Q4

(a)

|  |  |  |
| --- | --- | --- |
| n | Optimal sequence | M(n, 0) |
| 1 | [+ -] | 2 |
| 2 | [+ o -] | 3 |
| 3 | [+ o o -] | 4 |
| 4 | [+ + - -] | 4 |
| 5 | [+ + - o -] | 5 |
| 6 | [+ + o - -] | 5 |
| 7 | [+ + o - o -] | 6 |
| 8 | [+ + o o - -] | 6 |
| 9 | [+ + + - - -] | 6 |
| 10 | [+ + + - - o -] | 7 |
| 11 | [+ + + - o - -] | 7 |
| 12 | [+ + + o - - -] | 7 |
| 13 | [+ + + o - - o -] | 8 |
| 14 | [+ + + o - o - -] | 8 |
| 15 | [+ + + o o - - -] | 8 |
| 16 | [+ + + + - - - -] | 8 |
| 17 | [+ + + + - - - o -] | 9 |
| 18 | [+ + + + - - o - -] | 9 |
| 19 | [+ + + + - o - - -] | 9 |
| 20 | [+ + + + o - - - -] | 9 |
| 21 | [+ + + + o - - - o -] | 10 |

(b)

Base case (n = s2, s ∈ Z)

Each item represent the moving distance during each time steps.

The number of items represent number of time steps, that is M.

1 + 2 + 3 +… + s + (s-1) + (s-2) +…+ 1 + 0 = s2

‘+’ \* s ‘ – ‘ \* s

Let s2 = n, so s = , and M(n, 0) = 2 = 2

if s(s+1) < n <= (s+1)2 , s ∈ Z:

we change from the base case formula and avoid breaking the continuity.

That is s(s+1) +1

|  |  |
| --- | --- |
| formula | n |
| 1 + 2 + 3 +… + (s+1) + s + (s-1) + (s-2) +…+ 1 + 0 = s2 | (s+1)2 |
| -1 | (s+1)2 -1 |
| -1 -1 | (s+1)2 -2 |
| -1 -1 -1 | (s+1)2 -3 |
| … | … |
| -1 -1 -1 -1 -1 … -1 | (s+1)2 -s |

-1 \* s

The number of items is still 2(s+1), means M(n,0) = 2s+2

if s2 < n <= s(s+1), s ∈ Z:

That is s2 +1

|  |  |
| --- | --- |
| formula | n |
| 1 + 2 + 3 +… + (s+1) + s + (s-1) + (s-2) +…+ 1 + 0 = s2 | (s+1)2 |
| -(s+1) | (s+1)2 – (s+1) |
| -(s+1) -1 | (s+1)2 – (s+2) |
| -(s+1) -1 -1 | (s+1)2 – (s+3) |
| … | … |
| -(s+1) -1 -1 -1 -1 … -1 | (s+1)2 – (s+s) |

Remove this item !

The number of items is 2s+1, means M(n,0) = 2s+1

We conclude that:



M(n, 0) =

(c)

S’ S G



+ + + + …… + +

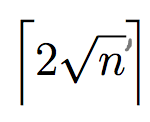
‘+’ \* k

k(k+1) 0 k n 0

We assume that the object start from S’ with a velocity of 0, and keep speeding up to S,when it arrived S,it has the speed of k, the location of S’ should be (1 + 2 + 3 +…+ k) = k(k+1) far away from S.

Let n’ = S’ to G, starts with 0 speed and ends with 0 speed.

M(n’, 0) = M(n, k) + k

We use the formula from part(b):

M(n’, 0) =

We already know that n’ = n + k(k+1)

 So M(n, k) = M(n’, 0) – k

=  2 - k

(d)

S’ S G G’



+ + + + …… + + - - - - - -

‘+’ \* k ‘-‘ \* k

- - -

k(k+1) 0 k n 0

If the distance between S and G is smaller than S to G’(the smallest distance for decelerating from volecity of k)=k(k-1):

The object will first arrived at G’ and turn back to G.

Let S’S = k(k+1); SG = n; SG’ = k(k-1);

So GG’ = SG’ – SG = k(k-1) – n

 M(SG’) = k

M(GG’, 0) =  2

Because the object will first arrived at G’ and turn back to G.

The real time steps is M(SG’, k) +M(GG’, 0)

That is M(n, k) = k + 2

(e)

h (r, c, u, v, rG, cG) = max(M(r-rG, u), M(c-cG, v))