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S. No. of Question Paper : 2743

Unique Paper Code : 32355402

GC-4

Name of the Paper : GE-4 Numerical Methods

Name of the Course : Mathematics : Generic Elective for
Honours

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

Use of Scientific Calculator is allowed.

1. (a) Explain significant digits and Local truncation error with examples. An approximate value of π is given by 3.1428571 and its true value is 3.1415926. Find the absolute and relative errors.

P.T.O.

- (b) Explain the Bisection method for computing the roots of equation $f(x) = 0$. Perform three iterations of the Bisection method in the interval (1, 2) to obtain roots of the equation $f(x) = x^3 - x - 1 = 0$.
- (c) Define rate of convergence. Determine the rate of convergence for the Secant method. 12
2. (a) Perform four iterations of the Regula-Falsi method to obtain a root of the equation :
- $$f(x) = 3x + \sin x - e^x = 0.$$
- (b) Perform three iterations of Newton's method to find the root of the equation $x^4 - x - 10 = 0$ and starting approximation as 1.5.
- (c) Perform two iterations of Newton's method to solve the non-linear system of equations with initial approximation (1, 1) :

$$f(x, y) = x^2 + y - 11 = 0 \text{ and}$$

$$g(x, y) = x + y^2 - 7 = 0. \quad 12$$

3. (a) Solve the linear system $Ax = b$ using Gaussian elimination with pivoting :

$$A = \begin{bmatrix} 6 & 2 & 2 \\ 6 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}.$$

- (b) Starting with initial vector $(x, y, z) = (0, 0, 0)$ perform three iterations of Gauss-Seidel method to solve the following system of equations :

$$2x - y = 7, -x + 2y - z = 1, -y + 2z = 1.$$

- (c) Explain Thomas Algorithm and solve the following Tridiagonal system $Ax = b$ using the Thomas Method :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \\ 0 & 3 & 10 \end{bmatrix}, b = \begin{bmatrix} 10 \\ 17 \\ 22 \end{bmatrix}.$$

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P.T.O.

4. (a) Find the unique polynomial $P(x)$ of degree 2 or less using Lagrange interpolating formula for the following data :

$$x = [1, 3, 4]$$

$$f(x) = [1, 27, 64]$$

Also, estimate $P(1.5)$.

- (b) Prove the following relations :

$$(i) \quad \mu = \sqrt{1 + \frac{\delta^2}{4}}$$

$$(ii) \quad \Delta = \frac{1}{2}\delta^2 + \delta \sqrt{1 + \frac{1}{4}\delta^2}$$

- (c) By use of Richardson extrapolation, find $f'(3)$ using the approximate formula :

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

with $h = 4, 2$ and 1 , from the following values :

x	$f(x)$
-1	1
1	1

2	16
3	81
4	256
5	625
7	2401

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5. (a) The following data represents the function $f(x) = e^x$:

x	$f(x)$
1	2.7183
1.5	4.4817
2.0	7.3891
2.5	12.1825

Estimate the value of $f(2.25)$ using the Newton's forward difference interpolation and compare with the exact value.

P.T.O.

- (b) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the given data :

x	$f(x)$
0	1
1	2
2	5
3	10

and interpolate at $x = 0.5$ and 1.5 .

- (c) Find the approximate value of :

$$I = \int_0^1 \frac{dx}{1+x}$$

using :

(i) Trapezoidal rule

(ii) Simpson's rule.

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6. (a) Apply Euler's modified method to approximate the solution of the initial value problem and calculate $y(1.3)$ by using $h = 0.1$:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \quad y(1) = 1.$$

- (b) Apply mid-point method (R.K. Second order) to solve the initial value problem :

$$\frac{dy}{dx} = yx^3 - 1.5y$$

from $x = 0$ to 2 where $y(0) = 1$ by using $h = 1$.

- (c) Apply finite difference method to solve the given problem :

$$\frac{d^2y}{dx^2} = y + x(x-4), \quad 0 \leq x \leq 4,$$

$$y(0) = 0, \quad y(4) = 0 \quad \text{with } h = 1.$$

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