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An improved car-following model considering influence of other factors on traffic jam

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ABSTRACT

In the Letter, a modified car-following model is presented, in which, the effects of vehicles (or non-motor vehicles) on other lanes without isolation belts are taken into account. The stability condition of the model is obtained by using the control theory method. To check the validity of the present theoretical scheme, the numerical simulation is carried out for the new car-following model, and the simulation result is consistent with the theoretical analysis.

1. Introduction

So far, a variety of approaches have been found to describe the properties of traffic flow jams, and many of traffic models have been developed, including macroscopic models in which traffic flow is viewed as a compressible fluid formed by vehicles, and microscopic models where an individual vehicle is proposed by a particle and the vehicle traffic is regarded as a system of interacting particles driven far from equilibrium [1–3].

Among these models, car-following model has been received much interesting research. It is well known that in the car-following models, velocity, headway, and relative velocity are the main variables to determine the vehicle's accelerations [4–10]. Some new car-following models were put forward to describe the traffic nature successfully. Some of them were extended by introducing multiple information of headway or relative velocity [11–13], and others were considered the two factors at the same time [14,15]. These car-following models mentioned above can reproduce many complex actual traffic phenomena, but cannot be used to study the effects of interruption of vehicles (or non-motor vehicles) on other lanes without isolation belts on traffic flow. In reality, on urban roads without isolation belts, the headway between the preceding and following vehicles with the considered vehicle on the current lane needs to be considered, and the influence of vehicles on other lanes (the transverse distance and the longitudinal distance) also needs to be emphasized.

Inspired by the previous work, we propose a new car-following model considering the effects of the traffic vehicles (or non-motor vehicles) on other lanes without isolation belts. In Section 2 the new model is given and the control method is used to analyze the stability conditions in Section 3. In Section 4, the numerical simulation is presented to confirm the theoretical results. Finally, Section 5 gives a summary.

2. Model

To describe the dynamic of the new model clearly, the variables for cars are given in Fig. 1, where $\tilde{x}_n$ is a distance between the nth motor vehicle and the non-motor vehicle, and $\bar{x}_n$ is the minimum lateral distance between the nth motor and non-motor vehicles.

A new car-following model is presented as follows

$$\frac{d^2x_n(t)}{dt^2}=a\big[V^{op}\big(\Delta x_n(t)\big)+\kappa\big(p\bar{V}^{op}\big(w_n(t)\big)+q\bar{V}^{op}\big(l_n(t)\big)\big)-v_n(t)\big]+\lambda\Delta v_n(t) \tag{1}$$

where $\Delta x_n(t)=x_{n+1}(t)-x_n(t)$ and $\Delta v_n(t)=v_{n+1}(t)-v_n(t)$, are the headway and the velocity difference between the nth considering vehicle and the preceding vehicle, respectively; a is the sensitivity of a driver and k is the influence coefficient between motor vehicle and non-motor vehicle; λ is the reaction coefficient of the relative velocity Δv_n; $V^{op}(\Delta x_n(t))$ is the optimal velocity function which can be defined as follows [3,5]

$$V^{op}\big(\Delta x_n(t)\big)=\frac{v_{\max}}{2}\big[\tanh\big(\Delta x_n(t)-h_c\big)+\tanh(h_c)\big] \tag{2}$$

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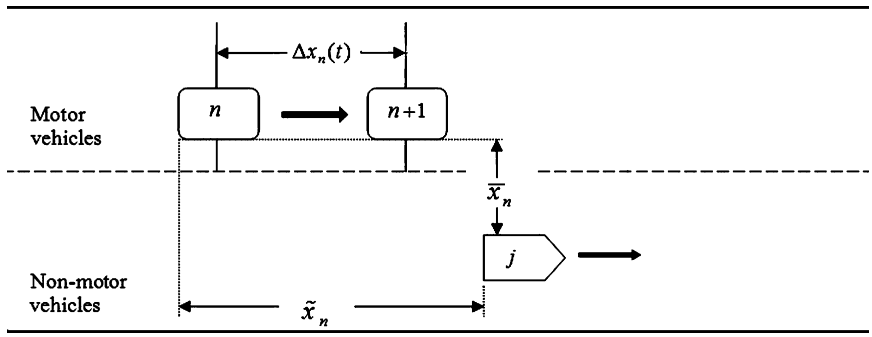

