Geometry

Regular Polygons → All sides and angles are equal.

Convex polygon → All diagonals lay inside the figure.

SUM OF ALL ANGLES OF A POLYGON →
$(n-2) \times 180^\circ$
↑ no. of sides

SUM OF EXTERIOR ANGLES OF A POLYGON – 360°

TRIANGLE

* Sum of two sides is always greater than the third side.

* difference of two sides is always less than the third side.

* side opposite to greatest angle is the greatest.

* side opposite to the smallest angle is the smallest.

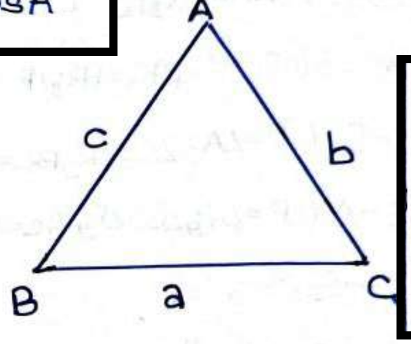
AREA FORMULAS

1. $\frac{1}{2} \times \text{base} \times \text{height}$
2. $\sqrt{s(s-a)(s-b)(s-c)}$

   $s = \frac{a+b+c}{2}$
3. $r \times s$ ($r \rightarrow$ inradius)
4. $\frac{abc}{4R}$ ($R \rightarrow$ circumradius)
5. $\frac{1}{2} \times$ product of two sides $\times$ sine of included angle

Rule of cosines $\rightarrow$

$a^2 = b^2 + c^2 - 2bc\cos A$



Rule of sines $\rightarrow$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = R$$

circumradius

RULES FOR CONGRUENCY $\Delta 1 \cong \Delta 2$

① S-S-S ($AB=PQ$; $AC=PR$; $BC=RQ$)

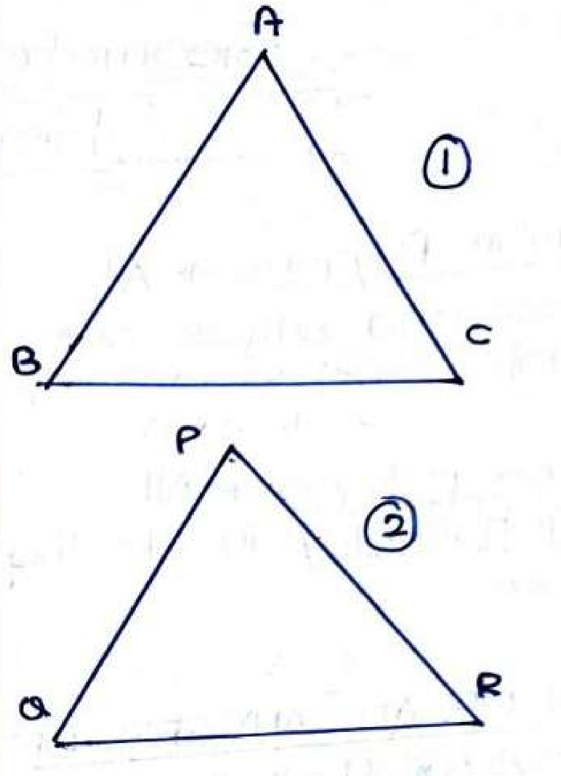
② S-A-S ($AB=PQ$; $AC=PR$; $\angle P=\angle A$)

③ A-A-S ($\angle P=\angle A$; $\angle C=\angle R$; $BC=QR$)

④ A-S-A ($\angle P=\angle A$; $\angle C=\angle R$; $AC=PR$)

⑤ A-S-S*

<u>Note</u> → The 5^{th} rule for congruency is true only if the equal angle in both the triangles lie opposite to the equal side.



RULES FOR SIMILARITY $\Delta 1 \sim \Delta 2$

① A-A-A or A-A ($\angle A=\angle P$)

② S-S-S (Ratio of sides)

$\left(\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}\right)$

③ S-A-S

$\left(\frac{AB}{PQ}=\frac{AC}{PR}; \angle A=\angle P\right)$

*all congruent triangles are similar.

