# Simulating ZF, MMSE, MLD, SIC, OSIC Receivers in a Rayleigh Fading, MIMO Environment

Project #3

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# 1 Implementation

## 1.1 SIC (Successive Interference Cancellation)

매 stage마다 하나의 signal(하나의 transmit antenna가 송신하는 신호)이 검출된다. 초반의 신호검출 오류가 이후의 신호 검출에 영향을 준다는 단점이 있다. (Error propagation)

$$H = \begin{bmatrix} h_{11} & \dots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R1} & \dots & h_{N_RN_T} \end{bmatrix} = \begin{bmatrix} h_1 \dots h_{N_T} \end{bmatrix} \tag{1}$$

$$W = \begin{bmatrix} w_{11} & \dots & w_{1N_R} \\ \vdots & \ddots & \vdots \\ w_{N_T1} & \dots & w_{N_TN_R} \end{bmatrix} = \begin{bmatrix} w_1 \\ \vdots \\ w_{N_T} \end{bmatrix} \tag{2}$$

```
NormalizationFactor = sqrt(2/3*(M-1) * Nt); % size(H,1) = Nt
for ii = 1:Nt
    if strcmp(ReceiverType, 'zf')
        w = NormalizationFactor * sqrt(Nt) * pinv(H);
    else
        w = NormalizationFactor * sqrt(Nt) * inv(H' * H + size(H,2) / EsN0
            * eye(size(H,2))) * H';
    end
    % Regard only one transmit antenna
    w = w(1,:);
    DetectedSymbol = w * ReceivedSymbolSequence;

    % Signal Detection
    DetectedSignal = qamdemod(DetectedSymbol, M);

    %% Remove the effect of the regarded transmit antenna
    RemodulatedSignal = qammod(DetectedSignal, M); % Remodulate Detected
        Signal
    ReceivedSymbolSequence = ReceivedSymbolSequence - H(:,1) *
        RemodulatedSignal / NormalizationFactor;
    H(:,1) = []; % remove first column
end
```

### 1.2 OSIC (Successive Interference Cancellation)

SIC와 같이 하나의 stage에서 단 하나의 신호가 검출된다. 매 stage마다 SINR의 값이 가장 큰 signal이 검출된다. SINR은 매 stage마다 다시 계산된다.

$$\begin{aligned} SINR_p &= \frac{\sigma_s^2 |w_p h_p|^2}{\sigma_s^2 \sum_{q \neq p} |w_p h_q|^2 + \sigma_n^2 \|w_p\|^2} \\ &= \frac{\sigma_s^2/\sigma_n^2 |w_p h_p|^2}{\sigma_s^2/\sigma_n^2 \sum_{q \neq p} |w_p h_q|^2 + \|w_p\|^2} \end{aligned} \tag{3}$$

우리는 실험을 $E_s/N_0$에 대해 설정한 뒤 실험을 하려한다. 그렇기에 $\sigma_s^2/\sigma_n^2$를 $E_s/N_0$에 대한 식으로 나타낼 필요가 있다.

$$\frac{\sigma_s^2}{\sigma_n^2} = \left( \sqrt{\frac{E_s}{N_0}} \frac{1}{\sqrt{\frac{2}{3}(M-1)\sqrt{N_T}}} \right)^2 = \frac{Es}{N0} \frac{1}{\frac{2}{3}(M-1)N_T} \tag{4}$$

```
DetectedSignalSequence = zeros(Nt,1);
NormalizationFactor = sqrt(2/3*(M-1) * Nt);

snr = EsN0 / (NormalizationFactor^2);
HasValue = false(Nt,1);

for ii = 1:Nt
    if strcmp(ReceiverType, 'zf')
        w = NormalizationFactor * pinv(H); % pinv(H) = inv(H' * H) * H'
    else
        w = NormalizationFactor * inv(H' * H + size(H,2) / EsN0 * eye(size
            (H,2))) * H';
    end
```

```
    wH_squared = abs(w*H).^2;

    %% Get Biggest SINR
    sinr = snr*diag(wH_squared)./(snr*(sum(wH_squared,2) - diag(wH_squared
        ))+sum(abs(w).^2,2));
    [val,idx] = max(sinr);
    DetectedSymbol = w(idx, :) * ReceivedSymbolSequence;
    DetectedSignal = qamdemod(DetectedSymbol, M);

    %% Relative index to absolute index (i.e. original index)
    OriginalIndex = get_original_index(HasValue, idx);
    DetectedSignalSequence(OriginalIndex, 1) = DetectedSignal;
    HasValue(OriginalIndex) = true;

    %% Remove the effect of the regarded transmit antenna
    RemodulatedSignal = qammod(DetectedSignal, M);
    ReceivedSymbolSequence = ReceivedSymbolSequence - H(:,idx) *
        RemodulatedSignal;
    H(:,idx) = []; % remove column
end

function OriginalIndex = get_original_index(HasValue, idx)
    OriginalIndex = 0;
    while idx
        OriginalIndex = OriginalIndex + 1;
        if ~HasValue(OriginalIndex)
            idx = idx - 1;
        end
    end
end
```

SIC와 다르게 OSIC에서는 처리되는 신호의 순서가 순차적이지 않다. 다시말해, 무조건 $s = [s_1 ... s_{N_T}]^T$일때 k 번째 stage에서 $s_k$ 신호가 검출되는 것은 아니다.