Fig. 1. Schematic of the new car-following model.

where h_c is the safe headway distance and $v_{\max}$ is the maximum velocity; $w_n = \bar{x}_n - d_1$, $l_n = \tilde{x}_n - d_2$. d_1, d_2 are the lateral and longitudinal safety distances respectively; p, q $(p+q=1)$ are the reaction coefficient of the lateral and longitudinal distances respectively

$$\bar{V}^{op}\big(w_n(t)\big) = \begin{cases} 0, & w_n(t) \geqslant d_1, \\ V^{op}(w_n(t)), & 0 \leqslant w_n(t) < d_1, \end{cases} \tag{3}$$

$$\bar{V}^{op}\big(l_n(t)\big) = \begin{cases} 0, & l_n(t) \geqslant d_2, \\ V^{op}(l_n(t)), & 0 \leqslant l_n(t) < d_2. \end{cases} \tag{4}$$

3. Stability analysis

Based on the stability analysis method presented by Konishi et al. [16,17], we describe the stable condition of the improved car-following model as follows

$$\begin{cases} \dfrac{dv_n(t)}{dt} = a\big[V^{op}\big(y_n(t)\big) + \kappa\big(p\bar{V}^{op}\big(w_n(t)\big) \\ \qquad + q\bar{V}^{op}\big(l_n(t)\big)\big) - v_n(t)\big] + \lambda\Delta v_n(t), \\ \dfrac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t), \end{cases} \tag{5}$$

where $y_n(t) = \Delta x_n(t)$, and we assume that the leading vehicle runs constantly with speed v_0, and then the following vehicles have the following steady states

$$\big[v_n^*(t), y_n^*(t)\big]^T = \big[v_0, V^{op^{-1}}(v_0)\big]^T. \tag{6}$$

The traffic system (5) can be linearized at the steady state (6), that is

$$\begin{cases} \dfrac{d\delta v_n(t)}{dt} = a\big[\delta y_n(t)\Lambda_1 + \kappa\big(p\delta w_n(t)\Lambda_2 + q\delta l_n(t)\Lambda_3\big) \\ \qquad - \delta v_n(t)\big] + \lambda\big(\delta v_{n+1}(t) - \delta v_n(t)\big), \\ \dfrac{d\delta y_n(t)}{dt} = \delta v_{n+1}(t) - \delta v_n(t) \end{cases} \tag{7}$$

where $\delta v_n(t) = v_n(t) - v_0$, $\delta y_n(t) = y_n(t) - V^{op^{-1}}(v_0)$, and $\delta w_n(t) = \delta\bar{x}_n - d_1^*$, $\delta l_n(t) = \delta\tilde{x}_n - d_2^*$, $\Lambda_1 = \frac{dV^{op}(y_n(t))}{dy_n(t)}|_{y_n(t)=V^{op^{-1}}(v_0)}$, $\Lambda_2 = \frac{d\bar{V}^{op}(w_n(t))}{dw_n(t)}|_{w_n(t)=d_1^*}$, $\Lambda_3 = \frac{d\bar{V}^{op}(l_n(t))}{dl_n(t)}|_{l_n(t)=d_2^*}$.

