Number Systems

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NATURAL NUMBERS →

All numbers used for counting, starting with 1.

$\{1, 2, 3, 4 - - -\}$

WHOLE NUMBERS →

All natural numbers with inclusion of 0.

$\{0, 1, 2, 3, - - -\}$

NEGATIVE NUMBERS →

All numbers less than 0.

$\{-1, -2, -3, - - -\}$

POSITIVE NUMBERS →

All non negative numbers

$\{0, 1, 2, 3 - - -\}$

FACTORS → Factors of a number x are those numbers that can completely divide that number.

* every number is a factor of itself.

COMPOSITE NUMBERS →

* A number with more than two factors are known as composite numbers.

{6, 10, 27, ---}

* 1 is neither prime nor composite.
* 0 is composite.

RATIONAL NUMBERS →

Those numbers whose decimals exhibit one of the following properties:

* they are terminating
* they are non-terminating, but recurring

are known as rational numbers.

{1.23, $1.2\overline{4}$, 97, ---}

IRRATIONAL NUMBERS →

Those numbers whose decimals exhibit the following properties:

* they are non terminating
* they are non recurring

are known as irrational numbers.

* Square roots of all non-perfect squares are irrational.

$\{\sqrt{2}, \sqrt{10}, \sqrt{35}, ---\}$

HIGHEST COMMON FACTOR →

* The HCF of two numbers is the greatest number that completely divides both the numbers.
* It's also known as Greatest Common Divisor (GCD).

LEAST COMMON MULTIPLE →

* The L.C.M. of two numbers is the smallest number that is completely divisible by both the numbers.

PRODUCT OF TWO NUMBERS = LCM × HCF

CO-PRIMES → Two numbers whose HCF is 1 are always co-primes

* LCM of co-primes is the product of co-primes.

* Two consecutive natural numbers ARE always co-primes {(5, 6), (10, 11) ---}

* Two prime numbers are always co-prime.

* A prime number and a composite number are always co-prime.

* Two consecutive odd numbers are always co-prime.

$$\text{HCF of fraction} = \frac{\text{HCF of numerator}}{\text{LCM of denominator}}$$

$$\text{LCM of fraction} = \frac{\text{LCM of numerator}}{\text{HCF of denominator}}$$

DIVISIBILITY

We know that

DIVIDEND = DIVISOR × QUOTIENT + REMAINDER

$\Rightarrow \boxed{D = dQ + R}$

* If divisor completely divides dividend leaving no remainder, we say that dividend is divisible by divisor.

* If a is divisible by b, then ac is divisible by b.

* If two numbers are divisible by each other, they are equal.

* If both m and n are divisible by d, then (m+n) and (m−n) are both divisible by d.

STANDARD FORM OF NUMBERS

* Standard form of numbers refers to writing the number as product of its prime factors.

* For example,

$$240 = 2^4 \times 3^1 \times 5^1$$

Number of factors

* the no of factors of a given number is determined by its standard form.

* if standard form of a number is $a^P b^Q r^S$, then number of factors $= (P+1)(Q+1)(S+1)$

* the sum of factors is given by

$$(a^0 + a^1 + a^2 + \_\_\_ + a^P) \times (b^0 + b^2 + b^3 + \_\_\_ + b^Q) \times (r^0 + r^2 + r^3 + \_\_\_ + r^S)$$

Even and Odd factors

Suppose we've to find no. of even factors of 240.

No. of factors $= (4+1)(1+1)(1+1)$

Now, to find even factors, we've to ensure that every expansion contains 2. So, we drop 2^0.

No. of even factors $= (4)(1+1)(1+1)$

For no. of odd factors we drop everything except 2^0.

No. of odd factors $= (1)(1+1)(1+1)$

Remainder Theorem

* Suppose we've to find the remainder of a×b, when divided by c, then we can find remainder of a when divided by c multiplied by remainder of b multiplied by c.

* Mathematically,

  remainder of $(a \times b) \div c$

  $=$ (remainder of $a \div c$) $\times$ (remainder of $a \div b$)

* Similar is the case for addition and subtraction.

* The remainders can sometimes be negative. To convert to a positive remainder, we add divisor to the negative remainder.

LAST 2-DIGITS OF A NUMBER

[Can be used for any number of digits]

* We can use remainder theorem to find last x digits of a number.

* For e.g., last two digits of a number is nothing but remainder of the number when divided by 100.

* The last two digits of a number $22 \times 31 \times 44$ are

  $=$ remainder of $(30008 \times 44) \div 100$

  $=$ remainder of $(8 \times 44) \div 100 = 52$

Power cycles

* for x^n, if x is constant and we keep on increasing n, the unit's place of x^n tends to repeat.

* For 0, 1, 5 and 6, x^n always ends with the same digit x ends in.

* For 4, if n is even, x^n ends in 6 else x^n ends in 4.

* For 3, 7, 8, 9, the following table is followed. ($n\%4$ implies the remainder when n is divided by 4.

LAST DIGIT OF x	LAST DIGIT OF x^n			
	$n\%4=0$	$n\%4=3$	$n\%4=2$	$n\%4=1$
3	1	7	9	3
7	1	3	9	7
8	6	2	4	8

* For 9, if n is even, x^n ends in 1 else it ends in 9.

Special Formulas

① The greatest number that divides x and y and leaves remainders a and b.

HCF of $(x-a), (y-b)$

② The least number that divides a, b, c, d and leaves remainder 4.

LCM of $a, b, c, d - 4$

③ The greatest number which when divided by x, y, z leaves the same remainder in each case

HCF of $(x-y), (y-z), (x-z)$

④ The least number that divides a, b, c, d to leave remainder x, y, z, w such that

$(a-x) = (b-y) = (c-z) = (d-w) = p$

LCM of $(x, y, z, w) - p$

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