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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1126**

Unique Paper Code : **235203** **G**

Name of the Paper : **Analysis-II [MAHT-202]**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts of each question.

All questions are compulsory.

1. (*a*) Use the $\in - \delta$ definition of the limit to show that :

$$\lim_{x \to c} x^2 = c^2, \; c \in \mathbb{R}.$$

(*b*) Let $A \subseteq \mathbb{R}$, let $f : A \to \mathbb{R}$ and let c be a cluster point of A. If

$$\lim_{x \to c} f(x) < 0$$

then prove that there exists a neighbourhood $V_\delta(c)$ of c such that $f(x) < 0$ for all $x \in A \cap V_\delta(c)$, $x \neq c$.

P.T.O.

(c) Using Sequantial Criterion for limits, prove that :

$$\lim_{x \to 0} \sin(1/x^2)$$

does not exist in R.

(d) Using definition, prove that :

(i) $\lim_{x \to 0} \left(\frac{1}{x^2}\right) = \infty$

(ii) $\lim_{x \to \infty} 1/x = 0.$ 5,5,5,5

2. (a) Prove that a function $f : A \to R$ is continuous at a point $c \in A$ if and only if for every sequence (x_n) in A that converges to c, the sequence $(f(x_n))$ converges to $f(c)$.

(b) Let $A \subseteq R$, let $f : A \to R$ be continuous at a point $c \in R$. Show that for any $\in > 0$ there exists a neighbou..ood $_\delta(c)$ of c such that if $x, y \in A \cap V_\delta(c)$ then :

$$|f(x) - f(y)| < \in.$$

(c) Let f and g be continuous from R to R and suppose that $f(r) \geq g(r)$ for every rational numbers then prove that $f(x) \geq g(x)$ for all $x \in$ R.

(d) State Bolzano's Intermediate value theorem and hence prove that $xe^x = 2$ for some x in $[0, 1]$. 5,5,5,5

3. (a) Let $I = [a, b]$ be an interval and let $f : I \rightarrow R$ be continuous on I. Then prove that f is uniformly continuous on I.

(b) Show that the function $f(x) = 1/x$ is uniformly continuous on $[a, \infty[$, $a > 0$ but is not uniformly continuous on $]0, \infty[$.

(c) Given that the function $f(x) = x^3 + 2x + 1$ for $x \in$ R has an inverse f^{-1} on R, find the value of $(f^{-1})'(y)$ at the points corresponding to $x = 0, 1$.

(d) Prove that if $f : I \rightarrow R$ has a derivative at $c \in I$, then f is continuous at c. Is the converse true ? Justify your answer. 5,5,5,5

4. (*a*) Let c be an interior point of the interval I at which $f : \mathrm{I} \to \mathrm{R}$ has a relative extremum. If the derivative of f at c exists then prove that :

$$f'(c) = 0.$$

Can f has a relative extremum at 'c' without being differentiable at 'c' ? Justify.

(*b*) Using the mean value theorem, prove that :

$$\frac{x-1}{x} < \log(x) < x - 1 \text{ for } x > 1.$$

(*c*) Find the point of relative extrema of the function :

$$f(x) = x(x-8)^{1/3} \text{ for } 0 < x < 9.$$

(*d*) State Darboux theorem. Let $f : [0, 2] \to \mathrm{R}$ is continuous on $[0, 2]$ and differentiable on $]0, 2[$ and that $f(0) = 0$, $f(1) = 2$, $f(2) = 2$ then show that there exists $c \in]0, 2[$, such that :

$$f'(c) = 3/2.$$

5,5,5,5

5. (*a*) Obtain Maclaurin's series expansion for the function $\sin(4x)$.

(*b*) Using Taylor's theorem prove that :

$$1 + \frac{x}{2} - \frac{x^2}{8} < \sqrt{1 + x} < 1 + \frac{x}{2} \text{ for } x > 0.$$

(*c*) Define Radius of convergence of the power series :

$$\sum_{n=0}^{\infty} a_n x^n.$$

Find the Radius of convergence and the exact interval of convergence for the power series :

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

(*d*) Check which of the following functions are convex :

(*i*) $|x|, x \in [-2, 5]$

(*ii*) $ax^3 + 2x + 3, a < 0, x \in [-1, 1].$ 5,5,5,5