In similar Δs,

RATIO OF MEDIANS =
RATIO OF HEIGHTS =
RATIO OF CIRCUMRADIUS =
RATIO OF INRADIUS =
RATIO OF ANGLE BISECTORS

EQUILATERAL TRIANGLE

① $h=\frac{a\sqrt{3}}{2}$

② All angles are $60°$

③ $R=\frac{2h}{3}=\frac{a}{\sqrt{3}}$

④ $r=\frac{h}{3}=\frac{a}{2\sqrt{3}}$

R → Circumradius
r → Inradius

* The orthocenter, circumcenter and incenter of equilateral Δ lie on the same point.

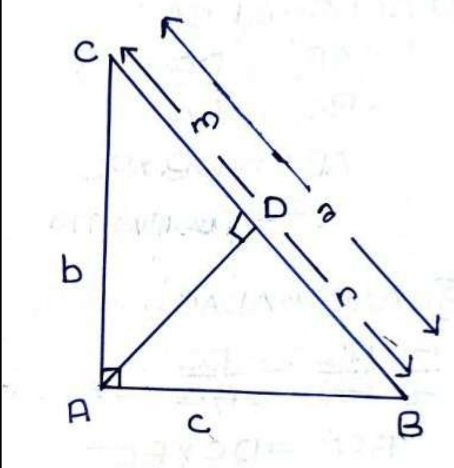
* The median is the same as altitude.

* Circumradius = 2×inradius

* Within a given perimeter, equilateral Δ has maximum area.

* Equilateral Δ has maximum area compared to other triangles in a circle.

Imp. fig. →



RIGHT-ANGLED TRIANGLE

Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

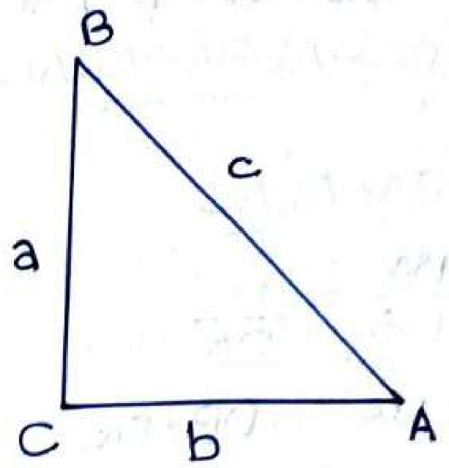
Derivation →

We know from rule of cosines that

$c^2 = a^2 + b^2 + 2ab\cos A$

$c^2 = a^2 + b^2 + 2ab\cos 90°$

$c^2 = a + b^2$



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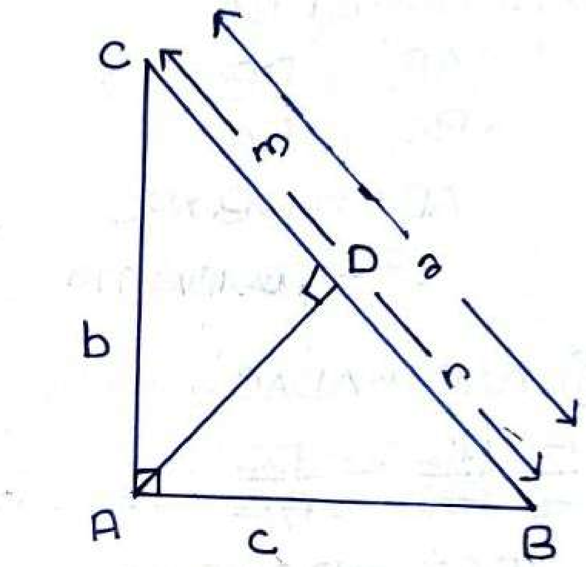
Imp. fig. →



