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[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **1823** **GC-4**

Unique Paper Code : 32351201

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Real Analysis

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(c) **All** questions are compulsory.

(b) Attempt any **two** parts from each question.

1. (a) Define Infimum and Supremum of a non-empty subset of R.

Find infimum and supremum of the set

$$S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}.$$ 5

(b) Prove that a number u is the supremum of a non-empty subset S or R if and only if :

(i) $S \leq u \quad \forall s \in S$.

(ii) For any $\in > 0$, there exists $s_{\in} \in S$ such that $u - \in < S_{\in}$. 5

(c) State Archimedean Property of Real numbers. Prove that if $S = \left\{\frac{1}{n} : n \in N\right\}$, then inf S=0. 5

2. (a) Let A and B be bounded non-empty subsets of R. Define :

$A + B = \{a + b : a \in A \text{ and } b \in B\}$

Prove that inf (A+B) = inf A+inf B. 5

(b) State Density Theorem. Show that if x and y are real numbers with x < y, then there exists an irrational number z such that $x < z < y$. 5

(c) Define limit point of a set. Find limit points of] 0, 1[. 5

3. (a) Define the convergence of a sequence (x_n) of real numbers. Show that if (x_n) is a convergent sequence of real numbers such that $x_n \geq 0 \ \forall n \in N$, then $x = \lim x_n \geq 0$.

(b) Using the definition of the limit of a sequence, find the following limits :

(i) $\lim_{n\to\infty}\left(\frac{3n+1}{2n+5}\right)$.

(ii) $\lim_{n\to\infty}\left(\frac{\sqrt{n}}{n+1}\right)$. 5

(c) Prove that $\lim_{n\to\infty} n^{1/n} = 1$.

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4. (a) Let (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0 \forall n \in N$. Show that the sequence $\sqrt{x_n}$ converges to $\sqrt{x}$. 5

(b) Prove that every monotonically increasing bounded above sequence is convergent. 5

(c) If $x_1 < x_2$ are arbitrary real numbers and $x_n = \frac{1}{2}(x_{n-2} + x_{n-1})$ for n>2, show that (x_n) is convergent. What is its limit ? 5

5. (a) Define a Cauchy Sequence. Is the sequence (x_n) a Cauchy Sequence, where $x_n = 1 + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!}$? Justify your answer. $7\frac{1}{2}$

(b) State and prove Bolzano Weierstrass Theorem for sequences. Justify the theorem with an example. $7\frac{1}{2}$

(c) (i) Show that if (x_n) is unbounded, then there exists a subsequence (x_{nk}) such that : $\lim\left(\frac{1}{x_{nk}}\right) = 0$. 5

(ii) Show that the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \ldots\ldots\right)$ is divergent.

$2\frac{1}{2}$

6. (a) If $\sum_{n=1}^{\infty} x_n$ converges, then prove that $\lim_{n\to\infty} x_n = 0$. Does the converse hold ?

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(b) Test the convergence of any **two** of the following series : 5

(i) $\sum \frac{n+1}{n2^n}$

(ii) $\sum\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

P.T.O.

(iii) $\sum \frac{n^2}{n!}$

(c) State the Alternating Series Test. Show that the alternating series $\sum \frac{(-1)^n}{n}$ is convergent. 5

7. (a) Let $0 \le a_n \le b_n \forall n$. Show that : 5

(i) If $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges, then so does $\sum_{n=1}^{\infty} b_n$.

(b) Show that every absolutely convergent series is convergent but the converse is not true. 5

(c) State Integral Test. Find the condition of convergence of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^p}$. 5