

At 15/05/2023 Regular (1-5)
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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4523

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Unique Paper Code : 32341401

Name of the Paper : Design and Analysis of Algorithms

Name of the Course : **B.Sc. (II) Computer Science**

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. Question No. 1 is compulsory.
 3. Attempt any **four** of Questions Nos. 2 to 7.
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1. (a) Use the Master's Theorem to give tight asymptotic bounds for the recurrence $T(n) = 8T(n/2) + O(n^2)$.
(3)

P.T.O.

- (b) Discuss the running time of the following snippet of code :

```
count = 0
```

```
for (i=1, i<=n, i++)
```

```
    for (j=1, j<=n, j = 2 * j)
```

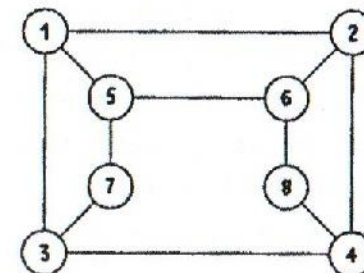
```
        count++
```

(3)

- (c) A team of explorers is visiting the Sahara desert. As the weather is very hot, they are having n bottles of different sizes to carry water and keep them hydrated. In covering few kilometres, they used all of their water but fortunately found a water source nearby. This water source is having only L litres of water which is way lesser than the capacity of all bottles. They want to fill L litres of water in minimum number of bottles. Describe a greedy algorithm to help them fill U litres of water in minimum number of bottles. (3)

- (d) Will the greedy strategy with the greedy parameter being value per unit weight of the items yield an optimal solution for the 0-1 knapsack problem? Justify. (3)

- (e) Can dynamic programming be applied to all optimization problems? Why or why not? (3)
- (f) Let G be a tree-graph. Further, let T_B and T_D be the trees produced by performing BPS and DFS respectively on G . Can T_B and T_D be different trees? Why or why not? (4)
- (g) Why is the worst-case running time for bucket sort $\theta(n^2)$? What changes would you make to the algorithm so that its worst-case running time becomes $O(n \lg n)$? (4)
- (h) Consider the following graph: (4)



Specify whether the above graph is bipartite or not. If yes, give the partition, else justify.

- (i) We are given a weighted graph G in which edge weights are not necessarily distinct. Can graph G have more than one minimum spanning tree (MST)? If yes, give an example, else justify. (4)
- (j) Show that in any subtree of a max-heap, root of the subtree contains the largest value occurring anywhere in that subtree. (4)
2. (a) Consider the scheduling problem wherein you are given a single resource and a set of requests having deadlines. A request is said to be late if it misses the deadline. Your goal is to minimize the maximum lateness. With respect to a schedule S , idle time is defined as the time during which the resource is idle, in between two requests. S is said to have an inversion when request i has been scheduled before j and $d(i) > d(j)$, where $d(i)$ and $d(j)$ are the deadlines of the requests i and j respectively. Argue that all schedules with no idle time and no inversions have the same maximum lateness. (6)
- (b) For each of the following sorting algorithms, merge sort and insertion sort, discuss whether or not it is
- (i) stable
 - (ii) in-place
- (4)

3. (a) Let $G = (V, E)$ be a directed unweighted graph. Given two vertices s and t in V , what is the time required to determine if there exists at least one s - t path in G ? Can we use the DFS algorithm to find the shortest-path distance from the s to t ? If yes, justify, otherwise give a counter example. (6)
- (b) Suppose we perform a sequence of stack operations on a stack whose size never exceeds k . After every k operations, we make a copy of the entire stack for backup purposes. Show that the cost of n stack operations, including copying the stack, is $O(n)$ by assigning suitable amortized costs to the various stack operations. (4)
4. (a) Show that for an n -element max heap (having distinct elements) represented through an array, the leaves are the nodes indexed by $\text{floor}(n/2 + 1)$, $\text{floor}(n/2 + 2)$, ..., n . What would be the location of the minimum element in the above heap? (6)
- (b) Given an array A of n integers, you need to find the maximum sum of any contiguous subarray. For instance, the maximum sum of any contiguous

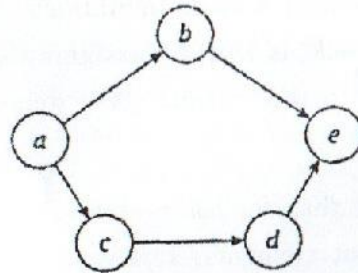
subarray in the array $-1, 2, 3, -2, 5, -6, 7, -8$ is 9 (which is the sum of the subarray $2, 3, -2, 5, -6, 7$). Complete the following Dynamic Programming solution for the above problem :

$$DP[0] = A[0]$$

For $i = 1$ to n

$$DP[i] = \max(A[i], \text{---}) \quad (4)$$

5. (a) How many topological orderings does the following graph have? Specify all of them. (6)



- (b) A student was asked to sort a list of n numbers in decreasing order. The student writes an algorithm which works iteratively as follows. In every iteration, the following two steps are done :

- (i) Linear search is used to find the maximum element in the portion of the array which is not yet sorted.

- (ii) The maximum element found in step 1 is placed at the beginning of the not-yet-sorted portion of the array.

This algorithm is given as input a list already sorted in decreasing order. What would be the time complexity of the algorithm on this input? Explain. (4)

6. (a) (i) What is the smallest possible depth of a leaf in a decision tree for a comparison sort? Name a sorting technique to which this smallest depth would correspond. (6)

- (ii) What is the minimum number of leaves in the decision tree for a comparison sort? Use this observation to derive a lower bound on the number of comparisons performed by a comparison sort in the worst case.

- (b) Show that at most $3 * \text{floor}(n/2)$ comparisons are sufficient to find both the minimum and maximum in a given array of size n . (4)

7. (a) The BFS algorithm has been used to produce the shortest paths from a node s to all other nodes in a graph G . Can the Dijkstra's algorithm be used in place of BFS? In a different scenario, the Dijkstra's algorithm has been used to produce the shortest paths from a node s to all other nodes in a graph G' . Can BFS be used in place of the Dijkstra's algorithm? Explain your answers for both the scenarios. (6)
- (b) Write a pseudocode for the memorized recursive algorithm to compute the n th Fibonacci number. What would be its time complexity? (4)