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Effect of bottleneck on route choice in two-route traffic system with real-time information

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HIGHLIGHTS

- We studied the effect of a bottleneck on the route choice in two-route traffic system.
- We proposed the extended two-route traffic model to take into account the bottleneck on a route.
- We clarified the dependence of the travel time and density on the bottleneck' strength.

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ABSTRACT

We study the traffic behavior in the case that there exists a bottleneck on a route in the two-route traffic system with real-time information. We introduce the bottleneck into the two-route dynamic model proposed by Wahle et al. When there is a bottleneck on route A, a traffic jam occurs behind the bottleneck on route A. The drivers try to avoid the congestion by the use of real-time information. We derive the dependence of the travel time and mean density on the bottleneck's strength. We show where, when, and how the traffic jam occurs by the bottleneck. We clarify the effect of the bottleneck on the traffic behavior in the route choice. We show that the dynamic transition occurs from the oscillating jam to the stationary jam with increasing blocking probability.

1. Introduction

Recently, physicists have been attracted much attention to traffic flow [1–5]. Physicists have proposed the simplified traffic models including a few factors at most to clarify the cause and effect. The traffic systems include so many factors that it is difficult to discover the essential factors affecting the traffic behavior. The physical models and concepts have been applied to transportation systems [6–15]. The traffic flow and pedestrian flow have been studied from the point of view of statistical mechanics and nonlinear dynamics [16–25].

Information is a key commodity in many socio-economic systems like traffic systems. In traffic flow, advanced traveler information systems provide real-time information about the traffic conditions to road users by means of communication such as variable message signs, radio broadcasts or on-board computers. The real-time information helps the individual road users to minimize their personal travel time. The information has an important effect on the traffic dynamics.

Wahle et al. have proposed the dynamic model for traffic flow with real-time information [26]. The traffic flow with real-time traffic information has been studied using a route choice scenario. The route-choice strategy has been extended to the three-route and crossing traffic systems [26–29]. In the two-route and three-route traffic systems, there are no traffic signals. However, it is important and necessary not only to obtain the real-time traffic information but also to know the control strategy of signals because the city traffic is generally controlled by many signals [30]. Tobita and Nagatani have

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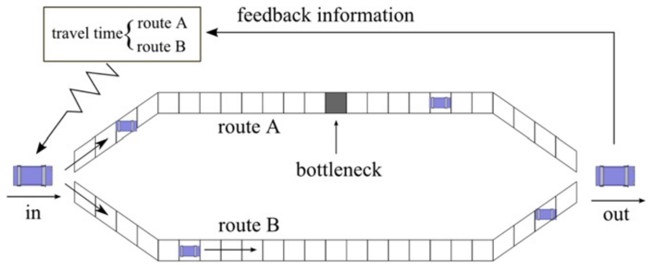

Fig. 1. Schematic illustration of the two-route traffic system with a bottleneck on route A. Two types of vehicles are introduced: dynamic and static vehicles. When vehicles enter the system, a so-called dynamic driver will make a choice on the basis of the travel-time feedback, while a static driver enters route A (B) with probability 1/2 (probability 1/2) ignoring any advice.

extended the two-route traffic system with real-time traffic information to that controlled by signals [31]. They have clarified the effect of signals on the two-route traffic flow. The vehicular motion varies not only with the route choice but also with signal's characteristic in the two-route traffic system using real-time traffic information.

Tang et al. have studied the effect of heterogeneity on the traffic flow on a road to take into account two kinds of vehicles [32]. Also, Tang et al. have investigated the effect of the road structure on traffic flow on a single road [33–37]. The road structure affects the route-choice behavior. Until now, it is unknown how the road structure with a bottleneck affects the route-choice behavior. The traffic information affects not only the route-choice behavior but also the traffic flow on a road. Tang et al. have studied the effect of the driver's forecast and honk on the road traffic [38–41]. The driver's individual property affects not only the route-choice behavior but also the traffic flow on a road. Tang et al. have studied the effect of the driver's individual property on the road traffic [42]. Tang et al. have investigated the effect of the road condition on traffic flow on a road [43,44]. The road condition affects the route-choice behavior.

