 [This question paper contains 8 printed pages.]

Your Roll No.....

**B**

Sr. No. of Question Paper : 749

Unique Paper Code : 32341202

Name of the Paper : Discrete Structures

Name of the Course : B.Sc. (Hons.) Computer  
Science

(For Admissions of 2019  
& 2020)

Semester : II

Duration : 3 hours

Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. 1 is compulsory in Section A.
3. Attempt any **four** questions from Section B.
4. Parts of a question should be attempted together.

P.T.O.



## SECTION A

1. (a) A class of 30 students comprises of boys who can play Cricket (C), Hockey (H) and Football (F). The following table shows how many students play each single game and in their various combinations:

| Game | C | H | F | CH | CF | HF | CHF |
|------|---|---|---|----|----|----|-----|
| Play | 3 | 3 | 2 | 2  | 11 | 10 | 4   |

Find how many students play at east one game? (5)

- (b) Consider a set  $A = \{U, V, W, X\}$ . Let  $R$  be the relation defined on  $A$  as

$$R = \{(U, U), (V, V), (Y, W), (W, Y) (X, Y), (X, X)\}$$

Answer the following :

- (i) Draw a Digraph for the given relation  $R$ .
- (ii) Is  $R$  an Equivalence Relation or a Partial order relation? Justify your answer. (5)



- (c) A graph has  $e$  edges and  $v$  vertices. Show that the given expression holds true in any connected planar graph with at least 2 edges and no loops

$$e \leq 3v - 6$$

Prove that  $K_5$  is non-planar using the above expression. (5)

- (d) Ashish, Aryan and their friend Abhay were playing inside the house. One of them broke a vase. When Mrs. Sharma, their mother, asked them about the vase, she gets these replies

Aryan : "I didn't break it"

Ashish : "Neither did I"

Abhay : "Aryan broke the window"

With the help of truth table find who broke the window, if two of the children lied? (5)

- (e) Find the particular solution for the given Recurrence Relation

$$a_r - 5 a_{r-1} = 3, r \geq 1$$

with the boundary conditions  $a_0 = 1$ . (5)



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- (f) Use Master method to find asymptotic bounds for the following recurrence relation :

$$T(n) = 27 T(n/3) + \theta(n^3) \quad (5)$$

- (g) Convert the following statement in symbolic form:

"If you send me the URL, then I will finish configuring the software. If you do not send me the URL, then I will go for a walk and If I go for a walk, then I will keep my blood pressure in control leads to the conclusion If I do not configure the software, then I will keep my blood pressure in control."

(5)

## SECTION B

2. (a) Use mathematical induction to prove that :

$$2n < n! \quad \forall n \text{ with } n \geq 4. \quad (5)$$

- (b) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$ , where  $\mathbb{R}$  is the set of real numbers.

Given  $f(x) = x^2$  and  $g(x) = x + 5$ .

Find  $f \circ g$  and  $g \circ f$ .

State whether  $f(x)$  is bijective. (5)



3. (a) Consider the word "MATHEMATICS". Calculate the number of ways these letters can be arranged. Calculate the number of ways the letters can be arranged such that the vowels should occur together. (5)

(b) Let  $R$  be a relation defined by  $xRy$  if and only if  $|x - y|$  is even. Show that  $R$  is an equivalence relation. (2)

(c) Let  $f_1$  and  $f_2$  be functions from  $R$  to  $R$  such that :

$$f_1(x) = x^2 \text{ and } f_2(x) = x - x^2. \text{ Compute } 0(f_1.f_2) \quad (3)$$

4. (a) How many vertices and edges are there in each of the following graphs?

(i)  $K_3$

(ii)  $C_5$

(iii)  $W_4$

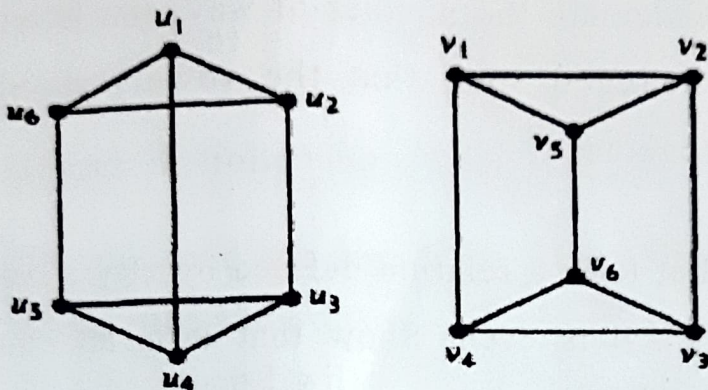
(iv)  $K_{3,4}$

(v)  $Q_3$

(5)



- (b) Define Isomorphism. Are the following pair of graphs isomorphic? Justify. (5)



5. (a) Given that:

$$T(n) = 2 T(n/2) + Cn$$

Using substitution method, prove that  $T(n)$  is  $\theta(n \lg n)$ . (5)

- (b) Show all the steps of Insertion Sort to put the following list of items in an ascending order :

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| 5 | 2 | 7 | 4 | 9 | 3 | 6 | 1 | 8 |
|---|---|---|---|---|---|---|---|---|

6. (a) "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed".



Show that the above premises leads to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed." (5)

(b) Solve the given recurrence relation for the sequence defined by :

$$C_n = 5C_{n-1} - 6C_{n-2}$$

with initial conditions  $C_1 = 2, C_3 = 1$ . (5)

7. (a) Given a relation  $R$  on set  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$  such that

$$R = \{(a, b) : a \text{ is divisor of } b \text{ and } a \in A, b \in A\}$$

Show that  $R$  is a POSET. Draw its Hasse Diagram.

(5)

(b) Prove that  $(\neg p \wedge ((\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$ .

(3)

(c) Find the inverse and contra-positive for the statement :

"If you send me the URL, then I will finish configuring the software." (2)



8. (a) Given that the value of  $p \rightarrow q$  is false, determine the value of  $(p' \vee q') \rightarrow q$ . (5)
- (b) Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane? (3)
- (c) How many edges does a full binary tree with 1000 internal vertices have? (2)