매 단계에서 어떤 index의 signal이 검출될지 계산된다. 이때 계산되는 index는 **상대적인 index**이다. 그렇기 때문에 이 **상대적인 index**를 다시 본래의 index로 바꾸어 data sequence의 순서를 맞춰줄 필요가 있다. 코드에서 함수 **get_original_index**가 이 역할을 해 준다.

**Discussion**

**OSIC-ZF**의 경우, $W_{ZF} = \sqrt{N_T/E_s}(H^H H)^{-1} H^H$이다. $W_{ZF}H$의 경우, 주대각선 성분은 모두 $\sqrt{N_T/E_s}$의 값을 가지며 그 외의 성분의 값은 0이다.

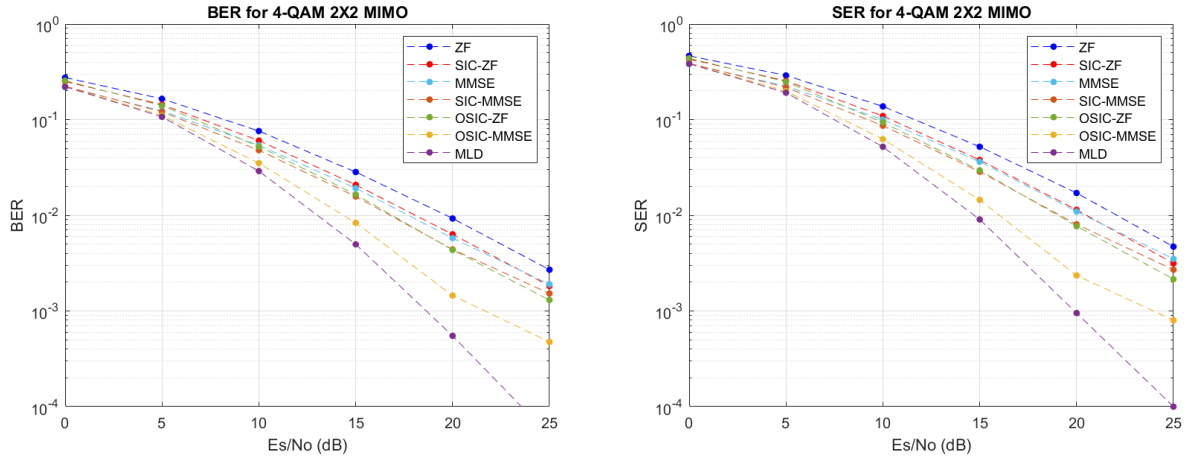
$$W_{ZF}H = \begin{bmatrix} \sqrt{N_T/E_s} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{N_T/E_s} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \vdots & 0 & \sqrt{N_T/E_s} & \dots & 0 \\ 0 & 0 & 0 & \dots & \sqrt{N_T/E_s} \end{bmatrix} \tag{5}$$

식(3)에서의 $|w_p h_p|^2$는 모든 $p$에 대하여 동일하며, $|w_p h_q|^2 = 0$ $(p \neq q)$이다. 즉, SINR 크기의 순서에 영향을 미치는 요소는 $\|w_p\|^2$뿐이다. 즉, 각각의 $\frac{1}{\|w_p\|^2}$만을 이용하여 SINR의 크기를 비교할 수 있다.