In real traffic, there exist bottlenecks on roads. The bottleneck induces a traffic jam. The traffic jam by bottlenecks has been studied extensively by many researchers [45–48]. The bottleneck on a road will have the important effect on the route choice. For example, drivers try to avoid the congestion by the use of real-time information. The traffic jam induced by the bottleneck varies with the use of real-time information. As a result, the traffic jam at the bottleneck occurs or disappears with time. However, the effect of bottlenecks on the route choice has not been investigated in the physical traffic models.

The combination of the two-route traffic with the bottleneck is not a simple problem of the bottleneck' effect superimposed on the two-route traffic. The travel-time feedback with real-time information changes by the bottleneck's effect, while the bottleneck' effect changes by the travel-time feedback contrary. Then, the traffic in the two-route traffic with a bottleneck shows the traffic behavior different definitely from the original two-route traffic proposed by Wahle et al.

In this paper, we study the effect of a bottleneck on the two-route traffic system at the travel-time feedback strategy. We introduce a bottleneck on a route into the dynamic model proposed by Wahle et al. We study how the bottleneck affects the traffic behavior. We investigate how the traffic jam induced by the bottleneck varies with time due to the real-time information. We derive the dependence of the travel time and the density on the bottleneck's strength on the route. We clarify the effect of bottleneck on the route choice for the two-route traffic system with real-time information.

2. Dynamic model for two-route system with a bottleneck

We consider the two-route traffic system in which there exists a bottleneck on route A. In order to demonstrate the effect of the feedback loop, Wahle et al. have studied the two-route traffic system using the scenario with dynamic information. We extend the dynamic model to the two-route system with a bottleneck. When vehicles enter the system, vehicles move either on route A or B. We apply the travel-time feedback strategy to the route choice.

Fig. 1 shows the schematic illustration of the two-route traffic system with a bottleneck on route A. Two types of vehicles are introduced: dynamic and static vehicles. When vehicles enter the system, a so-called dynamic driver will make a choice on the basis of the travel-time feedback, while a static driver enters route A (B) with probability 1/2 (probability 1/2) ignoring any advice. The dynamic driver always chooses the route with the shortest travel time at the entrance. The densities of dynamic and static drivers are S_{dyn} and $1-S_{dyn}$ respectively. Here, we introduce the bottleneck on route A into the original model proposed by Wahle et al. The bottleneck is set at a position on route A. Vehicles pass through the bottleneck with probability $1-p_b$ and are stopped at the bottleneck with probability p_b. The blocking probability p_b represents the strength of the bottleneck on route A. If the blocking probability is zero, the dynamic model is consistent with that proposed by Wahle et al. The static driver enters route A (B) with probability 1/2. The density of the static drivers on route A equals that on route B.

However, if the blocking probability is not zero, a part of vehicles are blocked at the bottleneck. The traffic jam occurs at the bottleneck. The drivers try to avoid the congestion by the use of real-time information and go to route B. In time, route B results in a congested state. When route B is congested, the drivers with the real-time information go to route A. The process is repeated. Thus, the traffic flow on route A is strongly affected by the bottleneck. Also, the congestion by the bottleneck will be reduced by the use of real-time information. The traffic flow on route A shows the different behavior from that on route

