Practical Assignment
Automated Reasoning 2IMF25

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The programs to be used

Programs to be used:

- `Z3`: `http://z3.codeplex.com/`.
- `Yices`: `http://yices.csl.sri.com/`.

  Both `Z3` and `Yices` are programs for satisfiability modulo theories (SMT). They accept standard SMT format, in particular boolean SAT format.
- `NuSMV`: `http://nusmv.fbk.eu/`.

  This is a symbolic model checker based on BDDs
- `Prover9` and `Mace4`: `https://www.cs.unm.edu/~mccune/mace4/`

  These are tools for predicate and equational logic: `Prover9` for giving proofs based on resolution, and `Mace4` for finding counterexamples.

Each of the problems should be solved using one of these tools. The tools should do the job: manual modifications of the problems should be avoided.

For further information about the course we refer to
`http://www.win.tue.nl/~hzantema/ar.html`

The assignment

The practical assignment has to be executed by one or two persons. It consists of two parts.

The results of the assignment have to be described in two reports that should be handed in on paper (printed two-sided, preferably less than 20 pages). For the report on the first part the deadline is **December 11, 2017**, for the report on the second part the deadline is **January 11, 2018**.

For all used formulas an extensive documentation is required, explaining the approach and the overall structure. A generic approach is preferred, since this may result in clearer descriptions, increasing the confidence in the correctness of the results. Formulas of more than half a page should not be contained in the report, instead the structure of the formula should be explained. From the output of the programs relevant parts should be contained in the report, and observations on computation time should be reported. The answers on the problems should be motivated. Every report should contain name, student number and email address of each of the authors. In case of two authors each of them is considered to be responsible for the full text and all results.

Guidelines for grading:

- Clear and generic descriptions are appreciated, both of the formulas themselves and the way they were designed. An example of appreciated style is given at

  `http://www.win.tue.nl/~hzantema/prvb.pdf`.

- For both parts at least 3 out of the 4 solutions should be correct to obtain a 7.
- Not giving a solution at all for one problem is preferred over giving a wrong solution.
- Reasons for obtaining higher than a 7 may be:
  - all problems correctly solved,
  - remarkably clear and structured descriptions,
  - approaches allowing generalizations,
  - original approaches and solutions.

The problems for the first part

For the first part (deadline December 11, 2017) you have to find and describe solutions of the following 4 problems using the indicated programs.

1. Eight trucks have to deliver pallets of obscure building blocks to a magic factory. Every truck has a capacity of 8000 kg and can carry at most eight pallets. In total, the following has to be delivered:

   - Four pallets of nuzzles, each of weight 800 kg.
   - A number of pallets of prittles, each of weight 1100 kg.
   - Eight pallets of skipples, each of weight 1000 kg.
   - Ten pallets of crottles, each of weight 2500 kg.
   - Twenty pallets of dupples, each of weight 200 kg.

   Skipples need to be cooled; only three of the eight trucks have facility for cooling skipples.

   Nuzzles are very valuable; to distribute the risk of loss no two pallets of nuzzles may be in the same truck.

   (a) Investigate what is the maximum number of pallets of prittles that can be delivered, and show how for that number all pallets may be divided over the eight trucks.

   (b) Do the same, with the extra information that prittles and crottles are an explosive combination: they are not allowed to be put in the same truck.

2. Give a chip design containing two power components and ten regular components satisfying the following constraints:

   - Both the width and the height of the chip is 30.
   - The power components have width 4 and height 3.
   - The sizes of the ten regular components are 4×5, 4×6, 5×20, 6×9, 6×10, 6×11, 7×8, 7×12, 10×10, 10×20, respectively.
   - All components may be turned 90°, but may not overlap.
   - In order to get power, all regular components should directly be connected to a power component, that is, an edge of the component should have at least one point in common with an edge of the power component.
   - Due to limits on heat production the power components should be not too close: their centres should differ at least 18 in either the x direction or the y direction (or both).

   What if this last distance requirement of 18 is increased to 20? And what if it increased to 22?