After Laplace transformation, we have

$$\begin{bmatrix} V_n(s) \\ Y_n(s) \end{bmatrix} = \frac{1}{p(s)} \begin{bmatrix} s & a\Lambda_1 \\ -1 & s+a+\lambda \end{bmatrix} \begin{bmatrix} a\kappa p\Lambda_2 & a\kappa q\Lambda_3 & \lambda \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} W_n(s) \\ L_n(s) \\ V_{n+1}(s) \end{bmatrix} \tag{8}$$

where $V_n(s) = L(\delta v_n(t))$, $Y_n(s) = L(\delta y_n(t))$, $W_n(s) = L(\delta w_n(t))$, $L_n(s) = L(\delta l_n(t))$, $L(.)$ denotes the Laplace transform and s is a complex variable; $p(s) = s^2 + (a+\lambda)s + a\Lambda_1$. From $\delta v_{n+1}(t)$ to $\delta v_n(t)$, we have the transfer function vector as follows

$$G(s) = [\,1 \quad 0\,]\frac{1}{p(s)} \begin{bmatrix} s & a\Lambda_1 \\ -1 & s+a+\lambda \end{bmatrix} \begin{bmatrix} a\kappa p\Lambda_2 & a\kappa q\Lambda_3 & \lambda \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \frac{1}{p(s)}\,[\,sa\kappa p\Lambda_2 \quad sa\kappa q\Lambda_3 \quad s\lambda + a\Lambda_1\,].$$

According to our analysis, the traffic jams will never occur in the traffic flow system if $p(s)$ is stable and $\|G(s)\|_\infty \leqslant 1$.

In order to make $p(s)$ be stable, we should confirm that $a+\lambda > 0$, and $a\Lambda_1 > 0$ according to Hurwitz stability criterion. Besides, because of the OV function characteristic of monotonously increase (i.e. $\Lambda_1 > 0$) and $a > 0$, the condition for $p(s)$ to be stable is $\lambda > 0$. Then, consider $\|G(s)\|_\infty \leqslant 1$, that is

$$\big\|G(s)\big\|_\infty = \sup_{\omega\in[0,\infty)} \big|G(j\omega)\big| \leqslant 1,$$

$$\big|G(j\omega)\big|^2 = \big|G(-j\omega)G(j\omega)\big|$$
$$= \frac{(a\kappa p\Lambda_2\omega)^2 + (a\kappa q\Lambda_3\omega)^2 + (a\Lambda_1)^2 + (\lambda\omega)^2}{(a\Lambda_1 - \omega^2)^2 + (a+\lambda)^2\omega^2} \leqslant 1.$$

The sufficient condition is given as

$$\omega^2 + a^2 + 2a\lambda - 2a\Lambda_1 - (a\kappa p\Lambda_2)^2 - (a\kappa q\Lambda_3)^2 \geqslant 0, \quad \omega \in [0,\infty). \tag{9}$$

The influence coefficient k between motor vehicle and non-motor vehicle should satisfy

$$0 \leqslant \kappa \leqslant \sqrt{\frac{2\lambda - 2\Lambda_1 + a}{a[(p\Lambda_2)^2 + (q\Lambda_3)^2]}} \quad \left(\lambda - \Lambda_1 + \frac{1}{2}a \geqslant 0\right). \tag{10}$$

4. Numerical simulation

In the simulations, the parameters for the new car-following vehicles are set $h_c = 7.02$ m, $d_1 = 5.5$ m, $d_2 = 6.5$ m, $a = 2$ s^{-1} and $T = 0.1$. It is assumed all vehicles have the same parameters. The initial condition is the steady state of the model. The initial positions and speeds are set to be $x_n(0) = \sum y_n^*$, $y_n(0) = y_n^*(t)$, $v_n(0) = v_n^*(t)$, $n = 1, 2, \ldots, N$, and $N = 120$ is the car number. We consider a situation where the leading vehicle stops suddenly for a short time.

Figs. 2(a) and 3(a) illustrate that when the stability condition in Eq. (10) is met for the new car-following model, the vehicle groups can adjust themselves to the steady state gradually although the headway fluctuation appears in the beginning. However, the whole state is broken when the effect of vehicles (or non-motor vehicles) on other lane is considered. Compared Figs. 3(a)–3(b) with Figs. 4(a)–4(b), as the velocity of the vehicles is increased, the fluctuation is aggravated and the arrival of the steady state will be delayed.

