Lib-Copy - 27/05/22 (Eve)

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1943 **A**

Unique Paper Code : 32355402

Name of the Paper : GE-4 Numerical Methods

Name of the Course : **Generic Elective CBCS (LOCF)** B.Sc(H)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.

1. (a) Find the interval in which the smallest positive root of the equation $x^3 - x - 4 = 0$ lies. Perform three iterations of the bisection method to determine the root of this equation.

(b) Perform three iterations of the secant method to find a root of the equation $xe^x = \cos x$ by taking $p_0 = 0$, $p_1 = 1$.

(c) Perform four iterations of the Newton-Raphson's method to obtain the approximate value of $(17)^{\frac{1}{3}}$ starting with the initial approximation $x_0 = 2$.

(d) Define Floating-point representation, Truncation error, and Global error with examples.

2. (a) Define the order of convergence of an iterative method. Determine the order of convergence of the Regula-Falsi method.

(b) Perform three iterations of the bisection method to find the smallest positive root of the equation $x^3 - 4x - 9 = 0$.

(c) Perform three iterations of the method of false position, to find the fourth root of 32.

(d) Perform three iterations of the Newton Raphson method to find the root of $f(x) = x \sin x + \cos x = 0$, assuming that the root is near $x = \pi$.

3. (a) Generate the forward difference table for the data

x	0	0.2	0.4	0.6	0.8
f(x)	0.12	0.46	0.74	0.9	1.2

Hence interpolate the values of $f(0.1)$ by using Gregory Newton forward differences Interpolation formulae.

(b) Define the average difference operator and the central difference operator. Prove that $\mu = \left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}}$.

(c) Given the following system of equations

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$

$$3x_1 + 5x_2 + 3x_3 = 4$$

Perform three iterations of the Gauss-Jacobi method starting with $X^{(0)} = (1,1,1)$.

(d) Find the unique polynomial of degree 3 or less such that $f(0) = -1$, $f(1) = 0$, $f(2) = 15$, $f(3) = 80$ using the Newton interpolation. Interpolate at $x = 1.5$.

4. (a) Perform three iterations of the Gauss-Seidel method starting with $X^{(0)} = (1,1,0)$ to solve the

following system of equations:

$$10x_1 + x_2 + 4x_3 = 31$$

$$x_1 + 10x_2 - 5x_3 = -23$$

$$3x_1 - 2x_2 + 10x_3 = 38.$$

(b) Solve the following system using the Gauss-Jordan method.

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22.$$

(c) Calculate the second order divided difference of $\frac{1}{x^2}$ based on the points x_0, x_1, x_2.

(d) Determine the Lagrange form of the interpolating polynomials for the following data set

x	-1	0	1	2
y=f(x)	5	1	1	11

Hence estimate the value of f(1.5).

5. (a) For the following data, find f′(2.3) and f″(2.3) by using Central difference formulas

$$f'(x_i) = \frac{f(x_i + h) - f(x_i - h)}{2h},$$

$$f''(x_i) = \frac{f(x_i + h) - 2f(x_i) + f(x_i - h)}{h^2}$$

x	2	2.3	2.6
y = f(x)	0.6932	0.7885	0.9556

(b) The following table of values is given :

x	-1	1	2	3	4	5	7
y=f(x)	1	1	16	81	256	625	2401

Find f′(3) by Richardson Extrapolation with h = 4, h = 2 and h = 1 using the following approximate formula

$$f'(x_i) = \frac{f(x_i + h) - f(x_i - h)}{2h}$$

(c) Find an approximate value of the integral

$$I = \int_0^1 \frac{1}{1+x}\, dx$$

Using: (i) Trapezoidal Rule, (ii) Composite Trapezoidal rule for n = 4.

Also calculate the error in each case.

(d) Find an approximate value of the integral

$$I = \int_1^2 \frac{1}{1+x^2}\,dx$$

Using (i) Simpson's rule, (ii) Composite Simpson's rule for n=6.

Also calculate the error in each case.

6. (a) Approximate the value of $\pi/4$ by using Simpson's $1/3^{rd}$ rule.

(b) Apply Euler's method to approximate the solution of the Initial value Problem (IVP)

$$\frac{dy}{dx} = 2 + \frac{3y}{x}, \quad 1 \le x \le 6, \; y(1) = 1, \text{ using 5 steps.}$$

(c) Apply Modified Euler's method to approximate the solution of the IVP and calculate y(0.3) by using h=0.1

$$\frac{dy}{dx} = 1 + xy, \quad 0 \le x \le 1, \; y(0) = 2.$$

(d) Use the Midpoint method to approximate the solution of

$$\frac{dy}{dx} = 2x + 3y, \quad y(0) = 0$$

with h=0.1. Determine y(0.2) and y(0.3).