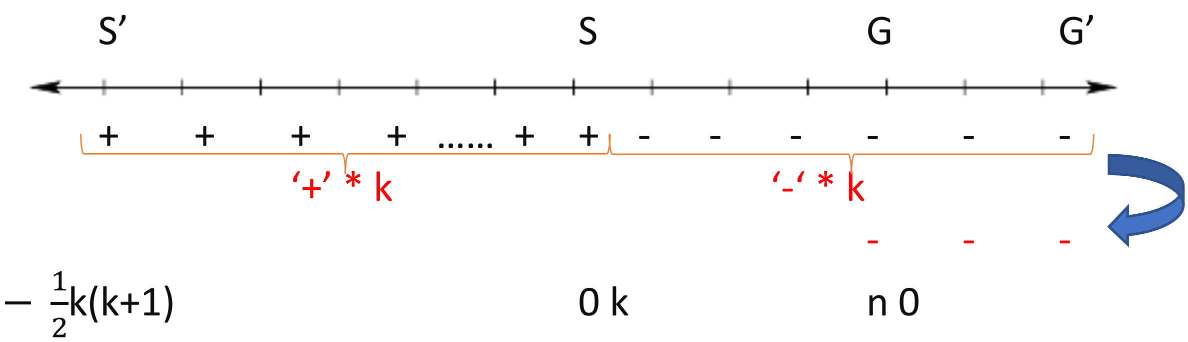
M(n’, 0) = $\left\lceil 2\sqrt{n'} \right\rceil$

We already know that n’ = n + $\frac{1}{2}$k(k+1)

So M(n, k) = M(n’, 0) – k

$= \left\lceil 2\sqrt{n + \frac{1}{2}\mathrm{k}(\mathrm{k}+1)} \right\rceil - \mathrm{k}$

(d)



If the distance between S and G is smaller than S to G’(the smallest distance for decelerating from volecity of k)= $\frac{1}{2}$k(k-1):

The object will first arrived at G’ and turn back to G.

Let S'S = $\frac{1}{2}$k(k+1); SG = n; SG' = $\frac{1}{2}$k(k-1);

So GG' = SG' – SG = $\frac{1}{2}$k(k-1) – n

M(SG') = k

M(GG', 0) = $\left\lceil 2\sqrt{\frac{1}{2}\mathrm{k}(\mathrm{k}-1)-\mathrm{n}} \right\rceil$

Because the object will first arrived at G' and turn back to G.

The real time steps is M(SG', k) +M(GG', 0)

That is M(n, k) = k + $\left\lceil 2\sqrt{\frac{1}{2}\mathrm{k}(\mathrm{k}-1)-\mathrm{n}} \right\rceil$

(e)

h (r, c, u, v, rG, cG) = max(M(r-r_G, u), M(c-c_G, v))