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This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper	:	**1127**
Unique Paper Code	:	**235204** **G**
Name of the Paper	:	**Probability and Statistics—MAHT 203**
Name of the Course	:	**B.Sc. (Hons.) Mathematics**
Semester	:	**II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

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In all there are *six* questions.

Question No. 1 is compulsory and

it contains five parts of 3 marks each.

In Question No. 2 to 6, attempt any *two* parts from three parts.

Each part carries 6 marks.

Use of Scientific calculator is allowed.

1. (*a*) If C_1 and C_2 are events such that $C_1 \subseteq C_2$, then prove that $P(C_1) \leq P(C_2)$.

(*b*) Let X be a random variable with mean μ, variance σ^2 and μ_2' as the second moment about the origin, then show that $\sigma^2 = \mu_2' - \mu^2$.

(*c*) Let X have the probability density as follows :

$$f(x) = \begin{cases} k\, e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find k and $P(0.5 \leq X \leq 1)$.

(*d*) If the probability density of X is given by :

$$f(x) = \begin{cases} 2(1 - x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that :

$$E(X^r) = \frac{2}{(r + 1)(r + 2)}.$$

(*e*) A five card poker hand dealt from a deck of 52 playing cards is said to be a full house if it consists of three of a kind and a pair. If all the five card hands are equally likely, what is the probability of being dealt a full house ?

2. (*a*) Let $\{C_n\}$ be an increasing sequence of events, then show that :

$$\lim_{n \to \infty} P(C_n) = P(\lim_{n \to \infty} C_n) = P\left(\bigcup_{n=1}^{\infty} C_n\right).$$

(*b*) Define a negative binomial distribution with parameters k and θ. If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it ?

(*c*) If A, B, C are any three events in a sample space, then show that :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

3. (*a*) Define a normal distribution and show that its moment generating function is given by :

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

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(b) For a binomial distribution find its moment generating function and hence find its mean and variance.

(c) Find the moment generating function of the geometric distribution with parameter θ and use it to show that its mean is $\frac{1}{\theta}$ and variance is $\frac{1-\theta}{\theta^2}$.

4. (a) Let X_1 and X_2 be two random variables with joint pdf as :

$$f(x_1, x_2) = \begin{cases} 4x_1x_2 & 0 < x_1 < 1, \\ & 0 < x_2 < 1, \\ 0 & \text{elsewhere} \end{cases}$$

Is $E(X_1X_2) = E(X_1)\,E(X_2)$?

(b) Let the random variables X and Y have a joint pdf :

$$f(x, y) = \begin{cases} x + y & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the correlation coefficient of X and Y.

(c) Let :

$$f(x_1, x_2) = \begin{cases} 21x_1^2x_2^3 & 0 < x_1 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

be the joint pdf of X_1 and X_2.

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Find :

(i) the conditional mean of X_1 given $X_2 = x_2$, $0 < x_2 < 1$, and

(ii) the distribution of $Y = E(X_1|X_2)$.

5. (a) Find the marginal density of X and Y if the pair of random variables (X, Y) has a bivariate normal distribution.

(b) If the regression of Y on X is linear, then show that :

$$\mu_{Y|x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1).$$

(c) Given the two random variables X and Y that have the joint density :

$$f(x, y) = \begin{cases} x \cdot e^{-x(1+y)} & \text{for } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the regression equation of Y on X and sketch the regression curve.

6. (a) If an individual is in state 1 (lower income class) then there is a probability of 0.65 that any offspring will be in the lower income class, a probability of 0.28 that offspring will be in state 2 (middle income class), and a probability of 0.07 that offspring will be in the state 3 (upper income class).

Write the transition matrix P for the said process. If a parent is in state 3 (upper income class), find the probability that a grandchild will be in state 2 (middle class).

(*b*) State and prove Chebyshev's Theorem.

(*c*) Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that X = 20. Use the normal approximation and then compare it to the exact solution.

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