In ΔABC and ΔDBA

$\angle B = \angle B$

$\angle D = \angle A = 90^\circ$

$\therefore \Delta ABC \sim \Delta DBA$

In ΔABC and ΔDAC

$\angle A = \angle D = 90^\circ$

$\angle C = \angle C$

$\therefore \Delta ABC \sim \Delta DAC$

Hence we can say that

$\Delta ABC \sim \Delta DBA \sim \Delta DAC$

① $\Delta DBA \sim \Delta DAC$

$$\frac{DA}{DB} = \frac{DC}{DA}$$

$$DA^2 = DB \times DC$$

$$(AD)^2 = mn$$

② $\Delta ABC \sim \Delta DBA$

$$\frac{AB}{BC} = \frac{DB}{BA}$$

$$AB^2 = DB \times BC$$

$$c^2 = na$$

③ $\Delta ABC \sim \Delta DAC$

$$\frac{AC}{BC} = \frac{DC}{AC}$$

$$AC^2 = DC \times BC$$

$$b^2 = ma$$

INCENTER

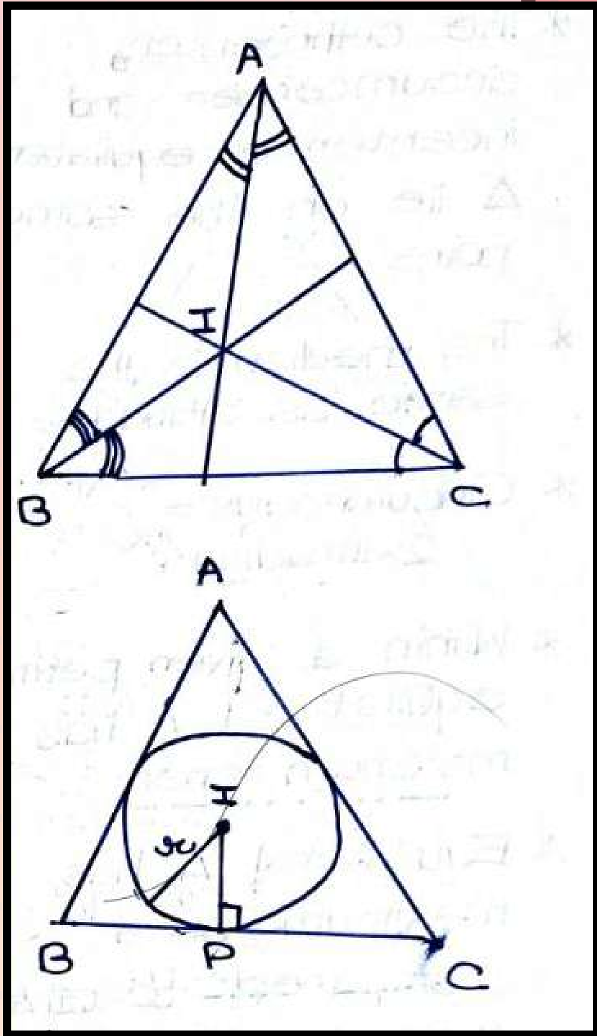
* It's the center of incircle.
* The radius of the circle is inradius (r)
* Incenter is the meeting point of all angle bisectors of triangle.

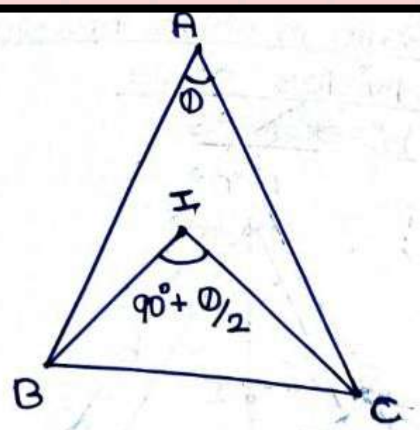
* Inradius touches side of Δ at 90°.

* Incenter is equidistant from all sides of Δ.

* Incenter always lies inside the Δ.

* AREA OF Δ = INRADIUS $\times$ SEMI-PERIMETER





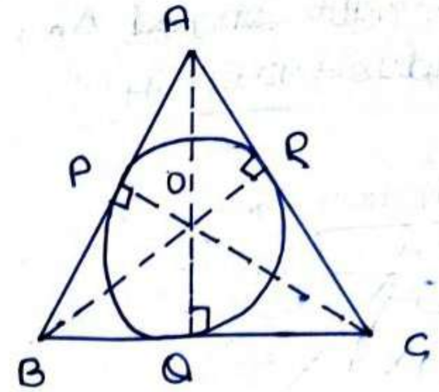
* Angle substended by two vertices of Δ on incenter is 90° more than half the angle substended by them on the third vertice.

* RATIO BETWEEN INRADIUS AND HEIGHTS OF Δ:

$$\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$$

* Incenter is NOT meeting point of all altitudes.

* Derivation of area = inradius × semiperimeter.



