Simulating Different Receivers in a Rayleigh, SISO Environment

Winter Intern Seminar (Project #1)

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모든 실험은 다음의 조건하에 진행되었다.

- Es/N0는 -2dB 20dB(2dB 간격)
- SISO; Single Input, Single Output

1 Binary Moduation

공통조건

```
NumberIteration = 10^4;
LengthBitSequence = 10^2;

EbN0_dB = -2 : 1 : 20;
EbN0 = db2pow(EbN0_dB);
```

1.1 ZF(Zero-forcing)

```
ErrorCount_ZF = zeros(1, length(EbN0_dB));

for iTotal = 1 : NumberIteration
    BitSequence = randi([0 1], 1, LengthBitSequence); % Bit Generation (BitSequence = rand(1, LengthBitSequence) > 0.5;)
    % SymbolSequence = qammod(BitSequence, 2, 'bin'); % 1 and -1
    SymbolSequence = 2 * BitSequence - 1; % Symbol (s) Generation; consisting of 1 and -1
    NoiseSequence = (randn(1, length(SymbolSequence)) + 1j * randn(1, length(SymbolSequence))) ./ sqrt(2); % Noise (n) Generation
    H = (randn(1, length(SymbolSequence)) + 1j * randn(1, length(SymbolSequence))) ./ sqrt(2); % Channel (h) Generation
    for indx_EbN0 = 1 : length(EbN0)
        ReceivedSymbolSequence_H = H .* SymbolSequence + NoiseSequence * sqrt(1 / EbN0(indx_EbN0)); % Received Signal (y = hs + n) Generation

        % ZF Receiver
        DetectionSymbolSequence_ZF = ReceivedSymbolSequence_H ./ H; % Detection (Zero-Forcing: y / h)

        % MMSE Receiver
```

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        rho = EbN0(indx_EbN0);
        w_mmse = conj(H) ./ (H.*conj(H)+1/rho);
        DetectionSymbolSequence_MMSE = ReceivedSymbolSequence_H .* w_mmse;

        % MLD Receiver
        arg = ([1 1]' * ReceivedSymbolSequence_H) - ([-1 1]' * H);
        arg = arg .* conj(arg);
        DetectionSymbolSequence_MLD = (arg(1, :)-arg(2, :)) > 0; % TODO: could possibly simplify it more

        % Symbol Sequence -> Bit Sequence
        DetectionBitSequence_ZF = real(DetectionSymbolSequence_ZF)>0;
        DetectionBitSequence_MMSE = real(DetectionSymbolSequence_MMSE)>0; % not sure if this is right
        DetectionBitSequence_MLD = DetectionSymbolSequence_MLD;

        ErrorCount_ZF(1, indx_EbN0) = ErrorCount_ZF(1, indx_EbN0) + biterr(DetectionBitSequence_ZF, BitSequence);
        ErrorCount_MMSE(1, indx_EbN0) = ErrorCount_MMSE(1, indx_EbN0) + biterr(DetectionBitSequence_MMSE, BitSequence);
        ErrorCount_MLD(1, indx_EbN0) = ErrorCount_MLD(1, indx_EbN0) + biterr(DetectionBitSequence_MLD, BitSequence);
    end
end

scatterplot(SymbolSequence)

BER_Simulation_ZF = ErrorCount_ZF / (LengthBitSequence * NumberIteration);
BER_Simulation_MMSE = ErrorCount_MMSE / (LengthBitSequence * NumberIteration);
BER_Simulation_MLD = ErrorCount_MLD / (LengthBitSequence * NumberIteration);

BER_Theory2_H = berfading(EbN0_dB, 'psk', 2, 1);