# 2 결과 및 분석

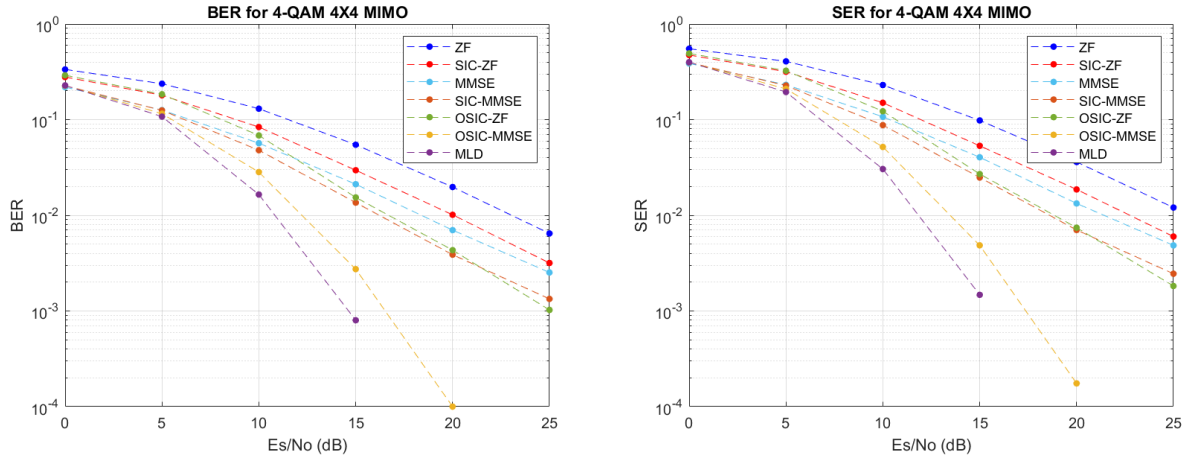
## 2.1 Simulation SER, BER Result

### 2.1.1 Es/N0: 0∼25dB



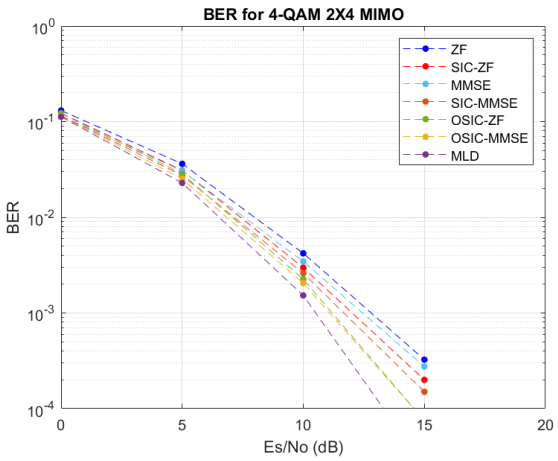

(a) BER (b) SER

Figure 1: 4-QAM 2×2 MIMO

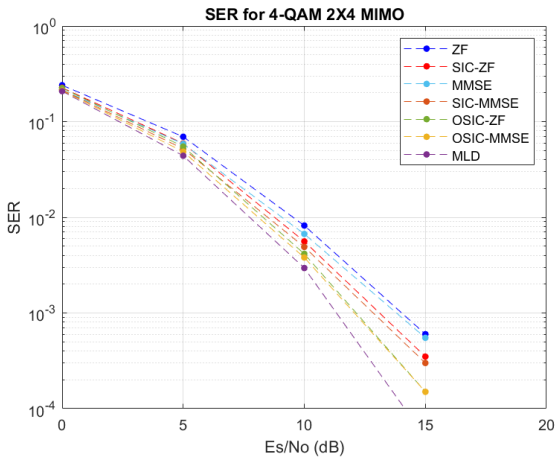



(a) BER (b) SER

Figure 2: 4-QAM 4×4 MIMO

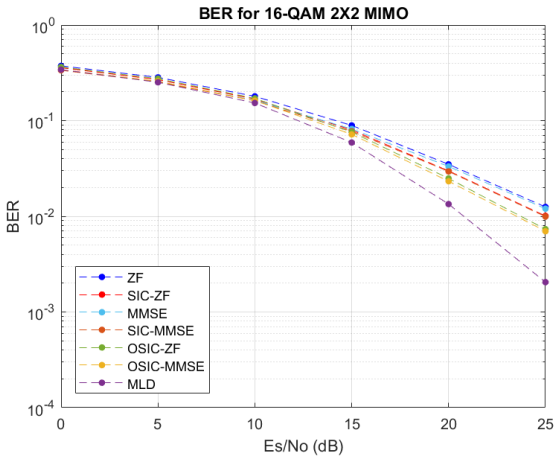



(a) BER

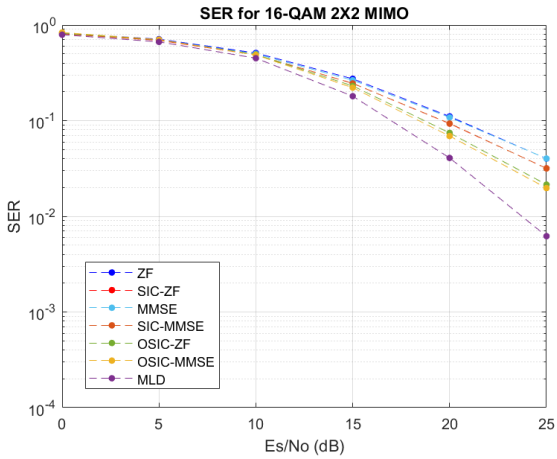



(b) SER

Figure 3: 4-QAM 2×4 MIMO

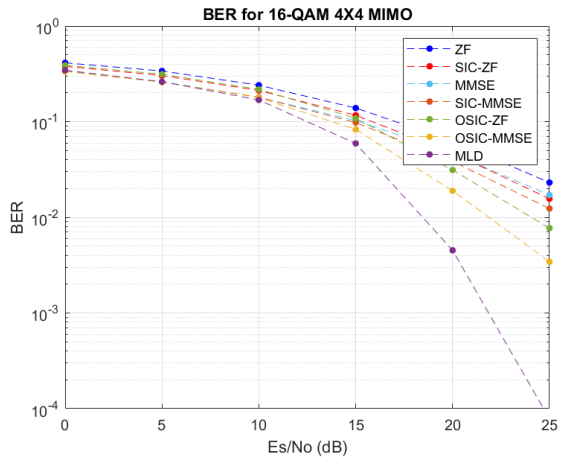



(a) BER

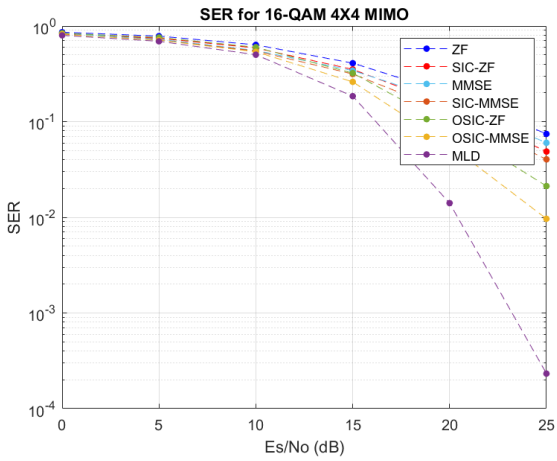



(b) SER

Figure 4: 16-QAM 2×2 MIMO

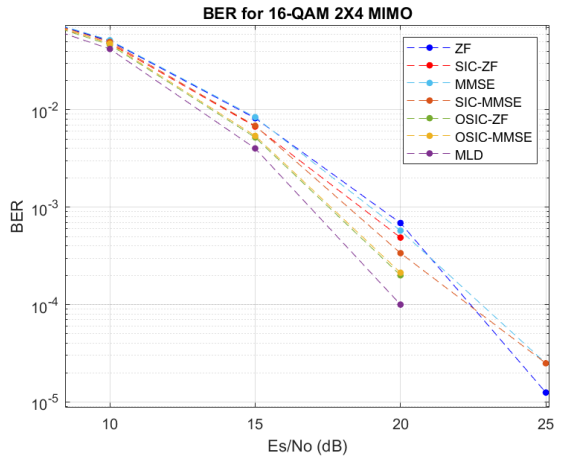



(a) BER