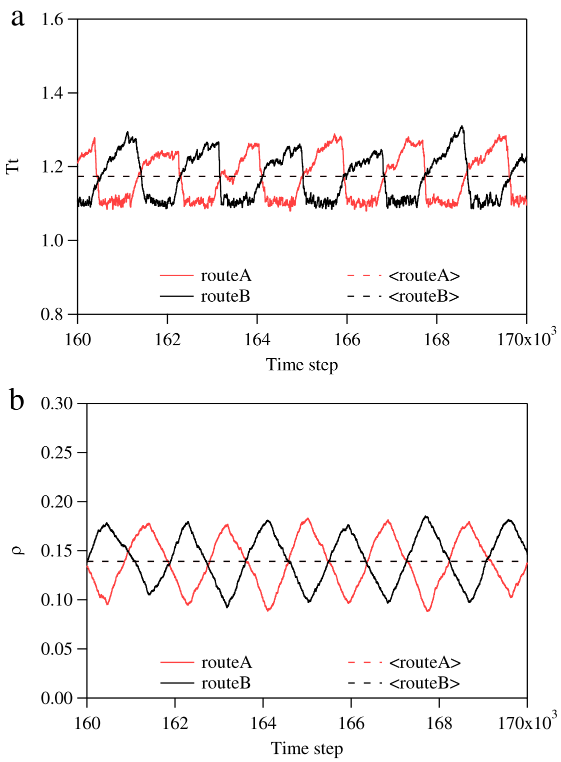

Fig. 2. (a) Plots of dimensionless travel times T_t against time t on two routes at $p_b = 0.0$ (without a bottleneck). Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. (b) Plots of densities ρ against time t on two routes at $p_b = 0.0$ (without a bottleneck).

B. By introducing the bottleneck, the traffic behavior in the two-route system changes greatly and the traffic flow displays the dynamic behavior different from the original model by Wahle et al.

The simulations are performed in the following steps. First, we set two routes and board empty (see Fig. 1). Second, after the vehicles enter the entrance, they become the dynamic driver with probability S_{dyn} and the static driver with probability $1 - S_{dyn}$. Third, the static driver chooses route A with probability 1/2 and route B with probability 1/2. Finally, the travel time information will be generated, transmitted, and displayed on the board at each time step, according to the travel-time feedback strategy. The dynamic drivers will choose the route with the shortest travel time at the entrance of two routes. After entering the route, the vehicle moves through the system according the dynamics of the cellular automaton model proposed by Nagel and Schreckenberg (NS model) [6].

For late convenience, the definition of the NS model is briefly reviewed for single-lane traffic. The road is subdivided into cells with a length. Each cell is either empty or occupied by only one vehicle with an integer velocity v_i where the maximum velocity is v_{max}. Let be N the total number of vehicles on a single route of length L. Then, the vehicular density is $\rho = N/L$. The motion of vehicles is described by the following rules (parallel update)

Rule 1. Acceleration: $v_i \leftarrow \min(v_i + 1, v_{\max})$

Rule 2. Deceleration: $v_i' \leftarrow \min(v_i, gap)$

Rule 3. Randomization: with a certain probability p do $v_i'' \leftarrow \max(v_i' - 1, 0)$

Rule 4. Movement: $x_i \leftarrow x_i + v_i''$

The variable *gap* denotes the number of empty cells in front of the vehicle at cell i. The maximum velocity is set as $v_{\max} = 3$ throughout this paper. Also, the randomization probability is set as $p = 0.25$.

For generalization, we use the dimensionless travel time, $T_t = t_t v_{\max}/L$.

3. Simulation result

We carry out numerical simulation for the traffic flows on routes A and B in the above model. We study how the travel time and density on routes A and B vary with time for various values of the blocking probability. The bottleneck is positioned at the center on route A. The parameters are set to road length $L = 2000$, maximum velocity $v_{\max} = 3$, randomization probability $p = 0.25$, and density $S_{dyn} = 0.5$ of dynamic drivers. All simulation results shown here are obtained by 2,000,000 iterations. The travel times and numbers of vehicles on two routes are calculated at each time step. The mean values of travel time and density are averaged over 1,000,000 time steps.