In ΔAOB

$$ar(\Delta AOB) = \frac{1}{2} \times OP \times AB$$

In ΔAOC

$$ar(\Delta AOC) = \frac{1}{2} \times AO \times OC$$

In ΔBOC

$$ar(\Delta BOC) = \frac{1}{2} \times QO \times BC$$

$$ar(\Delta ABC) = ar(\Delta AOB) + ar(\Delta AOC) + ar(\Delta BOC)$$

$$= \frac{1}{2} OP \cdot AB + \frac{1}{2} AO \cdot AC + \frac{1}{2} QO \cdot BC$$

$$= \frac{1}{2} \begin{bmatrix} r \cdot AB + \\ r \cdot AC + \\ r \cdot BC \end{bmatrix}$$

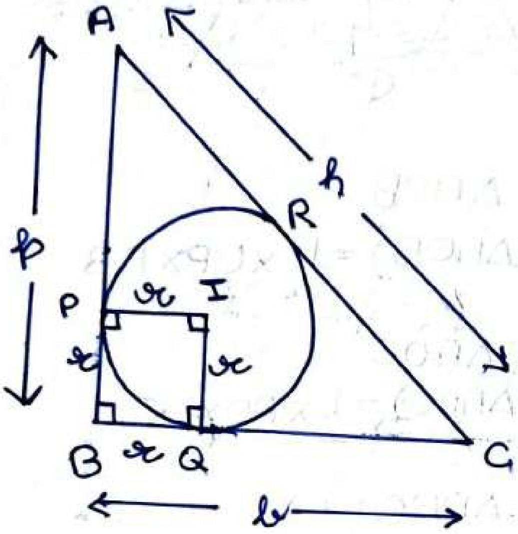
$$= r \frac{(AB + AC + BC}{2}$$

$$= sr$$

* In a right angled Δ,

$$\text{Inradius} = \frac{P+B-H}{2}$$

Derivation →



We can see that PBQI is a rectangle.

$PB = QI = r$

$BQ = PI = r$

~~ABQAR~~

$AR = AP$ (Tangents from a point)

$= p - r$

$RC = QC$ (Tangents from a point)

$= b - r$

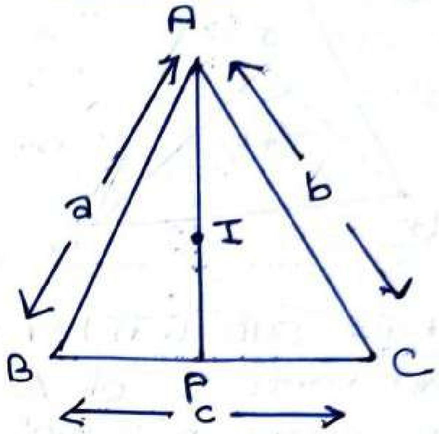
$AC = AR + RC$

$= p - r + b - r$

$h = p + b - 2r$

$$r = \frac{p+b-h}{2}$$

*Ratio in which inradius divides angle bisectors



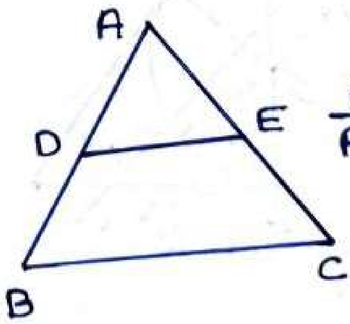
$$\frac{AI}{IP} = \frac{a+b}{c}$$

$$= \frac{\text{sides with } A}{\text{side without } A}$$

IMPORTANT THEOREMS

① BASIC PROPORTIONALITY THEOREM →

If a line is drawn ∥ to one side of a Δ, it divides the other two sides in the same ratio.



If $DE \parallel BC$,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

② CONVERSE OF BPT →

If a line divides Δ's ~~into the~~ two sides in same ratio then the line must be ∥ to the third side.

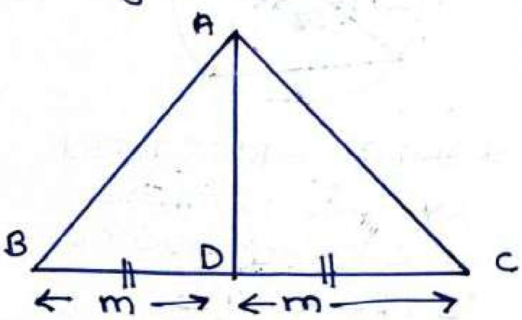
③ The ratio of areas of two similar Δs is equal to the square of ratio of its corresponding sides.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

Given $\Delta ABC \sim \Delta PQR$

④ APPOLONIUS THEOREM

This is used to find length of the median.