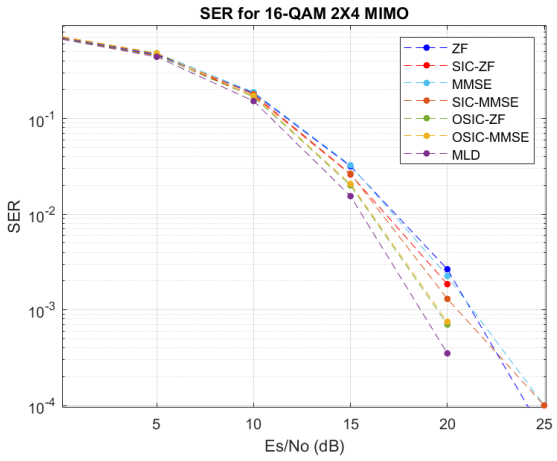



(b) SER

Figure 5: 16-QAM 4×4 MIMO



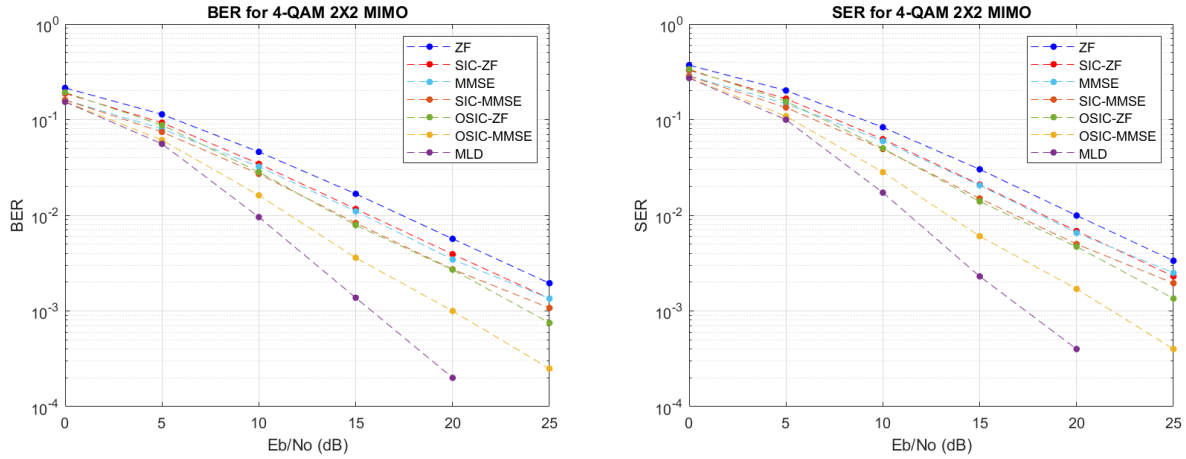

(a) BER




(b) SER

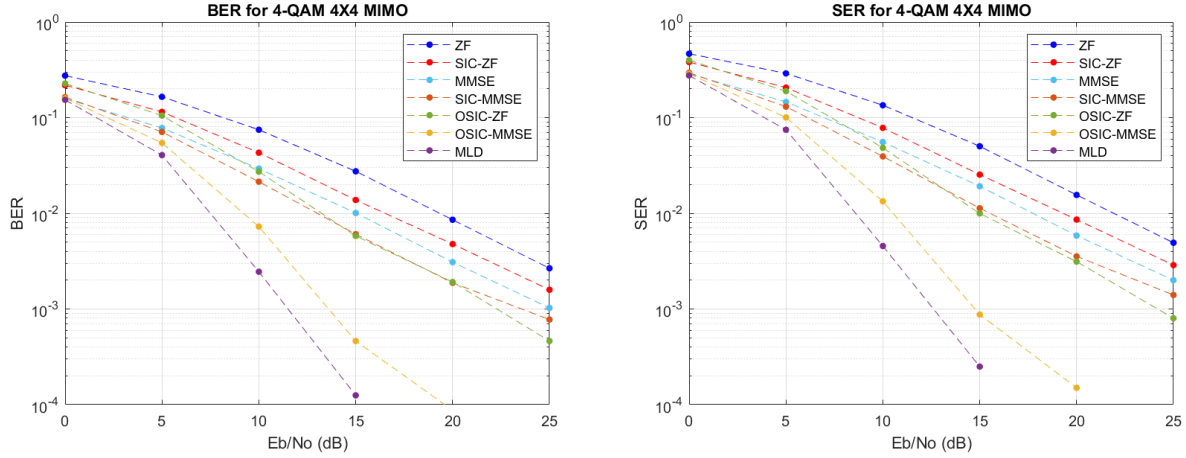
Figure 6: 16-QAM 2×4 MIMO

### 2.1.2 Eb/N0: 0∼25dB



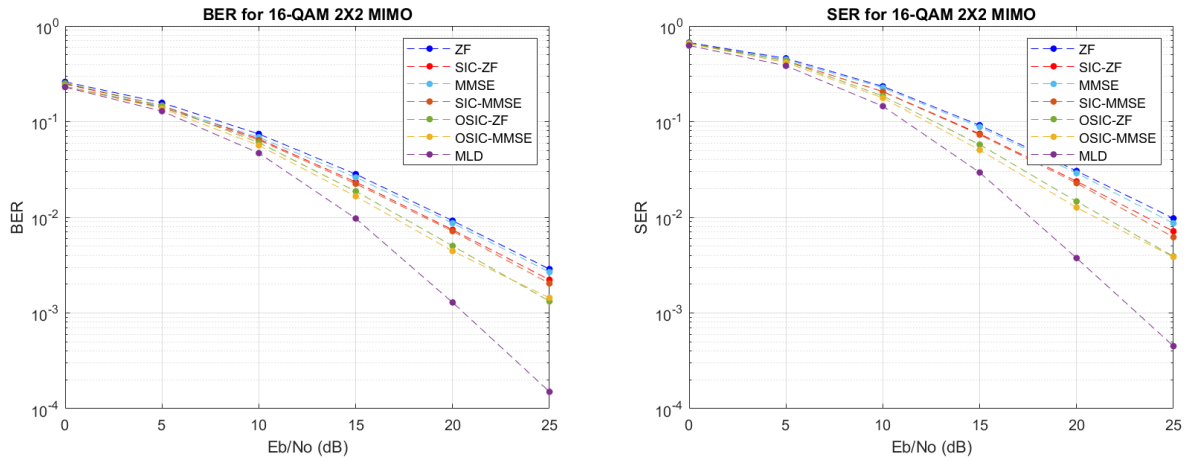

(a) BER (b) SER

Figure 7: 4-QAM 2×2 MIMO



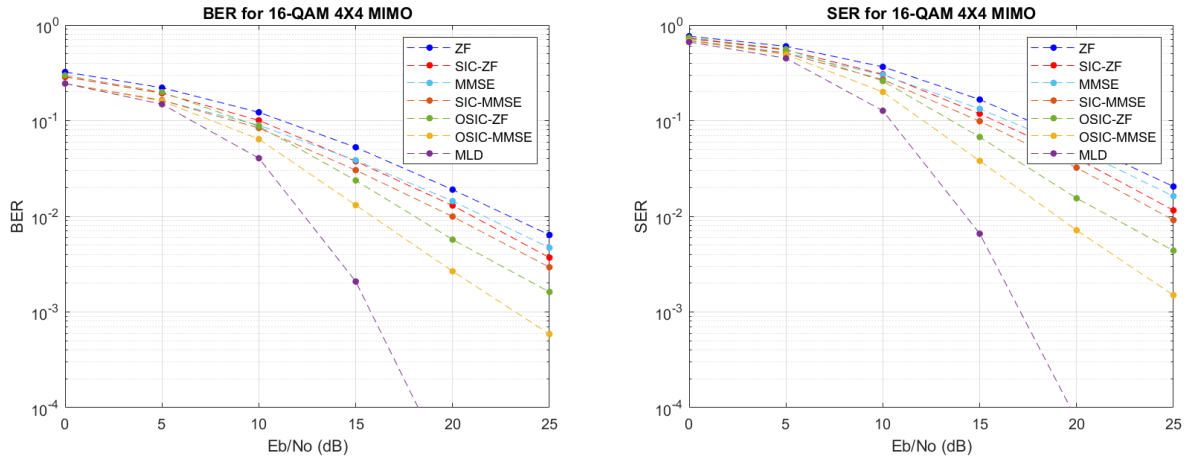

(a) BER (b) SER

Figure 8: 4-QAM 4×4 MIMO




(a) BER (b) SER

Figure 9: 16-QAM 2×2 MIMO




(a) BER (b) SER

Figure 10: 16-QAM 4×4 MIMO

### 2.1.3 Discussion

$N_T$, 즉, transmit antenna의 개수가 많을수록 OSIC의 장점이 더욱 잘 들어난다. $N_T = 2$일 때, 0.5의 확률로 SIC와 동일하게 작동하기 때문인 것으로 보인다.

## 2.2 SIC의 Error Propagation

다음은 **SIC(MMSE)에서 $s_1$의 검출이 틀렸을 때** $s_n$의 검출이 틀릴 확률을 실험적으로 얻은 결과이다.

| | $2\times 2$ **MIMO** | $2\times 4$ **MIMO** | $4\times 4$ **MIMO** | | |
|---|---|---|---|---|---|
| $E_s/N_0$**(dB)** | $n=2$ | $n=2$ | $n=2$ | $n=3$ | $n=4$ |
| 0 | 0.8055 | 0.7094 | 0.8232 | 0.8176 | 0.7989 |
| 5 | 0.7182 | 0.5435 | 0.7291 | 0.7309 | 0.7234 |
| 10 | 0.6420 | 0.3972 | 0.6336 | 0.6347 | 0.6238 |
| 15 | 0.5700 | 0.3314 | 0.5545 | 0.5536 | 0.5586 |
| 20 | 0.5164 | 0.3422 | 0.5219 | 0.5213 | 0.5029 |
| 25 | 0.5000 | 0.1667 | 0.5303 | 0.5182 | 0.5043 |