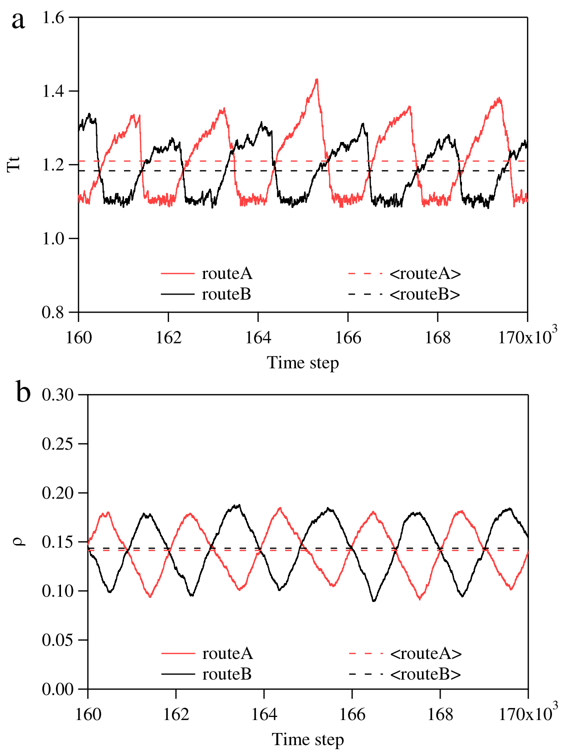

Fig. 3. (a) Plots of dimensionless travel times T_t against time t on two routes at blocking probability $p_b = 0.2$ (with a bottleneck). Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. (b) Plots of densities ρ against time t on two routes at blocking probability $p_b = 0.2$.

For comparison, we show the variations of travel times and densities on two routes for the system with blocking probability $p_b = 0.0$ (without a bottleneck) on route A. Fig. 2(a) shows the plots of dimensionless travel times T_t against time t on two routes at $p_b = 0.0$. Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. The travel time on each route varies periodically with time. The travel time on two routes varies alternately. Fig. 2(b) shows the plots of densities ρ against time t on two routes at $p_b = 0.0$. Red and black lines indicate densities on routes A and B respectively. Red and black dashed lines represent the mean density on routes A and B respectively. Density on each route also varies periodically and alternately.

We calculate the travel times and densities on two routes with a bottleneck for nonzero values of blocking probability on route A. We study the effect of the blocking probability on travel times and densities. Fig. 3(a) shows the plots of dimensionless travel times T_t against time t on two routes at blocking probability $p_b = 0.2$. Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. Fig. 3(b) shows the plots of densities ρ against time t on two routes at blocking probability $p_b = 0.2$. Red and black lines indicate densities on routes A and B respectively. Red and black dashed lines represent the mean density on routes A and B respectively. Fig. 3 is compared with Fig. 2. By changing the blocking probability from $p_b = 0.0$ to $p_b = 0.2$, the travel times and densities on two routes change from Fig. 2 to Fig. 3. The peak value of the travel time on route A is higher than that on route B. Also, the mean value of the travel time on route A is higher than that on route B.

We derive the traffic pattern to investigate the traffic behavior induced by the bottleneck. Fig. 4 shows the plot of vehicular position versus time step at blocking probability $p_b = 0.2$ corresponding to Fig. 3. Diagrams (a) and (b) represent, respectively, the traffic patterns (trajectories of vehicles) on routes A and B. Vehicles with the dynamic and static drivers are indicated by black and red dots respectively. The regions with high density just behind the center on route A represent the traffic jams which form the triangular shape. When the traffic jam is formed at the bottleneck, the travel time on route A increases due to the congestion of the bottleneck. As a result, the dynamic drivers on route A go to route B. In time, there are no dynamic drivers on route A, only static drivers exist on route A, and the traffic jam disappears on route A. When route B is congested, the dynamic drivers on route B go to route A. As a result, the traffic jam occurs on route A, the travel time on route A increases. The process is repeated. The travel times and densities oscillate. Also, the traffic jam occurs and disappears alternately. The jam oscillates. We call the jam as the oscillating jam.