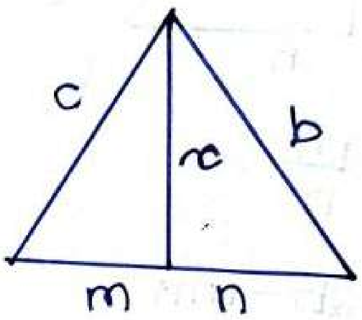
$$\frac{(AB)^2 + (AC)^2}{2} = (AD)^2 + m^2$$

5) STEWART THEOREM

Appolonius theorem is a special case of stewart theorem.

Stewart theorem ca be used to find length of any line, not just median.

$$x^2 = \frac{b^2m + c^2n}{m+n} - mn$$



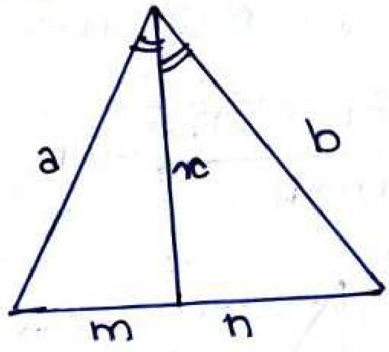
6) Relation between sides of Δ and median

Suppose a, b, c are sides of Δ and x, y, z are medians.

$$\frac{a^2+b^2+c^2}{x^2+y^2+z^2} = \frac{4}{3}$$

$$1 < \frac{a+b+c}{x+y+z} < 4/3$$

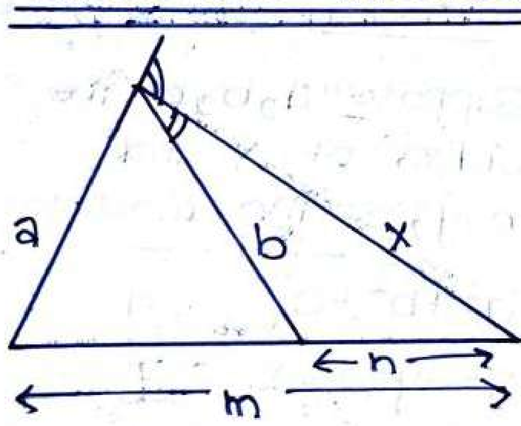
8) INTERIOR ANGLE BISECTOR THEOREM



$$\frac{a}{b} = \frac{m}{n}$$

$$x^2 = ab - mn$$

9) EXTERIOR ANGLE BISECTOR THEOREM



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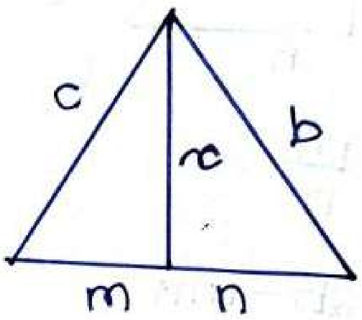

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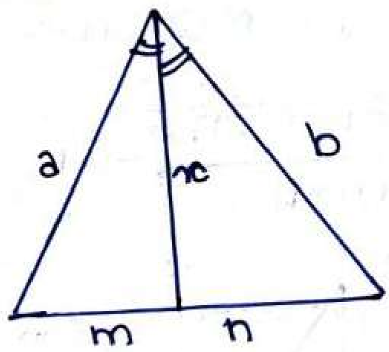

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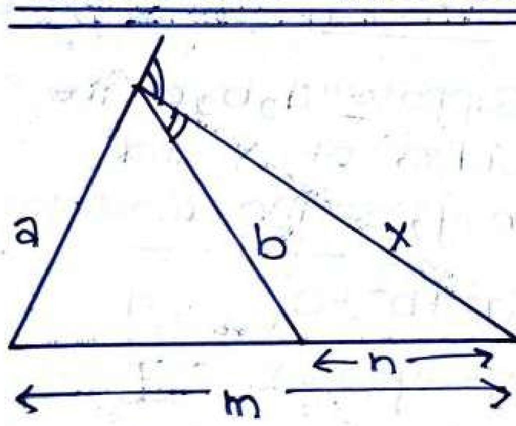


$$\frac{a}{b} = \frac{m}{n}$$

CIRCUMCENTRE

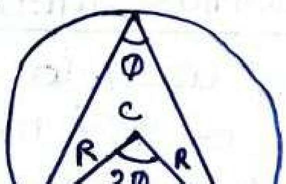
* It's the point where all ~~medians~~ ⊥ bisector meet.