Table 1: 16-QAM $s_n$의 검출이 틀릴 확률

| | $2\times 2$ **MIMO** | $2\times 4$ **MIMO** | $4\times 4$ **MIMO** | | |
|---|---|---|---|---|---|
| $E_s/N_0$**(dB)** | $n=2$ | $n=2$ | $n=2$ | $n=3$ | $n=4$ |
| 0 | 0.4705 | 0.3310 | 0.4189 | 0.4327 | 0.4568 |
| 5 | 0.4304 | 0.2801 | 0.3615 | 0.3624 | 0.3689 |
| 10 | 0.4084 | 0.2055 | 0.3571 | 0.3667 | 0.3484 |
| 15 | 0.4763 | 0.3333 | 0.3984 | 0.3516 | 0.3438 |
| 20 | 0.4729 | NaN | 0.3881 | 0.4030 | 0.3731 |
| 25 | 0.5385 | NaN | 0.4222 | 0.5111 | 0.3556 |

Table 2: 4-QAM $s_n$의 검출이 틀릴 확률

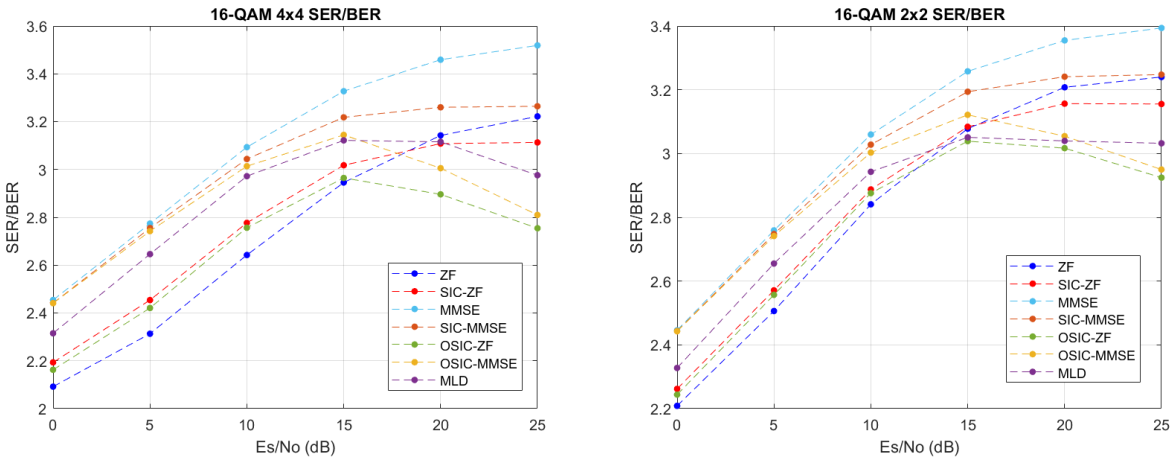
| | $2\times 2$ **MIMO** | $4\times 4$ **MIMO** | | |
|---|---|---|---|---|
| $E_s/N_0$**(dB)** | $n=2$ | $n=2$ | $n=3$ | $n=4$ |
| 0 | 0.9460 | 0.9519 | 0.9480 | 0.9440 |
| 5 | 0.9027 | 0.9206 | 0.9133 | 0.9055 |
| 10 | 0.8246 | 0.8611 | 0.8545 | 0.8413 |
| 15 | 0.7222 | 0.7568 | 0.7667 | 0.7526 |
| 20 | 0.6440 | 0.6628 | 0.6734 | 0.6682 |
| 25 | 0.5978 | 0.6038 | 0.6077 | 0.6148 |

Table 3: 64-QAM $s_n$의 검출이 틀릴 확률

$N_R > N_T$일 때, *error propagation*의 영향이 확연히 줄어든 것을 확인할 수 있었다.

## 2.3 SNR에 따른 SER/BER 변화 추이

다음은 $10^5$번의 iteration을 반복한 결과이다. (하나의 iteration에서는 모든 송신 안테나가 동시에 각각 단 하나의 신호 송신 & frequency flat fading)




(a) BER (b) SER

Figure 11: 16-QAM 4×4 MIMO

**Discussion**

높은 SER/BER은 signal error당 bit error는 적다는 것이다. Signal error가 있다고 하더라도 해당 signal에서 발생하는 bit error는 적다는 의미로 분석된다. 반대로, SER/BER이 낮다는 것은 signal error당 bit error가 높다는 의미로 분석된다.

SNR이 높아질수록 noise의 간섭(interference)가 작어지기 때문에 SER/BER이 높아지는 것은 예상할 수 있는 결과이다.

**OSIC와 MLD에서 SER/BER이 증가하다가 떨어지는** 원인에 대해서는 추가적인 분석이 필요할 것 같다.

# 3 미해결 & 추가연구 필요 내용

- **Table 2**를 보면 SNR이 증가했음에도 불구하고 $s_n$의 검출에 오류가 있을 확률이 증가하기도 하는 모습을 볼 수 있다. 이것이 단순히 낮은 iteration에 의한 결과인지, 다른 이유에 의한 결과인지 추가적으로 연구해볼 필요가 있을 것 같다.
- 어떤 receiver를 사용하는지와 상관없이 SER/BER은 지속적으로 오르다가 $\log_2 M$에 수렴할 것으로 예상했다. OSIC의 경우 SER/BER은 값이 오르다가 다시 떨어지는 모습을 보였다. 그 원인에 대해 연구가 필요하다.