Fig. 5(a) shows the plots of dimensionless travel times T_t against time t on two routes at blocking probability $p_b = 0.4$. Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. Fig. 3(b) shows the plots of densities ρ against time t on two routes at blocking probability $p_b = 0.4$. Red and black lines indicate densities on routes A and B respectively. Red and black dashed lines

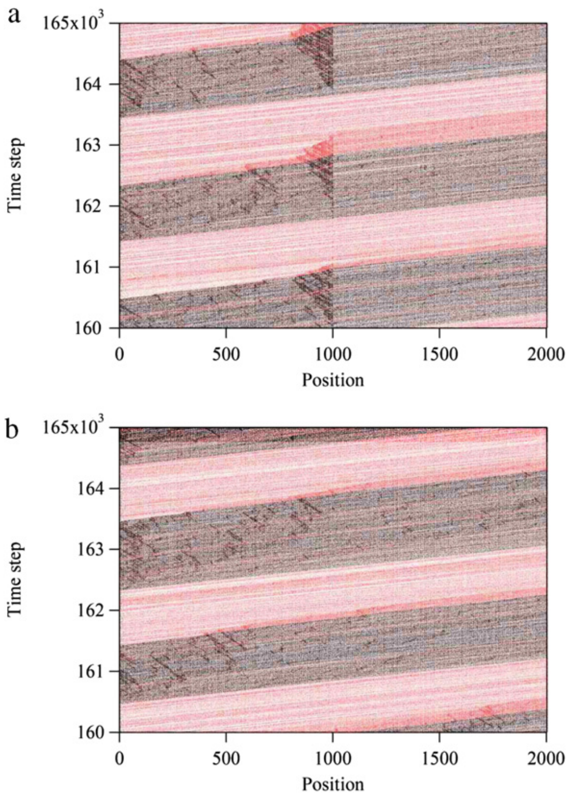

Fig. 4. Plot of vehicular position versus time step at blocking probability $p_b = 0.2$ corresponding to Fig. 3. Diagrams (a) and (b) represent, respectively, the traffic patterns (trajectories of vehicles) on routes A and B. Vehicles with the dynamic and static drivers are indicated by black and red dots respectively.

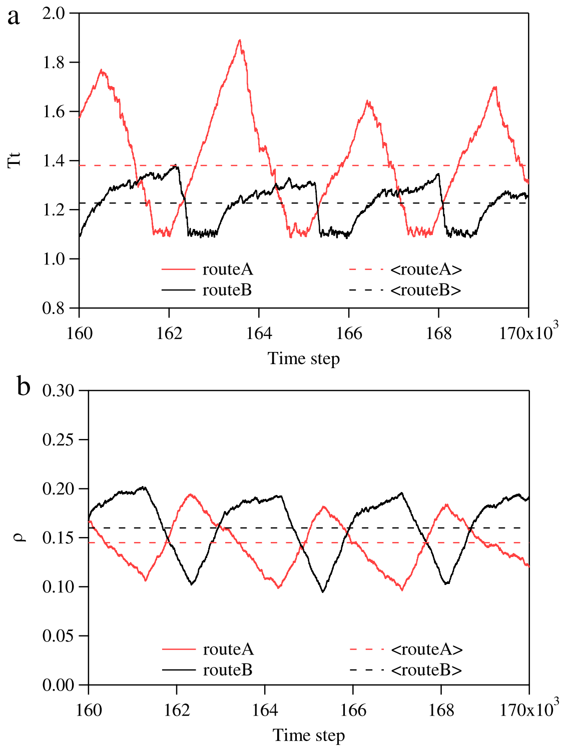

Fig. 5. (a) Plots of dimensionless travel times T_t against time t on two routes at blocking probability $p_b = 0.4$. Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. (b) Plots of densities ρ against time t on two routes at blocking probability $p_b = 0.4$.