* It's the centre of the circumcircle.

* In acute Δ_s circumcentre is inside.

* In obtuse Δ_s circumcentre is outside.

* In right Δ_s circumcentre is ON hypotenuse.



* Angle substended by two vertices of Δ on orthocenter is double the angle substended by them on the third vertice.

* In right Δ_s circumcentre lies exactly in middle of hypotenuse.

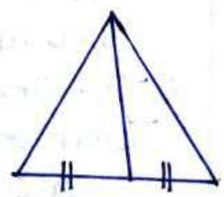
* DISTANCE BETWEEN INCENTER & CIRCUMCENTER

$$d = \sqrt{R^2 - Rr}$$

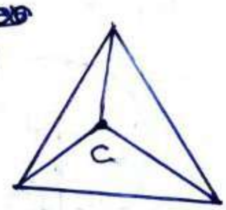
* Area of $\Delta = \frac{abc}{4R}$

* Some points related to areas

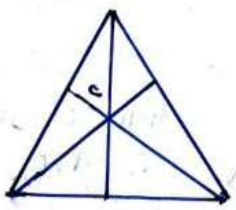
→ A median divides a Δ into two equal halves.



→ Centroid divides Δ into three equal thirds.



→ All three medians divide Δ into six equal parts.



→ If we join centres of sides of a Δ_s it will divide Δ into four equal quarters.

* similar to circumcenter (intersecting point of ⊥ bisectors) is the centroid (intersecting point of the medians)

* centroid divides median in 2:1.

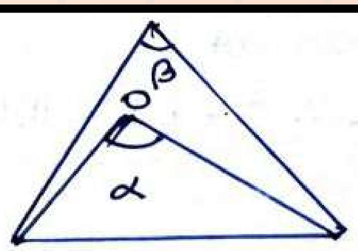
ORTHOCENTER

* intersection point of all three altitudes

RATIO OF SIDES OF Δ: RATIO OF INVERSE OF HEIGHTS

$\Rightarrow a:b:c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3}$

* in acute Δ, orthocenter lies inside.
* in obtuse Δ, orthocenter lies outside.
* in right Δ, orthocenter lies on intersection of base and height.



* $\alpha + \beta = 180°$

* Distance between incenter and orthocenter $= x\sqrt{2}$

CIRCLES

* Area $= \pi r^2$

* Circumference $= 2\pi r$

* Length of arc $= \dfrac{\theta}{360} \times 2\pi r$

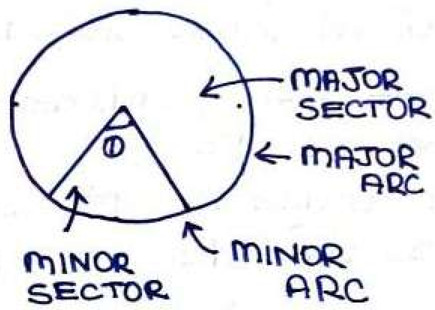
* Area of sector

$$= \frac{\theta}{360} \times \pi r^2$$

* Perimeter of segment

= Length of arc + length of segment

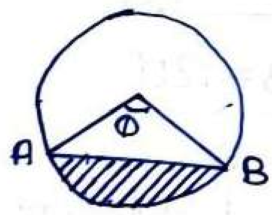
$$= \frac{\theta}{360} \times 2\pi r + 2r\sin\left(\frac{\theta}{2}\right)$$



* Area of segment

= Area of sector − Area of Δ

$$= \frac{\theta}{360}\pi r^2 - \frac{1}{2} r^2 \sin\theta$$

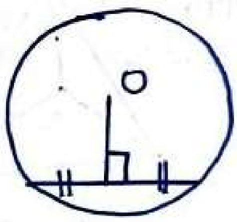


Important Theorems →

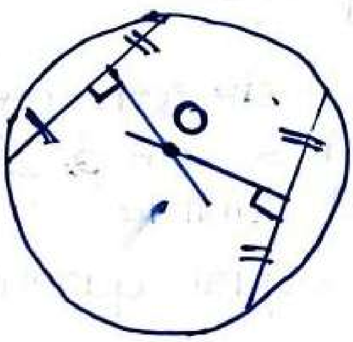
* ⊥ from center of the circle to a chord bisects the chord.