# 4 Entire Code [1]

**ZF, MMSE, MLD에 대한 코드 생략...**

[1] Uploaded on https://github.com/lightwick/ICS_project/tree/main/Successive_Cancellation

**main.m**

```
close all
clear all
clc
dbstop if error
dbstop if warning

addpath('../tools/')

% Environment Varible
M = 16
Nt = 4
Nr = 4
NumberIteration = 10^4;

% Simulation
LengthBitSequence = Nt * log2(M); % log2(M) bits per signal
LengthSignalSequence = Nt;

% EbN0_dB = 0:5:25;
% EbN0 = db2pow(EbN0_dB);
%
% EsN0 = EbN0 * log2(M);
% EsN0_dB = pow2db(EsN0);

EsN0_dB = 0:5:25;
EsN0 = db2pow(EsN0_dB);

EbN0 = EsN0 / log2(M);
EbN0_dB = pow2db(EbN0);

% BitErrorCount_ZF = zeros(1, length(EsN0_dB));
% SignalErrorCount_ZF = zeros(1, length(EsN0_dB));
% BitErrorCount_MLD = zeros(1, length(EsN0_dB));
% SignalErrorCount_MLD = zeros(1, length(EsN0_dB));
%
% BitErrorCount_MMSE = zeros(1, length(EsN0_dB));
% SignalErrorCount_MMSE = zeros(1, length(EsN0_dB));

BitErrorCount = zeros(7, length(EsN0_dB));
SignalErrorCount = zeros(7, length(EsN0_dB));

NormalizationFactor = sqrt(2/3*(M-1)*Nt);

BEC_tmp = zeros(size(BitErrorCount));
SEC_tmp = zeros(size(SignalErrorCount));

FivePercent = ceil(NumberIteration/20);

for iTotal = 1 : NumberIteration
    if mod(iTotal-100, FivePercent)==0
        tic
    end
    % Bit Generation
    SignalSequence = randi([0 M-1], Nt, 1);
    SignalBinary = de2bi(SignalSequence, log2(M), 'left-msb');
    SymbolSequence = qammod(SignalSequence, M) / NormalizationFactor;

    NoiseSequence = (randn(Nr, 1) + 1j * randn(Nr, 1)) / sqrt(2); % Noise (n) Generation
    H = (randn(Nr, Nt) + 1j * randn(Nr, Nt)) ./ sqrt(2); % Receiver x Transmitter

    for indx_EbN0 = 1 : length(EsN0)
```

```
        BEC = zeros(size(BitErrorCount,1), 1);
        SEC = zeros(size(SignalErrorCount,1), 1);
        % Received Signal (y = hs + n) Generation
        ReceivedSymbolSequence = H * SymbolSequence + NoiseSequence * sqrt(1 / EsN0(
            indx_EbN0)); % log2(M)x1 matrix

        [Error, General_Error] = error_propagation_2(ReceivedSymbolSequence,
            SignalSequence, SignalBinary,  M, H, EsN0, ReceiverType)

        % ZF
        [BEC(1), SEC(1)] = simulate_zf(ReceivedSymbolSequence, SignalSequence,
            SignalBinary, M, H);

        % ZF - SIC
        [BEC(2), SEC(2)] = simulate_sic(ReceivedSymbolSequence, SignalSequence,
            SignalBinary,  M, H, EsN0(indx_EbN0), 'zf');

        % MMSE
        [BEC(3), SEC(3)] = simulate_mmse(ReceivedSymbolSequence, SignalSequence,
            SignalBinary, M, H, EsN0(indx_EbN0));

        % MMSE - SIC
        [BEC(4), SEC(4)] = simulate_sic(ReceivedSymbolSequence, SignalSequence,
            SignalBinary,  M, H, EsN0(indx_EbN0), 'mmse');

        % ZF - OSIC
        [BEC(5), SEC(5)] = simulate_osic(ReceivedSymbolSequence, SignalSequence,
            SignalBinary,  M, H, EsN0(indx_EbN0), 'zf');

        % MMSE - OSIC
        [BEC(6), SEC(6)] = simulate_osic(ReceivedSymbolSequence, SignalSequence,
            SignalBinary,  M, H, EsN0(indx_EbN0), 'mmse');

        % MLD Receiver
        [BEC(7), SEC(7)] = simulate_mld(ReceivedSymbolSequence, SignalSequence,
            SignalBinary,  M, H);

        BEC_tmp(:, indx_EbN0) = BEC;
        SEC_tmp(:, indx_EbN0) = SEC;
    end
    BitErrorCount = BitErrorCount + BEC_tmp;
    SignalErrorCount = SignalErrorCount + SEC_tmp;

    if mod(iTotal-100, FivePercent)==0
        ElapsedTime = toc;
        EstimatedTime = (NumberIteration-iTotal)*ElapsedTime;
        disp(sprintf("%d%%, estimated wait time %d minutes %d seconds", round(iTotal/
            NumberIteration*100), floor(EstimatedTime/60), floor(mod(EstimatedTime, 60)))
            )
    end
end

BER = BitErrorCount / (LengthBitSequence * NumberIteration);
SER = SignalErrorCount / (LengthSignalSequence * NumberIteration);

% Plot
BER_Title = sprintf("BER for %d-QAM %dX%d MIMO", M, Nt, Nr);
SER_Title = sprintf("SER for %d-QAM %dX%d MIMO", M, Nt, Nr);
x_axis = "Es/No (dB)";

legend_order = ["ZF", "SIC-ZF", "MMSE", "SIC-MMSE", "OSIC-ZF", "OSIC-MMSE", "MLD"];
myplot(EsN0_dB, BER, BER_Title, x_axis, "BER", legend_order);
```

```
ylim([10^(-4) 1])
myplot(EsN0_dB, SER, SER_Title, x_axis, "SER", legend_order);
ylim([10^(-4) 1])
```