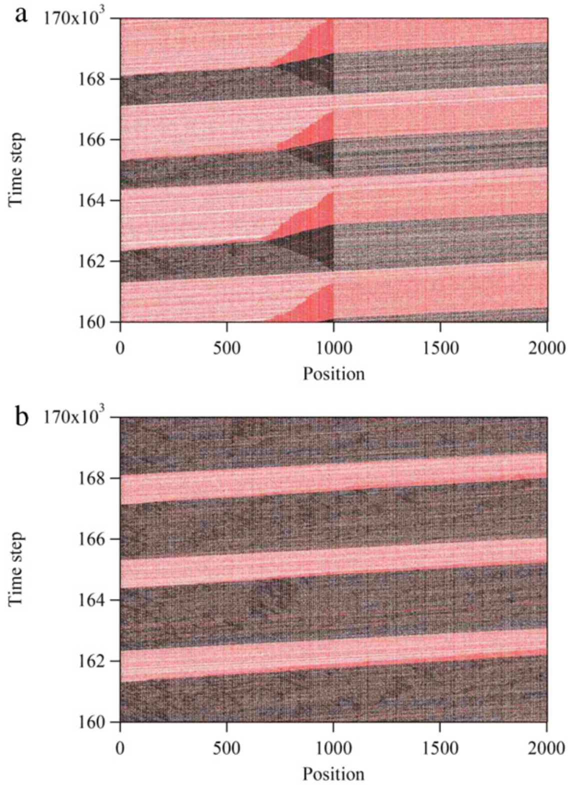

Fig. 6. Plot of vehicular position versus time step at blocking probability $p_b = 0.4$ corresponding to Fig. 5. Diagrams (a) and (b) represent, respectively, the traffic patterns (trajectories of vehicles) on routes A and B. Vehicles with the dynamic and static drivers are indicated by black and red dots respectively. Fig. 6 is compared with Fig. 4.

represent the mean density on routes A and B respectively. Fig. 5 is compared with Fig. 3. By changing the blocking probability from $p_b = 0.2$ to $p_b = 0.4$, the travel times and densities on two routes change from Fig. 3 to Fig. 5. The difference between travel times on routes A and B increases with increasing blocking probability. Also, the mean value of the travel time on route A is higher than that on route B. Similarly, the difference between the densities on routes A and B increases with increasing blocking probability.

Fig. 6 shows the plot of vehicular position versus time step at blocking probability $p_b = 0.4$ corresponding to Fig. 5. Diagrams (a) and (b) represent, respectively, the traffic patterns (trajectories of vehicles) on routes A and B. Vehicles with the dynamic and static drivers are indicated by black and red dots respectively. Fig. 6 is compared with Fig. 4. The traffic jam in Fig. 6 becomes stronger than that in Fig. 4. The jam is oscillating with time.

Fig. 7(a) shows the plots of dimensionless travel times T_t against time t on two routes at blocking probability $p_b = 0.6$. Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. Fig. 7(b) shows the plots of densities ρ against time t on two routes at blocking probability $p_b = 0.6$. Red and black lines indicate densities on routes A and B respectively. Red and black dashed lines represent the mean density on routes A and B respectively. Fig. 7 is compared with Figs. 3 and 5. By changing the blocking probability from $p_b = 0.2$, through $p_b = 0.4$, to $p_b = 0.6$, the travel times and densities on two routes change from Fig. 3, through Fig. 5, to Fig. 7. The travel times and densities on routes A and B do not vary with time.

Fig. 8 shows the plot of vehicular position versus time step at blocking probability $p_b = 0.6$ corresponding to Fig. 7. Diagrams (a) and (b) represent, respectively, the traffic patterns (trajectories of vehicles) on routes A and B. Vehicles with the dynamic and static drivers are indicated by black and red dots respectively. Fig. 8 is compared with Fig. 6. The strong jam is formed behind the bottleneck. The traffic jam extends to the entrance. The jam is not oscillating with time but stationary. All dynamic drivers go to route B because the travel time on route A is always higher than that on route B due to the strong jam. Thus, the dynamic transition from the oscillating jam to the stationary jam occurs with increasing blocking probability.