% Plot
figure()
semilogy(EbN0_dB, BER_Theory2_H, 'k--');
hold on
%hold on don't know what it does
semilogy(EbN0_dB, BER_Simulation_ZF, 'bo');
semilogy(EbN0_dB, BER_Simulation_MMSE, 'rx');
semilogy(EbN0_dB, BER_Simulation_MLD, 'g^');
axis([-2 20 10^-5 0.5]) % axis([a b c d]) x-axis from a to b, y-axis from c to d
grid on
legend('Theory', 'ZF', 'MMSE', 'MLD');
xlabel('Eb/No [dB]');
ylabel('BER');
title('BER for Binary Modulation');
```

1.2 ZF(Zero-forcing)

1.3 MMSE(Minimum Mean Square Error)

1.4 MLD(Maximum Likelihood Detection)

2 M-ary QAM

$M = 2^2n$ $(n = 1, 2, 3, ...)$의 상황을 가정하였다. 한 가지 생각해볼 만한 사항은 Normalization Factor이다. 이 Normalization Factor를 사용하여 평균 전력이 1W가 되게끔 둘 수 있다.

QAM의 일반적인 Constellation Diagram을 살펴보면 실수 $\sqrt{M}$개, 허수 $\sqrt{M}$개의 point를 갖는 것을 알 수 있다.
하나의 지점을 하나의 alphabet이라고하자. M개의 alphabet은 다음과 같다.

$$alphabet = \pm(2n-1) \pm j \cdot (2n-1) \qquad n \in 1, 2, ..., \sqrt{M} \tag{1}$$

신호의 평균전력은 다음과 같이 일반화 가능하다.

$$\begin{aligned} E_s = E[|s|^2] &= \frac{1}{M}\sum_{n=1}^{M}|s_n|^2 \\ &= \frac{1}{M}\sum_{n=1}^{M}\sum_{m=1}^{M}|(2m-1)^2 + (2n-1)^2| \\ &= \frac{1}{M} \cdot 4\sum_{n=1}^{\frac{\sqrt{M}}{2}}[(2n-1)^2 \cdot \sqrt{M}] \\ &= \frac{4}{M}\sum_{n=1}^{\frac{\sqrt{M}}{2}}[4n^2 - 4n + 1] \\ &= \frac{2}{3}(M-1) \end{aligned} \tag{2}$$

(2)에서의 결과를 토대로 normalization이 이뤄진 alphabet을 구할 수 있다.

$$normalized \quad alphabet = \Big[\pm\frac{2n-1}{\sqrt{\frac{2}{3}(M-1)}} \pm j \cdot \frac{2n-1}{\sqrt{\frac{2}{3}(M-1)}}\Big] \qquad n \in \{1, 2, ..., \sqrt{M}\} \tag{3}$$

해당 결과를 토대로 다시 평균 전력을 구한다면 E_s가 1W임을 확인할 수 있다.

참고자료
다음은 16-QAM의 Constellation이다.

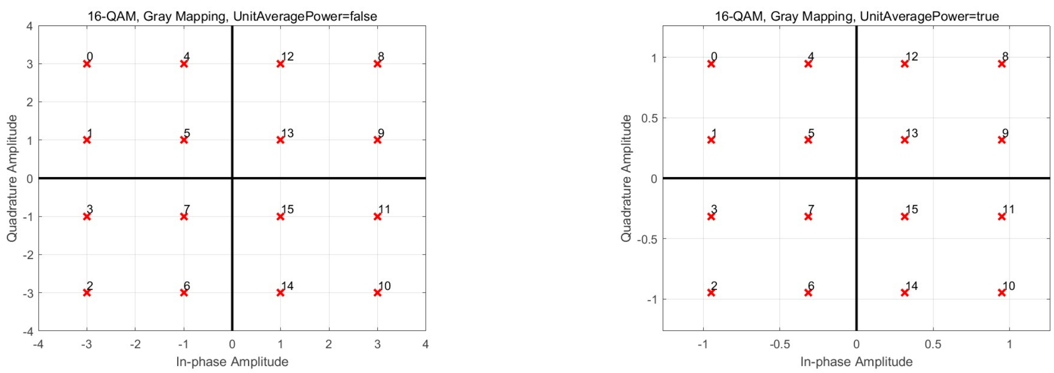
2.1 ZF(Zero-forcing)

2.2 MMSE(Minimum Mean Square Error)

2.3 MLD(Maximum Likelihood Detection)

3 과제 외적 의문점

- 왜 $M = 4$ 일때의 *Error Count*는 ZF와 MMSE의 경우 다르지만, $M = 16$일 때는 왜 모든 값이 같은가?



(a) Non-normalized Constellation

(b) Normalized Constellation

Figure 1: 16-QAM Constellation