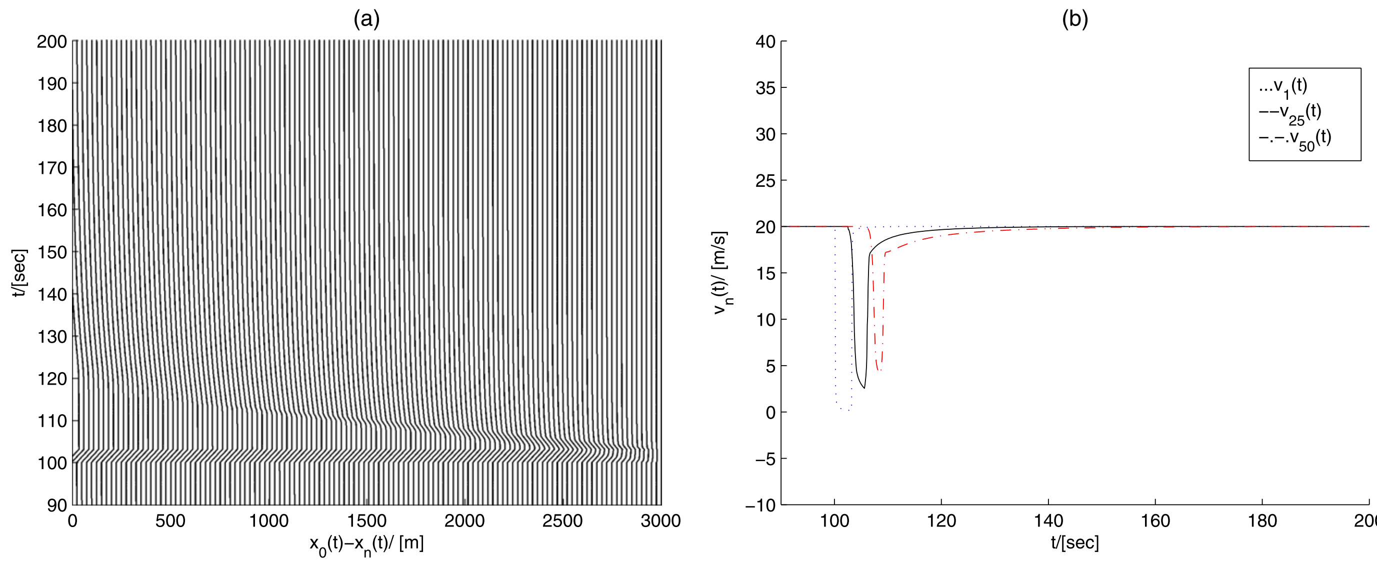

Fig. 2. (a) Space–time plot of the traffic system. (b) Temporal velocity behavior of the first, 25th and 50th vehicles ($p = q = 0$, $a = 2\ \text{s}^{-1}$, $v_{\max} = 20.6$ m/s).