We derive the relationship between the mean travel time and the blocking probability. We study the relationship between the mean density and the blocking probability. Also, we study the dependence of the mean travel time and the mean density on the position l_b of the bottleneck. Fig. 9(a) shows the plots of mean travel time $\langle T_t \rangle$ against blocking probability p_b. Circles, triangles and squares indicate, respectively, the mean travel times for the positions $l_b = 1000$, 500, and 1500 in which the bottleneck is set. Fig. 9(b) shows the plots of mean density $\langle \rho \rangle$ against blocking probability p_b. The difference between the mean travel times on routes A and B increases little by little with increasing blocking probability. When

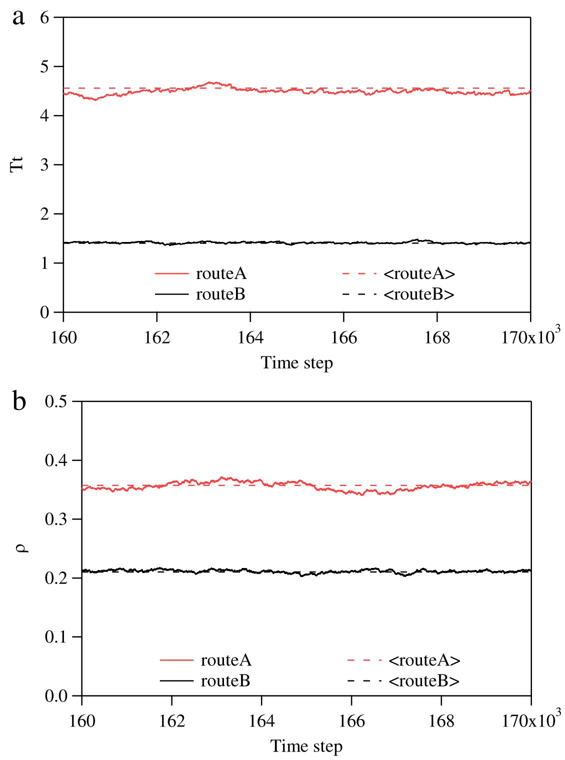

Fig. 7. (a) Plots of dimensionless travel times T_t against time t on two routes at blocking probability $p_b = 0.6$. Red and black lines indicate travel times on routes A and B respectively. Red and black dashed lines represent the mean travel time on routes A and B respectively. (b) Plots of densities ρ against time t on two routes at blocking probability $p_b = 0.6$.

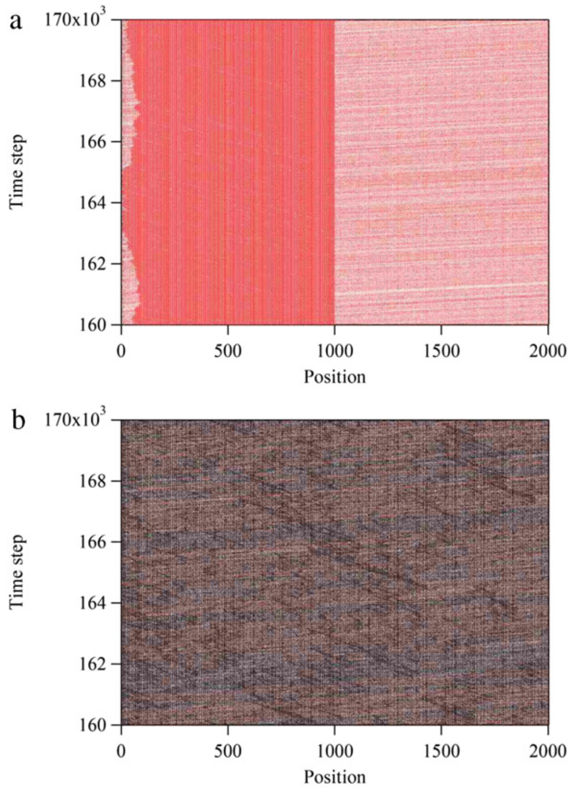

Fig. 8. Plot of vehicular position versus time step at blocking probability $p_b = 0.6$ corresponding to Fig. 7. Diagrams (a) and (b) represent, respectively, the traffic patterns (trajectories of vehicles) on routes A and B. Vehicles with the dynamic and static drivers are indicated by black and red dots respectively. Fig. 8 is compared with Fig. 6.

