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(M)

*This question paper contains 6 printed pages.*

Your Roll No. ....

*Sl. No. of Ques. Paper : 6511* **HC**  
*Unique Paper Code : 32341202*  
*Name of Paper : Discrete Structures*  
*Name of Course : B.Sc. (Hons.) Computer Science*  
*Semester : II*  
*Duration : 3 hours*  
*Maximum Marks : 75*

*(Write your Roll No. on the top immediately  
on receipt of this question paper.)*

*Question No. 1 is compulsory in Section A.  
Attempt any four questions from Section B.  
Parts of a question should be attempted together.*

## SECTION A

1. (a) Of a group of 20 students, 10 are interested in music, 7 are interested in photography, and 4 like swimming. Furthermore 4 are interested in both music and photography, 3 are interested in both music and swimming, 2 are interested in both photography and swimming and 1 is interested in music, photography and swimming. How many students are interested in photography but not in music and swimming? 3

(b) Prove that the given Boolean expression is a tautology using equivalence rules: (5)

$$(p \wedge q) \rightarrow ((\neg q \rightarrow p))$$

- (c) Given  $f(x)=x^2+1$  and  $g(x)=x+2$ . Find  $f \circ g$  and  $g \circ f$  where  $f$  and  $g$  are functions from  $\mathbb{R}$  to  $\mathbb{R}$ . 3

- (d) Consider the following advertisement of a game:

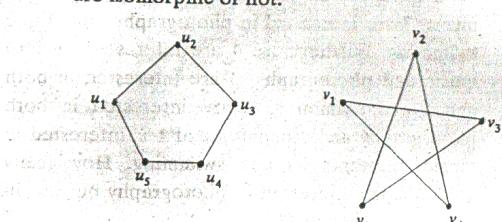
- (i) There are three statements in this advertisement.
- (ii) Two of them are not true.
- (iii) The average increase in IQ scores of people who learned this game is more than 20 points.

Prove that the statement (iii) is true using truth table.

4

- (e) A dance pair means a woman and man dancing together. How many such dance pairs can be formed from a group of 6 women and 10 men? 3

- (f) Determine whether the given graphs  $G_1$  and  $G_2$  are isomorphic or not. 4



- (g) Does there exist a simple graph with seven vertices having following given degree sequence:

$(1, 3, 3, 4, 5, 6, 6)$

3

- (h) Use Master method to find asymptotic bounds for the following recurrence relation:

$$T(n) = T(9n/10) + n$$

5

- (i) Solve the following recurrence relation using generating function method:

$$a_r = 3a_{r-1} + 2,$$

where  $r \geq 1$  with the boundary condition  $a_0 = 1$ . 5

### SECTION B

2. (a) A jigsaw puzzle consists of a number of pieces. Two or more pieces with matched boundaries can be put together to form a "big" piece. Finally, when all pieces are put together as one single block, the jigsaw puzzle is set to be solved. Putting two blocks with matched boundaries together is counted as one move. Use principle of Mathematical Induction to prove that for a jigsaw puzzle with  $n$  pieces it will take  $n-1$  moves to solve the puzzle. 5

- (b) What is a Poset? Draw a Hasse Diagram for the given Poset:

$$\{(2, 4, 5, 10, 12, 20, 25); |\}$$

5

3. (a) Consider two sets  $A$  and  $B$ ,  $A=\{1, 2, 3, 4\}$  and  $B=\{3, 4, 5, 6\}$ . Find the elements of each relation  $R$  stated below. Also, find the domain and range of  $R$ .

- (i)  $a \in A$  is related to  $b \in B$ , i.e.,  $aRb$  if and only if  $a < b$

P. T. O.

- (ii)  $a \in A$  is related to  $b \in B$ ; i.e.,  $aRb$ , if and only if  $a$  and  $b$  are both odd numbers. 5
- (b) Translate "Everybody has somebody who is his or her mother" into predicate calculus. 2
- (c) Give Big-O notation for factorial function. 3
4. (a) Show all the steps of Bubble Sort to put the following list of items in increasing order:

4	9	3	6	2	5	1
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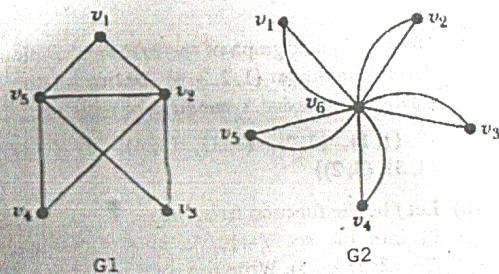
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- (b) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with  $n$  cars made in the  $n$ th month.
- (i) Set up a recurrence relation for the number of cars produced in the first  $n$  months by this factory.
  - (ii) Use recurrence equation to solve how many cars are produced in the first year.
  - (iii) Find an explicit formula for the number of cars produced in the first  $n$  months by this factory. 5

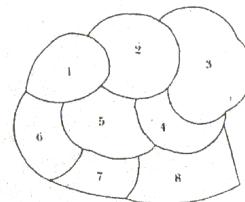
5. (a) Show that the following argument is valid:

"If Mohan is a lawyer then he is ambitious. If Mohan is an early riser, then he does not like rice. If Mohan is ambitious then he is early riser. Then if Mohan is a lawyer, then he does not like rice." 4

- (b) State the condition for Eulerian path and Eulerian circuit. Determine whether the given graphs  $G_1$  and  $G_2$  have Eulerian circuit or Eulerian path. 4



- (c) Define chromatic number for a graph. Determine  $\chi$  (chi) for the given graph  $G$ . 2



G.

6. (a) Find  $S^6a$  and  $S^{-3}a$  for the following numeric function where  $r$  is 0 for  $r=0, 1, 2, i-1$  and is  $a_{r-i}$  for  $r \geq i$ :

P. T. O.

$$a_r = \begin{cases} 1 & 0 \leq r \leq 9 \\ 2 & r \leq 10 \end{cases}$$

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- (b) Consider  $H_0=0$ ,  $H_1=1$  and  $H_n=H_{n-1}+2H_{n-2}$ .  
Give an explicit solution for  $H_n$ .

5

7. (a) Draw a directed graph of the following relations R defined on the set  $\{1, 2, 3, 4\}$ . Decide whether the relation is reflexive, symmetric, or transitive.

$$R=\{1, 1\}, \{2, 2\}, \{3, 3\}, \{4, 4\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{3, 2\}$$

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- (b) Let  $f$  be the function from the set  $X=\{2, 3, 4, 5, 6, 7\}$  into the set  $Y=\{0, 1, 2, 3, 4\}$  defined by  $f(x)=2x \pmod{5}$ . Write  $f$  as a set of ordered pairs.  
Is  $f$  one-one or onto  $Y$ ?

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