**simulate_sic.m**

```
function [BitErrorCount, SignalErrorCount] = simulate_sic(ReceivedSymbolSequence,
    SignalSequence, SignalBinary,  M, H, EsN0, ReceiverType)
    Nt = size(H,2);
    Nr = size(H,1);
    NormalizationFactor = sqrt(2/3*(M-1) * Nt); % size(H,1) = Nt
    persistent alphabet;
    if isempty(alphabet)
        alphabet = qammod([0:M-1], M) / NormalizationFactor;
    end
    DetectedSignalSequence = zeros(Nt,1);
    for ii = 1:Nt
        if strcmp(ReceiverType, 'zf')
            w = NormalizationFactor * pinv(H); % pinv(H) = inv(H' * H) * H'
        else
            w = NormalizationFactor * inv(H' * H + size(H,2) / EsN0 * eye(size(H,2))) *
                H';
        end
        w = w(1,:);
        DetectedSymbol = w * ReceivedSymbolSequence;
        DetectedSignal = qamdemod(DetectedSymbol, M);
        DetectedSignalSequence(ii, 1) = DetectedSignal;

        %% Remove the effect of the regarded transmit antenna
        RemodulatedSignal = alphabet(DetectedSignal+1);
        ReceivedSymbolSequence = ReceivedSymbolSequence - H(:,1) * RemodulatedSignal;
        H(:,1) = []; % remove first column
    end
    DetectedBinary = de2bi(DetectedSignalSequence, log2(M), 'left-msb');
    BitErrorCount = sum(SignalBinary~=DetectedBinary, 'all');
    SignalErrorCount = sum(SignalSequence~=DetectedSignalSequence, 'all');
end
```

**simulate_osic.m**

```
function [BitErrorCount, SignalErrorCount] = simulate_osic(ReceivedSymbolSequence,
    SignalSequence, SignalBinary,  M, H, EsN0, ReceiverType)
    Nt = size(H,2);
    Nr = size(H,1);
    NormalizationFactor = sqrt(2/3*(M-1) * Nt);
    persistent alphabet
    if isempty(alphabet)
        alphabet = qammod([0:M-1], M) / NormalizationFactor;
    end
    snr = EsN0 / (NormalizationFactor^2);
    DetectedSignalSequence = zeros(Nt,1);

    HasValue = false(Nt,1);

    for ii = 1:Nt
        if strcmp(ReceiverType, 'zf')
            w = NormalizationFactor * pinv(H); % pinv(H) = inv(H' * H) * H'
        else
            w = NormalizationFactor * inv(H' * H + size(H,2) / EsN0 * eye(size(H,2))) *
                H';
        end
        wH_squared = abs(w*H).^2;

        %% Get Biggest SINR
```

```
        sinr = snr*diag(wH_squared)./(snr*(sum(wH_squared,2) - diag(wH_squared))+sum(abs
            (w).^2,2));
        [~,idx] = max(sinr);
        DetectedSymbol = w(idx, :) * ReceivedSymbolSequence;
        DetectedSignal = qamdemod(DetectedSymbol, M);

        OriginalIndex = get_original_index(HasValue, idx);
        DetectedSignalSequence(OriginalIndex, 1) = DetectedSignal;
        HasValue(OriginalIndex) = true;

        %% Remove the effect of the regarded transmit antenna
        RemodulatedSignal = alphabet(DetectedSignal+1);
        ReceivedSymbolSequence = ReceivedSymbolSequence - H(:,idx) * RemodulatedSignal;
        H(:,idx) = []; % remove column
    end
    DetectedBinary = de2bi(DetectedSignalSequence, log2(M), 'left-msb');
    BitErrorCount = sum(SignalBinary~=DetectedBinary, 'all');
    SignalErrorCount = sum(SignalSequence~=DetectedSignalSequence, 'all');
end

function OriginalIndex = get_original_index(HasValue, idx)
    OriginalIndex = 0;
    while idx
        OriginalIndex = OriginalIndex + 1;
        if ~HasValue(OriginalIndex)
            idx = idx - 1;
        end
    end
end
```

**myplot.m**

```
function fig = myplot(x, y, title_in, xlabel_in, ylabel_in, legend_in)
    colors = ["#0000FF", "#FF0000", "#4DBEEE", "#D95319", "#77AC30", "#EDB120", "#7E2F8E
        "];

    fig = figure();
    semilogy(x, y(1,:), '.--', 'Color', colors(1),'MarkerSize', 15);
    hold on
    for ii=2:size(y,1)
        semilogy(x, y(ii,:), '.--','Color', colors(ii), 'MarkerSize', 15);
    end
    ylabel(ylabel_in);
    title(title_in);
    grid on
    legend(legend_in);
    xlabel(xlabel_in);
end
```