* ⊥ bisector of a chord passes through the center.

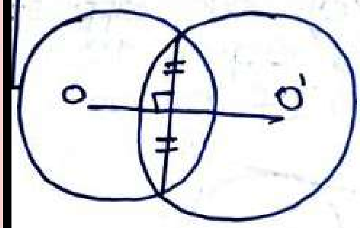
* The line from the center bisecting a chord is ⊥ to it.



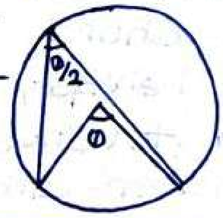
* ⊥ bisectors of two chords meet at the center.



If two circles intersect at two points, then the line through the center is ⊥ bisector of common chord.



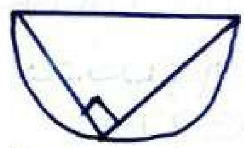
Angle substended by an arc on the center is twice the angle substended by it on any other part of the circle.



Angle substended by a diameter on the circle is a right angle.

OR

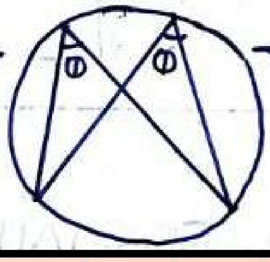
if a chord substends a right angle on the circle, then the chord is the diameter.

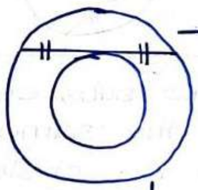


Secant → touches circle at 2 points

Tangent → touches circle at 1 point.

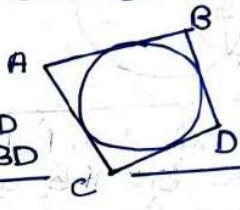
Angle substended by the same arc on the circle are equal.





A cocentric circle divides a chord into two halves, if chord touches the smaller circle only at one point.

If a circle touches all four sides of a quadrilateral, then the sum of two opposite sides is equal to the sum of other two.



AB+CD
=AC+BD

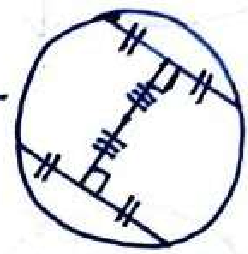
Sum of opposite angles of cyclic quadrilaterals is 180°.

Equal chords of a circle substend equal angles at the center.

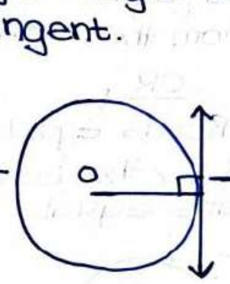
Equal chords of a circle are equidistant from the center

OR

chords equidistant from the center are equal.

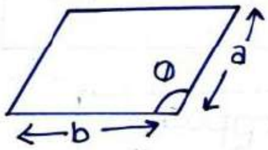


The radius of the circle makes a right angle with tangent.



||GM AND RECTANGLES

* Area = Base x height
  $= ab \sin\theta$



* Perimeter $= 2(a+b)$
* Diagonals of ||gm bisect each other.

* ||gm inscribed in a circle is rectangle.

* ||gm circumscribed on a circle is a rhombus.
* Opposite ∠s are equal.
* sum of squares of diagnols = sum of squares of sides.
* In a rectangle diagonals are equal.

RHOMBUS & SQUARE

* Diagonals bisect at 90°.
* In a square, diagonals are equal.
* Area = Base x height
  = Product of adjacent sides x Sine of angle between them.

TRAPEZIUM

* Area $= \frac{1}{2} \times$ (sum of || sides) x height
* if non-|| sides of trapezium are equal, then diagonals are also equal.