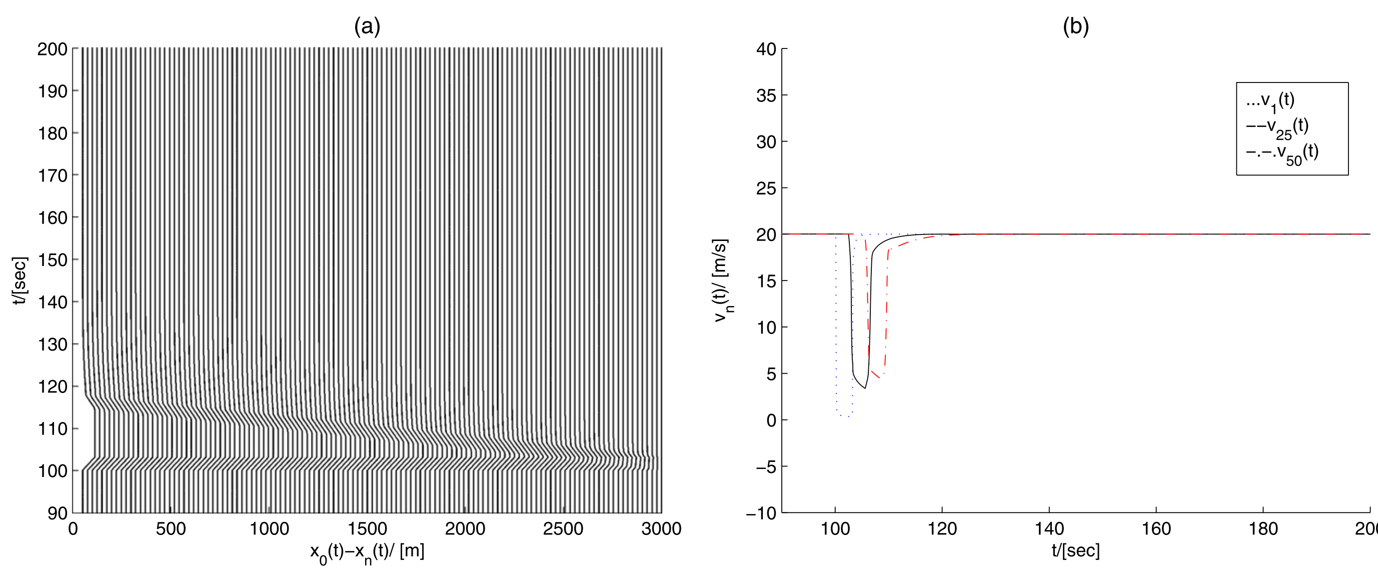

Fig. 3. (a) Space–time plot of the traffic system. (b) Temporal velocity behavior of the first, 25th and 50th vehicles ($p = q = 0.5$, $a = 2\ \text{s}^{-1}$, $k = 0.8$, $v_{\max} = 20.6$ m/s).

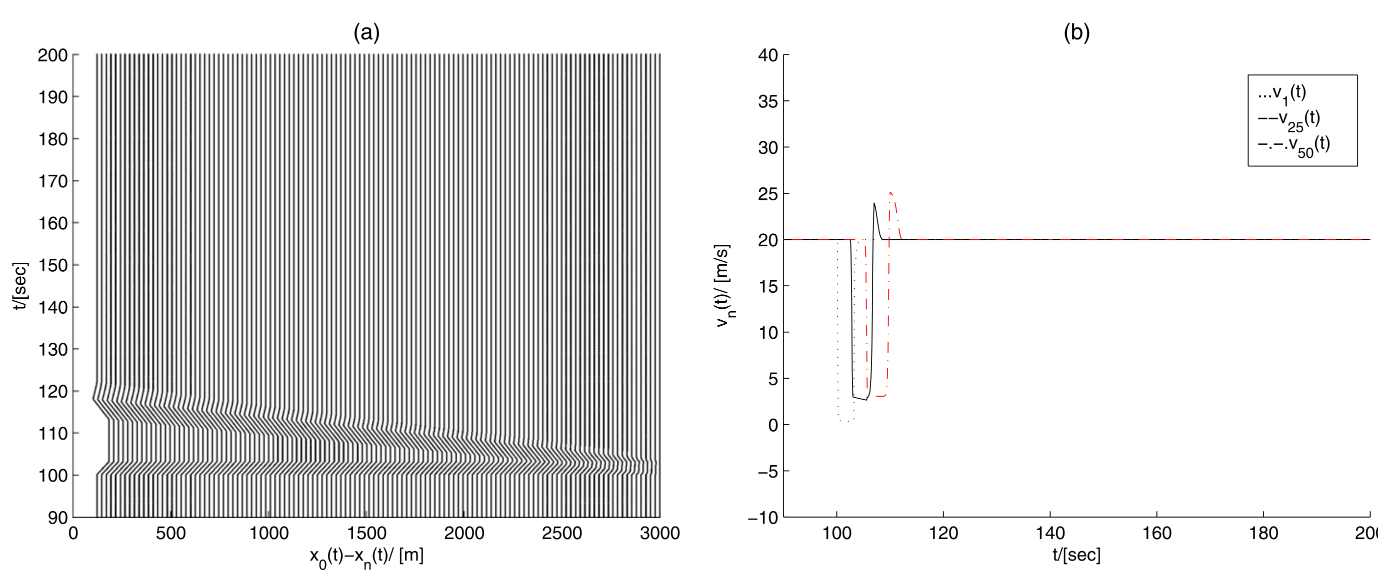

Fig. 4. (a) Space–time plot of the traffic system. (b) Temporal velocity behavior of the first, 25th and 50th vehicles ($p = q = 0.5$, $a = 2\ \text{s}^{-1}$, $k = 0.8$, $v_{\max} = 25.6$ m/s).