3. Twelve jobs numbered from 1 to 12 have to be executed satisfying the following requirements:

   - The running time of job i is $i + 5$, for $i = 1, 2, \ldots, 12$.
   - All jobs run without interrupt.
   - Job 3 may only start if jobs 1 and 2 have been finished.
   - Job 5 may only start if jobs 3 and 4 have been finished.
   - Job 7 may only start if jobs 3, 4 and 6 have been finished.
   - Job 8 may not start earlier than job 5.
   - Job 9 may only start if jobs 5 and 8 have been finished.
   - Job 11 may only start if job 10 has been finished.
   - Job 12 may only start if jobs 9 and 11 have been finished.
   - Jobs 5,7 en 10 require a special equipment of which only one copy is available, so no two of these jobs may run at the same time.

   Find a solution of this scheduling problem for which the total running time is minimal.

4. Take two copies of the process

   for $i := 1$ to 20 do
   if $c < 5$ then $c := c + i$ else $c := i - 2c$

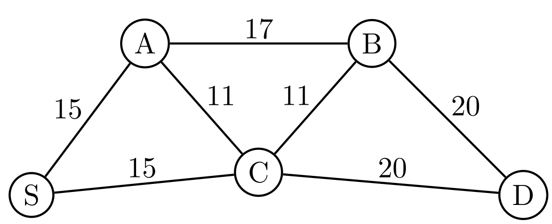
   in which i is a local variable in each of the copies, but c is a shared global variable, initially having value 0. The two processes will run in parallel, that is, as long as not both processes have been finished, an unfinished process is chosen and the next of its 20 steps is executed. Show how it is possible to end in $c = -24$, in $c = -7$, in $c = 3$ and in $c = 24$, and also try some other final values for c.

   For this problem both try to use an SMT solver and `NuSMV`, and discuss how they perform.

The problems for the second part

For the second part (deadline January 11, 2018) you have to find and describe solutions of the following 4 problems using the indicated programs.

1. Four non-self-supporting villages A, B, C and D in the middle of nowhere consume one food package each per time unit. The required food packages are delivered by a truck, having a capacity of 235 food packages. The truck has to pick up its food packages at location S containing an unbounded supply. The locations of this supply location and the villages and the roads between them are shown in the following picture. Here every number indicates a distance, more precisely, the number of time units the truck needs to travel from one village to another, including loading and delivering. The villages only have a limited capacity to store food packages: for A and C this capacity is 110, for B and D it is 160. Initially, the truck is in S and is fully loaded, and both in A and D there are 50 food packages, while B and C are fully loaded by 160 and 110 packages, respectively.

   (a) Show that it is impossible to deliver food packages in such a way that each of the villages consumes one food package per time unit forever.

   (b) Show that this is possible if the capacity of the truck is increased to 262 food packages. (Note that a finite graph contains an infinite path starting in a node v if and only if there is a path from v to a node w for which there is a non-empty path from w to itself.)

2. Let an NFA be defined as in wikipedia

   `https://en.wikipedia.org/wiki/Nondeterministic_finite_automaton`

   Note that empty steps do not occur in this kind of NFAs. Find an NFA M over $\Sigma = \{a, b\}$ with the least possible number of states for which the only non-empty words in $L(M)$ of length < 5 are aa, aba, baa, $abab$ and $babb$, and for which $baaaa \in L(M)$, $aaaba \in L(M)$, and $abaaa \notin L(M)$.

3. In mathematics, a *group* is defined to be a set G with an element $I \in G$, a binary operator $*$ and a unary operator inv satisfying

   $$x * (y * z) = (x * y) * z, \quad x * I = x \text{ and } x * inv(x) = I,$$

   for all $x, y, z \in G$.

   (a) Determine whether in every group each of the four properties

   $$I * x = x, \quad inv(inv(x)) = x, \quad inv(x) * x = I \text{ and } inv(x * y) = inv(x) * inv(y)$$

   holds for all $x, y \in G$. If a property does not hold, determine the size of the smallest finite group for which it does not hold.

   (b) A group is called *Abelian* if $x * y = y * x$ for all $x, y \in G$. Find the size of the smallest finite group that is not Abelian.

   (c) Define $x^1 = x$ and $x^{n+1} = x * x^n$ for $n \geq 1$. For $n = 2, 3, 4$ do the following. Establish whether every group G satisfying $x^n = I$ for all $x \in G$ is Abelian. If not, determine the size of the smallest corresponding non-Abelian finite group.

4. Give a precise description of a non-trivial problem of your own choice, and encode this and solve it by one of the given programs.

   Here 'trivial problems' include both minor modifications of earlier problems and single solutions of simply specified puzzles like sudokus. In case of doubt please contact the lecturer.