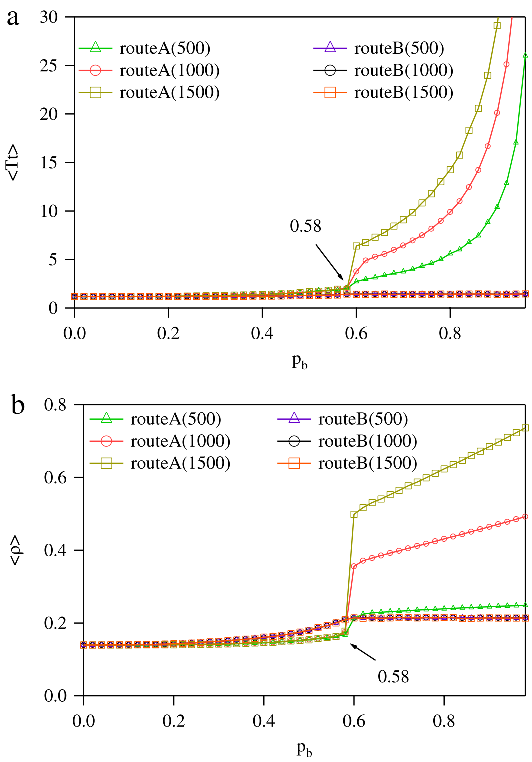

Fig. 9. (a) Plots of mean travel time $\langle T_t \rangle$ against blocking probability p_b. Circles, triangles and squares indicate, respectively, the mean travel times for the positions $l_b = 1000, 500$, and 1500 in which the bottleneck is set. (b) Plots of mean density $\langle \rho \rangle$ against blocking probability p_b. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the blocking probability is higher than $p_b = 0.58$, the difference increases abruptly with increasing blocking probability. Similarly, the profiles of the mean density display the same behavior as those of the mean travel time. For $p_b < 0.58$, the traffic jam just behind the bottleneck is oscillating, while the traffic jam is stationary and extends to the entrance for $p_b > 0.58$. At $p_b = 0.58$, the traffic jam changes from the oscillating jam to the stationary jam. Thus, the dynamic transition between two jams occurs at $p_b = 0.58$. The dynamic transition does not depend on the position of the bottleneck.

4. Summary

In real city traffic network, there are bottlenecks. Frequently, the bottlenecks induce the traffic jams and congestion. Generally, drivers try to avoid the congestion by the bottleneck by the use of real-time information. For the route choice of drivers, it is important and necessary to know the bottleneck effect on the travel time. In the two-route traffic with a bottleneck, drivers make a route choice using the travel-time information feedback because they do not want the traffic jam or congestion. This variation of the traffic jam is not due to the original bottleneck's effect but is induced by interaction of the bottleneck's jam with the route choice. We have extended the two-route traffic model proposed by Wahle et al. to take into account the bottleneck on a route. The traffic behavior changes greatly by the competition between the bottleneck's jam and driver's avoidance to the jam. We have investigated where, when, and how the traffic jam occurs by the bottleneck. We have clarified the effect of the bottleneck on the route choice in the two-route traffic system with real-time information. We have derived the dependence of the mean travel time and mean density on the blocking probability. We have shown that there are two kinds of traffic jams: one is the oscillating jam and the other is the stationary jam. We have found that the dynamic transition occurs between the oscillating jam and the stationary jam.

The study of the bottleneck effect on the route choice is the first and this study will be useful for the route choice in city traffic network.

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