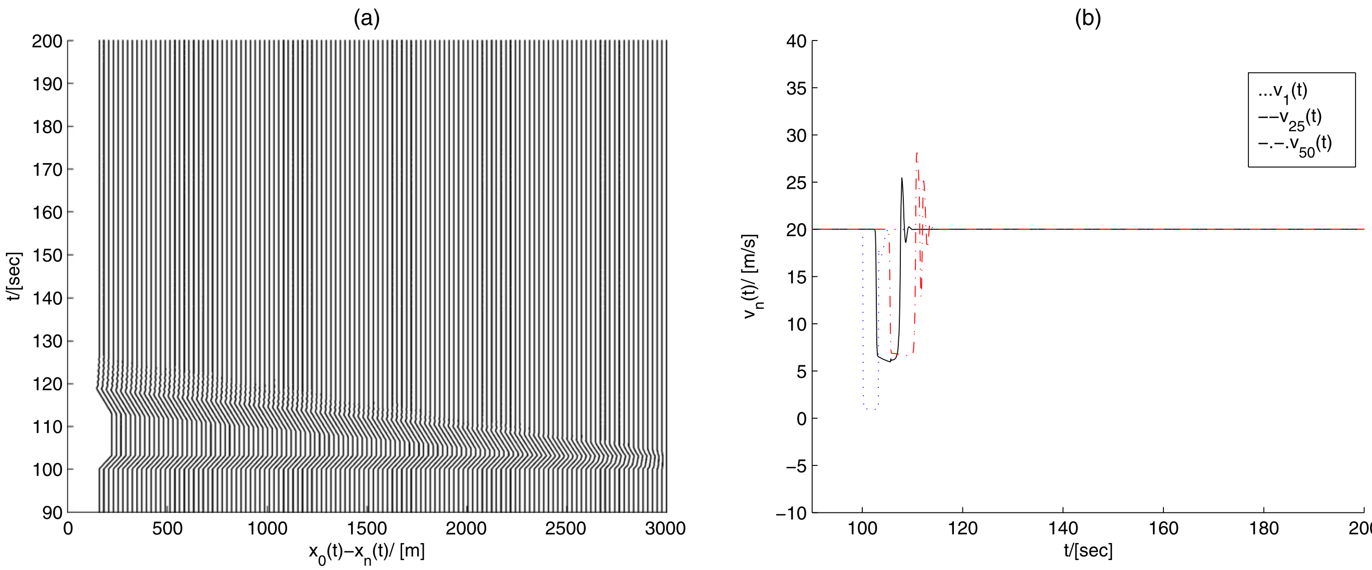

Fig. 5. (a) Space–time plot of the traffic system. (b) Temporal velocity behavior of the first, 25th and 50th vehicles ($p = q = 0.5$, $a = 2s^{-1}$, $k = 2.5$, $v_{\max} = 20.6$ m/s).

From Figs. 5(a)–5(b) and Figs. 3(a)–3(b), what deserves our attention is that as the influence coefficient κ increases, the stability of the traffic system weakens, which illustrates that the other factors (or non-motor vehicles) have produced significant influence for the stability of the traffic flow.

5. Summary

In this Letter, we have proposed a new car-following model considering the influence of other factors (or non-motor vehicles) on traffic flow. We have obtained the stability condition for the new model by applying the control method. Finally, the numerical simulations are given. The analytical results are in good agreement with the simulation results and show that the effect of other factors (or non-motor vehicles) can stabilize the traffic flow effectively.

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