Name of Course	: **Generic Elective CBCS (Other Than Maths. (H))**
Unique Paper Code	: **32355402_OC**
Name of Paper	: **GE-4 Numerical Methods**
Semester	: **IV**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Evaluate $S = \sqrt{102} - \sqrt{101}$ up to four significant digits and find its absolute error and relative error. If δ denotes the central difference operator and μ denotes the averaging operator , then establish the following relations:

(i) $\sqrt{1 + \delta^2\mu^2} = 1 + \frac{\delta^2}{2}$

(ii) $\mu^2 = 1 + \frac{1}{4}\delta^2$.

2. Perform three iterations of the secant method to find an approximate value to the root of the equation $x^2 - 2x + 1 = 0$ starting with initial approximations $x_0 = 2.6$ and $x_1 = 2.5$. Obtain the absolute error in each of the three iterations.
Perform three iterations of Newton-Raphson method to find an approximate value of $17^{1/2}$. Take initial approximation $x_0 = 4$.

3. Solve the following system of equations using the Gaussian elimination method with row pivoting:

$$\begin{aligned} 2x + y + 3z &= 1 \\ 4x - 3y + 5z &= -7 \\ -3x + 2y + 4z &= -3 \;. \end{aligned}$$

Starting with the initial vector $(x_1, x_2, x_3) = (0,0,0)$, perform three iterations of the Jacobi method to solve the following system of equations:

$$\begin{aligned} 2x_1 - x_2 &= 7 \\ -x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 2x_3 &= 1. \end{aligned}$$

4. Find the Lagrange form of interpolating polynomial for the function $f(x) = e^x$ passing through the points $(-1, e^{-1})$, $(0,1)$ and $(1, e)$. Hence estimate $\sqrt{e}$.
Find the interpolating polynomial using the Newton's forward difference interpolation for the following data :

x	0.1	0.2	0.3
$f(x)$	-1.27	-0.98	-0.63

Hence estimate $f(0.15)$.

Obtain the divided difference $f[a, b, c]$ for $f(x) = x^{-2}$.

5. Obtain the piecewise linear interpolation polynomial for the function defined by the given data:

x	0	1	16	81
$f(x)$	0	1	2	3

Hence interpolate at $x = 15$. Compare the interpolated value of $f(15)$ with $\sqrt[4]{15}$.

Find $f'(1)$ using the Richardson extrapolation and the approximate formula:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

with $h = 1$ and 0.5 from the following values:

x	0	0.5	0.75	1	1.25	1.5	2
$f(x)$	1	0	-0.7071	-1	-0.7071	0	1

Compare the extrapolated value of $f'(1)$ with $\frac{d}{dx}(\cos \pi x)$ at $x = 1$.

6. Solve the following initial value problem over the interval from $t = 0$ to $t = 1$ with step size $h = 0.5$:

$$\frac{dy}{dt} = 3e^{-t} - 0.4y\,, \qquad y(0) = 5$$

i. Using Euler's method
ii. Using Heun's method (without iteration).

Given that the exact solution of the given problem is $y(t) = 5e^{-t}(2e^{\frac{3t}{5}} - 1)$, verify which method gives better approximation to the solution by computing absolute error in each case.